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VCE Mathematical Methods ¾
AOS 2 Revision [2.0]
SAC 5

49 Marks. 10 Minute Reading. 70 Minutes Writing.

Section A: SAC Questions (49 Marks)

Question 1 (12 marks)

Let $g : [0, 2] \to \mathbb{R},\ g(x) = x^3 - 3x^2 + 2x$.

a.

i. Find the derivative function g'. (1 mark)

ii. Hence, state the x-values of the stationary points. (2 marks)

b. State the absolute maximum value of $g(x)$. (1 mark)

c. The function $g(x)$ is mapped to $y = g(x - k)$, where $k \in \mathbb{R}$. Find the possible value(s) of k such that $g(x - k)$ only has positive x-intercepts. (1 mark)

d. Another function is given by $f : [0, 2] \to \mathbb{R},\ f(x) = x^3 - 3x^2 + (2 + k)x$.

i. Find the coordinates of the stationary points in terms of k. (2 marks)

ii. Find the coordinates of the endpoints in terms of k. (1 mark)

iii. It is known that the absolute maximum and absolute minimum of $f(x)$ occur at endpoints. Find the possible value(s) of k. (4 marks)

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Question 2 (3 marks)

Consider the system of linear equations where k is a real constant:

$$\begin{cases} (k+2)x + 4y = 8 \\ 2x + (k+4)y = k+8 \end{cases}$$

Find the value of k if the system of linear equations has:

a. No solution. (1 mark)

b. Infinitely many solutions. (1 mark)

c. Unique solution. (1 mark)

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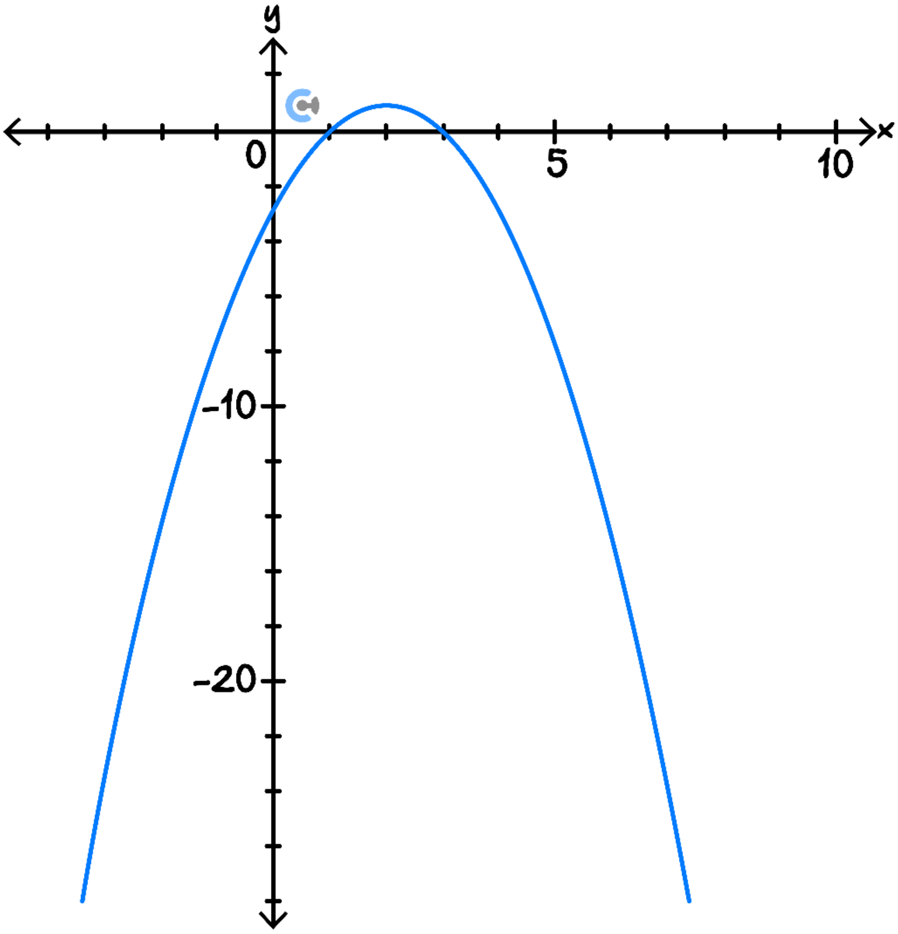
Question 3 (8 marks)

The graph of $f(x) = -x^2 + 4x - 3$ is shown.



a. Find the value(s) such that $f(x) = ax$ has more than one intersection. (3 marks)

b. Sketch one line from **part a.** on the graph. (1 mark)

c.

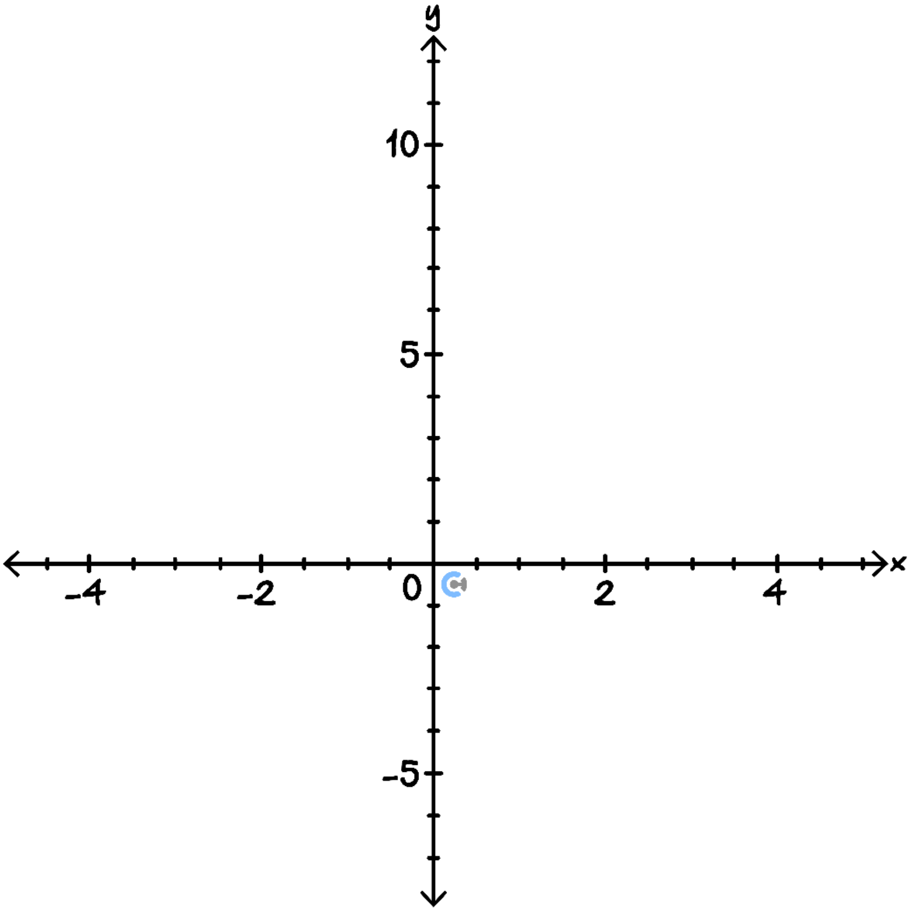
i. Verify that $(0, 0)$ does not lie on $f(x)$. (1 mark)

ii. Find the tangent line(s) to the curve $y = f(x)$ passing through $(0, 0)$. (3 marks)

Question 4 (16 marks)

Alex decides to go for a run on the path given by $a : (-\infty, 4] \to \mathbb{R}, a(x) = \sqrt{4 - x}$.

a. Sketch $a(x)$, labelling intercepts and endpoints. (3 marks)



Sam decides to follow a similar approach to Alex, yet he decides to take a different route given by the inverse of Alex's path.

b. Define Sam's path $s(x) = a^{-1}(x)$. (2 marks)

c. State the range of $s(x)$. (1 mark)

d. Sketch the graph of $s(x)$ on the same axes as Alex's path in **part a**. (1 mark)

Sam and Alex notice that they are running on paths rather similar to each other. As such, they get curious and decide to see what happens if they combine their routes.

e. The composition of their paths can be described as $a(s(x))$. Find the function $a(s(x))$. (2 marks)

After realising that this path is rather boring, Alex and Sam revert to their original paths.

f. Show that the point where Alex and Sam cross paths is given by $\left(\frac{\sqrt{17}-1}{2}, \frac{\sqrt{17}-1}{2}\right)$. (3 marks)

Due to their crossing paths, Sam and Alex realise that they are getting in each other's way when their paths cross. To avoid this problem, they both decide to offset their paths by a certain amount such that their paths never cross.

Alex applies the transformation to his path given by $y = a(x) + k$.

g. Find k such that $y = a(x) + k$ and $s(x)$ are tangential. Give your answer correct to two decimal places. (3 marks)

h. State the values of k such that the paths do not cross. (1 mark)

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Question 5 (10 marks)

Let $h(x) = x^2 + 2x$.

a.

i. Find $h'(x)$. (1 mark)

ii. Two tangents are drawn to $h(x)$ at points $\left(a, h(a)\right)$ and $(b, h(b))$, where $b > a$. These tangents intersect at $(0, -1)$. State the gradient of the line connecting $\left(a, h(a)\right)$ to $(0, -1)$ in terms of a. (1 mark)

iii. Hence (or otherwise), solve for the values of a and b. (3 marks)

b. Find the minimum distance between the graph of $h(x)$ and the point $(0, -1)$. (2 marks)

c. Consider the family of functions $f(x) = mx + c$.

Find the minimum distance between $f(x)$ and the point $(q, 0)$ in terms of p, m and q. (3 marks)

Space for Personal Notes

Section B: TI Solutions

Question Number	Solutions
2	$methods\_misc\backslash system\_solve((k+2)\cdot x+4\cdot y=8, 2\cdot x+(k+4)\cdot y=k+8, k)$ ▸ Solving: $\begin{bmatrix}(k+2)\cdot x+4\cdot y=8\\ 2\cdot x+(k+4)\cdot y=k+8\end{bmatrix}$ ▸ Unique Solution: $k\neq -6$ and $k\neq 0$ ▸ No Solutions: $k=-6$ ▸ Infinite Solutions: $k=0$
3(a)	Define $f(x)=-x^2+4\cdot x-3$ — *Done* $\text{solve}(f(x)=a\cdot x, x)$ $x=\frac{\sqrt{a^2-8\cdot a+4}-a+4}{2}$ or $x=\frac{-(\sqrt{a^2-8\cdot a+4}+a}{2}$ ▸ $\text{solve}(a^2-8\cdot a+4>0, a)$ $a<-2\cdot(\sqrt{3}-2)$ or $a>2\cdot(\sqrt{3}+2)$
3(c)	$\text{tangentLine}(f(x), x, a)$ — $-2\cdot(a-2)\cdot x+a^2-3$ $\text{solve}(a^2-3=0, a)$ — $a=-\sqrt{3}$ or $a=\sqrt{3}$ $\text{tangentLine}(f(x), x, -\sqrt{3})$ — $2\cdot(\sqrt{3}+2)\cdot x$ $\text{tangentLine}(f(x), x, \sqrt{3})$ — $-2\cdot(\sqrt{3}-2)\cdot x$
4(g)	Define $a(x)=\sqrt{4-x}$ — *Done* Define $s(x)=4-x^2$ — *Done* $methods\_diffcalc\backslash solve\_touch(a(x)+k, s(x), x, k)$ ▸ Derivative 1: $\frac{-1}{2\cdot\sqrt{4-x}}$ ▸ Derivative 2: $-2\cdot x$ ▸ Equating functions and derivatives. ▸ Solutions: $x=3.99609$ and $k=-12.03127$ $x=0.12703$ and $k=2.01588$

5(a)(i)	Define $dh(x)=\frac{d}{dx}(h(x))$ Done methods_misc\linear_info([a h(a)],[0 -1]) ▶ Point 1: $[a \quad a^2+2\cdot a]$ ▶ Point 2: $[0 \quad -1]$ ▶ Midpoint: $\left[\frac{a}{2} \quad \frac{a^2+2\cdot a-1}{2}\right]$ ▶ Distance: $\sqrt{a^4+4\cdot a^3+7\cdot a^2+4\cdot a+1}$ ▶ Gradient: $\frac{a^2+2\cdot a+1}{a}$ ▶ Linear Equation: $y=\frac{(a^2+2\cdot a+1)\cdot x}{a}-1$
5(a)(iii)	Define $t(x)=\frac{(a^2+2\cdot a+1)\cdot x}{a}-1$ Done methods_diffcalc\solve_touch(h(x),t(x),x,a) ▶ Derivative 1: $2\cdot x+2$ ▶ Derivative 2: $\frac{a^2+2\cdot a+1}{a}$ ▶ Equating functions and derivatives. ▶ Solutions: $x=1$ and $a=1$ $x=-1$ and $a=-1$
5(b)	Use output from linear_info program above Define $d(a)=\sqrt{a^4+4\cdot a^3+7\cdot a^2+4\cdot a+1}$ Done methods_func\analysed(d(a),a,?,?) Note: Asymptote finder disabled ▶ Start Point: $[-\infty \quad \infty]$ ▶ End Point: $[\infty \quad \infty]$

	▶ Maximal Domain: $-\infty<a<\infty$ ▶ No a –Intercepts Found ▶ Vertical Intercept: $[0 \quad 1]$ ▶ Derivative: $\dfrac{2\cdot a^3+6\cdot a^2+7\cdot a+2}{\sqrt{a^4+4\cdot a^3+7\cdot a^2+4\cdot a+1}}$ ▶ No Inflection Points Found ▶ Stationary Point: $[-0.41025 \quad 0.53784]$ (Local min.)
5(c)	$methods\_misc\backslash linear\_info(y=m\cdot x+c,[q \quad 0])$ ▶ x–int: $\left[\dfrac{-c}{m} \quad 0\right]$ ▶ y–int: $[0 \quad c]$ ▶ Perp. Line: $y=\dfrac{-(x-q)}{m}$ ▶ Intersection: $\left[\begin{cases}\dfrac{-(c\cdot m-q)}{m^2+1}, m\neq 0\end{cases} \quad \begin{cases}\dfrac{c+m\cdot q}{m^2+1}, m\neq 0\end{cases}\right]$ ▶ Shortest Distance: $\dfrac{\lvert c+m\cdot q\rvert}{\sqrt{m^2+1}}$

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