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VCE Mathematical Methods $\frac{3}{4}$
AOS 2 Revision [2.0]
SAC 3

45 Marks. 10 Minutes Reading. 65 Minutes Writing.

Section A: SAC Questions (45 Marks)**Question 1** (10 marks)

Let $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = x^3 - 3x^2 + 2$.

- a.** Find the coordinates of the stationary points of $f(x)$. (2 marks)

- b.** Using the turning points found in **part a.**, find the equation of the line segment joining these two points. State the length of this line segment. (3 marks)

Another function $g(x)$ exists such that $g(x) = f(x) + kx$, where k is a real constant.

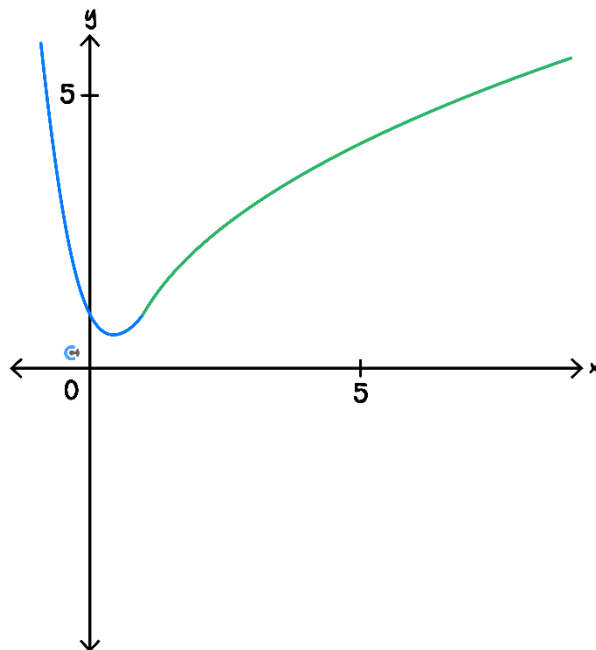
- c.** Find the x -values of the stationary points of $g(x)$ in terms of k as well as the coordinate of the point of inflection. (3 marks)

A tangent line is made at the inflection point found in **part c**.

d. Given that this tangent line passes through the coordinate $(5, 2)$, solve for the value(s) of k . (2 marks)

Question 2 (7 marks)

The hybrid function $f(x) = \begin{cases} -x^3 + 3x^2 - 2x + 1, & x \leq 1 \\ \sqrt{4x - 3}, & x > 1 \end{cases}$ is sketched on the graph below:



a. Define the derivative function $f'(x)$. (2 marks)

b. On the same axes, sketch the graph of the derivative function. (2 marks)

c. Find the values of x when $f(x)$ is strictly increasing. (1 mark)

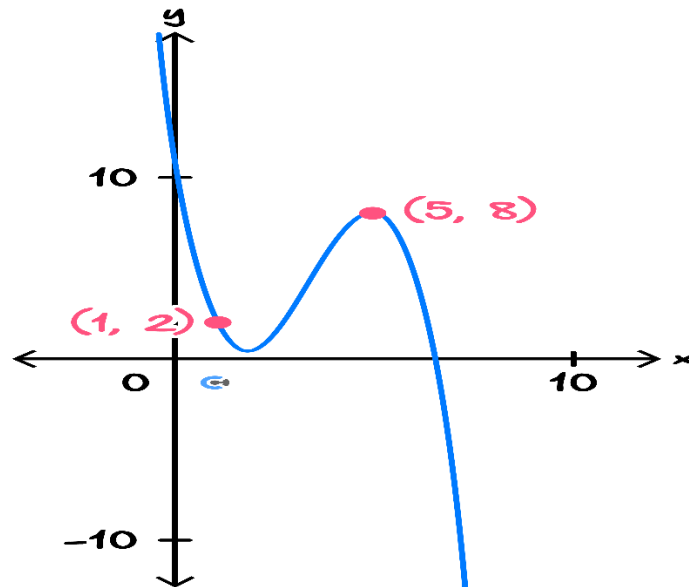
d. Let $g(x) = \begin{cases} -x^3 + 3x^2 - 2x + 1 + kx, & x \leq 1 \\ \sqrt{4x - 3}, & x > 1 \end{cases}$ with $k > 0$.

Find the values of x when $g(x)$ is strictly increasing in terms of k . (2 marks)

Space for Personal Notes

Question 3 (8 marks)

James is standing outside his house in Doreen at the point $(1, 2)$ and decides to take his dog Mango to the park located at $(5, 8)$. A map is shown below which depicts James current whereabouts as well as the local park. Mango is a rather impatient dog and instead of walking on the footpath he decides to walk in a straight line directly to the park.



- a. Calculate the distance travelled if Mango walks in a straight line from James' house to the park. (1 mark)

Once James and Mango arrive at the park, they decide to follow the exact path of the park given by:

$$f(x) = -\frac{1}{2}x^3 + 5x^2 - 13x + 10.5$$

Assume that they follow the path when they return home after a long walk.

- b. You could say that their path travelled is a relationship that is tangential to the park at every single point. If James and Mango are currently standing at $x = a$, state the equation of the path they are travelling in terms of a . (1 mark)

Whenever the gradient of the path is equal to 2, Mango's bladder kicks in and he suddenly needs to take a leak.

c.

- i.** State the values of x whenever Mango uses the toilet. (2 marks)

- ii.** Hence, state the equations of the tangent lines whenever Mango takes a leak. (2 marks)

After a while, Mango needs to do a number 2. The points at which Mango needs to do this occur whenever the gradient equals zero.

- d.** Find the coordinates of the points at which Mango does a number 2. (2 marks)

Space for Personal Notes

Question 4 (10 marks)

Consider the functions:

$$f(x) = \frac{x+2}{x+3}$$

$$g(x) = \sqrt{5-x}$$

where both functions are defined over their maximal domains.

- a. State the domains of $f(x)$ and $g(x)$. (2 marks)

- b. Determine whether $f(g(x))$ or $g(f(x))$ is defined. Give a reason why. (3 marks)

- c. For the function that was defined, find the range of that function. (2 marks)

- d. For the undefined function, state the maximal domain of that function such that the composite function exists. (3 marks)

Question 5 (10 marks)

A piecewise function is defined by:

$$f(x) = \begin{cases} ax^3 + x, & x \leq 1, \\ (3a + b)x - 2, & x > 1, \end{cases}$$

where a and b are constants with $a > 0$.

- a. State the values of a and b such that the function joins smoothly at $x = 1$. Hence, state the domain of the derivative function. (3 marks)

b.

- i.** Find the coordinates of the x -intercept of $f(x)$. (1 mark)

- ii.** Hence, state the angle in degrees that $f(x)$ makes with the x -axis at the x -intercept. (2 marks)

- c.** Find the coordinates of the intersection point(s) between $f(x)$ and $f^{-1}(x)$. (2 marks)

- d.** Hence, state the possible axis(es) of symmetry that exist for $f(x)$ and $f^{-1}(x)$. Explain why there is only one axis of symmetry or why there are multiple axes of symmetry. (2 marks)

Space for Personal Notes

Section B: TI Solutions

Question Number	Solutions
1(a)	Define $f(x)=x^3-3\cdot x^2+2$ Done <i>methods_func\analyse(f(x),x)</i> ▶ Start Point: $[-\infty \quad -\infty]$ ▶ End Point: $[\infty \quad \infty]$ ▶ Maximal Domain: $-\infty < x < \infty$ ▶ x-Intercepts: (3) $[-(\sqrt{3}-1) \quad 0], [1 \quad 0],$ $[\sqrt{3}+1 \quad 0]$ ▶ Vertical Intercept: $[0 \quad 2]$ ▶ Derivative: $3\cdot x^2-6\cdot x$ ▶ Inflection Point: $[1 \quad 0]$ (Decreasing) ▶ Stationary Points: (2) $[0 \quad 2]$ (Local max.) $[2 \quad -2]$ (Local min.)
1(b)	<i>methods_misc\linear_info([0 2],[2 -2])</i> ▶ Point 1: $[0 \quad 2]$ ▶ Point 2: $[2 \quad -2]$ ▶ Midpoint: $[1 \quad 0]$ ▶ Distance: $2\cdot\sqrt{5}$ ▶ Gradient: -2 ▶ Perp. Bisector: $y=\frac{x}{2}-\frac{1}{2}$ ▶ Linear Equation: $y=2-2\cdot x$ ▶ x-Intercept: $[1 \quad 0]$ ▶ y-Intercept: $[0 \quad 2]$

1(c)	$\text{methods_func\textit{analyse}}(g(x),x)$ ▶ Start Point: $[-\infty \quad -\infty]$ ▶ End Point: $[\infty \quad \infty]$ ▶ Maximal Domain: $-\infty < x < \infty$ ▶ x -Intercepts: (3) ▶ Vertical Intercept: $[0 \quad 2]$ ▶ Derivative: $3 \cdot x^2 - 6 \cdot x + k$ ▶ Inflection Point: $[1 \quad k]$ ▶ Stationary Points: (2) $\left[\frac{\sqrt{-3 \cdot (k-3)} + 3}{3} \quad \frac{2 \cdot (k-3) \cdot \sqrt{-3 \cdot (k-3)}}{9} + k \right]$ $\left[\frac{-(\sqrt{-3 \cdot (k-3)} - 3)}{3} \quad k - \frac{2 \cdot (k-3) \cdot \sqrt{-3 \cdot (k-3)}}{9} \right]$
1(d)	$\text{tangentLine}(g(x),x,1) \quad (k-3) \cdot x + 3$ Define $l(x) = (k-3) \cdot x + 3$ Done solve($l(5) = 2, k$) $k = \frac{14}{5}$
2(a)	Define $f(x) = \begin{cases} -x^3 + 3 \cdot x^2 - 2 \cdot x + 1, & x \leq 1 \\ \sqrt{4 \cdot x - 3}, & x > 1 \end{cases}$ Done Define $df(x) = \frac{d}{dx}(f(x))$ Done $df(x) = \begin{cases} -3 \cdot x^2 + 6 \cdot x - 2, & x < 1 \\ \frac{2}{\sqrt{4 \cdot x - 3}}, & x > 1 \end{cases}$
2(c)	Check the answer with the graph to make sure you are only including appropriate stationary points. solve($df(x) \geq 0, x$) $x \geq \frac{-(\sqrt{3} - 3)}{3}$ and $x \neq 1$

2(d)

$$\text{Define } g(x) = \begin{cases} -x^3 + 3 \cdot x^2 - 2 \cdot x + 1 + k \cdot x, & x \leq 1 \\ \sqrt{4 \cdot x - 3}, & x > 1 \end{cases}$$

Done

$$\text{Define } dg(x) = \frac{d}{dx}(g(x))$$

Done

$$dg(x) = \begin{cases} -3 \cdot x^2 + 6 \cdot x + k - 2, & x < 1 \\ \frac{2}{\sqrt{4 \cdot x - 3}}, & x > 1 \end{cases}$$

$$\text{solve}(dg(x) \geq 0, x) | k > 0$$

$$x \geq \frac{-(\sqrt{3 \cdot (k+1)} - 3)}{3} \text{ and } x \neq 1 \text{ and } k > 0$$

Note: We can rewrite this as the union of two intervals

3(a)

methods_misc *linear_info*([1 2],[5 8])

► Point 1: [1 2]

► Point 2: [5 8]

► Midpoint: [3 5]

► Distance: $2 \cdot \sqrt{13}$

► Gradient: $\frac{3}{2}$

► Perp. Bisector: $y = 7 - \frac{2 \cdot x}{3}$

► Linear Equation: $y = \frac{3 \cdot x}{2} + \frac{1}{2}$

► x-Intercept: $\left[\frac{-1}{3} \quad 0 \right]$

► y-Intercept: $\left[0 \quad \frac{1}{2} \right]$

Note: Convert decimal to fraction to obtain exact value of tangent.

3(b)

$$\text{Define } f(x) = \frac{-1}{2} \cdot x^3 + 5 \cdot x^2 - 13 \cdot x + \frac{21}{2} \quad \text{Done}$$

$$\text{tangentLine}(f(x), x, a)$$

$$\frac{2 \cdot a^3 - 10 \cdot a^2 + 21}{2} - \frac{(3 \cdot a^2 - 20 \cdot a + 26) \cdot x}{2}$$

3(c)

Compute Tangent Lines?

OK

Cancel

$$\text{methods_diffcalc}\backslash\text{solve_grad}(f(x), x, 2)$$

► Maximal Domain: $-\infty < x < \infty$

► Derivative: $\frac{-3 \cdot x^2}{2} + 10 \cdot x - 13$

► Found points with gradient 2 : (2)

"x"	"y"	"Eqn."	"x-Int."	"y-Int."
$-\frac{(\sqrt{10}-10)}{3}$	$\frac{227}{54} - \frac{28 \cdot \sqrt{10}}{27}$	$2 \cdot x - \frac{20 \cdot \sqrt{10} + 133}{54}$	$\frac{20 \cdot \sqrt{10} + 133}{108}$	$-\frac{(20 \cdot \sqrt{10} + 133)}{54}$
$\frac{\sqrt{10}+10}{3}$	$\frac{28 \cdot \sqrt{10}}{27} + \frac{227}{54}$	$2 \cdot x + \frac{20 \cdot \sqrt{10} - 133}{54}$	$-\frac{(20 \cdot \sqrt{10} - 133)}{108}$	$\frac{20 \cdot \sqrt{10} - 133}{54}$

3(d)

$$\text{methods_diffcalc}\backslash\text{solve_grad}(f(x), x, 0)$$

► Maximal Domain: $-\infty < x < \infty$

► Derivative: $\frac{-3 \cdot x^2}{2} + 10 \cdot x - 13$

► Found points with gradient 0 : (2)

$-\frac{(\sqrt{22}-10)}{3}$	$\frac{227}{54} - \frac{22 \cdot \sqrt{22}}{27}$
$\frac{\sqrt{22}+10}{3}$	$\frac{22 \cdot \sqrt{22}}{27} + \frac{227}{54}$

4(a)	Define $f(x)=\frac{x+2}{x+3}$ Done Define $g(x)=\sqrt{5-x}$ Done domain($f(x),x$) $x \neq -3$ domain($g(x),x$) $-\infty < x \leq 5$									
4(c)	Note: function is monotonic so we can substitute in the range of g to obtain the range of $f(g(x))$. $f(0)$ $\frac{2}{3}$ $f(\infty)$ undef									
4(d)	solve($f(x) \leq 5, x$) $x \leq \frac{-13}{4}$ or $x > -3$									
5(a)	$_diffcalc\text{solve_smooth}(f1(x),f2(x),x,\{a,b\},1)$ ► Left Derivative: $3 \cdot a \cdot x^2 + 1$ ► Right Derivative: $3 \cdot a + b$ <table><tr><td>"At x=1:"</td><td>"Left Func."</td><td>"Right Func."</td></tr><tr><td>"Value:"</td><td>$a+1$</td><td>$3 \cdot a + b - 2$</td></tr><tr><td>"Gradient:"</td><td>$3 \cdot a + 1$</td><td>$3 \cdot a + b$</td></tr></table> ► Solutions: $a=1$ and $b=1$	"At x=1:"	"Left Func."	"Right Func."	"Value:"	$a+1$	$3 \cdot a + b - 2$	"Gradient:"	$3 \cdot a + 1$	$3 \cdot a + b$
"At x=1:"	"Left Func."	"Right Func."								
"Value:"	$a+1$	$3 \cdot a + b - 2$								
"Gradient:"	$3 \cdot a + 1$	$3 \cdot a + b$								
5(b)(i)	Define $a=1$ Done Define $b=1$ Done zeros($f1(x),x$) $x \leq 1$ $\{0\}$ zeros($f2(x),x$) $x \geq 1$ $\{\}$									
5(b)(ii)	$\frac{d}{dx}(f1(x)) _{x=0}$ 1									



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