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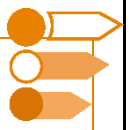
VCE Mathematical Methods ¾
Applications of Differentiation [2.4]
Homework

Homework Outline:

Compulsory Questions	Pg 2 - Pg 11
Supplementary Questions	Pg 12 - Pg 20

Section A: Compulsory Questions



Sub-Section [2.4.1]: Find Tangents and Normals

Question 1

Find the equation of the tangent to the graph of $f(x) = 2x^2 + 4x + 3$ at the point $x = 1$.

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Question 2

Let $f: \left(-\infty, -\frac{1}{2}\right] \to \mathbb{R},\ f(x) = (3x+2)^3$. Find the equation of the tangent to the graph of $y = f(x)$ which has a gradient of 9.

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Question 3

Find the equation of the tangent to the graph of $f : (0,5) \rightarrow \mathbb{R}, f(x) = x^2 - 4x$ at the point $x = a$. Hence, obtain the equation of the tangent that passes through the point $(6,3)$.

Question 4 Tech-Active.

Find the equation of the line tangent to $y = 2\log_e(2x - 1)$ when $x = 1$.

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Sub-Section [2.4.2]: Find Minimum and Maximum

Question 5

Find the maximum and minimum values of the function $f(x) = x^3 + 3x^2 - 9x - 10$ with domain $x \in [-3,2]$.

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Question 6

A farmer is building a rectangular pen for his pigs using 40 metres of fencing.

a. Write down a function $A(x)$ which gives the area of the pen.

b. Hence, determine the side lengths of the pen to maximise the area and give this maximum area.

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Question 7

Find the maximum positive rate of change for the function $f(x) = -x^3 + 9x^2 + 6x - 10$

Question 8 Tech-Active.

Find the maximum and minimum values of the function $f(x) = -x^3 + 6x^2 + 4x - 10$ for $x \in [-2, 7]$.

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Sub-Section [2.4.3]: Apply Newton's Method to Find the Approximation of a Root and its Limitations

Question 9

Approximate the root of the equation $x^3 - 5x + 3$ using Newton's method with an initial value of $x_0 = 1$ and a tolerance level of 0.01. Leave your answer correct to two decimal places.

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Question 10

Approximate a solution of the equation $e^x = \sin(x)$ using Newton's method with an initial value of $x_0 = -2$. Use a tolerance level of 0.01 and give your answer correct to two decimal places.

Question 11

Consider the function $f(x) = x^3 - 6x$. Determine two possible initial values which are not suitable to use in Newton's method to approximate the roots of f.

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Sub-Section: The 'Final Boss'

Question 12

A piece of wire is 16 metres long. The wire is cut into two pieces and used to form two squares.

a. If one piece of wire has length x metres, show that the combined area of the two squares is given by $A = \frac{1}{8}x^2 - 2x + 16 = \frac{1}{8}(x^2 - 16x + 128)$.

b. Find $\frac{dA}{dx}$.

c. Find the value of x that minimises A and gives this minimum value.

d. Find the maximum possible area of the two squares if $x \in [1, 12]$.

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Section B: Supplementary Questions

Sub-Section [2.4.1]: Find Tangents and Normals

Question 13

Find the equation of the normal to the graph of $f(x) = \cos(5x)$ at the point $x = \frac{\pi}{4}$.

Question 14

Find the equation of the normal to the graph of $f(x) = x^2 - 3x - 1$ which has a gradient of $-\frac{1}{5}$.

Question 15

Find the equation of the normal to the graph of $f: (2, \infty) \rightarrow \mathbb{R}$, $f(x) = x^2 - 2x$ at the point $x = a$. Hence by using a CAS, obtain the equation of the normal that passes through the point $(-1,4)$.

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Question 16

Consider the function given by $f(x) = e^{x^2} - \cos(x)$.

a. Find the equation of the tangent to the graph of $f(x)$ at the point $x = 1$.

b. Without needing to do any further differentiation/solving, find the equation of the normal that passes through the point $x = -1$.

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Sub-Section [2.4.2]: Find Minimum and Maximum

Question 17

Find the maximum and minimum values of the function $f(x) = x^3 - 3x^2 - 24x + 15$ with domain $x \in [0,5]$.

Question 18

Find the maximum area of a rectangle with a perimeter equal to $18\ m$.

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Question 19

Find the maximum rate of change of the function $f(x) = -x^3 + 6x^2 + 10x - 5$.

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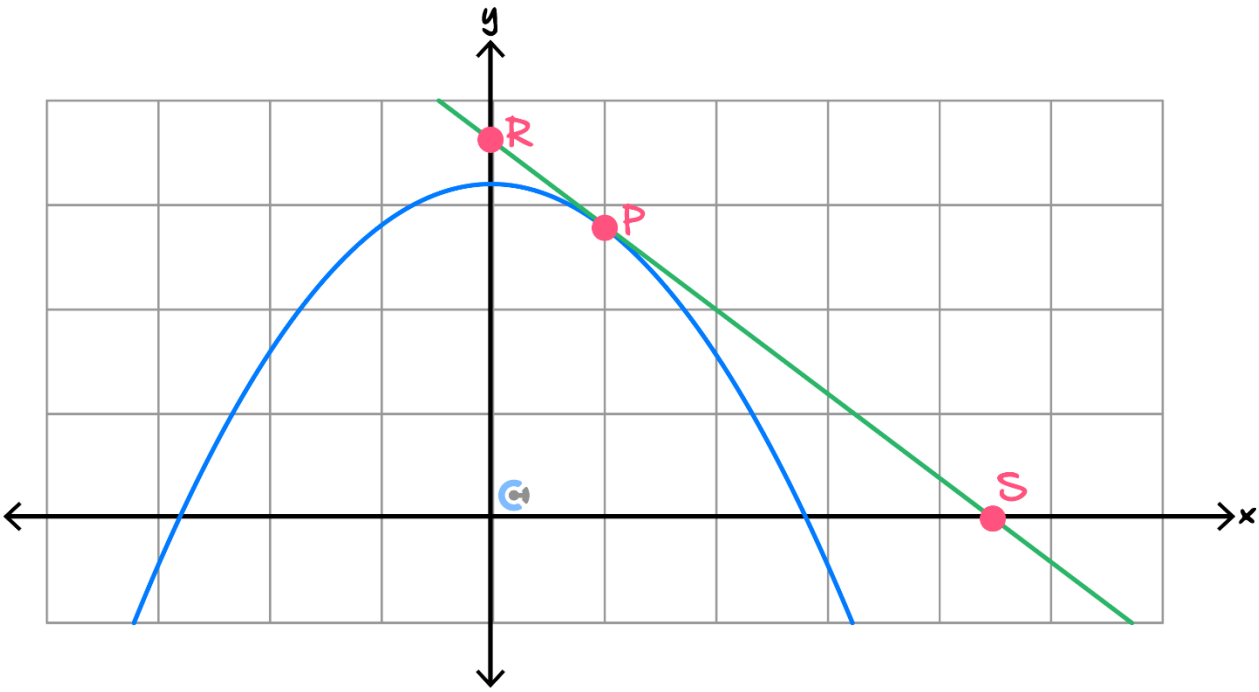
Question 20 Tech-Active.

The diagram below shows the graph of the function $f(x) = 16 - 2x^2$.



The graph of the tangent to the curve at the point $P(p.f(p))$, where $p \in \left[\frac{1}{2}, \frac{5}{2}\right]$ is also shown.

Determine the equation of the tangent line in terms of p.

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Sub-Section [2.4.3]: Apply Newton's Method to Find the Approximation of a Root and its Limitations

Question 21

Approximate the root of the equation $x^3 - 2x^2 + 5x - 6$ using Newton's method with an initial value of $x_0 = 1.2$ and a tolerance level of 0.01. Leave your answer correct to two decimal places.

Question 22

Approximate a solution of the equation $e^x = \cos(2x - 1)$ using Newton's method with an initial value of $x_0 = -2$. Use only one iteration for your approximation.

Question 23

Consider the function $f(x) = \sin(x) - e^{2x}$. Explain why it would be unsuitable to choose an initial value that solves the equation $\cos(x) - 2e^{2x}$.

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Question 24

An issue that can arise when using Newton's method is that the derivative may not be easy to calculate.

a. Explain why Newton's method is impractical for approximating the roots of $f(x) = \sin^{-1}(x^2 - \frac{\pi}{2})$ within the context of VCE Mathematical Methods Units 3 and 4.

b. Nevertheless, we can try to use a similar method known as the secant method with an initial guess of $x_0 = 1.5$. Approximate the tangent to $x_0 = 1.5$ by finding the equation of secant (i.e. the straight line) passing through the points $(1.5, 0.7467)$ and $(1.51, 0.7885)$. Your answer should be given to two decimal places.

c. Hence, obtain an approximation for a root of $f(x) = \sin^{-1}(x^2 - \frac{\pi}{2})$ based on the line obtained above.

Note that this method approximates the root which is $x = \sqrt{\pi/2} \approx 1.2533$.

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