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VCE Mathematical Methods ¾
AOS 2 Revision [2.0]
SAC 8 Solutions

54 Marks. 15 Minute Reading. 75 Minutes Writing.

Section A: SAC Questions (54 Marks)

Question 1 (11 marks)

Some differential functions have derivatives that just happen to be a transformation of themselves. Consider the following function:

$$f(x) = e^{ax+b} + c, \text{ where } a, b, c \in R$$

a. Show that the function $g(x) = ke^{ax} + c$ where $k \in R^+$ is identical to $f(x)$, and hence, write k in terms of b. (1 mark)

Since $k \in R^+$, we can stipulate that $k = e^b$, and hence, $g(x) = e^b \times e^{ax} + c = f(x)$.

b.

i. State the derivative of $f(x)$. (2 marks)

$f'(x) = ae^{ax+b}$

ii. Hence, state the sequence of transformation that maps $f(x)$ to $f'(x)$ in terms of a and c. (2 marks)

Translation c units down if $c \geq 0$ or translation $-c$ units up if $c < 0$.
Dilation by factor a from the x-axis.

Let $g(x) = (x+1)^2(x-2)^2$ and $h(x) = (x+1)(x-1)(x+2)(x-2)$

c. State the local maximum of $g(x)$. Hence, using **translations only**, describe a sequence of transformations of $h(x)$ such that its image has a local maximum at the same coordinates as that of $g(x)$. (3 marks)

$$\left(\frac{1}{2}, \frac{81}{16}\right)$$

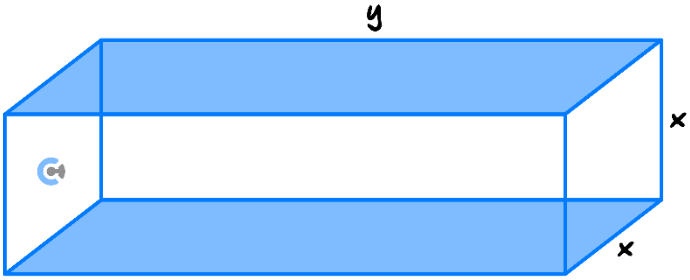
Translate 0.5 unit to the right and 1.0625 units up.

d. Using one **dilation and translations**, describe a different sequence of transformations of $h(x)$ such that its image has both local minima at the same coordinates as that of $g(x)$. (3 marks)

- Dilate by a factor of $\frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$ from the y-axis.
- Translate $\frac{1}{2}$ unit to the right.
- Translate $\frac{9}{4}$ units up.

Space for Personal Notes

Question 2 (7 marks)



Magical Cuboid

Physicists at Contour Labs have discovered new spacial dimensions that allow them to extend to negative lengths. This also implies that negative, areas and volumes are also possible. As a sacrifice a cube is sent through this new spacial dimension. The following diagram depicts its dimensions.

a. Express the total surface area of the cube in terms of x and y. (1 mark)

$2x^2 + 4xy$

b. Express the total volume of the cube in terms of x and y. (1 mark)

$x^2 y$

c. The box is configured in a way such that the numerical values of the surface area and the volume are the same.

i. Considering that, express the volume, V in terms of x only. (2 marks)

$$\frac{2x^3}{x-4}$$

ii. Provide the value of a such that there are two or more solutions for $V(x) = a$. (2 marks)

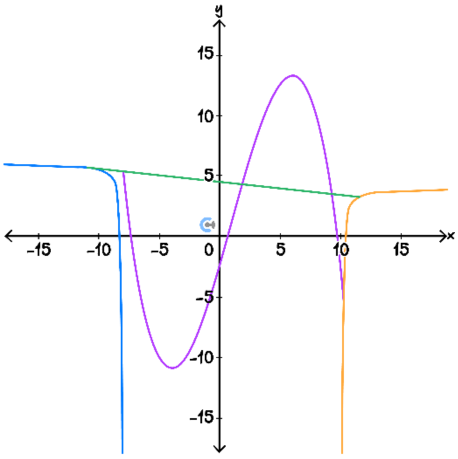
$$a >= 216$$

iii. Hence assuming that a volume of 0 doesn't make sense, what values of V can only be attained with a negative x-value? (1 mark)

V in $(0, \infty)$

Space for Personal Notes

Question 3 (11 marks)



Enter Bridge

The side support of a bridge can be modelled by a cubic function. The diagram above shows the schematic of a bridge, a road and the surrounding terrain. The bridge runs through a river where the water level is given by $y = 0$. All measurements are in metres.

a. The two cliff-sides can be modelled by one-half of the following family of functions:

$$c(x) = \pm\frac{1}{(x-a)} + b$$

i. Given that the model for the right side of the cliff is asymptotic around the line $y = 4$, and $x = 10$, state the function that represents it, alongside the restricted domain. (2 marks)

$$f(x) = -\frac{1}{(x-10)} + 4 \mid x > 10$$

ii. Given that the model for the left side of the cliff goes through $(-9, 5)$, and is asymptotic around $y = 6$, state the function that represents it, alongside the restricted domain. (2 marks)

$$\frac{1}{x+8} + 6 \mid x < -8$$

b. The bridge can be modelled by:

$$b(x) = -\frac{1}{20}(x^3 - 3x^2 - 70x + 40)$$

i. The lowest point of the bridge reaches the very bottom of the river. What is the depth of the river, correct to 2 decimal places? (2 marks)

$10.40\ m$

ii. The road is slanted, and the line representing the road is given by:

$$y = -0.1x + 4.5$$

One end of the bridge anchors itself to the right cliff face, and the other to the road. Find these two points, correct to 2 decimal places. (2 marks)

$(-8.01, 5.30), (10.14, -3{,}21)$

iii. Hence, state the restricted domain of $b(x)$, correct to 2 decimal places. (1 mark)

$[-8.01, 10.14]$

c. The ships going outbound travel through the right-hand side of the bridge, whilst the ships coming inbound travel through the left-hand side of the bridge. Assume that the ships travel on the surface of the water ($y = 0$) and that the heights of the ships can be ignored.

i. What is the widest ship that can travel inbound, correct to 2 decimal places? (1 mark)

7.88

ii. What is the widest ship that can travel outbound, correct to 2 decimal places? (1 mark)

$9.20\ m$

Space for Personal Notes

Question 4 (10 marks)

Wayne Watson wants to buy an oven that operates at just the right temperature to make his favourite butter popcorn. However, he wants to maximise the efficiency by operating at the lowest possible temperature, which cooks the popcorn in 3 minutes. The temperature of the popcorn, in t seconds can be modelled by the following:

$$T(t) = T_{oven} + (T_{room} - T_{oven})e^{-0.01t}$$

a. Will the popcorn ever be as hot as the oven? Justify. (1 mark)

Since $(T_{room} - T_{oven})e^{-0.01t}$ approaches to 0 over time, T_{oven} can be thought of as the asymptote of the function. Hence, the popcorn will never be as hot as the oven.

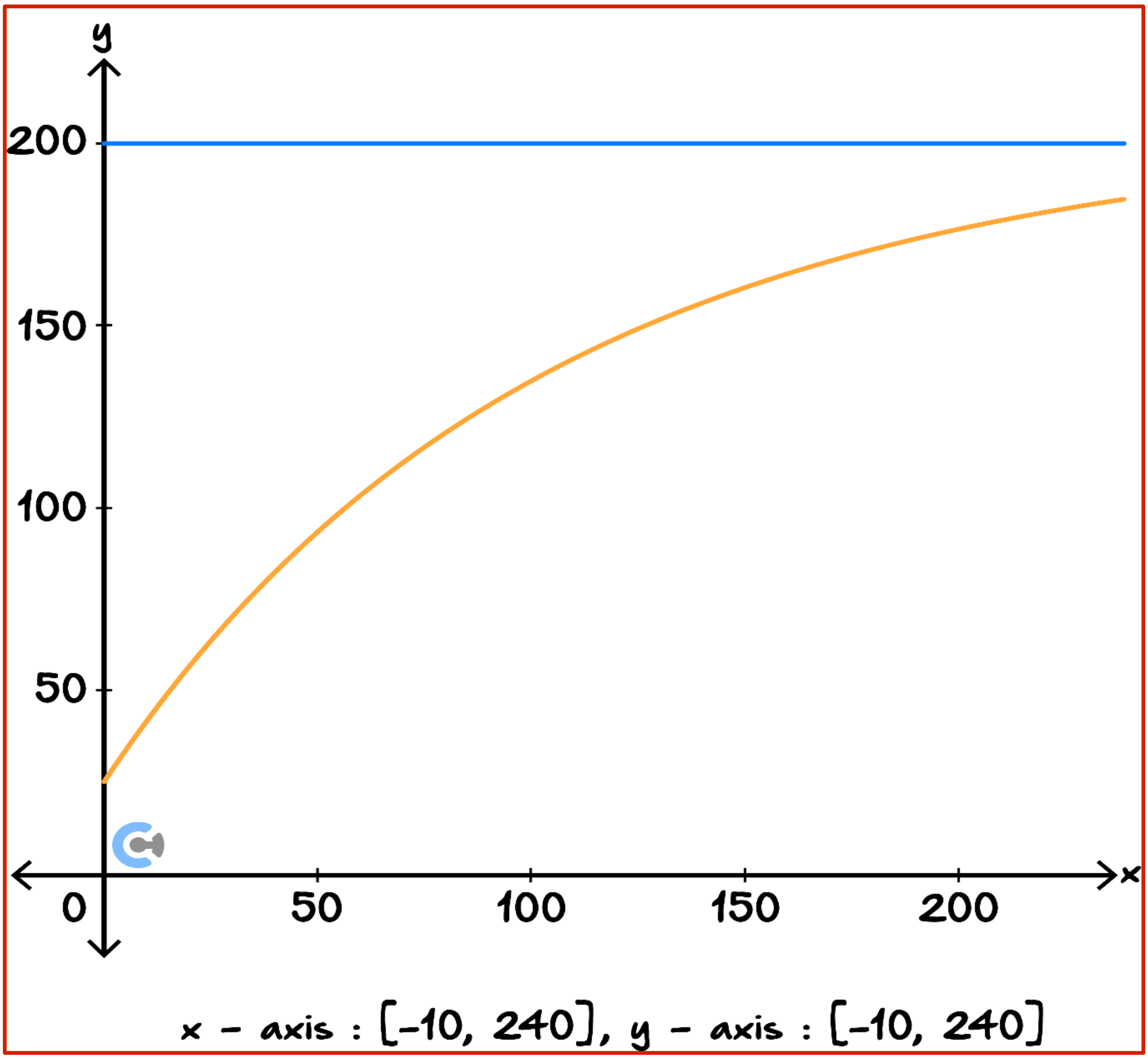
b. What is the initial temperature of the popcorn? (1 mark)

Same as the temperature of the room.

c. Given that the oven operates at 200 degrees Celsius, and the room temperature is 25 degrees Celsius, how long will it take for the popcorn to reach 150 degrees Celsius? Estimate your answer correct to 3 decimal places. (3 marks)

$t = 55.962\ s$

d. Using the equation in **part c**, sketch $T(t)$, over t. Label all endpoints and asymptotes. (3 marks)



e. Wayne becomes interested in a new oven given by the equation below. This oven also cooks the popcorn in exactly 3 minutes. For all the popcorn to pop, it needs to be above 150 degrees Celsius longer than 60 s. However, it cannot be above that temperature for more than 2 minutes or else, it will burn. What is the range of values for $r \in R^-$ in the following family of functions that meet this requirement? (2 marks)

$$T_r(t) = 200 - 175e^{rt}$$

Solve $T_r(t) = 150$,

$$t = \frac{-\log e(7) + \log e(2)}{r}$$

$60 < t < 120$ (such that popcorn cooks for longer than 60 secs but less than 2 minutes.

Hence, $r \in \left[\frac{\ln\left(\frac{2}{7}\right)}{60}, \frac{\ln\left(\frac{2}{7}\right)}{120}\right]$

Question 5 (15 marks)

A semicircle is above the x-axis, with a centre at the origin and a radius of 4.

a. State the equation of this semicircle, given that it is in the form of $f(x) = \sqrt{a - x^2}$. (1 mark)

$f(x) = \sqrt{16 - x^2}$

b. The domain of the semicircle is restricted such that only one quadrant of the circle remains and the inverse function exists.

i. State the possible value(s) of the domain of the function from **part a.** (2 marks)

$[-4, 0]$ or $[0, 4]$

ii. Assuming that $f(x)$ represents the positive quadrant of the semicircle, find the rule, domain and range for the inverse function. (2 marks)

$f^{-1}(x) = \sqrt{16 - x^2}, x \in [0,4]$ and $y \in [0, 4]$

iii. Hence, state the possible reflections that could be applied to $f^{-1}(x)$ such that it forms a semicircle with $f(x)$. (2 marks)

Reflection in x or y-axis.

c. Show that the tangent line at any point, a, on the semicircle has the gradient in the form, $-\frac{x}{y}$. (2 marks)

$$-\frac{a}{\sqrt{16-a^2}}$$

$f(x)$ is now in its original form as a semicircle, where $-4 \leq x \leq 4$.

d. Given that two points have tangent lines that are normal to each other, find the coordinates of these points and the equations of these lines. (3 marks)

Solve $\left(f'^{(x)} = \pm 1, x\right)$

$$x = \pm 2\sqrt{2}$$

Therefore, $y = \pm x + 4\sqrt{2}$

e. Hence, state the relationship between the y-coordinates of these points. (1 mark)

$$y_b = \sqrt{1 - {y^2}_a}$$

f. Can any tangent to the semicircle have 2 other tangent lines normal to it? Justify? (2 marks)

No. Only possible way is to take the horizontal line at $x = 0$, and the vertical lines. However, vertical lines, by definition are undefined as the derivative does not exist at that point. So, no.

$x = 0$. The tangent lines are $x = 4$ and $x = -4$

$$x = 0.$$

The tangent lines are $x = 4$ and $x = -4$.

Section B: TI Solutions

Question Number	Solutions
1(c)	Define $g(x)=(x+1)^2\cdot(x-2)^2$ *Done* Define $h(x)=(x+1)\cdot(x-1)\cdot(x+2)\cdot(x-2)$ *Done* *methods_func\analyse*$(g(x),x)$ ▸ Start Point: $[-\infty \quad \infty]$ ▸ End Point: $[\infty \quad \infty]$ ▸ Maximal Domain: $-\infty<x<\infty$ ▸ x –Intercepts: (2) $[-1 \quad 0]$, $[2 \quad 0]$ ▸ Vertical Intercept: $[0 \quad 4]$ ▸ Derivative: $2\cdot(x-2)\cdot(x+1)\cdot(2\cdot x-1)$ ▸ Inflection Points: (2) $\left[\frac{-(\sqrt{3}-1)}{2} \quad \frac{9}{4}\right]$ (Increasing) $\left[\frac{\sqrt{3}+1}{2} \quad \frac{9}{4}\right]$ (Decreasing) ▸ Stationary Points: (3) $[-1 \quad 0]$ (Local min.) $\left[\frac{1}{2} \quad \frac{81}{16}\right]$ (Local max.) $[2 \quad 0]$ (Local min.)

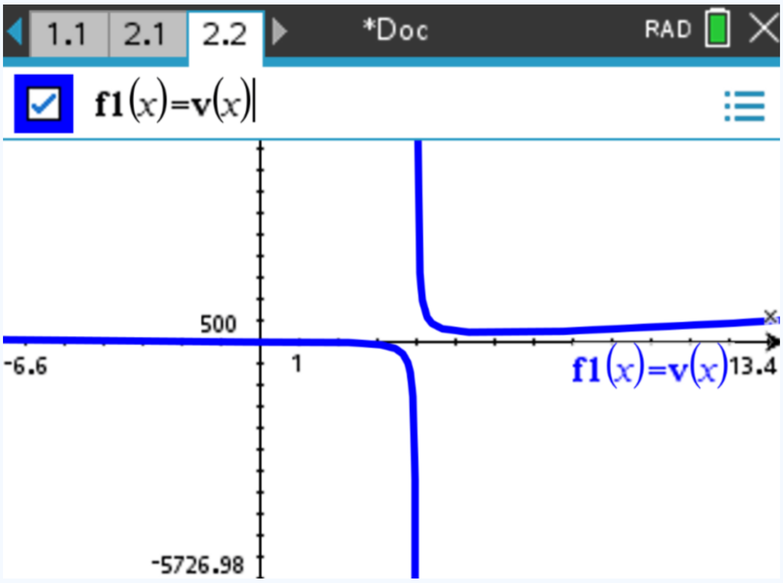
	$methods\_func\backslash analyse(h(x),x)$ ▶ Start Point: $[-\infty \quad \infty]$ ▶ End Point: $[\infty \quad \infty]$ ▶ Maximal Domain: $-\infty<x<\infty$ ▶ x –Intercepts: (4) $[-2 \quad 0],[-1 \quad 0],$ $[1 \quad 0],[2 \quad 0]$ ▶ Vertical Intercept: $[0 \quad 4]$ ▶ Derivative: $4\cdot x^3-10\cdot x$ ▶ Inflection Points: (2) $\begin{bmatrix} \frac{-\sqrt{30}}{6} & \frac{19}{36} \end{bmatrix}$ (Increasing) $\begin{bmatrix} \frac{\sqrt{30}}{6} & \frac{19}{36} \end{bmatrix}$ (Decreasing) ▶ Stationary Points: (3) $\begin{bmatrix} \frac{-\sqrt{10}}{2} & \frac{-9}{4} \end{bmatrix}$ (Local min.) $[0 \quad 4]$ (Local max.) $\begin{bmatrix} \frac{\sqrt{10}}{2} & \frac{-9}{4} \end{bmatrix}$ (Local min.) $\begin{bmatrix} \frac{1}{2} & \frac{81}{16} \end{bmatrix}-[0 \quad 4]$ $[0.5 \quad 1.0625]$
2(c)(i)	Define $v(x)=\frac{2\cdot x^3}{x-4}$ *Done*

2(c)(ii)

$methods\_func\backslash analyse(v(x),x)$

- Start Point: $[-\infty \quad \infty]$
- End Point: $[\infty \quad \infty]$
- Maximal Domain: $x \neq 4$ and $-\infty < x < \infty$
- Asymptote: $x=4$ (Vertical)
- x −Intercept: $[0 \quad 0]$
- Vertical Intercept: $[0 \quad 0]$
- Derivative: $\dfrac{4 \cdot x^2 \cdot (x-6)}{(x-4)^2}$
- Inflection Point:
  $[0 \quad 0]$ (Stationary)
- Stationary Points: (2)
  $[0 \quad 0]$ (Inflection)
  $[6 \quad 216]$ (Local min.)

Menu 4 8 to fit



Any value above the local minimum at 216 has two or more solutions.

3(b)(i)	methods_func\analyse$(b(x),x)$ ▸ Start Point: $[-\infty \quad \infty]$ ▸ End Point: $[\infty \quad -\infty]$ ▸ Maximal Domain: $-\infty<x<\infty$ ▸ x –Intercepts: (3.) $[-7.31576 \quad 0.]$, $[0.560481 \quad 0.]$, $[9.75528 \quad 0.]$ ▸ Vertical Intercept: $[0. \quad -2.]$ ▸ Derivative: $-0.15\cdot(x^2-2.\cdot x-23.3333)$ ▸ Inflection Point: $[1. \quad 1.6]$ (Increasing) ▸ Stationary Points: (2.) $[-3.93288 \quad -10.4033]$ (Local min.) $[5.93288 \quad 13.6033]$ (Local max.)
3(b)(ii)	Define $r(x)=-0.1\cdot x+4.5$ *Done* methods_func\intersectd$(b(x),r(x),x,-15,0)$ ▸ Intersection Points: (1) $[-8.01188 \quad 5.30119]$ Define $cr(x)=\frac{-1}{x-10}+4$ *Done* methods_func\intersectd$(b(x),cr(x),x,10,15)$ ▸ Intersection Points: (1) $[10.1388 \quad -3.20575]$
3(c)	zeros$(b(x),x)$ $\{-7.31576,0.560481,9.75528\}$ $0.560481--7.31576$ 7.87624 $9.75528-0.560481$ 9.1948

4(c)	Define $f(t)=200+(25-200)\cdot e^{-0.01\cdot t}$ *Done* solve$(f(t)=150,t)$ $t=125.276$
4(e)	Define $fr(t)=200-175\cdot e^{r\cdot t}$ *Done* solve$(fr(t)=150,t)$ $t=\frac{-\ln\left(\frac{7}{2}\right)}{r}$ solve$\left(\frac{-\ln\left(\frac{7}{2}\right)}{r}=60,r\right)$ $r=\frac{-\ln\left(\frac{7}{2}\right)}{60}$ solve$\left(\frac{-\ln\left(\frac{7}{2}\right)}{r}=120,r\right)$ $r=\frac{-\ln\left(\frac{7}{2}\right)}{120}$
5(a)	Define $f(x)=\sqrt{a-x^2}$ *Done* $\frac{d}{dx}(f(x))$ $\frac{-x}{\sqrt{a-x^2}}$
5(d)	Define $a=16$ *Done* *methods_diffcalc\solve_grad*$(f(x),x,1)$ ▸ Maximal Domain: $-4\le x\le 4$ ▸ Derivative: $\frac{-x}{\sqrt{16-x^2}}$ ▸ Found point with gradient 1 : $\begin{bmatrix} \text{"x"} & \text{"y"} & \text{"Eqn."} & \text{"x-Int."} & \text{"y-Int."} \\ -2\cdot\sqrt{2} & 2\cdot\sqrt{2} & x+4\cdot\sqrt{2} & -4\cdot\sqrt{2} & 4\cdot\sqrt{2} \end{bmatrix}$ *methods_diffcalc\solve_grad*$(f(x),x,-1)$ ▸ Maximal Domain: $-4\le x\le 4$ ▸ Derivative: $\frac{-x}{\sqrt{16-x^2}}$ ▸ Found point with gradient -1 : $\begin{bmatrix} \text{"x"} & \text{"y"} & \text{"Eqn."} & \text{"x-Int."} & \text{"y-Int."} \\ 2\cdot\sqrt{2} & 2\cdot\sqrt{2} & 4\cdot\sqrt{2}-x & 4\cdot\sqrt{2} & 4\cdot\sqrt{2} \end{bmatrix}$

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