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VCE Mathematical Methods $\frac{3}{4}$
AOS 2 Revision [2.0]
SAC 4

54 Marks. 15 Minutes Reading. 70 Minutes Writing.

Section A: SAC Questions (54 Marks)**Question 1 (20 marks)**

For the following question consider the following cubic function:

$$f(x) = x^3 - 4x^2 + x + 6$$

a.

- i.** Show that the equation of the tangent at any point $x = a$ is given by: (2 marks)

$$y_t = (3a^2 - 8a + 1)x - 2a^3 + 4a^2 + 6$$

- ii.** Find the equation of the normal at any point $x = a$. (2 marks)

- iii.** Find the value of a , for which the tangent line crosses the origin. Provide your answer correct to 3 decimal places. (3 marks)

- iv. Find the coordinates of the local maxima and minima of $f(x)$. (2 marks)

- v. Hence, for what value(s) of a will the normal to the function only intersect the function once, correct to 2 decimal places? (3 marks)

- b.** Consider the following family of functions:

$$f(x) = x^3 - 4x^2 + kx + 6$$

where $k \in \mathbb{R}$.

- i.** For what values of k will there be no stationary points? (2 marks)

- ii.** Find the coordinates of the stationary points in terms of k . (1 mark)

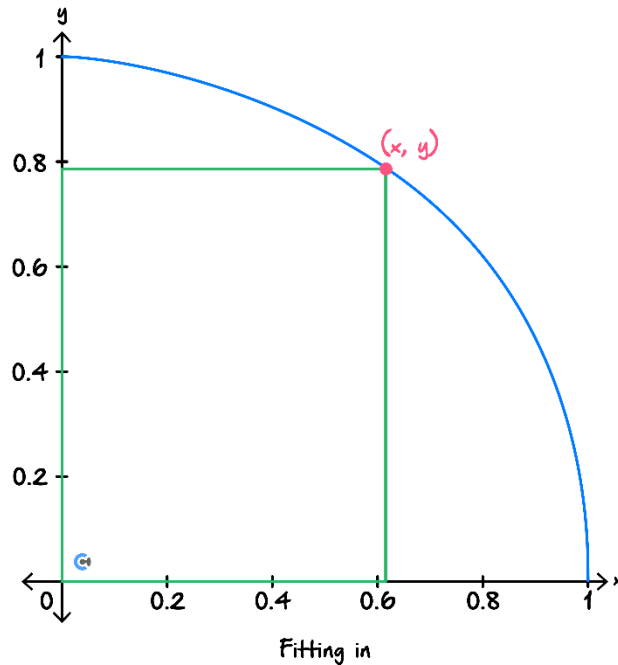
- iii.** Hence, for what values of k will there be one solution for the equation $f(x) = 3$? Provide your answer correct to 1 decimal place. (3 marks)

iv. Is it possible to have no real solutions for $f(x) = m$, where $m \in R$? Justify. (2 marks)

Space for Personal Notes

Question 2 (20 marks)

The figure below depicts a rectangle that is inscribed in a quarter of a unit circle. The equation of the quarter of a unit circle can be described by $f(x) = \sqrt{1 - x^2}$.



a.

- i.** Express the area of the rectangle in terms of x only. (2 marks)

- ii.** Express the perimeter of the rectangle in terms of x only. (1 mark)

- iii.** Restrict the domain to model the situation. (1 mark)

- b.** Show that the maximum area and perimeter occur for the same value of x . (3 marks)

- c.** Two parallel lines, with a gradient of -1 form two sides of a rectangle, inscribed in the same portion of a circle in **part a**. One of these lines is a secant line to the quarter-circle (a secant line intersects with a curve at atleast two points).

- i.** Find the equation of the line passing through the endpoints of the quarter-circle. (1 mark)

- ii.** Find the equation of the line that is tangent to the quarter circle. (1 mark)

- iii.** Hence, what are the possible values of the y -intercept of the secant line? (2 marks)

The other two sides of the rectangle originate from the intersection points of the secant line and the circle.

- iv. One of the intersection points between the secant line and the circle is given by $(a, f(a))$. State the coordinates of the other intersection point at point $(b, f(b))$, where $b > a$. (1 mark)

- v. Hence or otherwise, find the equation of these two lines in terms of a and b . (2 marks)

- vi. State the relationship between a and b . Hence, what do you notice about the two lines in **part c. v.**? (2 marks)

The area of the rectangle can be generalised in terms of a and b given by the equation below:

Area = _____

- vii. State the area in terms of a . (1 mark)

viii. Hence, find the maximum area possible for this rectangle. (3 marks)

Space for Personal Notes

Question 3 (14 marks)

a. For an ideal scenario for a pathogen, it will double its population every day. And hence, only needing 1 pathogen to start a potentially deadly infection. A possible equation to model the scenario can be given in the form $p(t) = 2^t$ where $p(t)$ represents the population of pathogens and t is the number of days.

i. The same equation is rewritten in the form of $p(t) = e^{\log_e(b)t}$. State the value of b . (1 mark)

ii. State $p'(t)$ in terms of $p(t)$. (1 mark)

iii. Hence, show that: (2 marks)

$$\frac{d}{dx}(a^x) = \ln(a)a^x$$

There are two main ways a pathogen is removed from the body by the immune system or medication.

- b.** The immune response is dependent on the time after a pathogen is found. The amount of immune cells per 200 ml of blood is given by:

$$i(t_2) = e^{t_2}$$

- i.** What is the number of immune cells per litre of blood at $t = 0$? (1 mark)

- ii.** Write $i(t_2)$ in terms of t , given that the number of immune cells only starts increasing n number of days after a pathogen is found. Give your answer in terms of n . (1 mark)

Taking into account the immune response, the population of the pathogen is given by: $p^*(t) = p(t) - i(t_2)$.

- iii.** Given that the pathogen is eliminated no later than a month after the initial infection, how many days did it take to detect it initially? (2 marks)

- iv.** The sickest day corresponds to the day with the highest number of pathogens. For what value of t does this occur, given that it took 5 days to detect the pathogen? Provide your answer upto 2 decimal places. (2 marks)

- c. Now, consider the effects of medication. The amount of medicine present in blood for one dosage is given by the following:

$$m(t_3) = e^{125t_3(a-t_3)}$$

- i. Given that it takes 6 days to reach the peak effect of the medication, find the value of a . (2 marks)

- ii. Express t_3 in terms of t and the time the patient took the medication, b . (2 marks)

Space for Personal Notes

Section B: TI Solutions

Question Number	Solutions
1(a)(i)	<p>Define $f(x)=x^3-4\cdot x^2+x+6$ Done</p> <p><code>methods_diffcalc\tangent_line(f(x),x,a)</code></p> <hr/> <ul style="list-style-type: none"> ▶ Derivative: $3\cdot x^2-8\cdot x+1$ ▶ Gradient: $3\cdot a^2-8\cdot a+1$ ▶ Passes Through: $\left[a \quad a^3-4\cdot a^2+a+6 \right]$ ▶ x -Intercept: $\left[\frac{2\cdot (a^3-2\cdot a^2-3)}{3\cdot a^2-8\cdot a+1} \quad 0 \right]$ ▶ Vertical Intercept: $\left[0 \quad -2\cdot (a^3-2\cdot a^2-3) \right]$ ▶ Tangent Line: $(3\cdot a^2-8\cdot a+1)\cdot x-2\cdot (a^3-2\cdot a^2-3)$
1(a)(ii)	<p><code>normalLine(f(x),x,a)</code></p> $\frac{3\cdot a^5-20\cdot a^4+36\cdot a^3+6\cdot a^2-46\cdot a+6}{3\cdot a^2-8\cdot a+1} - \frac{x}{3\cdot a^2-8\cdot a+1}$
1(a)(iii)	<p>Solve vertical intercept equal 0</p> <p><code>solve(-2\cdot (a^3-2\cdot a^2-3)=0,a)</code> $a=2.48558$</p>
1(a)(iv)	<p><code>methods_func\analyse(f(x),x)</code></p> <hr/> <ul style="list-style-type: none"> ▶ Start Point: $[-\infty \quad -\infty]$ ▶ End Point: $[\infty \quad \infty]$ ▶ Maximal Domain: $-\infty < x < \infty$ ▶ x -Intercepts: (3) $[-1 \quad 0], [2 \quad 0],$ $[3 \quad 0]$

	<p>► Vertical Intercept: $[0 \ 6]$</p> <p>► Derivative: $3 \cdot x^2 - 8 \cdot x + 1$</p> <p>► Inflection Point: $\left[\frac{4}{3} \ \frac{70}{27} \right]$ (Decreasing)</p> <p>► Stationary Points: (2) $\left[\frac{-(\sqrt{13}-4)}{3} \ \frac{26 \cdot \sqrt{13}}{27} + \frac{70}{27} \right]$ (Local max.) $\left[\frac{\sqrt{13}+4}{3} \ \frac{70}{27} - \frac{26 \cdot \sqrt{13}}{27} \right]$ (Local min.)</p>
1(a)(v)	<p>Define $n(x)=\text{normalLine}(f(x),x,a)$ Done</p> <p>$\text{methods_diffcalc}\backslash\text{solve_touch}(n(x),f(x),x,a)$</p> <p>► Derivative 1: $\frac{-1}{3 \cdot a^2 - 8 \cdot a + 1}$</p> <p>► Derivative 2: $3 \cdot x^2 - 8 \cdot x + 1$</p> <p>► Equating functions and derivatives.</p> <p>► Solutions: $x=2.52420$ and $a=-1.04840$ $x=1.95574$ and $a=0.08852$ $x=0.71093$ and $a=2.57815$ $x=0.14247$ and $a=3.71507$</p> <p>$\text{solve}\left(\frac{d}{dx}(f(x))=0,x\right)$ $x=0.13148$ or $x=2.53518$</p> <p>Use sliders to see when one point of intersection occurs. $a \in (-\infty, -1.05) \cup (0.09, 0.13] \cup [2.54, 2.58) \cup (3.71, \infty)$</p>
1(b)(i)	<p>Define $f(x)=x^3-4 \cdot x^2+k \cdot x+6$ Done</p> <p>Define $df(x)=\frac{d}{dx}(f(x))$ Done</p> <p>$\text{solve}(df(x)=0,x)$ $x=\frac{\sqrt{16-3 \cdot k}+4}{3}$ or $x=\frac{-(\sqrt{16-3 \cdot k}-4)}{3}$</p> <p>$\text{solve}(16-3 \cdot k < 0, k)$ $k > \frac{16}{3}$</p>

1(b)(ii)

methods_func\analyse(f(x),x)

- ▶ Start Point: $[-\infty \quad -\infty]$
- ▶ End Point: $[\infty \quad \infty]$
- ▶ Maximal Domain: $-\infty < x < \infty$
- ▶ x-Intercepts: (3)
- ▶ Vertical Intercept: $[0 \quad 6]$
- ▶ Derivative: $3 \cdot x^2 - 8 \cdot x + k$
- ▶ Inflection Point:

$$\left[\frac{4}{3} \quad \frac{4 \cdot k}{3} + \frac{34}{27} \right]$$
- ▶ Stationary Points: (2)

$$\left[\frac{\sqrt{16-3 \cdot k} + 4}{3} \quad \frac{-2 \cdot (16-3 \cdot k)^{\frac{3}{2}}}{27} + \frac{4 \cdot k}{3} + \frac{34}{27} \right]$$

$$\left[\frac{-(\sqrt{16-3 \cdot k} - 4)}{3} \quad \frac{2 \cdot (16-3 \cdot k)^{\frac{3}{2}}}{27} + \frac{4 \cdot k}{3} + \frac{34}{27} \right]$$

1(b)(iii)

Using a slider, we see that both stationary points must be above 3. Note that if a cubic has no stationary point, then it is one-to-one, so there is always a unique solution. Hence we must also include our answer to **part i**.

$$\text{solve} \left(\frac{2 \cdot (16-3 \cdot k)^{\frac{3}{2}}}{27} + \frac{4 \cdot k}{3} + \frac{34}{27} > 3, k \right)$$

$$k \leq \frac{16}{3}$$

$$\text{solve} \left(\frac{-2 \cdot (16-3 \cdot k)^{\frac{3}{2}}}{27} + \frac{4 \cdot k}{3} + \frac{34}{27} > 3, k \right)$$

$$2.60587 < k \leq \frac{16}{3}$$

2(b)

Define $f(x) = \sqrt{1-x^2}$ Done

Define $a(x) = x \cdot f(x)$ Done

Define $p(x) = 2 \cdot x + 2 \cdot f(x)$ Done

methods_func\analysed(a(x),x,0,1)

- ▶ Start Point: $[0 \ 0]$
- ▶ End Point: $[1 \ 0]$
- ▶ Maximal Domain: $0 \leq x \leq 1$
- ▶ x -Intercepts: (2)
 $[0 \ 0], [1 \ 0]$
- ▶ Vertical Intercept: $[0 \ 0]$
- ▶ Derivative:

$$\sqrt{1-x^2} - \frac{x^2}{\sqrt{1-x^2}}$$

- ▶ Inflection Point:
 $[0 \ 0]$ (Increasing)
- ▶ Stationary Point:
 $\left[\frac{\sqrt{2}}{2} \ \frac{1}{2} \right]$ (Local max.)

methods_func\analysed(p(x),x,0,1)

- ▶ Start Point: $[0 \ 2]$
- ▶ End Point: $[1 \ 2]$
- ▶ Maximal Domain: $0 \leq x \leq 1$
- ▶ No x -Intercepts Found
- ▶ Vertical Intercept: $[0 \ 2]$
- ▶ Derivative: $2 - \frac{2 \cdot x}{\sqrt{1-x^2}}$
- ▶ No Inflection Points Found
- ▶ Stationary Point:
 $\left[\frac{\sqrt{2}}{2} \ 2 \cdot \sqrt{2} \right]$ (Local max.)

2(c)(ii)

```
methods_diffcalc\solve_grad(f(x),x,-1,0,1)
```

▶ Maximal Domain: $0 \leq x \leq 1$

▶ Derivative: $\frac{-x}{\sqrt{1-x^2}}$

▶ Found point with gradient -1 :

"x"	"y"	"Eqn."	"x-Int."	"y-Int."
$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\sqrt{2} - x$	$\sqrt{2}$	$\sqrt{2}$

2(c)(v)

DelVar a

Done

```
methods_miscVinear_info([a f(a)],1)
```

▶ Point: $\left[a \sqrt{1-a^2} \right]$ Gradient: 1

▶ Linear Equation: $y = x + \sqrt{1-a^2} - a$

▶ x-Intercept: $\left[a - \sqrt{1-a^2} \quad 0 \right]$

▶ y-Intercept: $\left[0 \sqrt{1-a^2} - a \right]$

```
methods_miscVinear_info([b f(b)],1)
```

▶ Point: $\left[b \sqrt{1-b^2} \right]$ Gradient: 1

▶ Linear Equation: $y = x + \sqrt{1-b^2} - b$

▶ x-Intercept: $\left[b - \sqrt{1-b^2} \quad 0 \right]$

▶ y-Intercept: $\left[0 \sqrt{1-b^2} - b \right]$

2(c)(vii)

Height of the diagonal rectangle is distance between two points on the secant intersecting function

```
methods_miscVinear_info([a f(a)], [b f(b)])
```

Point 1: $\left[a \sqrt{1-a^2} \right]$

Point 2: $\left[b \sqrt{1-b^2} \right]$

Midpoint: $\left[\frac{a+b}{2} \quad \frac{\sqrt{1-a^2} + \sqrt{1-b^2}}{2} \right]$

Distance: $\sqrt{-2 \cdot (\sqrt{1-a^2} \cdot \sqrt{1-b^2} + a \cdot b - 1)}$

Width found by taking y intercept of perpendicular at (a,f(a)) then taking distance to (a,f(a)).

```
methods_miscVinear_info([a f(a)],1)
```

► Point: $\begin{bmatrix} a & \sqrt{1-a^2} \end{bmatrix}$ Gradient: 1

► Linear Equation: $y=x+\sqrt{1-a^2}-a$

► x-Intercept: $\begin{bmatrix} a-\sqrt{1-a^2} & 0 \end{bmatrix}$

► y-Intercept: $\begin{bmatrix} 0 & \sqrt{1-a^2}-a \end{bmatrix}$

```
methods_miscVinear_info([a f(a)],[0 sqrt(1-a^2)-a])
```

► Point 1: $\begin{bmatrix} a & \sqrt{1-a^2} \end{bmatrix}$

► Point 2: $\begin{bmatrix} 0 & \sqrt{1-a^2}-a \end{bmatrix}$

► Midpoint: $\begin{bmatrix} \frac{a}{2} & \frac{2\cdot\sqrt{1-a^2}-a}{2} \end{bmatrix}$

► Distance: $|a|\cdot\sqrt{2}$

Define $b=\sqrt{1-a^2}$

Done

©get height in terms of a

$$\sqrt{-2\cdot(\sqrt{1-a^2}\cdot\sqrt{1-b^2}+a\cdot b-1)}$$

$$\sqrt{-2\cdot(\sqrt{1-a^2}\cdot|a|+a\cdot\sqrt{1-a^2}-1)}$$

©define area

Define $area(a)=\sqrt{-2\cdot(\sqrt{1-a^2}\cdot|a|+a\cdot\sqrt{1-a^2}-1)}\cdot a\cdot\sqrt{2}$ Done

$$area(a)|a>0$$

$$2\cdot a\cdot\sqrt{1-2\cdot a\cdot\sqrt{1-a^2}}$$

2(c)(viii)

Note: The analyse program might freeze with this question. Hold [esc] or [home] to exit if required.

$$\text{solve}\left(\frac{d}{da}(area(a))=0,a\right)$$

$$a=\frac{\sqrt{2-\sqrt{2}}}{2}$$

$$area\left(\frac{\sqrt{2-\sqrt{2}}}{2}\right)$$

$$\frac{\sqrt{8}-2}{2}$$

3(a)	<div>Define $p(t)=2^t$ Done</div> <div>Define $dp(t)=\frac{d}{dt}(p(t))$ Done</div> <div>$dp(t)$ $\ln(2) \cdot 2^t$</div>
3(b)(iii)	<div>Define $i(t)=e^t$ Done</div> <div>solve($i(t)=p(30),t$) $t=30 \cdot \ln(2)$</div> <div>$30-30 \cdot \ln(2)$ $30-30 \cdot \ln(2)$</div>
3(b)(iv)	<div><code>methods_func\analyse(p2(t),t)</code></div> <hr/> <div> <p>► Start Point: $[-\infty \quad 0.00000]$</p> <p>► End Point: $[\infty \quad -\infty]$</p> <p>► Maximal Domain: $-\infty < t < \infty$</p> <p>► Asymptote: $y=0.00000$ (Horizontal)</p> <p>► t-Intercept: $[16.29350 \quad 0.00000]$</p> <p>► Vertical Intercept: $[0.00000 \quad 0.99326]$</p> <p>► Derivative:</p> <p>$0.69315 \cdot (2.00000)^t - 0.00674 \cdot (2.71828)^t$</p> <p>► Inflection Point:</p> <p>$[13.90465 \quad 7967.83240]$ (Increasing)</p> <p>► Stationary Point:</p> <p>$[15.09907 \quad 10769.69680]$ (Local max.)</p> </div>
3(c)	<div>Define $m(t)=e^{125 \cdot t \cdot (a-t)}$ Done</div> <div>Define $dm(t)=\frac{d}{dt}(m(t))$ Done</div> <div>solve($dm(6)=0,a$) $a=12$</div>



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