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VCE Mathematical Methods ¾

AOS 2 Revision [2.0]

SAC 6

49 Marks. 15 Minutes Reading. 75 Minutes Writing.

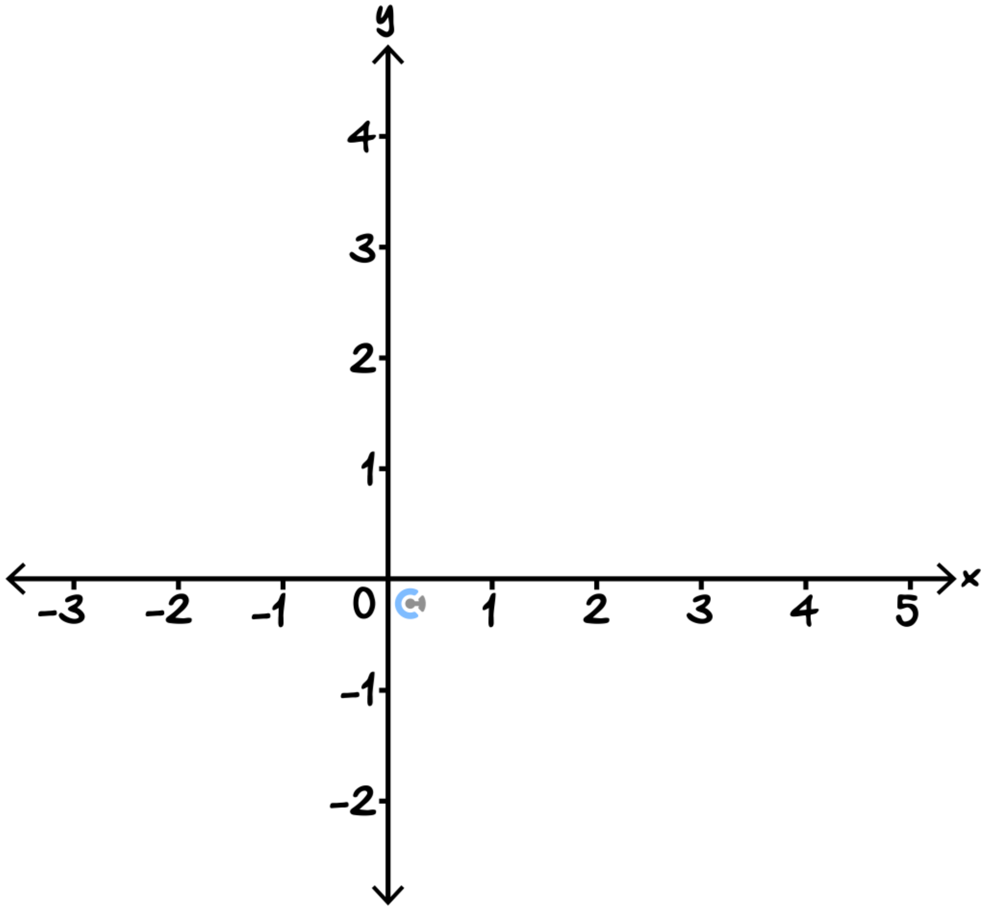
Section A: SAC Questions (49 Marks)

Question 1 (14 marks)

The front of a barn can be modelled by a parabola, given by:

$$f(x) = \frac{-2}{5}(x^2 - 2x - 8), x \in [-2,4]$$

a. Sketch the graph of $y = f(x)$, labelling all axes intercepts, and stationary points. (3 marks)

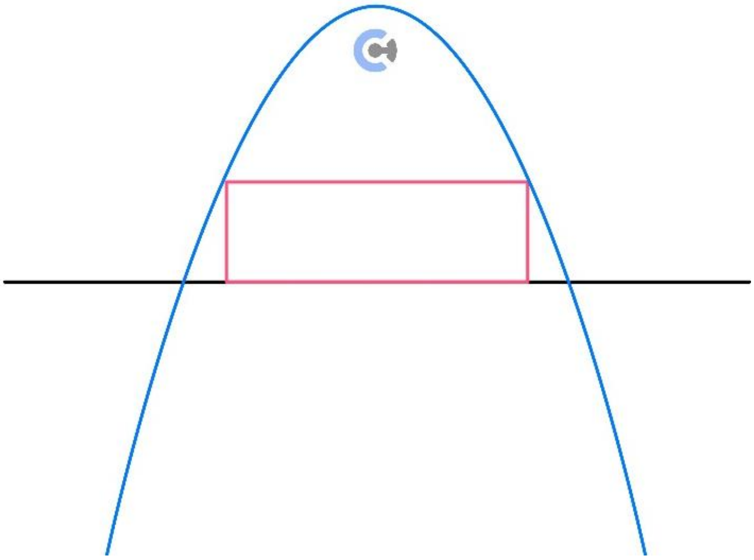


b. Two points are picked to be reinforced. They are symmetrical about the axis of symmetry. State the axis of symmetry. (2 marks)

c. The tangent lines at these two points are perpendicular to each other. What are the coordinates of these points? (3 marks)

d. A piece of wooden stilt is used to reinforce the barn. The stilt follows the same equations as the lines from **part c.** Given that the wooden stilt starts at the point it is tangent to the barn and ends at the ground, what is the length of this wooden stilt? (2 marks)

e. A new plan for the barn is proposed. A portion of the current parabolic shape will be replaced by a rectangular base, as shown below.



i. Express the area of the rectangle, given a corner lies on $(a, f(a))$. (2 marks)

ii. The vertices of the rectangle are picked such that it maximises the rectangle's area. Provide the coordinates of each vertex. (2 marks)

Space for Personal Notes

Question 2 (8 marks)

The average temperature (in °C) in Brisbane over the Jan 1900 – Jan 2023 period is given by the following function:

$$f(t) = 0.5\ln(t+1) + 0.015t + 20,$$

where t is the number of years since January 1900.

a. State the domain of $f(t)$. (1 mark)

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b. A trend line of such graphs is given by the straight line between the most recent and oldest points.

What is the equation of the trend line for this temperature history? Give the numerical values correct to 2 decimal places. (2 marks)

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c. Assuming that this model will continue into the future, what value does the rate at which the temperature changes with respect to time approach in the long run? (2 marks)

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d. A laboratory facility needs to be kept at a constant 20°C. It has been operational 24/7 since 2020. Consider the usual daily temperature fluctuations in a day, h hours since midnight:

$$T(h) = 10 \cdot e^{-\left(\frac{(h-14)^2}{18}\right)} + 15$$

i. State $d(h)$ the difference between the temperature inside and outside. (1 mark)

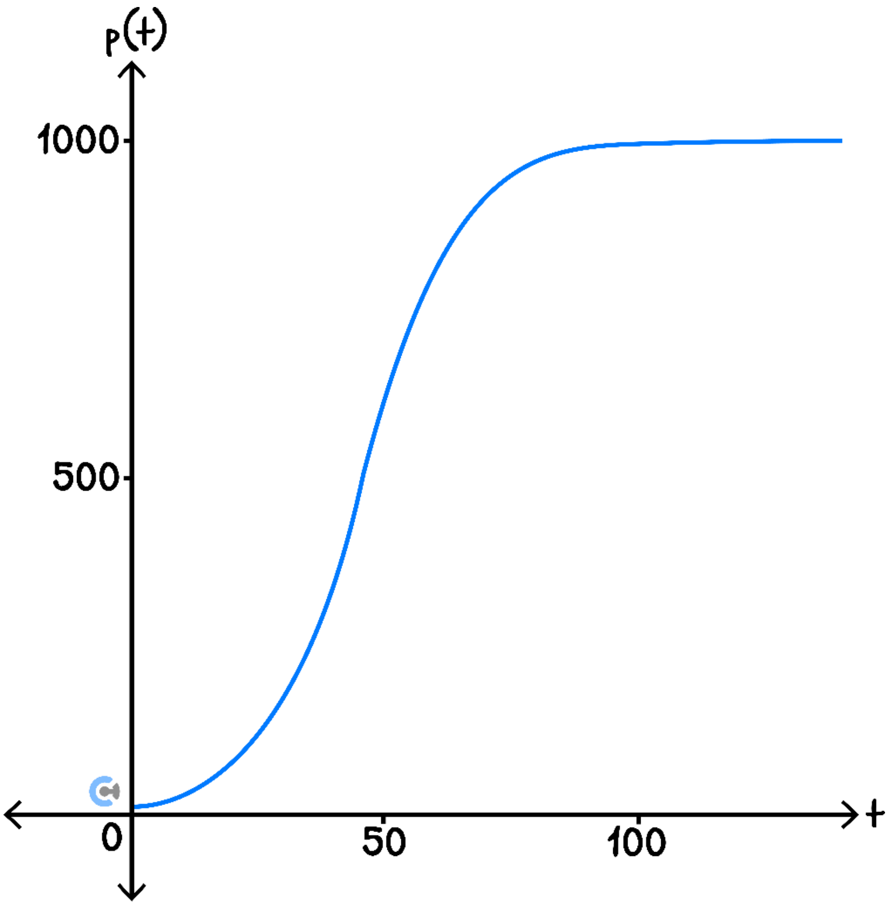
ii. The heater runs whenever the temperature outside is lower than the optimal temperature. On the other hand, the air conditioner is run whenever the temperature outside is higher than the optimal temperature. For what percentage of time is the heater running, correct to two decimal places. (2 marks)

Space for Personal Notes

Question 3 (7 marks)

The following is the population of the bugs at a zoo, t days since the start of the year.

$$p(t) = \frac{1000}{1 + 99e^{-0.1t}}$$



a. What is the population of the bugs at the start of the year? (1 mark)

b. In the long term, what is the population tending to? (2 marks)

c. What is the derivative of $p(t)$, $p'(t)$? (1 mark)

d. For every 2 bugs increase, its only predator in the habitat (a species of finch) will increase by one. Provide the relationship between $f'(t)$ (the change in the finch's population), and $p'(t)$. (1 mark)

e. When is the population of the bugs increasing the fastest, correct to two decimal places? (2 marks)

Space for Personal Notes

Question 4 (9 marks)

Consider the function:

$$f(x) = ae^{bx-c} + d$$

Note: e is Euler's number.

a. Find $f'(x)$. (1 mark)

b. State the transformation that maps the graph of $f(x)$ to $f'(x)$. (2 marks)

c. If the derivative of the function is given by $f'(x) = 2e^2e^x$, find a possibility for the coefficients, a, b, c, d. (2 marks)

d. A new function $k(x)$ is defined as follows:

$$k(x) = \frac{\ln(x)}{x} + 2$$

i. Find the iterative formula for the x-intercepts of $k(x)$ using Newton's Method. (2 marks)

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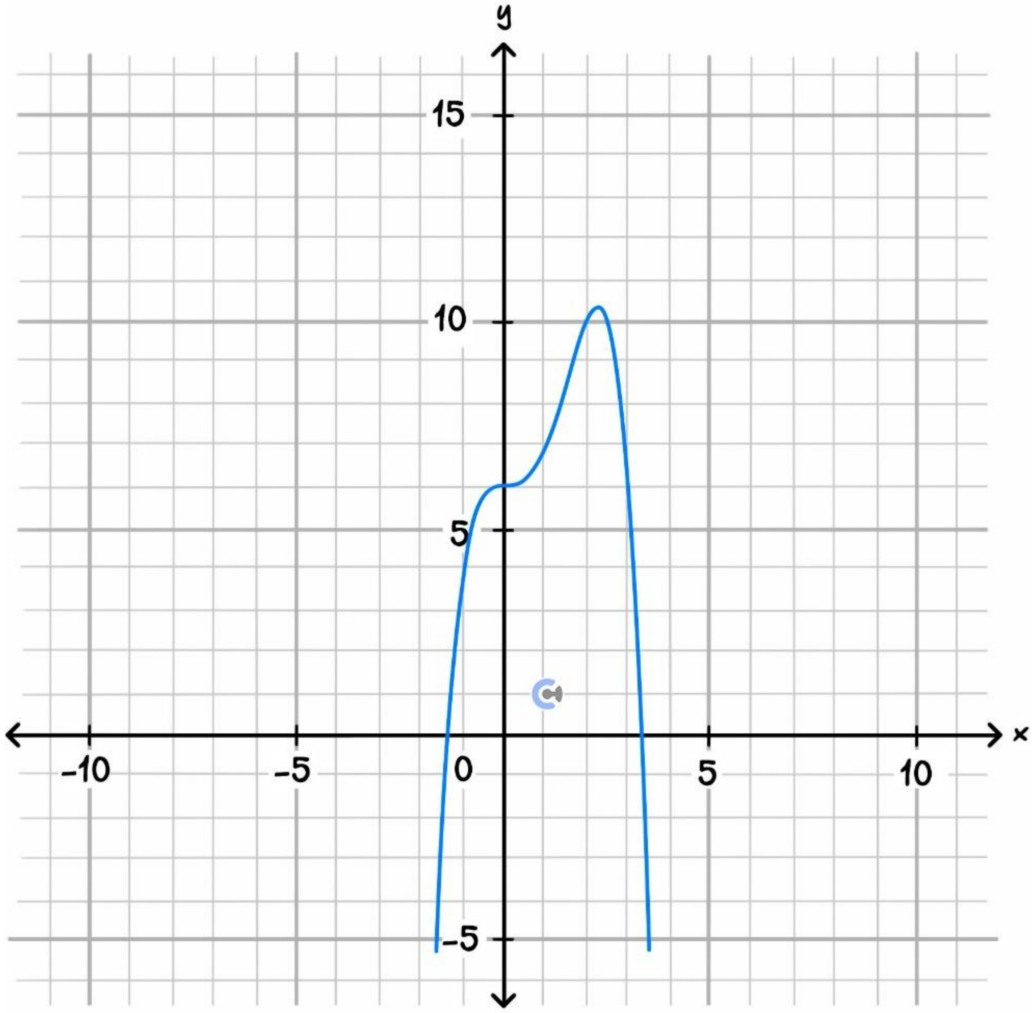
ii. Use the iterative function to approximate the root of $k(x) = 0$ with an accuracy correct to two decimal places. Choose an appropriate starting point. (2 marks)

Space for Personal Notes

Question 5 (11 marks)

The cross-section of a mountain is shown below. Take ground level to be $y = 0$.



$$m(x) = -0.5\,x^3(x-3) + 6$$

a. Find and classify all the stationary points. Provide your justifications. (3 marks)

b. A large straight-hanging bridge runs through the line that is tangential to two different points of the mountain. What is the equation of this line? Give your answer correct to 2 decimal places. (3 marks)

c. What are the coordinates of the contact points, correct to 1 decimal place? (2 marks)

d. For some, the hanging bridge is too easy of a hike. Another much steeper section is added, connecting the two stationary points through the mountain starting from the ground. What is the equation that represents this draw bridge? (1 mark)

e. Assume that hikers only travel in straight lines. For each degree increase in elevation, an extra 100 calories are burnt over the climb. How many extra calories will be burnt on the steeper climb compared to the climb in **part b.**? Provide your answer correct to the nearest whole number. (2 marks)

Section B: TI Solutions

Question Number	Solutions
1(c)	Define $f(x)=\frac{-2}{5}\cdot\left(x^2-2\cdot x-8\right)$ *Done* *methods_diffcalc\solve_grad*$(f(x),x,1)$ ▸ Maximal Domain: $-\infty<x<\infty$ ▸ Derivative: $\frac{-4\cdot(x-1)}{5}$ ▸ Found point with gradient 1 : $\begin{bmatrix} \text{"x"} & \text{"y"} & \text{"Eqn."} & \text{"x-Int."} & \text{"y-Int."} \\ \frac{-1}{4} & \frac{119}{40} & x+\frac{129}{40} & \frac{-129}{40} & \frac{129}{40} \end{bmatrix}$ *methods_diffcalc\solve_grad*$(f(x),x,-1)$ ▸ Maximal Domain: $-\infty<x<\infty$ ▸ Derivative: $\frac{-4\cdot(x-1)}{5}$ ▸ Found point with gradient -1 : $\begin{bmatrix} \text{"x"} & \text{"y"} & \text{"Eqn."} & \text{"x-Int."} & \text{"y-Int."} \\ \frac{9}{4} & \frac{119}{40} & \frac{209}{40}-x & \frac{209}{40} & \frac{209}{40} \end{bmatrix}$
1(d)	◂*hods_misc\linear_info*$\left(\begin{bmatrix} \frac{9}{4} & \frac{119}{40} \end{bmatrix},\begin{bmatrix} \frac{209}{40} & 0 \end{bmatrix}\right)$ ▸ Point 1: $\begin{bmatrix} \frac{9}{4} & \frac{119}{40} \end{bmatrix}$ ▸ Point 2: $\begin{bmatrix} \frac{209}{40} & 0 \end{bmatrix}$ ▸ Midpoint: $\begin{bmatrix} \frac{299}{80} & \frac{119}{80} \end{bmatrix}$ ▸ Distance: $\frac{119\cdot\sqrt{2}}{40}$

1(e)(ii)	Define $r(a)=2\cdot(1-a)\cdot f(a)$ *Done* *methods_func\analysed*$(r(a),a,-2,1)$ ▸ Start Point: $[-2\ \ 0]$ ▸ End Point: $[1\ \ 0]$ ▸ Maximal Domain: $-2\le a\le 1$ ▸ a –Intercepts: (2) $[-2\ \ 0],[1\ \ 0]$ ▸ Vertical Intercept: $\left[0\ \ \frac{32}{5}\right]$ ▸ Derivative: $\frac{12\cdot a^2}{5}-\frac{24\cdot a}{5}-\frac{24}{5}$ ▸ Inflection Point: $[1\ \ 0]$ (Decreasing) ▸ Stationary Point: $\left[-(\sqrt{3}-1)\ \ \frac{24\cdot\sqrt{3}}{5}\right]$ (Local max.) *Done* $f(-(\sqrt{3}-1))$ $\frac{12}{5}$ This gives the top left corner. The x-coordinates or the right corner is $\sqrt{3}+1$ by symmetry.
2(b)	◂*ods_misc\linear_info*$([0\ \ f(0)],[123\ \ f(123)])$ ▸ Point 1: $[0.00000\ \ 20.00000]$ ▸ Point 2: $[123.00000\ \ 24.25514]$ ▸ Midpoint: $[61.50000\ \ 22.12757]$ ▸ Distance: 123.07358 ▸ Gradient: 0.03459 ▸ Linear Equation: $y=0.03459\cdot x+20.00000$

2(c)	Define $df(t)=\frac{d}{dt}(f(t))$ — Done $df(t)$ — $\frac{0.50000}{t+1.00000}+0.01500$
2(d)(i)	Define $t(h)=10\cdot e^{\frac{-(h-14)^2}{18}}+15$ — Done Define $d(h)=20-t(h)$ — Done $d(h)$ — $5-10\cdot e^{\frac{-h^2}{18}+\frac{14\cdot h}{9}-\frac{98}{9}}$
2(d)(ii)	$\text{solve}(d(h)<0,h)\vert 0<h<24$ — $10.46777<h<17.53223$ $17.53223-10.46777$ — 7.06446 $\frac{7.06446}{24}\cdot 100$ — 29.43525 $100-29.43525$ — 70.56475
3(a)	Define $p(t)=\frac{1000}{1+99\cdot e^{-0.1\cdot t}}$ — Done $p(0)$ — 10.00000
3(b)	$\lim\limits_{t\to\infty}(p(t))$ — 1000.00000

3(c)	Switch to exact mode to answer this question. Define $dp(t)=\frac{d}{dt}(p(t))$ Done $dp(t)$ $\dfrac{9900\cdot e^{\frac{t}{10}}}{\left(e^{\frac{t}{10}}+99\right)^2}$
3(e)	$methods\_func\backslash analyse(dp(t),t)$ ▸ Start Point: $[-\infty \quad 0.00000]$ ▸ End Point: $[\infty \quad 0.00000]$ ▸ Maximal Domain: $-\infty<t<\infty$ ▸ Asymptote: $y=0.00000$ (Horizontal) ▸ No t –Intercepts Found ▸ Vertical Intercept: $[0.00000 \quad 0.99000]$ ▸ Derivative: ▸ Inflection Points: (2.00000) $[32.78189 \quad 16.66667]$ (Increasing) $[59.12127 \quad 16.66667]$ (Decreasing) ▸ Stationary Point: $[45.95158 \quad 25.00000]$ (Local max.)
4(a)	Define $f(x)=a\cdot e^{b\cdot x+c}+d$ Done Define $df(x)=\frac{d}{dx}(f(x))$ Done $df(x)$ $a\cdot b\cdot e^{b\cdot x+c}$
4(b)	$methods\_func\backslash transform(f(x),x,\{y-d,b\cdot y\})$ ▸ Translation $-d$ units along the y–dir. $a\cdot e^{b\cdot x+c}$ ▸ Dilation by factor of b from the x–axis $a\cdot b\cdot e^{b\cdot x+c}$

4(d)	Define $k(x)=\frac{\ln(x)}{x}+2$ *Done* *methods_diffcalc\newtons_method*$(k(x),x,0.2)$ ▶ Derivative: $\frac{1}{x^2}-\frac{\ln(x)}{x^2}$ ▶ Iterative Formula: $\frac{x\cdot(2\cdot\ln(x)+2\cdot x-1)}{\ln(x)-1}$ ▶ Number of Iterations: 5 "n" "xn" "|xn−xn-1|" "f(xn)" 0.00000 0.20000 _ -6.04719 1.00000 0.29270 0.09270 -2.19757 2.00000 0.37718 0.08448 -0.58512 3.00000 0.41932 0.04215 -0.07268 4.00000 0.42616 0.00684 -0.00147 5.00000 0.42630 0.00014 -6.34677E-
5(a)	Define $m(x)=-0.5\cdot x^3\cdot(x-3)+6$ *Done* *methods_func\analyse*$(m(x),x)$ ▶ Start Point: $[-\infty \quad -\infty]$ ▶ End Point: $[\infty \quad -\infty]$ ▶ Maximal Domain: $-\infty<x<\infty$ ▶ No x −Intercepts Found ▶ Vertical Intercept: $[0 \quad 6]$ ▶ Derivative: $\frac{9\cdot x^2}{2}-2\cdot x^3$ ▶ Inflection Points: (2) $[0 \quad 6]$ (Stationary) $\left[\frac{3}{2} \quad \frac{273}{32}\right]$ (Increasing) ▶ Stationary Points: (2) $[0 \quad 6]$ (Inflection) $\left[\frac{9}{4} \quad \frac{5259}{512}\right]$ (Local max.)

5(b)	Define $t(x)$=tangentLine$(m(x),x,a)$ *Done* *methods_diffcalc\solve_touch*$(m(x),t(x),x,a)$ ▸ Derivative 1: $4.50000\cdot x^2-2.00000\cdot x^3$ ▸ Derivative 2: $-2.00000\cdot a^2\cdot (a-2.25000)$ ▸ Equating functions and derivatives. ▸ Solutions: $x=2.04904$ and $a=-0.54904$ $x=-0.54904$ and $a=2.04904$ $x=$**c2** and $a=$**c2** tangentLine$(m(x),x,2.04904)$ $1.68749\cdot x+6.63284$
5(c)	$m(-0.5)$ 5.78125 $m(2)$ 10.00000
5(d)	*methods_misc\linear_info*$\left(\begin{bmatrix}0 & 6\end{bmatrix},\begin{bmatrix}\frac{9}{4} & \frac{5259}{512}\end{bmatrix}\right)$ ▸ Point 1: $\begin{bmatrix}0 & 6\end{bmatrix}$ ▸ Point 2: $\begin{bmatrix}\frac{9}{4} & \frac{5259}{512}\end{bmatrix}$ ▸ Midpoint: $\begin{bmatrix}\frac{9}{8} & \frac{8331}{1024}\end{bmatrix}$ ▸ Distance: $\frac{9\cdot\sqrt{75433}}{512}$ ▸ Gradient: $\frac{243}{128}$ ▸ Linear Equation: $y=\frac{243\cdot x}{128}+6$

5(e)	$\tan^{-1}\left(\frac{243}{128}\right)$ 1.08598 $\tan^{-1}(1.69)$ 1.03649 $1.09-1.04$ 0.05000 $\frac{0.05\cdot 180}{\pi}$ 2.86479 $2.8647889756541\cdot 100$ 286.47890

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