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VCE Mathematical Methods ¾
Differentiation I [2.1]
Workbook

Outline:

Introduction to Differentiation Pg 2-13
- Average Rate of Change
- Instantaneous Rate of Change
- Differentiation
- Understanding Differentiation

Advanced Differentiation Pg 14-30
- Product Rule
- Quotient Rule
- Chain Rule

Stationary Points and Strictly Increasing Pg 31-35
- Stationary Points
- Strictly Increasing and Decreasing

Graphs of the Derivative Function Pg 36-39
- Graphs of Derivative Function

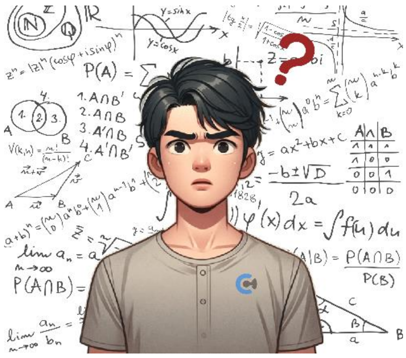
Learning Objectives:

- ☐ **MM34 [2.1.1]** - Find the instantaneous rate of change and average rate of change.
- ☐ **MM34 [2.1.2]** - Identify the nature of stationary points and trends (strictly increasing and decreasing).
- ☐ **MM34 [2.1.3]** - Graph Derivative Functions.

Section A: Introduction to Differentiation

What in the world is differentiation?

Context: Calculus and Sam's Weight



- Sam as usual loses control and eats 100 chocolates.
- He wants to find out how many kg he gained with respect to the chocolates.

How many kilograms did I gain with respect to 100 chocolates?

- Sam is getting even more curious and asks the following equation:

How many kilograms did I gain with respect to 10 chocolates?

- Even more curious!

How many kilograms did I gain with respect to 1 chocolate?

How many kilograms did I gain with respect to 0.1 chocolate?

How many kilograms did I gain with respect to 0.00001 chocolat

- Sam is finding the rate of change of weight with respect to choc..

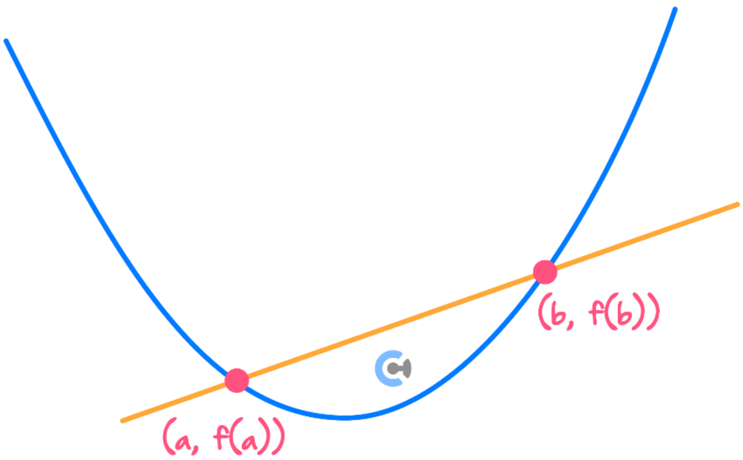
  I.e. Sam is differentiating his weight with respect to the number of chocolates.

- Today we will ask this question!

How much did the y value change with respect to tiny change in x

Sub-Section: Average Rate of Change

Average Rate of Change



- The average rate of change of a function $f(x)$ over $x \in [a, b]$ is given by:

$$\textit{Average rate of change} = \frac{f(b) - f(a)}{b - a}$$

- It is the gradient of the line joining the two points.

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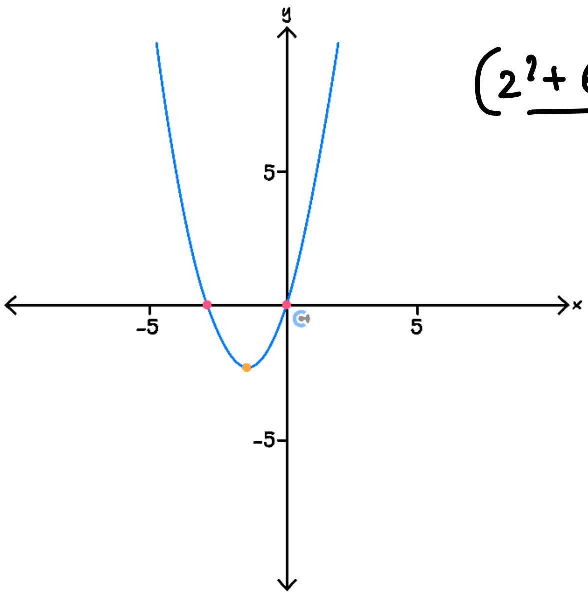
Question 1

Find the average rate of change of $y = x^2 + 3x$ over the interval $x \in [-1,2]$.



$$\frac{(2^2+6)-(1-3)}{2-(-1)}$$

$$= 4$$

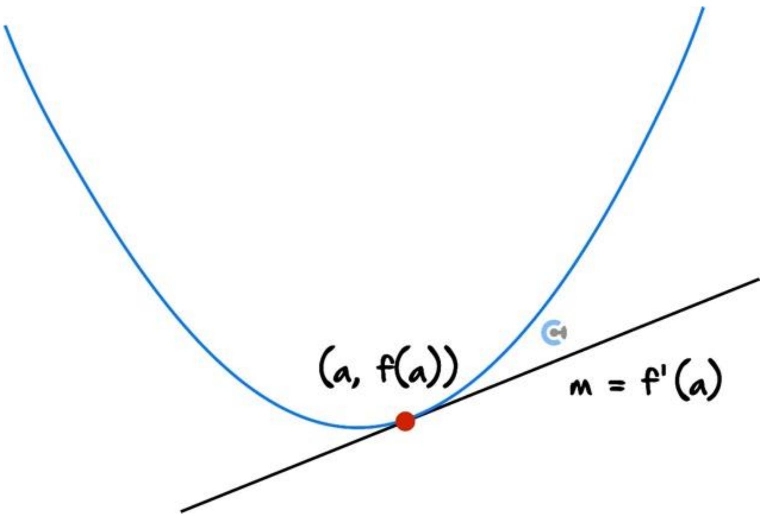
NOTE: We do **NOT** need differentiating to find the average rate of change!

Discussion: Is finding the average rate of change same as Sam finding weight change over the entire chocolates he ate or the tiny piece of the chocolate?

Sub-Section: Instantaneous Rate of Change

How can Sam find his weight change from the tiniest atom of chocolate?

Instantaneous Rate of Change



- **Instantaneous Rate of Change** is a gradient of a graph at a single point.

$$\textit{Instantaneous Rate of Change} = f'(a)$$

- **Differentiation** is the process of finding the derivative of a function.

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Question 2

Consider the function $f(x)$ and its derivative $f'(x)$. It is known that $f(2) = 4$, $f(3) = 9$, $f'(2) = 3$ and $f'(3) = -4$.

Find the gradient of the function $f(x)$ at $x = 3$.

$f'(3) = -4.$

NOTE: Derivative function gives us the gradient of a point.

<u>Alternative Notation for Derivative</u>

Definition

$$f'(x) = \frac{dy}{dx}$$

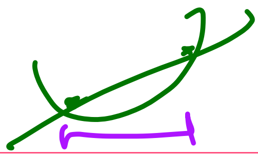
change
delta.
differ
differ

differential
= instantaneous change

<u>Discussion:</u> How does the notation $\frac{dy}{dx}$ make sense?

$\Delta x \to 0 \therefore dx$

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{dy}{dx}$$



$\Delta x \to 0.$

Sub-Section: Differentiation

How do we find derivative functions?

Definition

Derivatives of Functions

- The derivatives of many of the standard functions are in the summary table below:

$f(x)$	$f'(x)$
x^n	$n \cdot x^{n-1}$
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$\frac{1}{\cos^2(x)} = \sec^2(x)$
e^x 2.72	e^x
$\log_e(x)$	$\frac{1}{x}$

Question 3

Consider the function $f(x) = x^3 - 4x$.

Find the gradient of the function at $x = 2$.

$f'(x) = 3x^2 - 4$

$f'(2) = 12 - 4$

$= 8$

Question 4

Consider the function $f(x) = 2e^x - 4$.

Find the gradient of the function at $x = 3$.

$f'(x) = 2e^x$

$f'(3) = 2e^3$

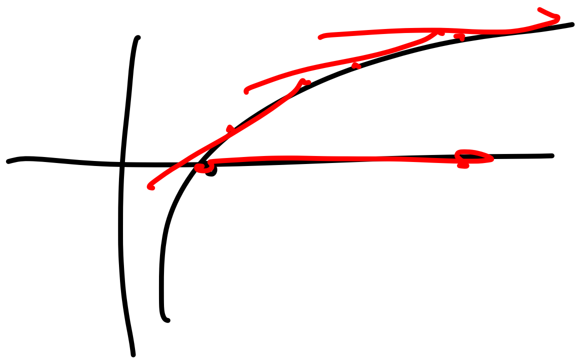
Question 5

Consider the function $f(x) = 2\log_e(x)$.

Find the gradient of the function at $x = 2e$.

$f'(x) = 2 \cdot \frac{1}{x} = \frac{2}{x}$

$f'(2e) = 2\frac{1}{2e} = \frac{1}{e}$



Question 6

Consider the function $f(x) = \cos(x) + \sin(x)$.

Find the gradient of the function at $x = \frac{\pi}{4}$.

$$f'(x) = -\sin(x) + \cos(x)$$

$$f'\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = 0$$

Calculator Commands: Finding Derivatives

Mathematica	TI	Casio
f'[x]	Shift Minus $\frac{d}{dx}(f(x))$	Math2 $\frac{d}{dx}(f(x))$

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Question 7 Tech-Active.

Consider the function $f(x) = \tan(x)$.

Find the gradient of the function at $x = \frac{\pi}{6}$.

ctrl $\boxed{\equiv}$

$$\frac{d}{dx}(\tan(x)) \Big| x = \frac{\pi}{6}$$

$$= \frac{4}{3}$$

$$\cancel{\frac{d}{dx}\left(\tan\left(\frac{\pi}{6}\right)\right)}$$

$$\cancel{= \frac{d}{dx}\left(\frac{1}{\sqrt{3}}\right)}$$

$$\cancel{= 0.}$$

NOTE: You must substitute the x value **after** finding the derivative function first.

<u>Discussion:</u> What would happen if you derived $f\left(\frac{\pi}{6}\right)$ instead on CAS?

0.

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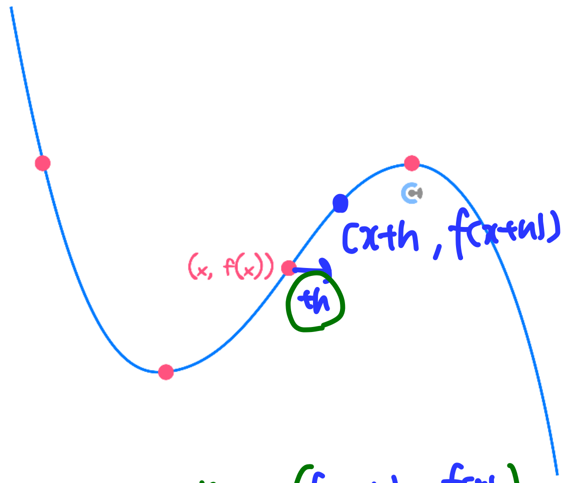
Sub-Section: Understanding Differentiation

How does this work?

First Principles

Definition



$$f'(x) = \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{x+h-x} \right)$$

- The fundamental method of diffing.

Pascal 4:

1

1 1

1 2 1

1 3 3 1

Question 8

Consider the function $f(x) = x^3$.

Find the derivative using the first principle.

$$f'(x) = \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{x+h-x} \right)$$

$$= 3x^2$$

$$f(x+h) = (x+h)^3 = x^3 + 3x^2h + 3xh^2 + 1h^3$$

$$f(x+h) - f(x) = 3x^2h + 3xh^2 + h^3$$

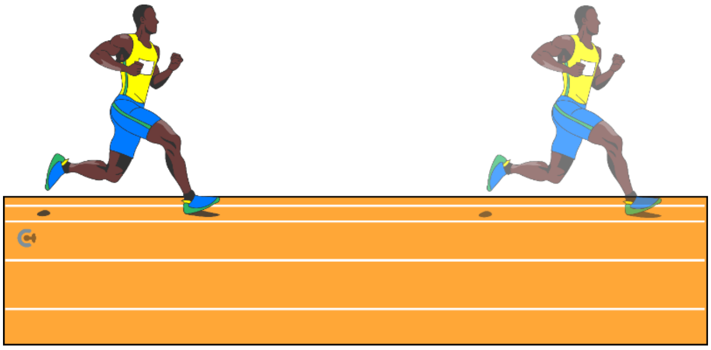
$$\frac{f(x+h) - f(x)}{h} = 3x^2 + \underbrace{3xh + h^2}_{\to 0}$$

NOTE: It's the same as the table above!

Analogy: Understanding the instantaneous rate of change.

- Usain Bolt ran the world record $100\ m$ race in 9.58 seconds.

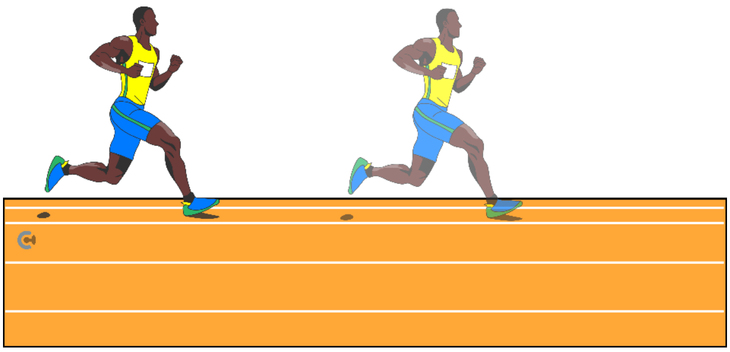
- Let's say we take a photo of Usain Bolt at the start and end of the race.

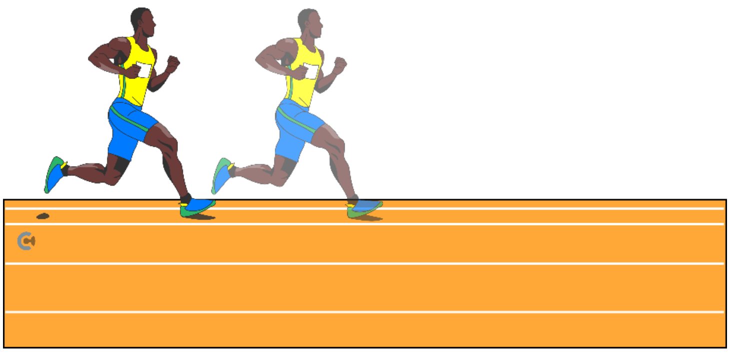


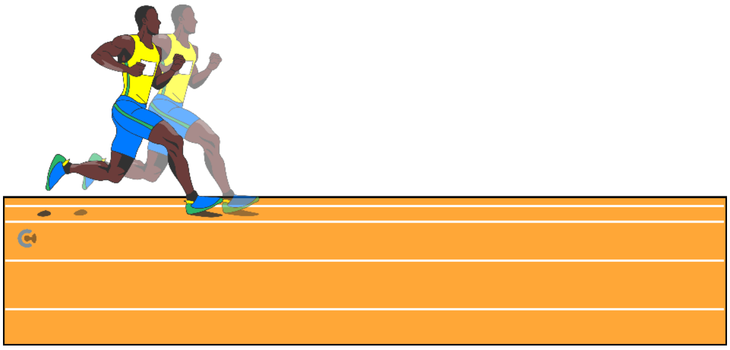
$$\boldsymbol{Average\ Speed = \frac{Distance}{Time\ Taken}}$$

- We can calculate the average speed of the entire race.

- Now let's say we take the two photos closer to each other.







- What would the average speed between two photos slowly approach to?

Speed at a given moment!

- In summary how do we find the speed of Usain Bolt at a single moment?

We take two photos so close to each other that the time between the snapshots becomes basically zero.

Discussion: If $x =$ Usain's distance travelled and $t =$ time travelled, what notation would represent his instantaneous speed?

dx/dt

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Section B: Advanced Differentiation

Sub-Section: Product Rule

How do we find the derivative when two functions are multiplied?
For example: $x^2 \sin(x)$.

The Product Rule

* Theorem: We never diff 2 funcs in 1 go.

- The derivative of $h(x) = f(x) \times g(x)$ is given by:

$$h'(x) = f'(x)\, g(x) + f(x) \cdot g'(x)$$

- Or, in another form:

$$\frac{d}{dx}(u \cdot v) = u'v + uv'$$

NOTE: Order does **not** matter.

Question 9 Walkthrough.

Find the derivative of $f(x) = x^3 \tan(x)$.

$$f'(x) = 3x^2 \cdot \tan(x) + x^3 \cdot \sec^2(x)$$

NOTE: We **never** differentiate **both** functions at the same time!

Your turn!

Question 10

Find the derivatives of:

a. $f(x) = x^2 e^x$

$f'(x) = 2xe^x + x^2 e^x$

b. $y = 3\sin(x)\cos(x)$ $\frac{3}{2}\sin(2x)$

$\frac{dy}{dx} = 3\cos(x)\cos(x) + 3\sin(x)\cdot -\sin(x)$

$= 3\cos^2(x) - 3\sin^2(x)$ $3\cos(2x)$

c. $g(x) = \log_e(x) \cdot x$

$g'(x) = \frac{1}{x}\cdot x + \log_e(x)\cdot 1$

$= 1 + \log_e(x)$

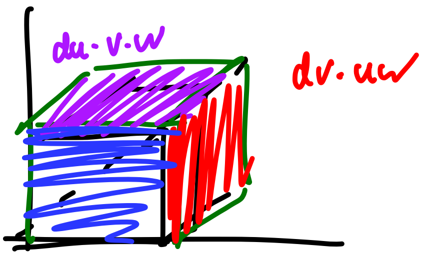
Question 11 Extension.

$u \cdot v \cdot w$

Find the derivative of $f(x) = x^3 \log_e(x) \sin(x)$.

x^2

$3x^2 \log_e(x) \sin(x) + x^3 \cdot \frac{1}{x} \sin(x) + x^3 \log_e(x) \cos(x)$



tor^{-2}

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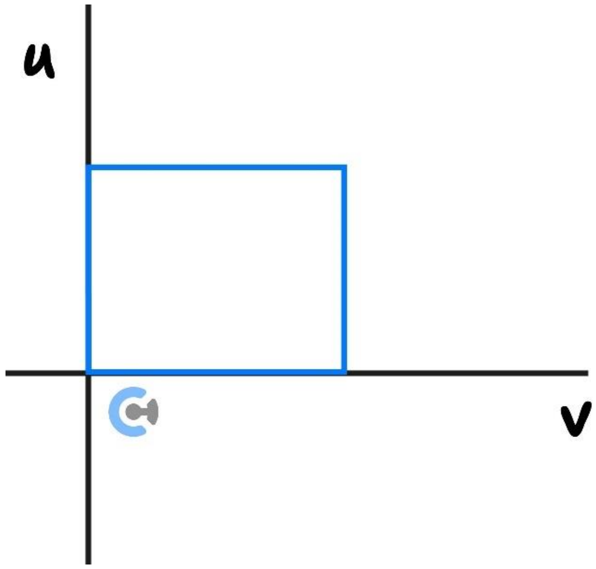
How does this work?

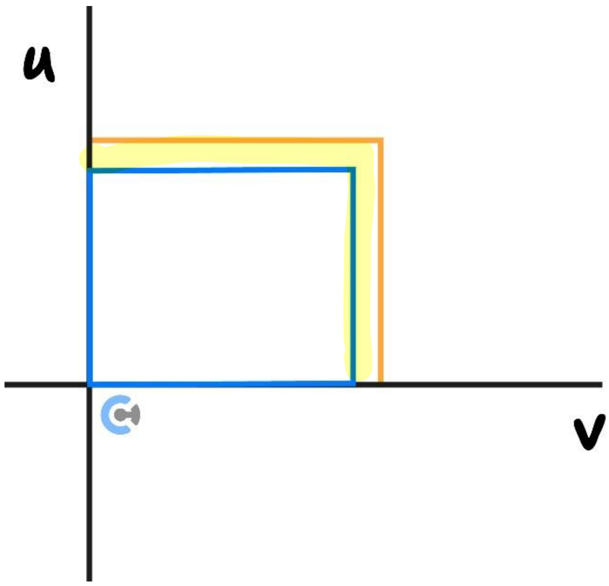
Extension: Understanding Product Rule

- Consider the rectangle in the diagram below. $\frac{d}{dx}(uv) = \frac{du}{dx} \cdot v + u \cdot \frac{dv}{dx}.$



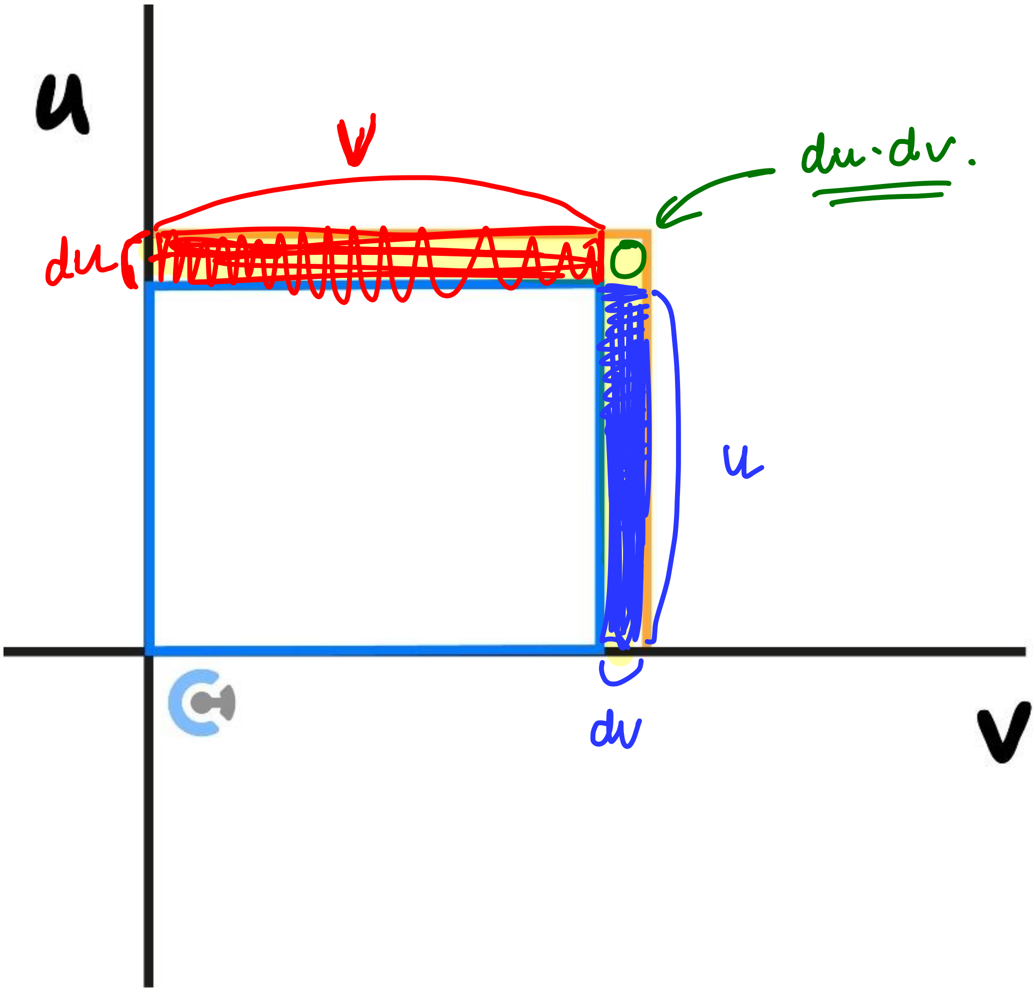
$$u \cdot v = \textit{area of the rectangle}$$

- Now let's say the rectangle grew in size!



- How do we find the rate of area of the rectangle?
- The image above shows two snapshots of u and v **very close to each other** while they are changing.

➤ How can we find the instantaneous **change of $u \cdot v$**?



$d(u \cdot v) =$ $v \cdot du + u\,dv$

➤ Therefore, how can we find $\frac{d(u \cdot v)}{dx} = \frac{d}{dx}(u \cdot v)$?

$$\frac{d(u \cdot v)}{dx} = v \cdot \frac{du}{dx} + u \cdot \frac{dv}{dx}$$

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Sub-Section: Quotient Rule

Definition

The Quotient Rule

- The derivative of a $h(x) = \frac{f(x)}{g(x)}$ is given by:

$$h'(x) = \frac{f'(x)g(x) - f(x) \cdot g'(x)}{(g(x))^2}$$

- Or, written in another form:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - uv'}{v^2}$$

Always differentiate the top function first.

Question 12 Walkthrough.

Find the derivative of $y = \frac{x^2}{\sin(x)}$.

$$\frac{dy}{dx} = \frac{2x\sin(x) - x^2\cos(x)}{\sin^2(x)}$$

NOTE: The order **matters** for the quotient rule! We differentiate the **top function first**.

Question 13

Find the derivatives of:

a. $\frac{e^x}{4x^3}$

$$\frac{d}{dx}\left(\frac{e^x}{4x^3}\right) = \frac{e^x \cdot 4x^3 - e^x \cdot 12x^2}{16x^6}$$

$$= \frac{4x^2 e^x (x-3)}{16x^6}$$

$$= \frac{e^x(x-3)}{4x^4}$$

b. $\frac{\log_e(x)}{x}$

$$\frac{d}{dx}\left(\frac{\log_e(x)}{x}\right) = \frac{\frac{1}{x} \cdot x - \log_e(x) \cdot 1}{x^2}$$

$$= \frac{1 - \log_e(x)}{x^2}$$

c. $g(x) = \frac{\sin(x)}{\cos(x)}$ tan

$$g'(x) = \frac{\cos(x)\cos(x) - \sin(x) \times -\sin(x)}{\cos^2(x)} = \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)}$$

$$= \frac{1}{\cos^2(x)}$$

NOTE: The last question is a derivative of tan.

Question 14 Extension.

Find the derivative of $y = \frac{x^2 e^x}{\log_e(x)}$.

$$\frac{dy}{dx} = \frac{(2xe^x + x^2e^x)\log_e(x) - x^2e^x \cdot \frac{1}{x}}{(\log_e(x))^2}$$

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Prove: $y = \frac{u}{v}$. $\Rightarrow$ $y' = \frac{u'v - uv'}{v^2}$

$yv = u$

$y'v + yv' = u'$

$y'v = u' - yv'$

$y' = \frac{u' - yv'}{v} = \frac{u' - \frac{u}{v} \cdot v'}{v}$

$= \frac{u'v - uv'}{v^2}$

Sub-Section: Chain Rule

The Chain Rule

$$y = f(g(x))$$

$$\frac{dy}{dx} = f'(g(x)) \times g'(x).$$

- The process for finding derivatives of **composite functions.**

How does the chain rule work?

Exploration: Understanding Chain Rule

- Consider the function we want to differentiate with respect to x is:

$$y = f(g(x))$$

- We can remove the composition by letting the inside function equal to u.

$$\text{Let } u = g(x)$$

$$\text{Then } y = f(u)$$

- We can now derive y respect to u.

$$\frac{dy}{du} = f'(u)$$

Note that we have $\frac{dy}{du}$ instead of $\frac{dy}{dx}$ as we derived in terms of u.

- To find $\frac{dy}{dx}$ we simply multiply by $\frac{dy}{du}$ with $\frac{du}{dx}$.

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = f'(u) \times \frac{du}{dx}$$

- Finally, we can substitute $u = g(x)$.

$$\frac{dy}{dx} = f'(g(x)) \times g'(x)$$

Question 15 Walkthrough.

Find the derivative of $f(x) = \sin(x^2)$.

1) let $y = \sin(u)$, $u = x^2$

2) $\frac{dy}{du} = \cos(u)$. $\frac{du}{dx} = 2x$

3) $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \cos(u) \times 2x$

$= \cos(x^2) \cdot 2x$

NOTE: Always let the inside function equal to u.

Your turn!

Question 16

Find the derivatives of:

a. $e^{x^2+\frac{1}{2}x}$

$y = e^u, \quad u = x^2 + \frac{1}{2}x.$

$\frac{dy}{du} = e^u, \quad \frac{du}{dx} = 2x + \frac{1}{2}$

$\frac{dy}{dx} = e^{x^2+\frac{1}{2}x} \cdot \left(2x + \frac{1}{2}\right)$

b. $\left(4x + \frac{1}{x}\right)^3$

$\frac{1}{x} = x^{-1}$ ⟶ $-1 \cdot x^{-2}$ (-) & square the bottom

let $y = u^3, \quad u = 4x + \frac{1}{x}$

$\frac{dy}{du} = 3u^2 \quad \frac{du}{dx} = 4 - \frac{1}{x^2}$

$\frac{dy}{dx} = 3\left(4x + \frac{1}{x}\right)^2 \cdot \left(4 - \frac{1}{x^2}\right)$

c. $\log_e(x^2)$

$\log_e(x^2) = 2\log_e(x)$

$y = \log_e(u), \quad u = x^2$

$\frac{dy}{du} = \frac{1}{u}, \quad \frac{du}{dx} = 2x$

$2\log_e(x)$

$2 \cdot \frac{1}{x}$

$\frac{dy}{dx} = \frac{1}{x^2} \times 2x = \frac{2}{x}$

Question 17 Extension.

Find the derivative of $f(x) = x^3 \log_e(x^2) \sin^2(x)$.

$\underbrace{\log_e(x^2)}_{2\log_e(x)}$

$y = (u)^2 \qquad u = \sin(x)$

$\frac{dy}{du} = 2u. \qquad \frac{du}{dx} = \cos(x)$

$$f'(x) = 3x^2 \log_e(x^2) \sin^2(x) + x^3 \cdot \frac{2}{x} \cdot \sin^2(x) + x^3 \cdot \log_e(x^2) \times 2\sin(x) \cdot \cos(x)$$

Is there a quicker way to do a chain rule?

Definition

Shortcut for Chain Rule

$$y = f(g(x))$$

$$\frac{dy}{dx} = f'(g(x)) \times g'(x).$$

- Derive the outside function only.
- Multiply the function by the derivative of the inside.

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Question 18 Walkthrough.

Using the quick method of chain rule, find the derivative of $f(x) = \cos(x^3)$.

$f'(x) = -\sin(x^3) \times 3x^2$

$= -3x^2 \sin(x^3)$

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Your turn!

Question 19

Using the quick method of chain rule, find the derivative of:

a. e^{3x^2-x}

$e^{3x^2-x} \times (6x-1)$

b. $\log_e(x^2+9x+6)$

$\frac{1}{x^2+9x+6} \times (2x+9)$

c. $g(x) = \tan(x^2)$

$\sec^2(x^2) \times 2x$

Question 20 Extension.

Find the derivative of $g(t) = \log_e(\cos(\sqrt{t+1}))$.

$\cancel{(\ )^{\frac{1}{2}} \Rightarrow \frac{1}{2}(\ )^{-\frac{1}{2}}}$

$\sqrt{\ } \Rightarrow \frac{1}{2\sqrt{\ }}$

$$\frac{1}{\cos(\sqrt{t+1})} \times -\sin(\sqrt{t+1}) \times \frac{1}{2\sqrt{t+1}} \times 1.$$

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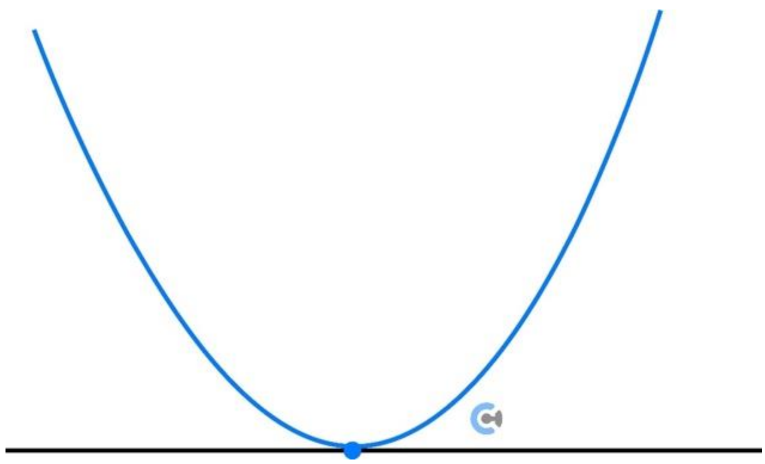
Section C: Stationary Points and Strictly Increasing

Sub-Section: Stationary Points

Discussion: What would be the gradient of a point that is neither increasing nor decreasing?

Zero

Stationary Points



- Point where the gradient of the function is zero.

$$f'(x) = 0, \qquad \frac{dy}{dx} = 0$$

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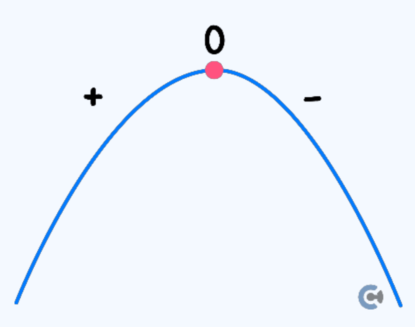
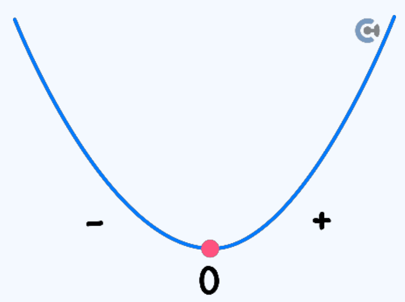
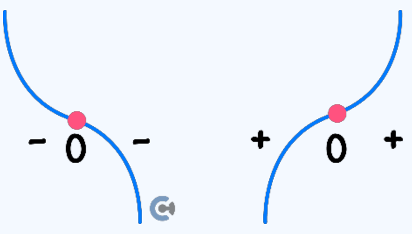
What are the types of stationary points?

Types of Stationary Points

Local Maximum	Local Minimum	Stationary Point of Inflection
		

Sign Test

- We can identify the nature of a stationary point by using the sign table.

x	Less than a	a	Bigger than a
$f'(x)$	Negative	0	Positive
Shape	∩ - Decreasing curve	Stationary Point	∪ - Increasing curve

- Find the gradient of the neighbouring points.

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Question 21 Walkthrough.

Find and identify the nature of the stationary points of $y = -e^{x^2+4}$.

$\frac{dy}{dx} = -e^{x^2+4} \times 2x = 0$

$2x = 0$

$x = 0$

$(0, -e^4)$

$\boxed{\text{Local Max}}$

x	-1	0	1
$\frac{dy}{dx}$	$+$	0	$-$
Shape	/	—	\

$\frac{dy}{dx}\Big|_{x=-1} = -e^5 \times -2 = 2e^5$

$\frac{dy}{dx}\Big|_{x=1} = -e^5 \times 2 = -2e^5$

Question 22

Find and identify the nature of the stationary points of $y = \log_e(x^2 + 4)$.

```
f[x_] := Log[x^2 + 4]
          |로그

Solve[f'[x] == 0, x]
|풀이 함수

{{x → 0}}

{f'[-1], f'[0], f'[1]}

{-2/5, 0, 2/5}

(* Local Minimum *)
```

Question 23 Extension.

$\frac{d}{dx}(x \times g(x))$

Consider the function $f(x) = xg(x)$.

It is known that, $g(0) = -5$, $g(1) = -2$ and $g(2) = 1$.
$g'(0) = -4$, $g'(1) = 2$ and $g'(2) = 3$ and that f has only one stationary point.

Show that $f(x)$ has a stationary point when $x = 1$ and identify its nature.

$f'(x) = 1 \cdot g(x) + x \cdot g'(x)$

$f'(1) = g(1) + 1 \cdot g'(1)$

$= -2 + 2 = 0$

Local Min

x	0	1	2
f'	-5	0	7
Shape	\	—	/

$f'(0) = g(0) + 0 \cdot g'(0) = g(0) = -5$

$f'(2) = g(2) + 2 \cdot g'(2) = 1 + 2 \times 3 = 7$

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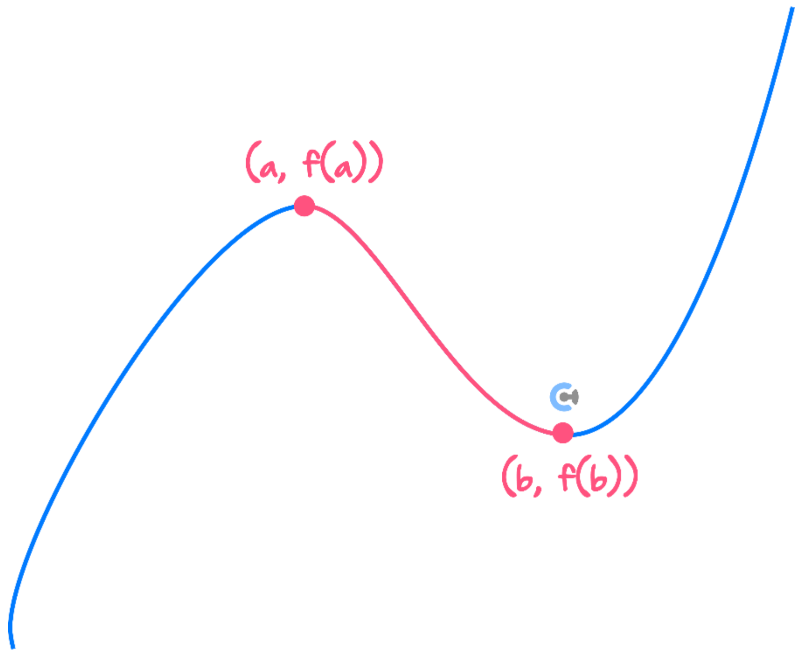
Sub-Section: Strictly Increasing and Decreasing

Strictly Increasing and Strictly Decreasing Functions



Strictly Increasing: $x \in (-\infty, a] \cup [b, \infty)$

Strictly Decreasing: $x \in [a, b]$

- Steps:

1. Find the turning points.
2. Consider the sign of the derivative between/outside the turning points.

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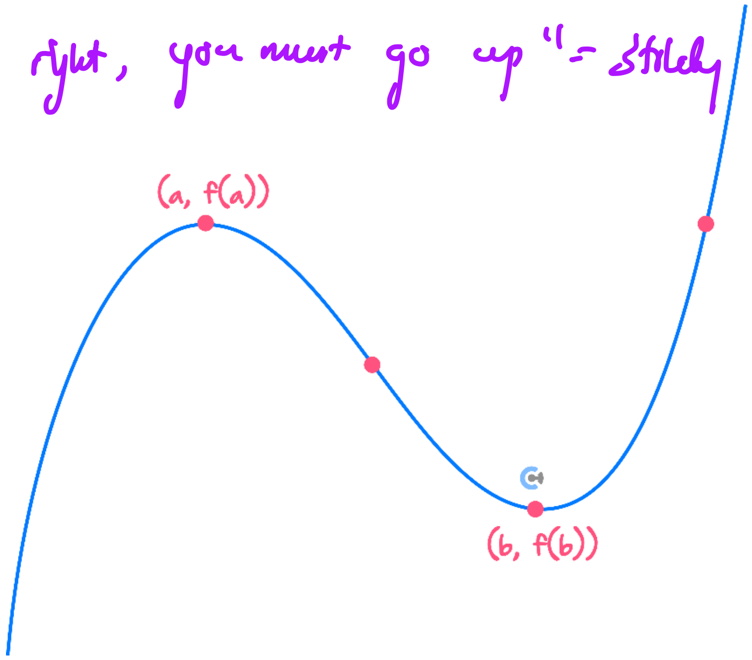
Question 24 Walkthrough.

State the value(s) of x for the function below for which it is strictly increasing and strictly decreasing.

"If you go right, you must go up" = strictly increasing



g: $(-\infty, a] \cup [b, \infty)$

g: $[a, b]$

TIP: The terminology is confusing. Simply remember the irony.

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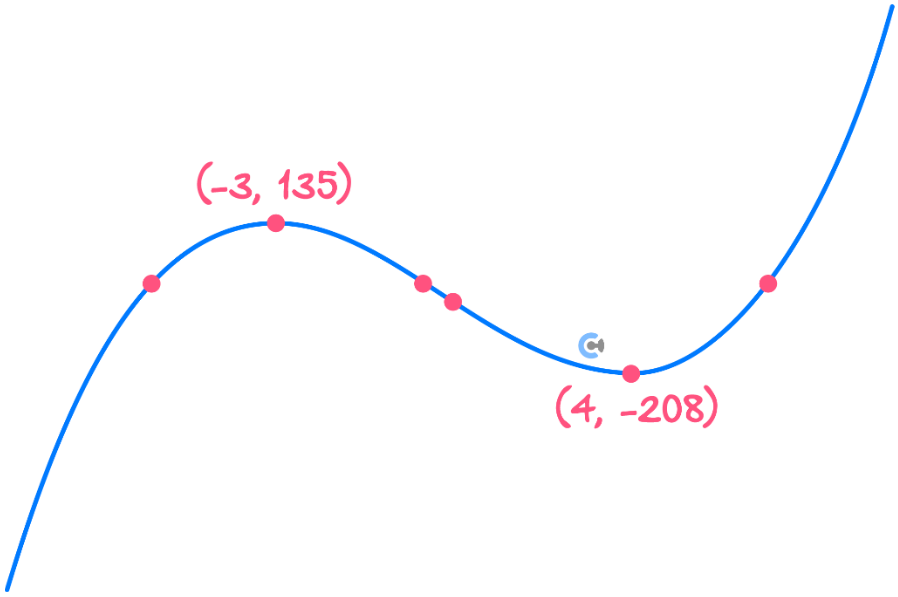
Question 25

State the value(s) of x for the function below for which it is strictly increasing and strictly decreasing.

$$y = -72x - 3x^2 + 2x^3$$



Increasing for $(-\infty, -3) \cup (4, \infty)$. Strictly Increasing $(-\infty, -3] \cup [4, \infty)$.
Decreasing $(-3, 4)$. Strictly Decreasing $[-3, 4]$.

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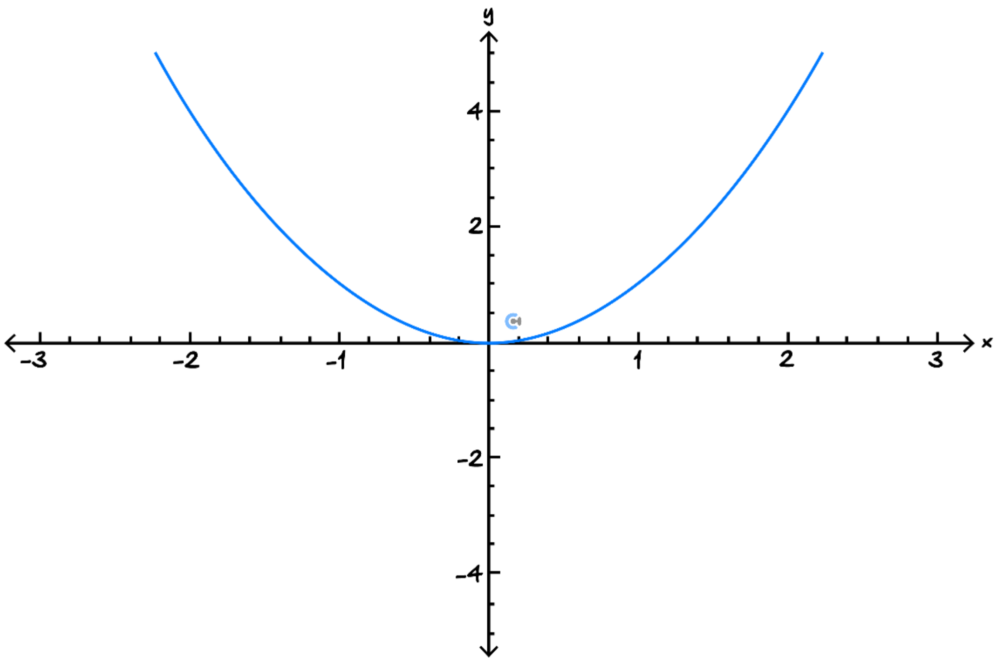
Section D: Graphs of the Derivative Function

Sub-Section: Graphs of Derivative Function

Exploration: Graph of Derivative Functions

- Consider the graph of $f(x) = x^2$ below.



- What is the derivative of $f(x)$?

  Sketch the derivative above.

$$f'(x) = 2x$$

- What do you notice about $f'(x)$: Derivative when $f(x)$ has a stationary point?

  It has an x-intercept.

- What do you notice about $f'(x)$ when $f(x)$ is increasing?

  It has a positive y value. (It is above the x-axis.)

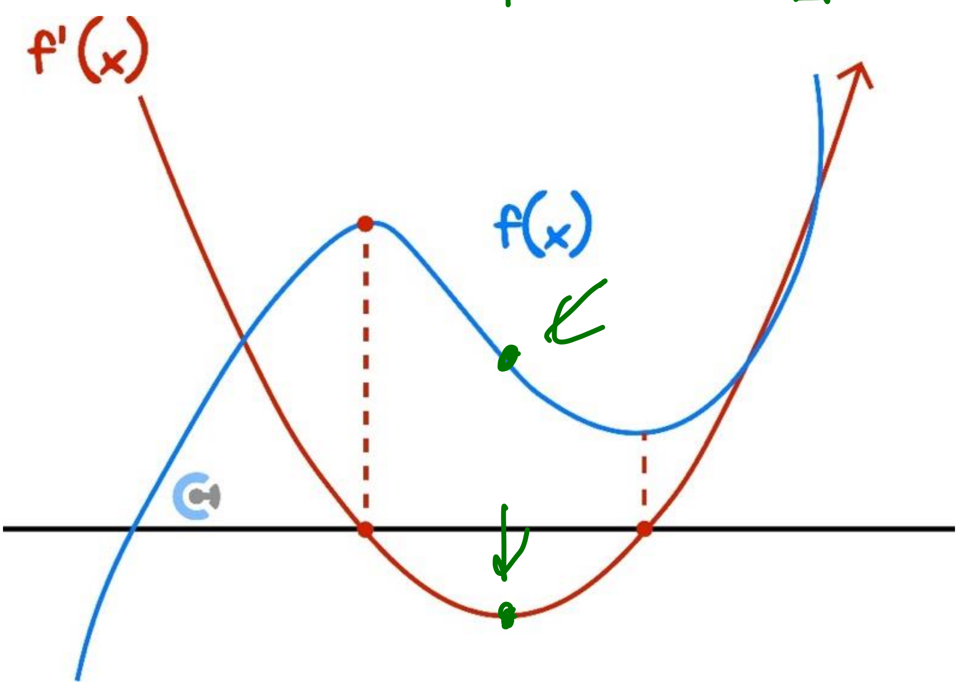
- What do you notice about $f'(x)$ when $f(x)$ is decreasing?

  It has a negative y value. (It is below the x-axis.)Type your text

In summary!

Graphs of the Derivative Function

Inflection → t.p



$f(x)$	$f'(x)$
Stationary Point	x intercept.
Increasing	Positive (Above x axis)
Decreasing	Negative (Below x axis)

y value of $f'(x)$ = gradient of the original.

- Steps

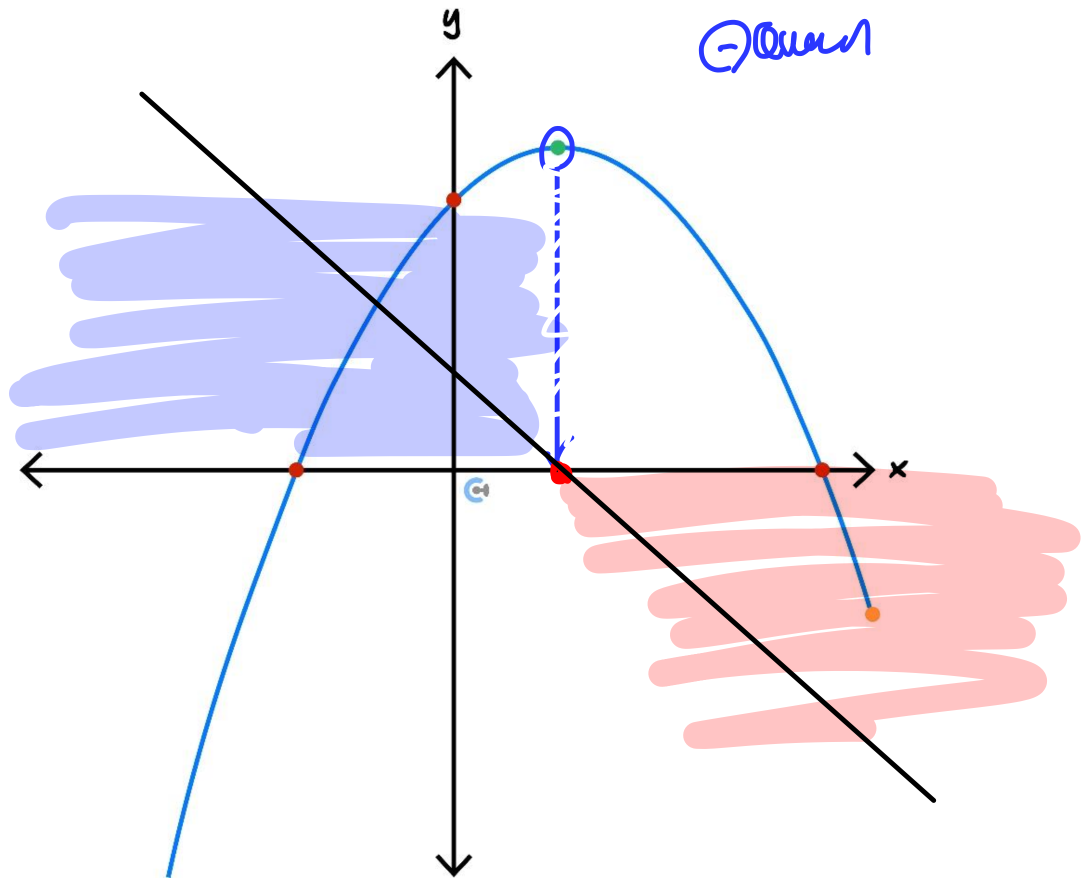
1. Plot x-intercept at the same x value as the stationary point of the original.
2. Consider the trend of the original function and sketch the derivative.

   Original is increasing → Derivative is above the x-axis.

   Original is decreasing → Derivative is below the x-axis.

Question 26 Walkthrough.

Sketch the derivative graph of the function shown below, on the same set of axes.



<u>Active Recall:</u> **Steps on sketching the derivative function**

1. Plot x-intercept at the same x value as the stationary point of the original.

2. Consider the trend of the original function and sketch the derivative.

   Original is increasing → Derivative is above the x-axis.

   Original is decreasing → Derivative is below the x-axis.

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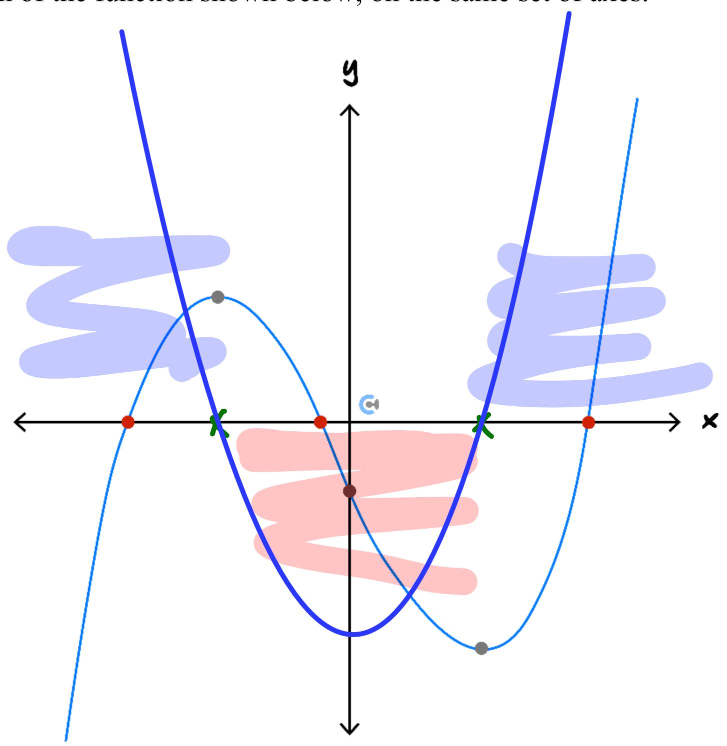
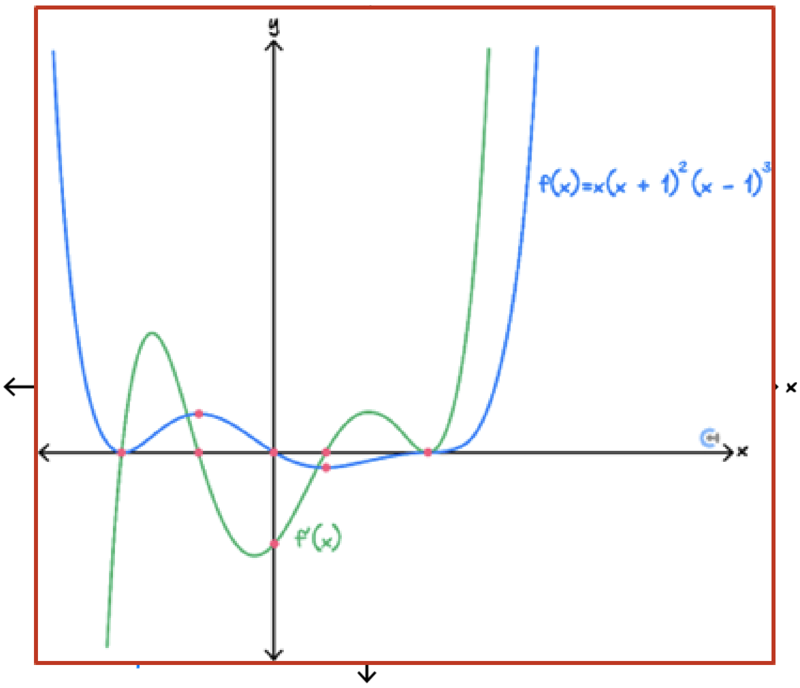
Question 27

Sketch the derivative graph of the function shown below, on the same set of axes.



Question 28 Extension.

Sketch the derivative graph of the function shown below, on the same set of axes.



Contour Check

Learning Objective: [2.1.1] - Find the Instantaneous Rate of Change and Average Rate of Change.

Key Takeaways

- The average rate of change of a function $f(x)$ over $x \in [a, b]$ is given by:

$$\textit{Average rate of change} = \frac{f(b)-f(a)}{b-a}$$

- It is the gradient of the line joining the two points.
- **Instantaneous Rate of Change** is a gradient of a graph at a single point.
- First Principles derivative definition:

$$f'(x) = \lim_{h \to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$$

- The Product Rule
  - The derivative of $h(x) = f(x) \times g(x)$ is given by:

$$h'(x) = f'(x)g(x) + f(x)g'(x)$$

  - Or, in another form:

$$\frac{d}{dx}(u \cdot v) = u'v + uv'$$

- **The Quotient Rule**
  - The derivative of a $h(x) = \frac{f(x)}{g(x)}$ is given by:

$$h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

  - Or, written in another form:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - uv'}{v^2}$$

    - Always differentiate the top function first.

- **The Chain Rule**

$$y = f(g(x))$$

$$\frac{dy}{dx} = f'(g(x)) \times g'(x).$$

- The process for finding derivatives of **composite functions.**

Learning Objective: [2.1.2] – Identify the Nature of Stationary Points and Trends. (Strictly Increasing and Decreasing.)

Key Takeaways

- Point where the gradient of the function is zero.

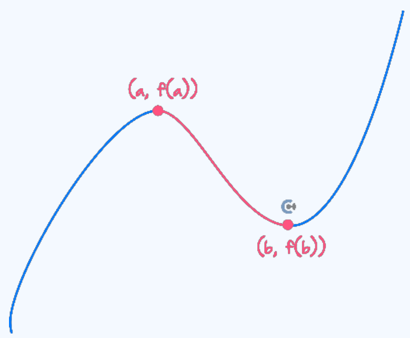
$$f'(x) = 0, \qquad \frac{dy}{dx} = 0$$

- We can identify the nature of a stationary point by using the sign table.

x	Less than a	a	Bigger than a
$f'(x)$	Negative	0	Positive
Shape	∩ - Decreasing curve	Stationary Point	∪ - Increasing curve

- Find the gradient of the neighbouring points.

- <u>Strictly Increasing and Strictly Decreasing Functions</u>



Strictly Increasing: $x \in (-\infty, a] \cup [b, \infty)$

Strictly Decreasing: $[a, b]$

- Steps:
  1. Find the stationary point.
  2. Consider the sign of the derivative between/outside the turning points.

Learning Objective: [2.1.3] – Graph Derivative Functions.

Key Takeaways

- **Steps on sketching the derivative function:**

1. Plot x-intercept at the same x value as the stationary point of the original.
2. Consider the trend of the original function and sketch the derivative.

   Original is increasing → Derivative is above the x-axis.

   Original is decreasing → Derivative is below the x-axis.

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