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VCE Mathematical Methods ¾
Differentiation II [2.2]
Homework

Homework Outline:

Compulsory Questions	Pg 2 – Pg 14
Supplementary Questions	Pg 15 – Pg 25

Section A: Compulsory Questions

Sub-Section [2.2.1]: Evaluate Limits and Find Points Where the Function is not Continuous

Question 1

Evaluate the following limits:

a. $\lim_{x \to 2}(x^2 - 3)$

b. $\lim_{x \to 3}(3^x - 2x^2 + 3)$

c. $\lim_{x \to 1}(f(x))$, where,

$$f(x) = \begin{cases} 2x + 1, & x < 1 \\ 5x - 2, & x \geq 1 \end{cases}$$

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Question 2

Find the points x for which the following functions are **discontinuous** and state a reason as to why they are discontinuous.

a. $f(x) = \begin{cases} 2x, & x < 2 \\ 2x + 1, & x \geq 2 \end{cases}$

b. $f(x) = \frac{6}{x^2 - x - 2}$

c. $f(x) = \begin{cases} 2x, & x > 0 \\ -2x, & x < 0 \end{cases}$

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Question 3

Consider the following function $f(x)$ with rule:

$$f(x) = \begin{cases} 2x^2 - 4x + 3, & x < 2 \\ ax + 4, & x \geq 2 \end{cases}$$

Find the value of a such that $f(x)$ is continuous for all $x \in \mathbb{R}$.

Question 4 Tech-Active.

Consider the following function $f(x)$ with rule:

$$f(x) = \begin{cases} x^2 - 3x + 2, & x < 2 \\ a^2 - ax - 3, & 2 \leq x < 4 \\ 2x - 14, & x \geq 4 \end{cases}$$

Find the value of a such that $f(x)$ is continuous for all $x \in \mathbb{R}$.

Sub-Section [2.2.2]: Apply Differentiability to Find Points Where Functions are not Differentiable, Domain of the Derivative and Unknowns of a Function

Question 5

Find the values of x such that the following functions are not differentiable.

a. $f(x) = \begin{cases} -x+2, & x < 1 \\ x, & x \geq 1 \end{cases}$

b. $f(x) = \frac{1}{x-3}$

c. $f(x) = \begin{cases} 2x-4, & x < -1 \\ 2x-6, & x \geq -1 \end{cases}$

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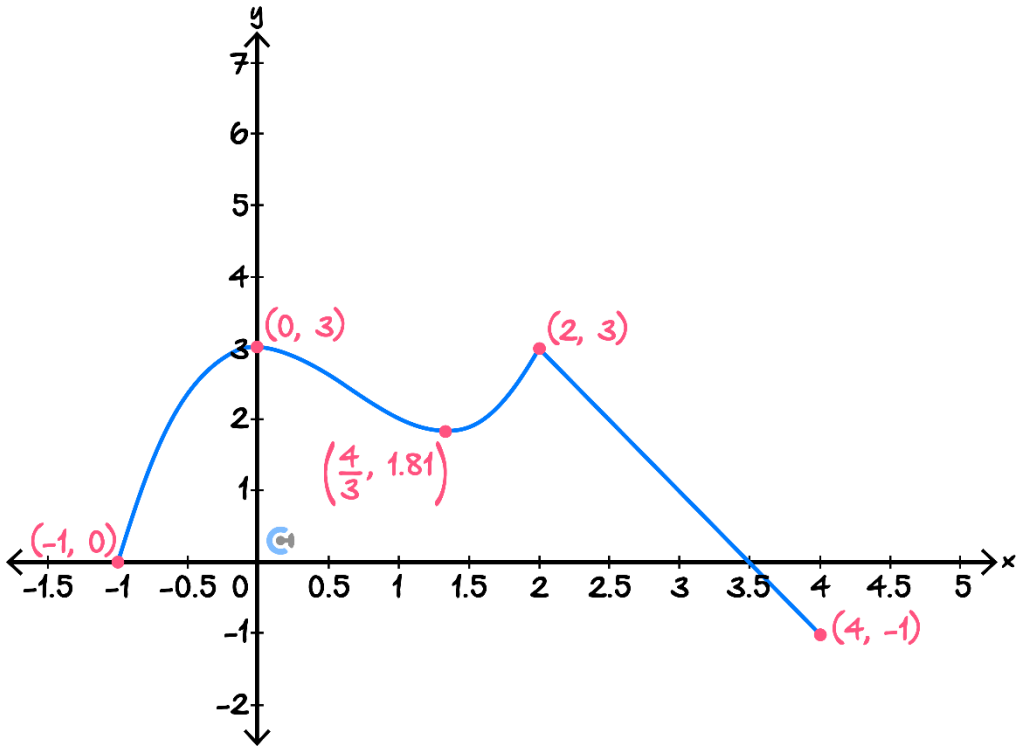
Question 6

Consider the following function. Sketch the corresponding derivative function on the same set of axes.



a. Sketch the corresponding derivative function on the same set of axes above.

b. Furthermore, state the domain of the derivative function.

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Question 7

Consider the following function $f(x)$ with rule:

$$f(x) = \begin{cases} x^2 - 4x + 3, & x < 1 \\ ax + b, & x \geq 1 \end{cases}$$

Find the value of a and b such that $f(x)$ is differentiable at $x = 1$.

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Question 8 Tech-Active.

Consider the following function $f(x)$ with rule:

$$f(x) = \begin{cases} x^3 - x^2 - 2x + 3, & x < 2 \\ -x^2 + bx + c, & x \geq 2 \end{cases}$$

Find the value of a and b such that $f(x)$ is differentiable at $x = 2$.

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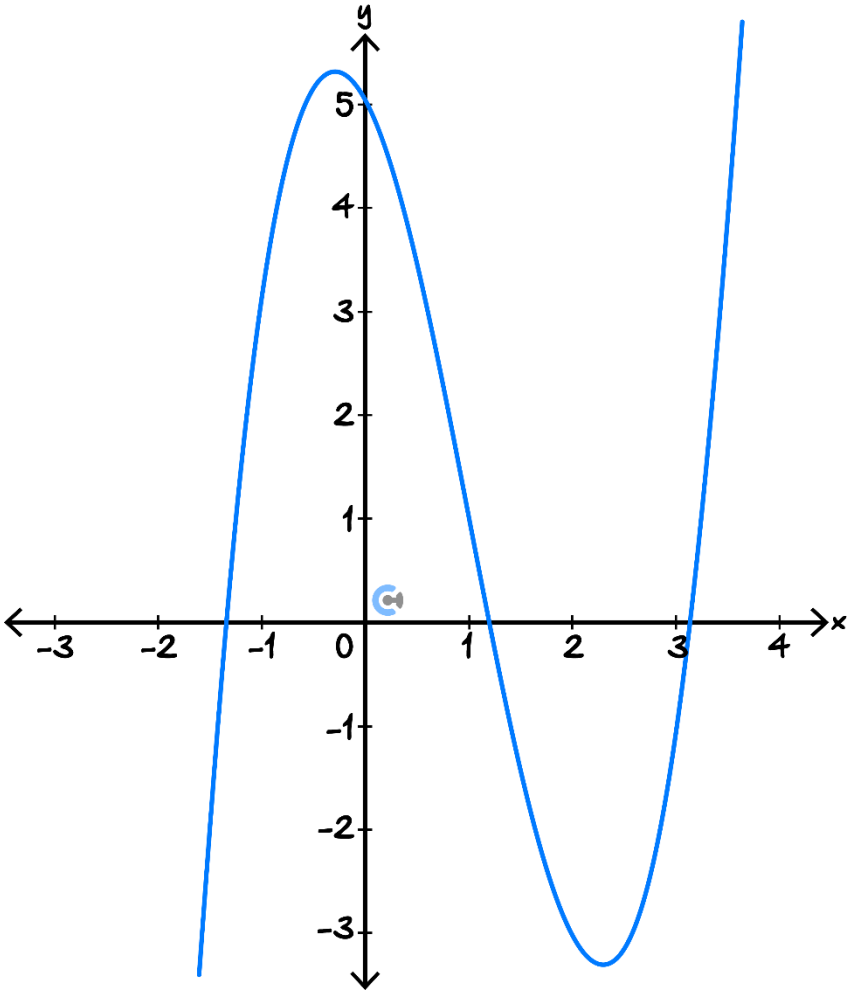
Sub-Section [2.2.3]: Identify Concavity and Find Inflection Points

Question 9

Consider the following graph for $f(x)$.



a. Circle the point of inflection on the above graph.

b. State the values of x such that the function is concave up.

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c. State the values of x such that the function is concave down.

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Question 10

Consider a function $f: \mathbb{R} \to \mathbb{R}, f(x) = x^4 + 2x^3 - 12x^2 + 6x + 4$.

a. Calculate the second derivative of the function $f(x)$.

b. Find the points of inflection of the function $f(x)$.

c. Find the values of x where the function is concave up.

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Question 11

Suppose that a function $f(x)$ is double differentiable for all $x \in (0,2)$, and satisfies the following properties:

- $f''(1) = 0$
- $f'(0) = 2$
- $f'(0.5) = 0$
- $f'(0.75) = -0.5$
- $f'(1) = -2$
- $f'(1.25) = -0.5$
- $f'(1.5) = 0$
- $f'(1.75) = 0.5$

Find the values of x such that the function is concave down.

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Question 12 Tech-Active.

Find a rule of a polynomial $f(x)$ so that $f(0) = 3, f(1) = 2, f'(2) = -8$, and so that there is a point of inflection when $x = 2$.

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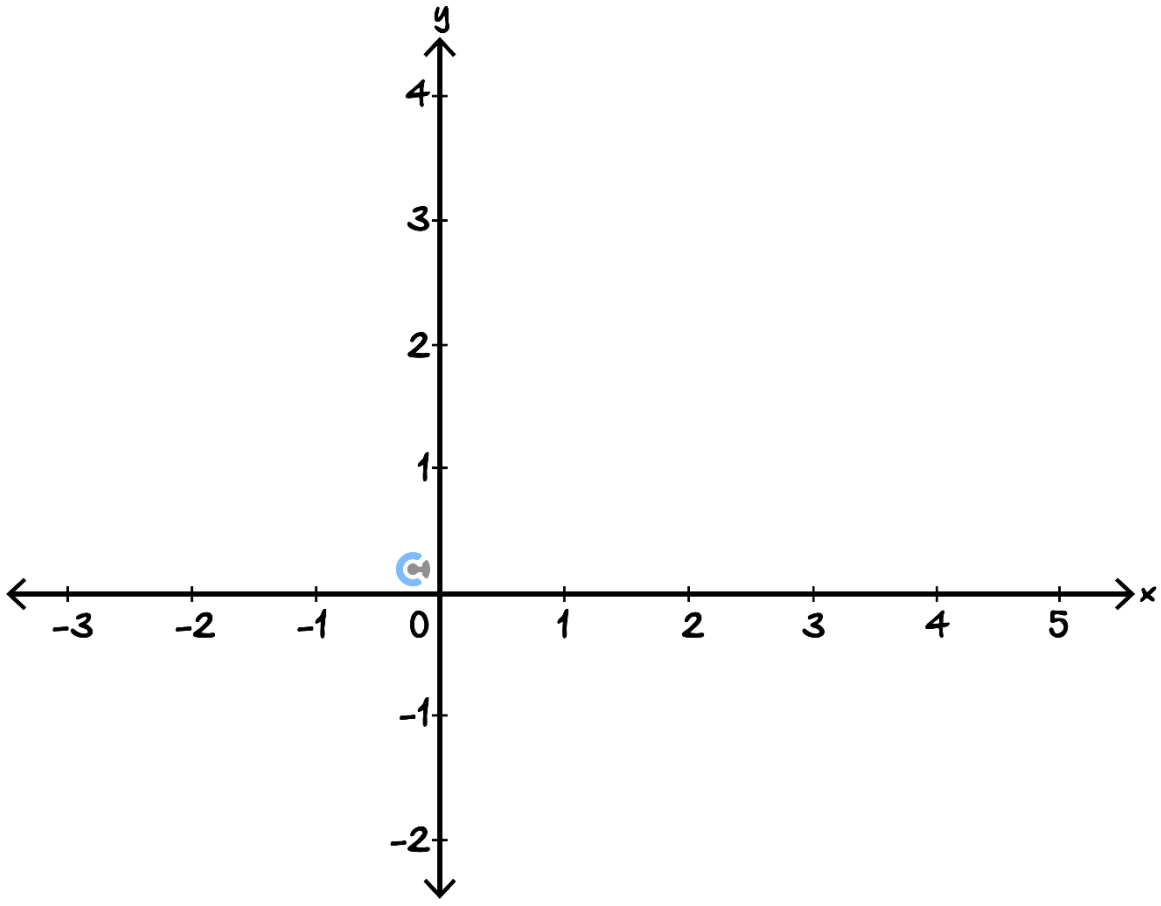
Sub-Section: The 'Final Boss'

Question 13

Consider the hybrid function:

$$f(x) = \begin{cases} x^3 - 3x + 1, & -2 \leq x \leq 1 \\ x - 3, & x > 1 \end{cases}$$

a. Sketch the graph of $y = f(x)$ on the axes below.



b. Define the derivative function $f'(x)$, specifying its domain.

c. State the point of inflection for the function $f_1(x) = x^3 - 3x + 1$.

d. The function:

$$g(x) = \begin{cases} x^3 - x^2 + a - 2, & x \leq 1 \\ x^2 + bx + 3, & x > 1 \end{cases}$$

is continuous and differentiable for all $x \in \mathbb{R}$. Find the values of a and b.

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Section B: Supplementary Questions

Sub-Section [2.2.1]: Evaluate Limits and Find Points Where the Function is not Continuous

Question 14

Evaluate the following limits:

a. $\lim_{x \to 3}(x^3 - 2x^2 + 5)$

b. $\lim_{x \to 4}(2^{\sqrt{x}} + \log_3(x^3 + 2x))$

c. $\lim_{x \to 3}(f(x))$, where,

$$f(x) = \begin{cases} 2x + 1, & x < 3 \\ 3x - 2, & x \geq 3 \end{cases}$$

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Question 15

Find the points x for which the following functions are **discontinuous** and state a reason as to why they are discontinuous.

a. $f(x) = \begin{cases} x, & x < 1 \\ x + 1, & x \geq 1 \end{cases}$

b. $f(x) = \frac{50}{x^2 - 7x + 6}$

c. $f(x) = \frac{x^2 - 4x + 3}{x - 3}$

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Question 16

Consider the following function $f(x)$ with rule:

$$f(x) = \begin{cases} 3^{x-2} + 5x, & x < 2 \\ ax + 6, & x \geq 2 \end{cases}$$

Find the value of a such that $f(x)$ is continuous for all $x \in \mathbb{R}$.

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Question 17

Consider the following function $f(x)$ with rule:

$$f(x) = \begin{cases} x^2 - 4x - 12, & x < 7 \\ a^2 - ax + 1, & 7 \leq x < 10 \\ -x - 5, & x \geq 10 \end{cases}$$

Find the value of a such that $f(x)$ is continuous for all $x \in \mathbb{R}$.

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Sub-Section [2.2.2]: Apply Differentiability to Find Points Where Functions are not Differentiable, Domain of the Derivative and Unknowns of a Function

Question 18

Find the values of x such that the following function are not differentiable.

a. $f(x) = \begin{cases} -x+5, & x < 2 \\ x+1, & x \geq 2 \end{cases}$

b. $f(x) = \frac{1}{x^2-4x+3}$

c. $f(x) = \begin{cases} 2x, & x < 0 \\ 2x+1, & x \geq 0 \end{cases}$

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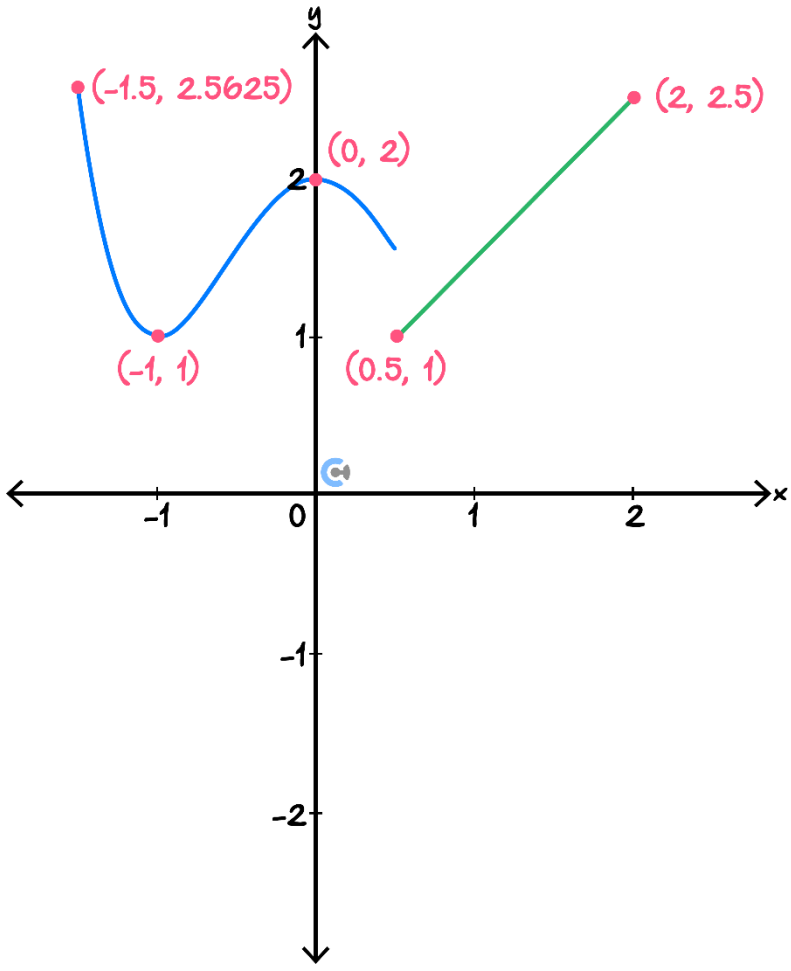
Question 19

Consider the following function.



a. Sketch the corresponding derivative function on the same set of axes above.

b. Furthermore, state the domain of the derivative function.

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Question 20

Consider the following function $f(x)$ with rule:

$$f(x) = \begin{cases} 2x^2 - 6x + 5, & x < 2 \\ ax + b, & x \geq 2 \end{cases}$$

Find the value of a and b such that $f(x)$ is differentiable at $x = 2$.

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Question 21

Consider the following function $f(x)$ with rule:

$$f(x) = \begin{cases} x^3 - 3x + 5, & x < -1 \\ g(x), & -1 \leq x < 1 \\ x^2 - 5x + 2, & x \geq 1 \end{cases}$$

The goal for this question is to find a suitable rule $g(x)$ making $f(x)$ differentiable for all $x \in \mathbb{R}$.

a. State the four equations that $g(x)$ and $g'(x)$ must satisfy at $x = 1$ and $x = -1$.

b. A natural choice would be to let $g(x)$ be a polynomial. As there are four equations that need to be satisfied, explain why it is suitable to set $g(x)$ to be a cubic polynomial.

c. Hence, find a suitable rule for $g(x) = ax^3 + bx^2 + cx + d$ assuming $g(x)$ is a polynomial. It may be necessary to use a CAS to solve the system of equations obtained in the working.

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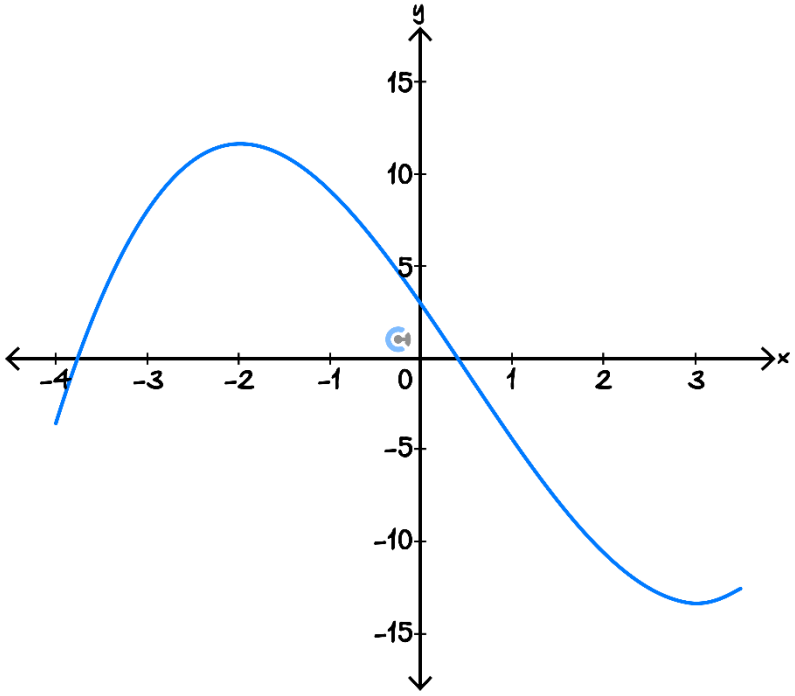
Sub-Section [2.2.3]: Identify Concavity and Find Inflection Points

Question 22

Consider the following graph for $f(x)$.



a. Circle the point of inflection on the above graph.

b. State the values of x such that the function is concave up.

_____

c. State the values of x such that the function is concave down.

_____

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Question 23

Consider a function $f: \mathbb{R} \to \mathbb{R}, f(x) = x^4 - 2x^3 - 36x^2 + 5x + 1$.

a. Calculate the second derivative of the function $f(x)$.

b. Find the points of inflection of the function $f(x)$.

c. Find the values of x where the function is concave up.

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Question 24

Suppose that a function $f(x)$ is double differentiable for all $x \in (0,2)$, and satisfies the following properties:

- $f''(1) = 0$
- $f'(0) = 1$
- $f'(0.5) = 0$
- $f'(0.75) = -0.71$
- $f'(1) = -1$
- $f'(1.25) = -0.71$
- $f'(1.5) = 0$

Find the values of x such that the function is concave up.

Question 25

Find a rule of a polynomial $g(x)$ so that $g(0) = 12,\ g(1) = 9,\ g(2) = 0$, and so that there is a point of inflection when $x = 2$.

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