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VCE Mathematical Methods $\frac{3}{4}$

AOS 2 Revision [2.0]

SAC 5 Solutions

49 Marks. 10 Minute Reading. 70 Minutes Writing.

Section A: SAC Questions (49 Marks)

Question 1 (12 marks)

Let $g : [0, 2] \rightarrow \mathbb{R}$, $g(x) = x^3 - 3x^2 + 2x$.

a.

- i. Find the derivative function g' . (1 mark)

$$\frac{d}{dx}(g(x)) \quad \left\{ 3 \cdot x^2 - 6 \cdot x + 2, 0 < x < 2 \right. \quad \text{Domain required (function)}$$

- ii. Hence, state the x -values of the stationary points. (2 marks)

$$\text{solve } \left(\frac{d}{dx}(g(x)) = 0, x \right) \\ x = \frac{-(\sqrt{3}-3)}{3} \text{ or } x = \frac{\sqrt{3}+3}{3}$$

- b. State the absolute maximum value of $g(x)$. (1 mark)

$$\begin{array}{l} \text{fMax}(g(x), x, 0, 2) \\ g\left(\frac{-(\sqrt{3}-3)}{3}\right) \end{array} \quad \begin{array}{l} x = \frac{-(\sqrt{3}-3)}{3} \\ \frac{2 \cdot \sqrt{3}}{9} \end{array} \quad \text{only y-value required.}$$

- c. The function $g(x)$ is mapped to $y = g(x - k)$, where $k \in \mathbb{R}$. Find the possible value(s) of k such that $g(x - k)$ only has positive x -intercepts. (1 mark)

$$k > 0$$

d. Another function is given by $f : [0, 2] \rightarrow \mathbb{R}$, $f(x) = x^3 - 3x^2 + (2 + k)x$.

i. Find the coordinates of the stationary points in terms of k . (2 marks)

$$\text{solve } \left(\frac{d}{dx}(f(x)) = 0, x \right)$$

$$x = \frac{\sqrt{-3 \cdot (k-1)} + 3}{3} \text{ or } x = \frac{-(\sqrt{-3 \cdot (k-1)} - 3)}{3}$$

$$f\left(\frac{\sqrt{-3 \cdot (k-1)} + 3}{3}\right) = \frac{2 \cdot (k-1) \cdot \sqrt{-3 \cdot (k-1)} + 9 \cdot k}{9}$$

$$f\left(\frac{-(\sqrt{-3 \cdot (k-1)} - 3)}{3}\right) = \frac{-(2 \cdot (k-1) \cdot \sqrt{-3 \cdot (k-1)} - 9 \cdot k)}{9}$$

Answer in coordinates

ii. Find the coordinates of the endpoints in terms of k . (1 mark)

$$(0, 0)$$

$$(2, 2k)$$

iii. It is known that the absolute maximum and absolute minimum of $f(x)$ occur at endpoints. Find the possible value(s) of k . (4 marks)

$$k \leq -2 \text{ or } k \geq 1/4$$

2 marks Answer

2 marks for working shown below (makes more sense after use of slider)

Critical values:

$$\text{solve } \left(f(2) = f\left(\frac{-(\sqrt{-3 \cdot (k-1)} - 3)}{3}\right), k \right) \quad k = \frac{1}{4}$$

$$\text{solve } \left(f(0) = f\left(\frac{-(\sqrt{-3 \cdot (k-1)} - 3)}{3}\right), k \right) \quad k = -2$$

Space for Person

Question 2 (3 marks)

[1. 5. 4] Find the unknown value for systems of linear equations.

Consider the system of linear equations where k is a real constant:

$$\begin{cases} (k+2)x + 4y = 8 \\ 2x + (k+4)y = k+8 \end{cases}$$

Find the value of k if the system of linear equations has:

a. No solution. (1 mark)

$$\frac{k+2}{2} = \frac{4}{k+4}$$

$$k = 0 \text{ or } k = -6$$

Also, $8 \neq k+8$

So, $k \neq 0$

So $k = -6$ for no solution

b. Infinitely many solutions. (1 mark)

$k = 0$

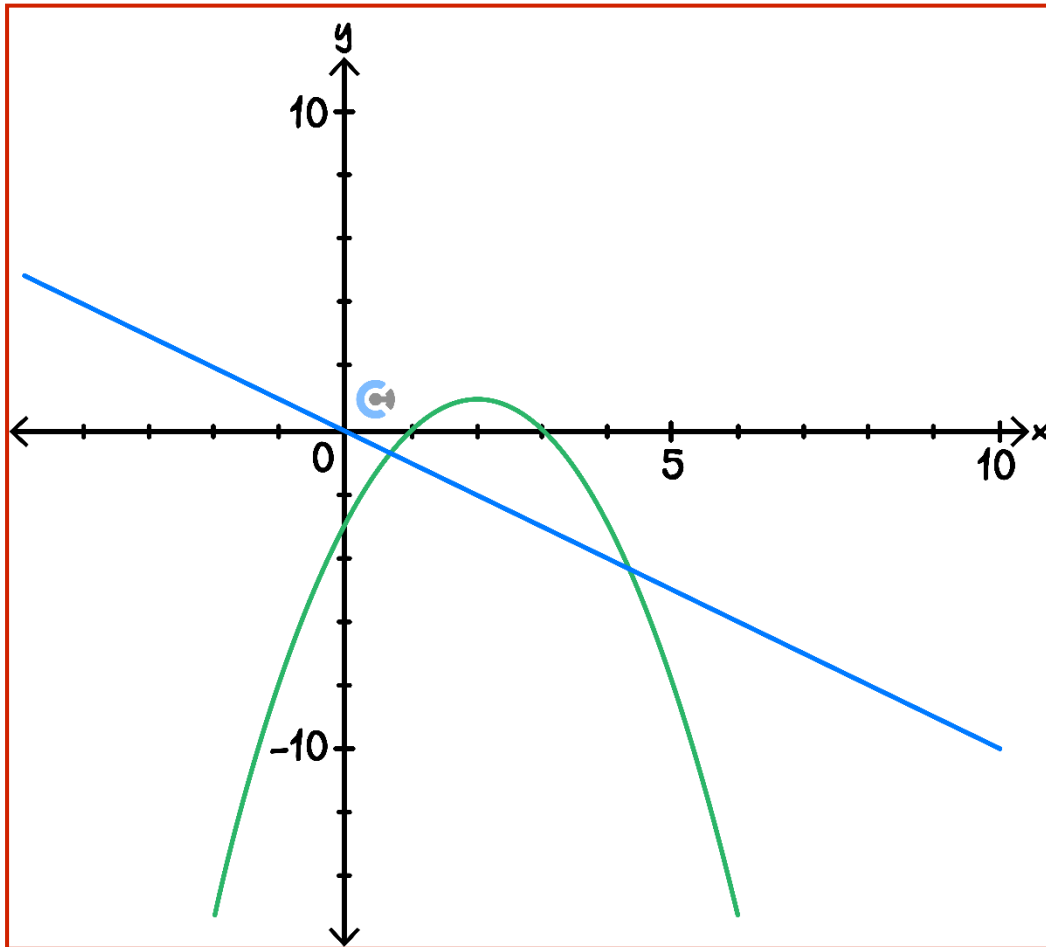
c. Unique solution. (1 mark)

$k \in \mathbb{R} \setminus \{0, -6\}$

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Question 3 (8 marks)

The graph of $f(x) = -x^2 + 4x - 3$ is shown.



- a.** Find the value(s) such that $f(x) = ax$ has more than one intersection. (3 marks)

$$\begin{aligned}
 & -x^2 + 4x - 3 = ax \\
 & x^2 + (a - 4)x + 3 = 0 \\
 & \Delta > 0 \text{ for more than one solution (two solutions)} \\
 & (a - 4)^2 - 12 > 0 \Rightarrow a < 4 - 2\sqrt{3} \text{ or } a > 4 + 2\sqrt{3} \\
 & \text{Alternatively, solve } f'(x) = a \text{ and } f(x) = ax \text{ simultaneously to find when they are tangential.}
 \end{aligned}$$

- b.** Sketch one line from **part a.** on the graph. (1 mark)

c.

- i. Verify that $(0,0)$ does not lie on $f(x)$. (1 mark)

$$f(0) = -(0)^2 + 4(0) - 3 = -3 \neq 0$$

So $(0,0)$ does not lie on $f(x)$.

- ii. Find the tangent line(s) to the curve $y = f(x)$ passing through $(0,0)$. (3 marks)

$$\text{solve} \left(\frac{f(x)-0}{x-0} = \frac{d}{dx}(f(x)), x \right) \quad x = -\sqrt{3} \text{ or } x = \sqrt{3}$$

$$\text{tangentLine}(f(x), x = -\sqrt{3}) \quad 2 \cdot (\sqrt{3} + 2) \cdot x$$

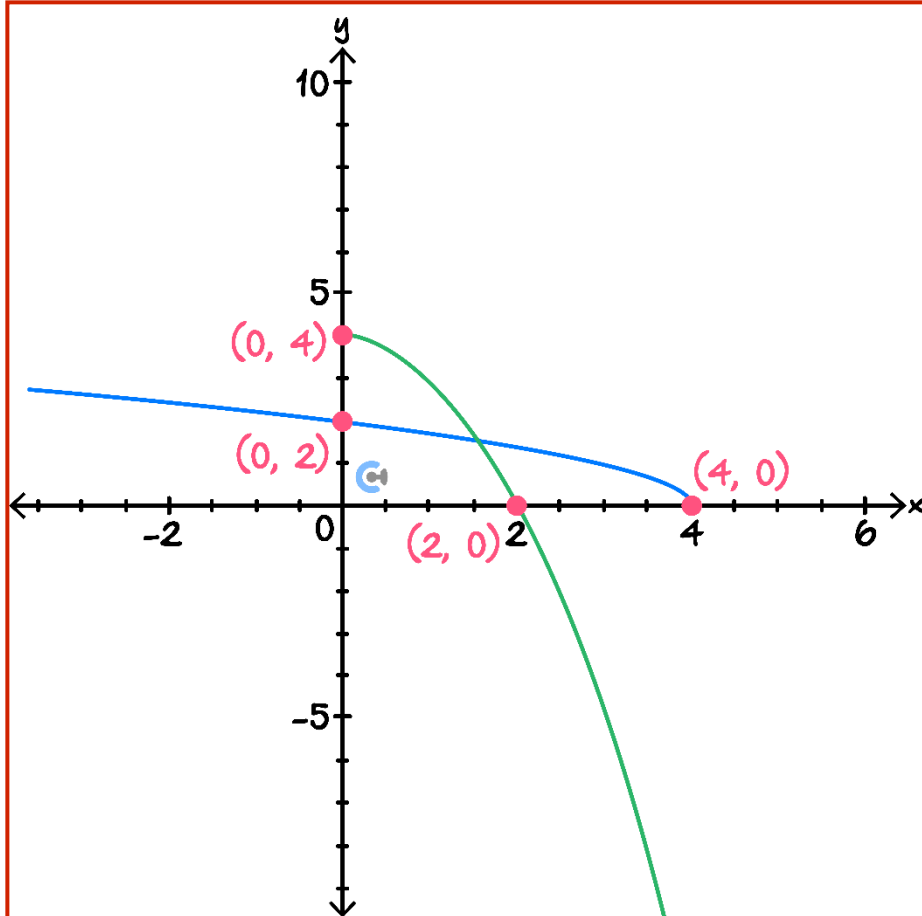
$$\text{tangentLine}(f(x), x = \sqrt{3}) \quad -2 \cdot (\sqrt{3} - 2) \cdot x$$

Must write $y =$ _____

Question 4 (16 marks)

Alex decides to go for a run on the path given by $a : (-\infty, 4] \rightarrow \mathbb{R}, a(x) = \sqrt{4-x}$.

- a. Sketch $a(x)$, labelling intercepts and endpoints. (3 marks)



Sam decides to follow a similar approach to Alex, yet he decides to take a different route given by the inverse of Alex's path.

- b. Define Sam's path $s(x) = a^{-1}(x)$. (2 marks)

$$s : [0, \infty) \rightarrow \mathbb{R}, s(x) = 4 - x^2$$

- c. State the range of $s(x)$. (1 mark)

$$\text{ran } s = (-\infty, 4]$$

- d. Sketch the graph of $s(x)$ on the same axes as Alex's path in **part a**. (1 mark)

Sam and Alex notice that they are running on paths rather similar to each other. As such, they get curious and decide to see what happens if they combine their routes.

- e. The composition of their paths can be described as $a(s(x))$. Find the function $a(s(x))$. (2 marks)

$$a \circ s : [0, \infty) \rightarrow \mathbb{R}, a(s(x))$$

After realising that this path is rather boring, Alex and Sam revert to their original paths.

- f. Show that the point where Alex and Sam cross paths is given by $\left(\frac{\sqrt{17}-1}{2}, \frac{\sqrt{17}-1}{2}\right)$. (3 marks)

$$\begin{aligned} s(x) &= x \\ 4 - x^2 &= x \\ x^2 + x - 4 &= 0 \\ \left(x + \frac{1}{2}\right)^2 - \frac{1}{4} - 4 &= 0 \\ \left(x + \frac{1}{2}\right)^2 &= \frac{17}{4} \\ x + \frac{1}{2} &= \frac{\sqrt{17}}{2} \\ x &= \frac{\sqrt{17}-1}{2} \end{aligned}$$

So they cross paths at $\left(\frac{\sqrt{17}-1}{2}, \frac{\sqrt{17}-1}{2}\right)$

Due to their crossing paths, Sam and Alex realise that they are getting in each other's way when their paths cross. To avoid this problem, they both decide to offset their paths by a certain amount such that their paths never cross.

Alex applies the transformation to his path given by $y = a(x) + k$.

- g. Find k such that $y = a(x) + k$ and $s(x)$ are tangential. Give your answer correct to two decimal places. (3 marks)

$$\frac{d}{dx}(a(x)+k) = \frac{d}{dx}(s(x)) \text{ and } a(x)+k=s(x), x, k$$

$$x=0.127033462704 \text{ and } k=2.0158770986$$

$$x=3.99608609442 \text{ and } k=-12.031265288$$

- h. State the values of k such that the paths do not cross. (1 mark)

$$k < -12.031 \text{ or } k > 2.016$$

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Question 5 (10 marks)

Let $h(x) = x^2 + 2x$.

a.

- i.**
- Find
- $h'(x)$
- . (1 mark)

$$h'(x) = 2x + 2$$

- ii.**
- Two tangents are drawn to
- $h(x)$
- at points
- $(a, h(a))$
- and
- $(b, h(b))$
- , where
- $b > a$
- . These tangents intersect at
- $(0, -1)$
- . State the gradient of the line connecting
- $(a, h(a))$
- to
- $(0, -1)$
- in terms of
- a
- . (1 mark)

$$\text{Gradient} = \frac{h(a) - (-1)}{a - 0} = \frac{a^2 + 2a + 1}{a} = \frac{(a+1)^2}{a}$$

- iii.**
- Hence (or otherwise), solve for the values of
- a
- and
- b
- . (3 marks)

$$\begin{aligned} \text{tangentLine}(h(x), x=a) & \quad 2 \cdot (a+1) \cdot x - a^2 \\ \text{tangentLine}(h(x), x=b) & \quad 2 \cdot (b+1) \cdot x - b^2 \\ \text{solve} \left(\begin{cases} y = 2 \cdot (a+1) \cdot x - a^2 \\ y = 2 \cdot (b+1) \cdot x - b^2 \\ x = 0 \\ y = -1 \end{cases}, \{x, y, a, b\} \right) & \quad | b > a \\ x=0 \text{ and } y=-1 \text{ and } a=-1 \text{ and } b=1 & \end{aligned}$$

- b.**
- Find the minimum distance between the graph of
- $h(x)$
- and the point
- $(0, -1)$
- . (2 marks)

$$\begin{aligned} & \sqrt{(x-0)^2 + (h(x)-(-1))^2} \\ & \sqrt{x^4 + 4 \cdot x^3 + 7 \cdot x^2 + 4 \cdot x + 1} \\ & \text{fMin}(\sqrt{x^4 + 4 \cdot x^3 + 7 \cdot x^2 + 4 \cdot x + 1}, x) \\ & \quad x = -0.410245487699 \\ & \sqrt{x^4 + 4 \cdot x^3 + 7 \cdot x^2 + 4 \cdot x + 1} | x = -0.410245487699 \\ & \quad 0.537841448698 \end{aligned}$$

c. Consider the family of functions $f(x) = mx + c$.

Find the minimum distance between $f(x)$ and the point $(q, 0)$ in terms of p , m and q . (3 marks)

$$\text{solve} \left(\frac{d}{dx} \left(\sqrt{(x-q)^2 + (f(x)-0)^2} \right) = 0, x \right)$$

$$x = \frac{-(c \cdot m - q)}{m^2 + 1}$$

$$\sqrt{(x-q)^2 + (f(x)-0)^2} \Big|_{x=\frac{-(c \cdot m - q)}{m^2 + 1}} = \frac{|c + m \cdot q|}{\sqrt{m^2 + 1}}$$

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Section B: TI Solutions

Question Number	Solutions
2	$\text{methods_misc}\backslash\text{system_solve}((k+2)\cdot x+4\cdot y=8, 2\cdot x+(k+4)\cdot y=k+8, k)$ <ul style="list-style-type: none"> ▶ Solving: $\begin{bmatrix} (k+2)\cdot x+4\cdot y=8 \\ 2\cdot x+(k+4)\cdot y=k+8 \end{bmatrix}$ ▶ Unique Solution: $k\neq -6$ and $k\neq 0$ ▶ No Solutions: $k=-6$ ▶ Infinite Solutions: $k=0$
3(a)	$\text{Define } f(x) = -x^2 + 4\cdot x - 3 \quad \text{Done}$ $\text{solve}(f(x)=a, x, x)$ $x = \frac{\sqrt{a^2 - 8\cdot a + 4} - a + 4}{2} \text{ or } x = \frac{-\left(\sqrt{a^2 - 8\cdot a + 4} + a\right)}{2}$ $\text{solve}(a^2 - 8\cdot a + 4 > 0, a)$ $a < -2\cdot(\sqrt{3} - 2) \text{ or } a > 2\cdot(\sqrt{3} + 2)$
3(c)	$\text{tangentLine}(f(x), x, a) \quad -2\cdot(a-2)\cdot x + a^2 - 3$ $\text{solve}(a^2 - 3 = 0, a) \quad a = -\sqrt{3} \text{ or } a = \sqrt{3}$ $\text{tangentLine}(f(x), x, -\sqrt{3}) \quad 2\cdot(\sqrt{3} + 2)\cdot x$ $\text{tangentLine}(f(x), x, \sqrt{3}) \quad -2\cdot(\sqrt{3} - 2)\cdot x$
4(g)	$\text{Define } a(x) = \sqrt{4-x} \quad \text{Done}$ $\text{Define } s(x) = 4-x^2 \quad \text{Done}$ $\text{methods_diffcalc}\backslash\text{solve_touch}(a(x)+k, s(x), x, k)$ <ul style="list-style-type: none"> ▶ Derivative 1: $\frac{-1}{2\cdot\sqrt{4-x}}$ ▶ Derivative 2: $-2\cdot x$ ▶ Equating functions and derivatives. ▶ Solutions: <ul style="list-style-type: none"> $x=3.99609$ and $k=-12.03127$ $x=0.12703$ and $k=2.01588$

5(a)(i)

Define $dh(x) = \frac{d}{dx}(h(x))$ Done

`methods_misc\linear_info([a h(a)], [0 -1])`

- ▶ Point 1: $[a \quad a^2 + 2 \cdot a]$
- ▶ Point 2: $[0 \quad -1]$
- ▶ Midpoint: $\left[\frac{a}{2} \quad \frac{a^2 + 2 \cdot a - 1}{2} \right]$
- ▶ Distance: $\sqrt{a^4 + 4 \cdot a^3 + 7 \cdot a^2 + 4 \cdot a + 1}$
- ▶ Gradient: $\frac{a^2 + 2 \cdot a + 1}{a}$
- ▶ Linear Equation: $y = \frac{(a^2 + 2 \cdot a + 1) \cdot x}{a} - 1$

5(a)(iii)

Define $t(x) = \frac{(a^2 + 2 \cdot a + 1) \cdot x}{a} - 1$ Done

`methods_diffcalc\solve_touch(h(x), t(x), x, a)`

- ▶ Derivative 1: $2 \cdot x + 2$
- ▶ Derivative 2: $\frac{a^2 + 2 \cdot a + 1}{a}$
- ▶ Equating functions and derivatives.
- ▶ Solutions:
 - $x = 1$ and $a = 1$
 - $x = -1$ and $a = -1$

5(b)

Use output from linear_info program above

Define $d(a) = \sqrt{a^4 + 4 \cdot a^3 + 7 \cdot a^2 + 4 \cdot a + 1}$ Done

`methods_func\analysed(d(a), a, ?, ?)`

- Note: Asymptote finder disabled
- ▶ Start Point: $[-\infty \quad \infty]$
 - ▶ End Point: $[\infty \quad \infty]$

	<ul style="list-style-type: none"> ▶ Maximal Domain: $-\infty < a < \infty$ ▶ No a -Intercepts Found ▶ Vertical Intercept: $[0 \ 1]$ ▶ Derivative: $\frac{2 \cdot a^3 + 6 \cdot a^2 + 7 \cdot a + 2}{\sqrt{a^4 + 4 \cdot a^3 + 7 \cdot a^2 + 4 \cdot a + 1}}$ ▶ No Inflection Points Found ▶ Stationary Point: $[-0.41025 \ 0.53784] \text{ (Local min.)}$
5(c)	<p><i>methods_miscVinear_info</i>($y=m \cdot x+c, [q \ 0]$)</p> <hr/> <ul style="list-style-type: none"> ▶ x-int: $\left[\frac{-c}{m} \ 0 \right]$ ▶ y-int: $[0 \ c]$ ▶ Perp. Line: $y = \frac{-(x-q)}{m}$ ▶ Intersection: $\left[\left\{ \frac{-(c \cdot m - q)}{m^2 + 1}, m \neq 0 \right\} \left\{ \frac{c + m \cdot q}{m^2 + 1}, m \neq 0 \right\} \right]$ ▶ Shortest Distance: $\frac{ c + m \cdot q }{\sqrt{m^2 + 1}}$



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