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VCE Mathematical Methods ¾
Differentiation Exam Skills [2.3]
Homework

Homework Outline:

Compulsory Questions	Pg 2-Pg 17
Supplementary Questions	Pg 18-Pg 37

Section A: Compulsory Questions

Sub-Section [2.3.1]: Find General Derivatives With Functional Notation

Question 1

If f is a differentiable function, find $\frac{dy}{dx}$ for the following:

a. $y = \sin(f(x))$

b. $y = f(2x^3)$

Question 2

If f and g are differentiable functions, find $\frac{dy}{dx}$ for the following:

a. $y = \log_e(f(x)) \cdot \cos(x)$

b. $y = \frac{xf(x)}{g(x)}$

Question 3

If f and g are differentiable functions, find $\frac{dy}{dx}$ for the following:

a. $y = e^{f(x)g(x)} \cdot \sin(x^2)$

b. $y = \frac{\log_e\left(f(g(x))\right)}{[g(x)]^2}$

Sub-Section [2.3.2]: Apply Differentiability to Join Two Functions Smoothly

Question 4

A hybrid function is defined as:

$$f(x) = \begin{cases} ax + b, & x < 3 \\ x^2 - 3x + 4, & x \geq 3 \end{cases}$$

Find the values of a and b such that $f(x)$ is smooth and continuous at $x = 3$.

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Question 5

A function $f(x)$ is given by:

$$f(x) = \begin{cases} ax^2 + bx + 1, & x < 2 \\ x^3 - 2x + 3, & x \geq 2 \end{cases}$$

Find the values of a and b such that $f(x)$ is both continuous and differentiable at $x = 2$.

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Question 6

Consider the hybrid function:

$$f(x) = \begin{cases} 2\sin(x-a)+b, & x<1 \\ x^2-2x+2, & x\geq 1 \end{cases}$$

Find all possible values of a and b so that f is a smooth and continuous function for all $x \in \mathbb{R}$.

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Sub-Section: Exam 1 Questions

Question 7

a. Let $f: (-2, \infty) \to \mathbb{R}, f(x) = \frac{x+3}{x+2}$.

Find $f'(x)$.

b. Let $g(x) = (3 - x^3)^3$. Evaluate $g'(1)$.

c. Evaluate $\lim_{x \to 2} \frac{x^2 + 2x - 8}{x - 2}$.

Question 8

Let $f(x) = 2x^3 + 3x^2 - 12x + 12$.

a. Find the coordinates of all stationary points of f.

b. State the nature of any stationary points found in **part a.**

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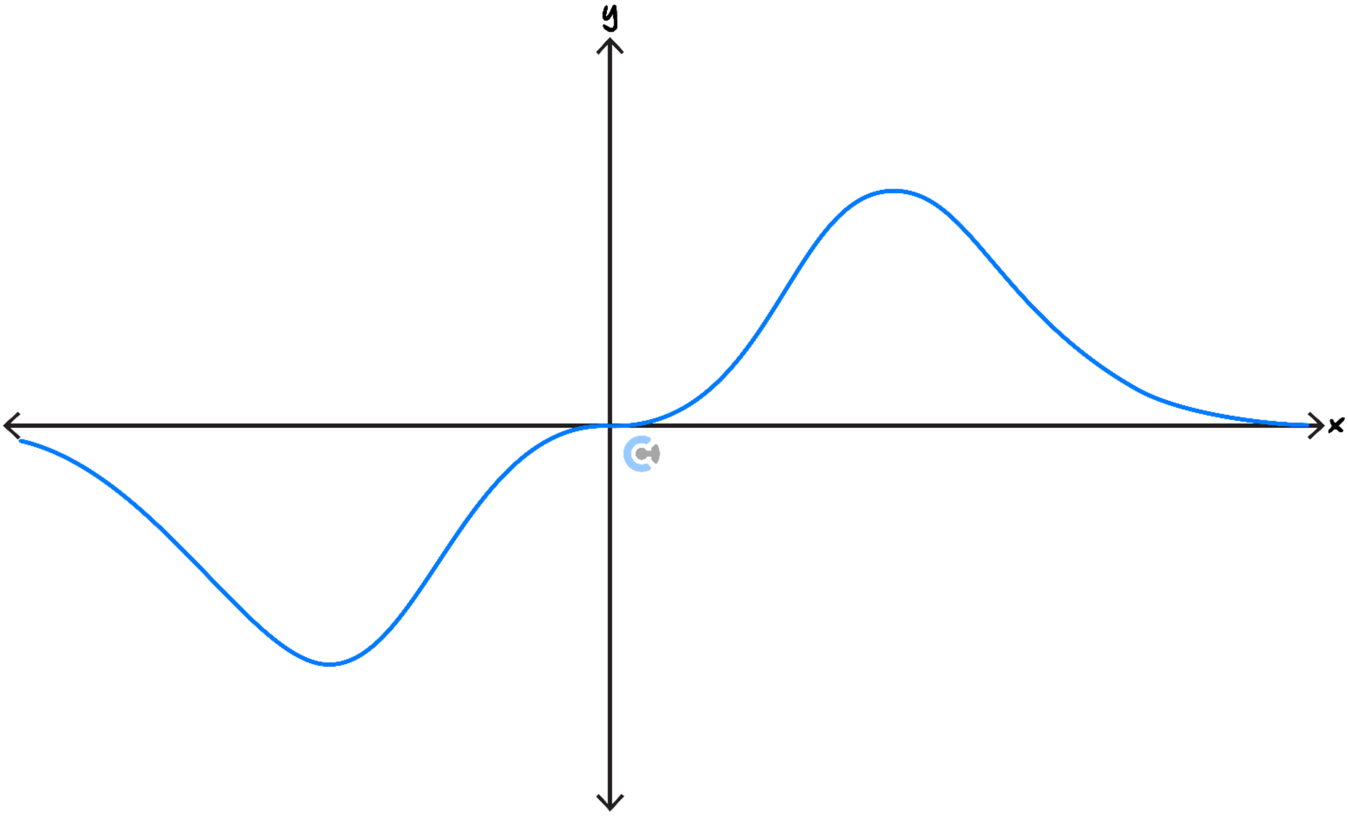
Question 9

The graph of f is shown on the axes below. Sketch the graph of f' on the same set of axes.



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Question 10

Consider the function $f(x) = x^3e^{-x^2}$.

a. Find $f'(x)$ in the form $ax^2e^{-x^2}(b - cx^2)$ for positive integers $a, b,$ and c.

b. Hence, find the coordinates for any stationary points of f.

c. Determine the nature of any stationary points of f.

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Sub-Section: Exam 2 Questions

Question 11

If $y = 2x^2 + 4x + 3$, the rate of change of y with respect to x at $x = k$ is:

A. $4k + 4$

B. $2k + 4$

C. $k^2 + 4$

D. $k^3 + 4$

Question 12

Let $f(x) = (ax + b)^3$ and let g be the inverse function of f.

Given that $f(0) = 1$, what is the value of $g'(1)$?

A. $\frac{3}{a}$

B. 1

C. $\frac{1}{3a}$

D. 0

Question 13

If $f(x) = e^{g(x^3)}$, where g is a differentiable function, then $f'(x)$ is equal to:

A. $3x^2e^{g(x^3)}$

B. $3x^2g(x^3)e^{g(x^3)}$

C. $3x^2g'(x^3)e^{g(x^3)}$

D. $3x^2g'(3x^2)e^{g(x^3)}$

Question 14

For two differentiable functions f and g the derivative of $f(3x) \times g(x^2)$ is:

A. $6xf'(3x)g'(x^2)$

B. $x^2f(3x)g'(x^2) + 3xf'(3x)g(x^2)$

C. $3f(3x)g'(x^2) + 3xf'(3x)g(x^2)$

D. $2xf(3x)g'(x^2) + 3f'(3x)g(x^2)$

Question 15

Consider the function:

$$f(x) = \begin{cases} 2x^2 + ax + 1 & x \leq 2 \\ x^2 + 3x + b & x > 2 \end{cases}$$

If f is a smooth and continuous function for all $x \in \mathbb{R}$ then the values of a and b are:

A. $a = 1, b = 3$

B. $a = -1, b = -3$

C. $a = 1, b = -3$

D. $a = -1, b = 3$

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Question 16

Tammy Jones is exploring the jungle looking for a lost civilisation when she is struck by a blowgun dart, fired by the local tribesman.

The dart is poisoned and the concentration of poison, in mg/L, in Tammy's blood t minutes, after she is hit, is given by the continuous function:

$$C(t) = \begin{cases} \dfrac{350}{70-t} & 0 \le t \le k \\ m, & k < t \le 60 \end{cases}$$

a. What is the initial concentration of poison in Tammy's blood?

b. Find an expression for m in terms of k.

c. Find the minimum and maximum values of m.

d. Find the domain and rule for the function $C'(t)$.

e. If the rate at which the concentration of poison in Tammy's blood was increasing was 2 mg/L per minute, find the value of t. Express your answer correct to two decimal places.

Tammy will not survive if the concentration of poison in her blood exceeds 12 mg/L.

f. Given that Tammy is unable to receive any treatment for 60 minutes, find the possible values of k in order for her to survive.

Question 17

a. Let $f(x) = (x^2 + bx + c)\sqrt{3x - 4}$.

i. Use calculus to find $f'(x)$.

ii. Hence, express the derivative of $(x^2 + bx + c)\sqrt{3x - 4}$ in the form $\frac{q(x)}{2\sqrt{3x-4}}$ where $q(x)$ is a quadratic function.

b. Let $g(x) = (x^2 + bx - 1)\sqrt{3x - 4}$.

i. State the domain of $g(x)$.

ii. Find the values of b for which g has a stationary point.

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Section B: Supplementary Questions

Sub-Section [2.3.1]: Find General Derivatives With Functional Notation

Question 18

If f is a differentiable function, find $\frac{dy}{dx}$ for the following:

a. $y = f(x)\tan(x)$

b. $y = \sqrt{f(x)}$

Question 19

If f and g are differentiable functions, find $\frac{dy}{dx}$ for the following:

a. $y = f(e^x) \cdot g(x)$

b. $y = f(g(\cos(3x)))$

Question 20

If f and g are differentiable functions, find $\frac{dy}{dx}$ for the following:

a. $y = \sqrt{f(3x^2) + g(2x + f(x))}$

b. $y = \frac{e^{f(x^2)}}{g(f(x^2)) + f(x^2)}$

Question 21

If f and g are differentiable increasing functions, with $g'(x)$ also being one-to-one, what is the maximum amount of stationary points that $y = f(x) + 3x + g(-f(x) - 3x)$ has?

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Sub-Section [2.3.2]: Apply Differentiability to Join Two Functions Smoothly

Question 22

A hybrid function is defined as:

$$f(x) = \begin{cases} e^{2x} - 2, & x < 0 \\ ax + b, & x \geq 0 \end{cases}$$

Find the values of a and b such that $f(x)$ is smooth and continuous at $x = 0$.

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Question 23

A hybrid function is defined as:

$$f(x) = \begin{cases} \log_e(ax), & x < 1 \\ bx^2, & x \geq 1 \end{cases}$$

Where $a > 0$. Find the values of a and b such that $f(x)$ is both continuous and differentiable at $x = 1$.

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Question 24

A hybrid function, $f : \mathbb{R} \to \mathbb{R}$, is defined as:

$$f(x) = \begin{cases} 2x + 4 & x < -2 \\ ax^3 + bx^2 + cx + d & -2 \leq x \leq 2 \\ x^2 - 6x + 10 & x > 2 \end{cases}$$

Find the values of a, b, c and d such that $f(x)$ is both continuous and smooth over its entire domain.

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Question 25 Tech-Active.

a. A hybrid function $f : \mathbb{R} \to \mathbb{R}$, is defined as:

$$f(x) = \begin{cases} \sin(x) + 3 & x < 0 \\ g_1(x) & 0 \leq x < 1 \\ g_2(x) & 1 \leq x < 2 \\ g_3(x) & 2 \leq x < 3 \\ \log_e\left(\frac{e^2 x^3}{27}\right) & x \geq 3 \end{cases}$$

Where g_1, g_2 and g_3 are cubic polynomials. Find g_1, g_2, g_3 if both f and f' are smooth on $\mathbb{R}$.

b. A different hybrid function, $h : \mathbb{R} \to \mathbb{R}$, is defined as:

$$h(x) = \begin{cases} \sin(x) + 3 & x < 0 \\ g_4(x) & 0 \leq x < 3 \\ \log_e\left(\frac{e^2 x^3}{27}\right) & x \geq 3 \end{cases}$$

Where g_4 is a polynomial. If both h and h' are smooth on $\mathbb{R}$, what is the minimum degree of $g_4(x)$?

Sub-Section: Exam 1 Questions

Question 26

a. Let $f: (1, \infty) \to \mathbb{R}, f(x) = \frac{x^2-2x}{(x-1)^2}$. Differentiate f with respect to x.

b. Let $g(x) = (x-3)^3(x+1)^2$. Solve $g'(x) = 0$ for x.

c. If $y = \frac{e^{x^3+2x}}{\sin(x^3+2x)}$, find $\frac{dy}{dx}$.

d. Let $h : [1, \infty) \to \mathbb{R}, h(x) = \sqrt{\log_e(x)}$. Evaluate $h'(e)$.

Question 27

Let $f : \mathbb{R} \to \mathbb{R}, f(x) = x^3 e^{kx}$.

Find the value of k for which $f(x)$ and $f'(x)$ have exactly one point of intersection.

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Question 28

Let $f : [-\pi, \pi] \to \mathbb{R}, f(x) = e^x \sin(x)$.

a. Find $f'(x)$.

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b. Show that $f(x)$ has a stationary point when $x = -\frac{\pi}{4}, \frac{3\pi}{4}$.

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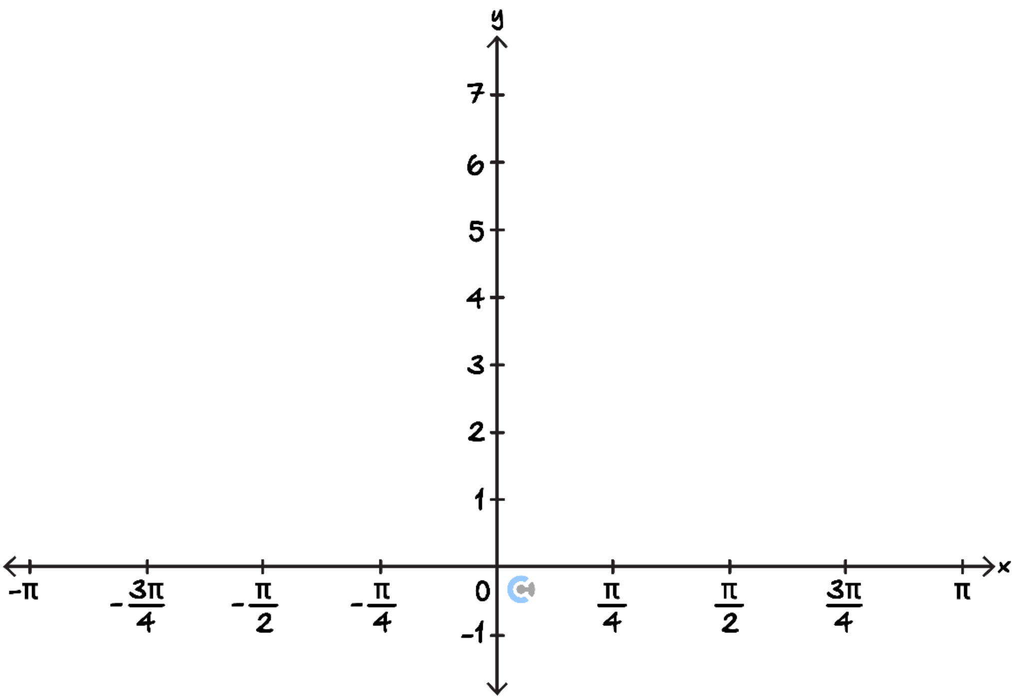
c. On the set of axes below, sketch the graph of $y = f'(x)$ on the domain $\left[-\pi, \frac{3\pi}{4}\right]$, labelling the endpoints and points of intersection with the graph of $y = f(x)$ with their co-ordinates.



Question 29

Consider the piecewise function, $f : \mathbb{R} \to \mathbb{R}$, with the following rule;

$$f(x) = \begin{cases} 0 & x \leq 0 \\ x^n \log_e(x) & 0 < x \leq 1 \\ ax + b & x > 1 \end{cases}$$

a. Find the values of a and b such that the graph of $y = f(x)$ is smooth at $x = 1$.

b. It is known that $\lim_{x \to 0^+} x^k \log_e(x) = 0$ if and only if $k > 1$.

i. For what values of n is the graph of $y = f(x)$ continuous at $x = 0$?

ii. For what values of n is the graph of $y = f(x)$ smooth at $x = 0$?

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Sub-Section: Exam 2 Questions

Question 30

The gradient of the graph of $y = \log_e(2x)$ at the point where the graph crosses the horizontal axis is equal to:

A. 1

B. 2

C. e

D. $2e$

Question 31

Let f and g be differentiable functions.

If $f'(2) = 3, f'(5) = -1, g(1) = 5$ and $g'(1) = 2$, then what is the value of $(f \circ g)'(1)$?

A. -2

B. -5

C. 15

D. There is insufficient information to determine the correct answer.

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Question 32

Consider the function:

$$f(x) = \begin{cases} e^{x-1} & x \leq 1 \\ ax^2 + bx & x > 1 \end{cases}$$

If the derivative of f is a smooth function for all $x \in \mathbb{R}$, then the values of a and b are:

A. $a = 0$ and $b = 1$.

B. $a = 1$ and $b = 1$.

C. $a = 2$ and $b = 1$.

D. $a = \frac{1}{2}$ and $b = 0$.

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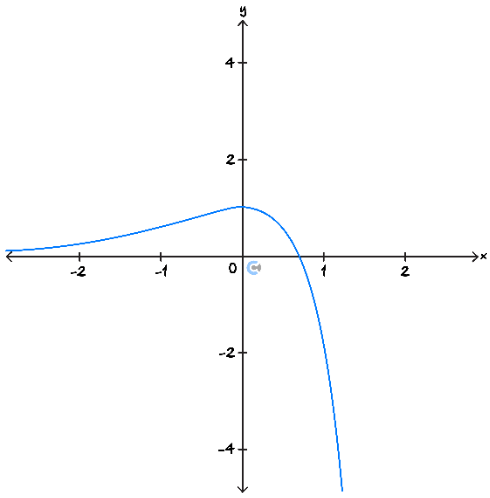
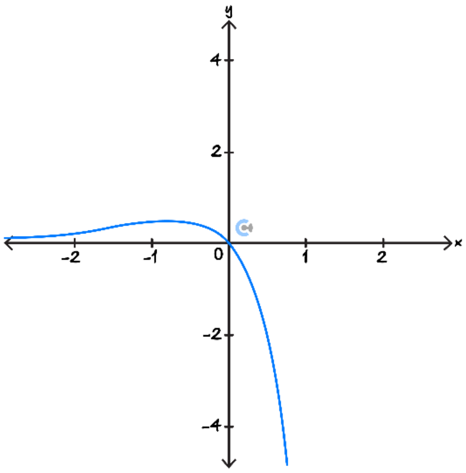
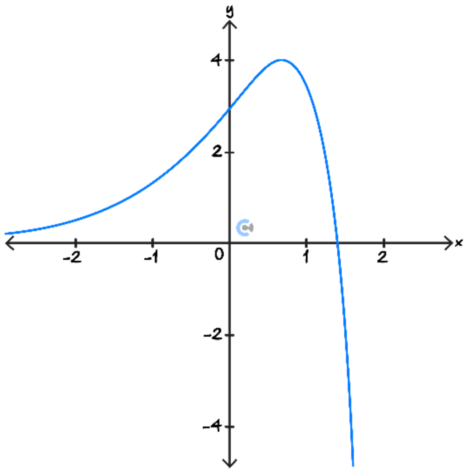
Question 33

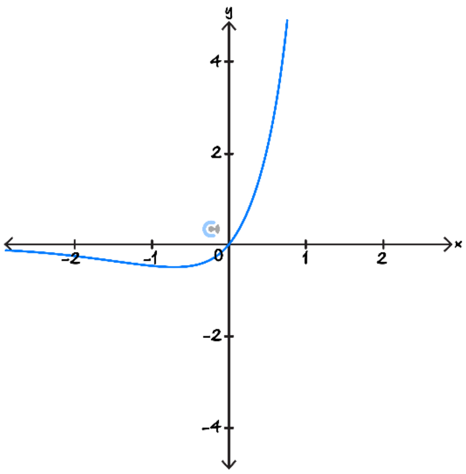
The graph of $y = f(x)$ is shown below.

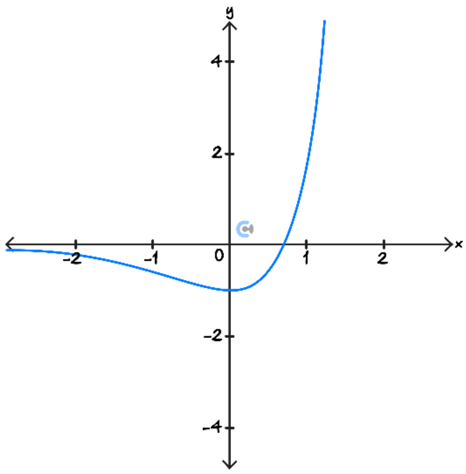


The graph of $y = f'(x)$, the first derivative of $f(x)$ with respect to x, could be:

A.


B.


C.


D.


Question 34

Suppose a function $f : [-2, 2] \to \mathbb{R}$ and its derivative $f' : [-2, 2] \to \mathbb{R}$ are defined and continuous in their domains.

If $f'(x) > f(x)$ for all x and $f'(0) = 0$, which one of these statements must be true?

A. f is increasing on $[0, 2]$.

B. f is increasing on $[-2, 0]$.

C. f is decreasing on $[0, 2]$.

D. f is decreasing on $[-2, 0]$.

Question 35

The population of cockatiels (in thousands) living in Australia, t years after 1800, is modelled by the smooth piecewise function:

$$f(t) = \begin{cases} 800 + 100\cos\left(\dfrac{\pi t + 10\pi}{30}\right) & 0 \le t < 100 \\ g(t) & 100 \le t < 145 \\ 1000e^{0.02(t-145)} + 120\sin\left(\dfrac{\pi t + 10\pi}{30}\right) & t \ge 145 \end{cases}$$

a. Write $f'(t)$ as a piecewise function in terms of $g'(t)$.

b. If $g(t) = a(t-100)^3 + b(t-100)^2 + c(t-100) + d$, construct simultaneous equations to show that, $a = \frac{48-\sqrt{3}\pi}{6075}$, $b = -\frac{42+4\sqrt{3}\pi}{135}$, $c = \frac{5\pi}{\sqrt{3}}$ and $d = 850$.

c. Let g be defined as in **part b.**

i. Solve $f'(t) = 0$ for $t \in [100, 145]$. Give your answer correct to 2 decimal places.

ii. Hence, state the values of $t \in [100, 145]$ for which $f(t)$ is strictly decreasing.

Give your answer correct to 2 decimal places.

d. Find the minimum value of $f'(t)$ for $t \in [100, 145]$ correct to 2 decimal places.

e. We can approximate $g(t)$ using an alternative approximation, with:

$$g(t) = 850 + p\sin\left(\frac{\pi t}{30}\right) + q\cos\left(\frac{\pi t}{30}\right) + r\sin\left(\frac{\pi t}{15}\right) + s\cos\left(\frac{\pi t}{15}\right)$$

Find the values of p, q, r and s correct to 2 decimal places.

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Question 36

Consider the composite function $g(x) = f(\tan(2x))$, where the function f is an unknown but differentiable function, whose derivative, $f'(x)$ is a decreasing function for all values of x.

Use the following table of values for f and f'.

x	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$
$f(x)$	-2	5	-7
$f'(x)$	2	0	-3

a. Express $g'(x)$ in terms of $f'(x)$.

b. Find two solutions to $g'(x) = \frac{16}{3}$ for all $x \in [0, \pi]$.

c.

i. Solve $g'(x) = 0$ for all $x \in \mathbb{R}$.

ii. Explain why there cannot be any more solutions to the equation $g'(x) = 0$, than those you have provided in the previous part.

iii. Find the maximal subset of $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$ for which $g(x)$ is decreasing.

d. How many solutions are there to the equation $g(x) = 0$ over the interval $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$? Explain your answer.

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