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VCE Mathematical Methods ¾

AOS 2 Revision [2.0]

SAC 2 Solutions

45 Marks. 10 Minute Reading. 65 Minutes Writing.

Section A: SAC Questions (45 Marks)

Question 1 (8 marks)

Let $f(x)$ be defined by the function $f(x) = e^{x+2}\ln(x+2)$.

a. State the maximal domain of $f(x)$. (1 mark)

$\text{dom } f = (-2, \infty)$

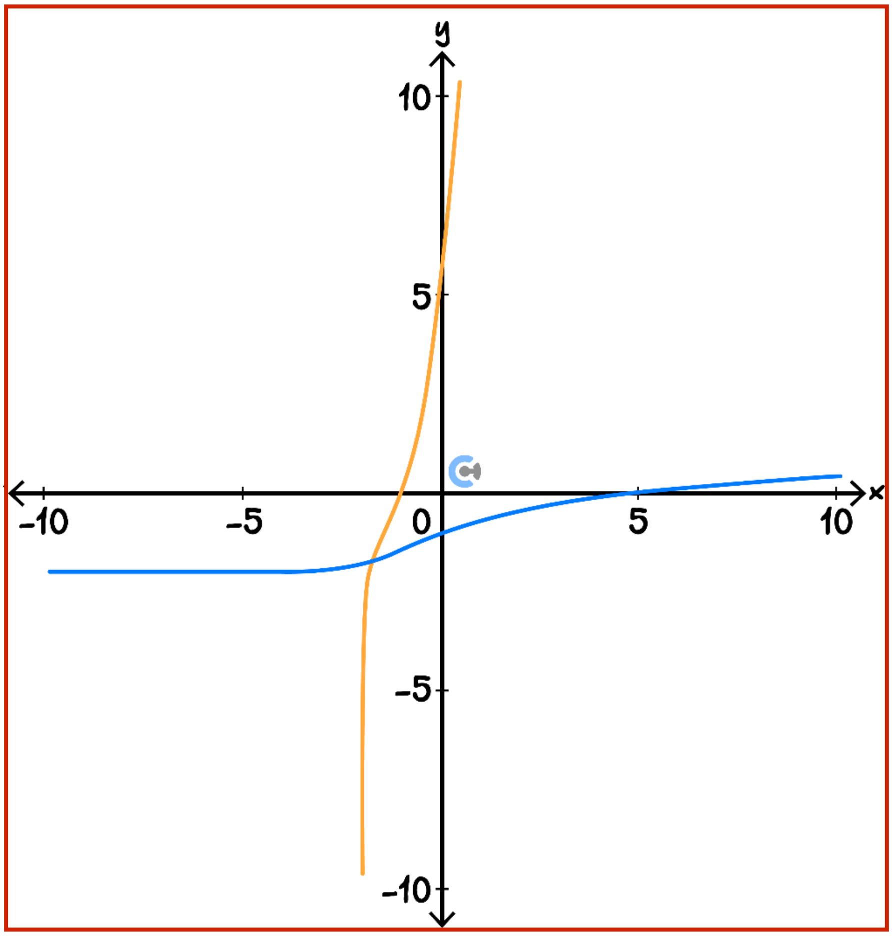
b. Find the coordinates of the x-intercepts of the function $f(x)$. (2 marks)

$(-1, 0)$

c. The y-intercept of $f(x)$ is found at the point (a, b). State the coordinates of (a, b). (2 marks)

$(0, e^2\ln(2))$

d. Part of the graph of $f(x)$ is shown below. On the same axes sketch the graph of the inverse function $f^{-1}(x)$. Label all asymptote(s) and intercept(s) with their equation(s). (3 marks)



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Inverse has asymptote $y = -2$ and x-intercept $(e^2 \ln(2), 0)$ Note CAS can't solve for inverse.

Question 2 (9 marks)

Let $P(3, -2)$ and the line $y_1 = 2x + 1$ be given.

Point Q lies on y_1 so that the distance between Q and P is minimised.

a. State the gradient of the line perpendicular to y_1. (1 mark)

$$-\frac{1}{2}$$

b. The line perpendicular to y_1 that passes through P can be written in the form

$$ax + by + c = 0$$

Show that $a = 1$, $b = 2$, $c = 1$. (2 marks)

$$y + 2 = -\frac{1}{2}(x - 3)$$

$$2y + 4 = -(x - 3) \implies x + 2y + 1 = 0.$$

Hence, $a = 1$, $b = 2$, $c = 1$

c. Find the intersection between y_1 and the perpendicular line. Hence, state the coordinates of Q and determine the minimum distance between P and Q. (3 marks)

$$Q\left(-\frac{3}{5}, -\frac{1}{5}\right)$$

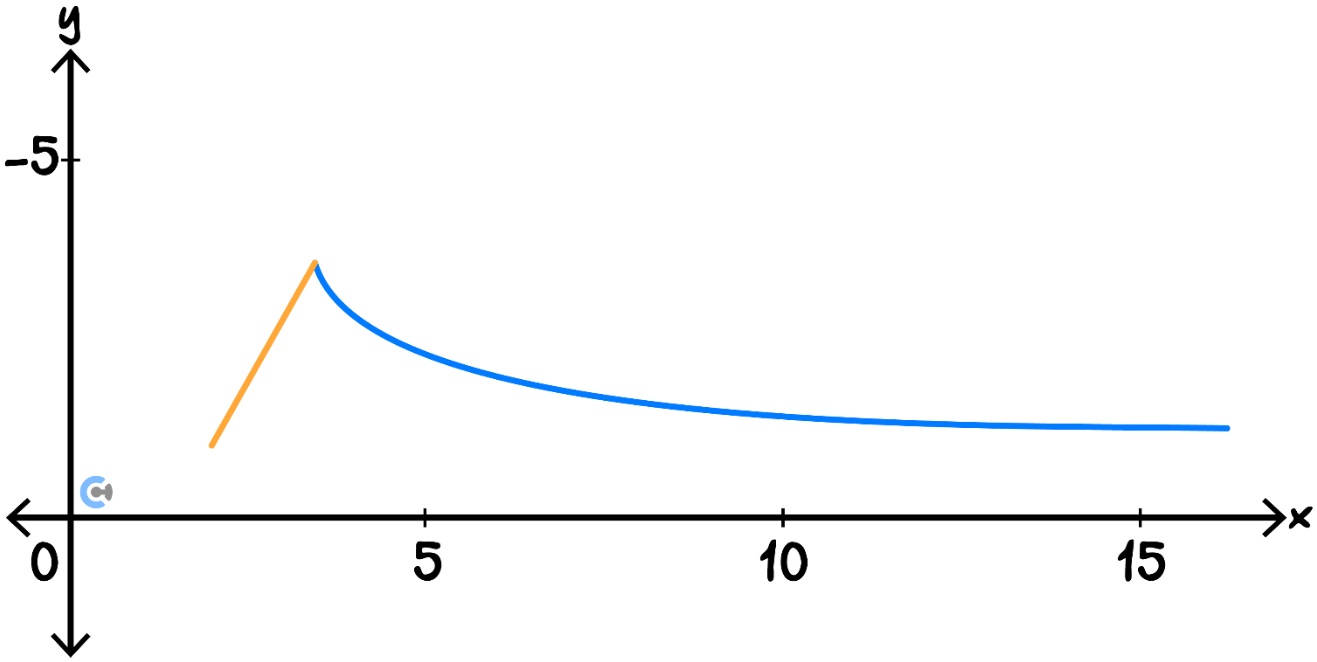
$$\sqrt{\left(3-\left(-\frac{3}{5}\right)\right)^2 + \left(-2-\left(-\frac{1}{5}\right)\right)^2} = \sqrt{\left(\frac{18}{5}\right)^2 + \left(-\frac{9}{5}\right)^2} = \frac{9\sqrt{5}}{5}$$

d. Another line is given by $y_3 = 2x - 4$, which is parallel to y_1. Find the minimum distance between y_1 and y_3 in the form $\frac{a\sqrt{b}}{c}$ (with a, b, c positive integers). (3 marks)

$\frac{1\sqrt{5}}{1}$ (matching the required form)

Question 3 (7 marks)

Contour Park is famously renowned for having one of the world's only infinite slides, the Contour Slide. The slide consists of two parts, one being the ladder to the top of the slide and the other being the slide itself. The image below shows a side view of the design of the slide:



It is known that the ladder component, due to safety reasons, makes an angle of 59.7 degrees with the ground.

a. State the gradient of the slide m_s. Give your answer correct to 2 decimal places. (1 mark)

$m_s = 1.71$

b. It is known that the base of the ladder is positioned at the coordinate $(2, 1)$. Find the equation that models the ladder $l(x)$. Give all coefficients correct to two decimal places. (2 marks)

$$l(x) = 1.71x - 2.42$$

The actual slide component can be modelled by the equation:

$$s(x) = \frac{25x + 41}{25(x - 2)}$$

c. $s(x)$ can be expressed in the form $s(x) = k + \frac{a}{x-h}$. State the values of a, h and k. (1 mark)

$$a = 3.64, h = 2, k = 1$$

The gradient of the slide is the steepest at the very top of the slide. It is known that children feel scared when the slide slope exceeds a magnitude of 45 degrees.

d.

i. Find the equation for $s'(x)$. (1 mark)

$$s'(x) = -\frac{91}{25(x - 2)^2}$$

ii. Hence, state the possible values of x correct to three decimal places where the children feel scared. (2 marks)

$$3.458 \leq x < 3.908$$

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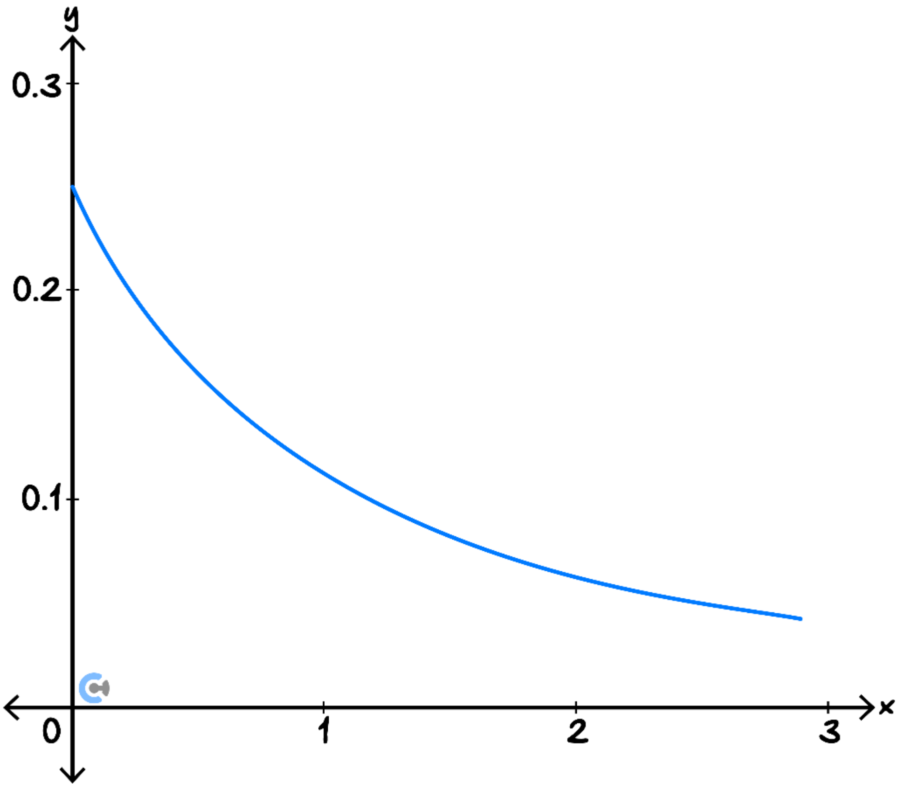
Question 4 (13 marks)

The graph of $y = f(x)$, where $f : [0, \infty) \to R, f(x) = \frac{1}{(x+2)^2}$ is shown below:



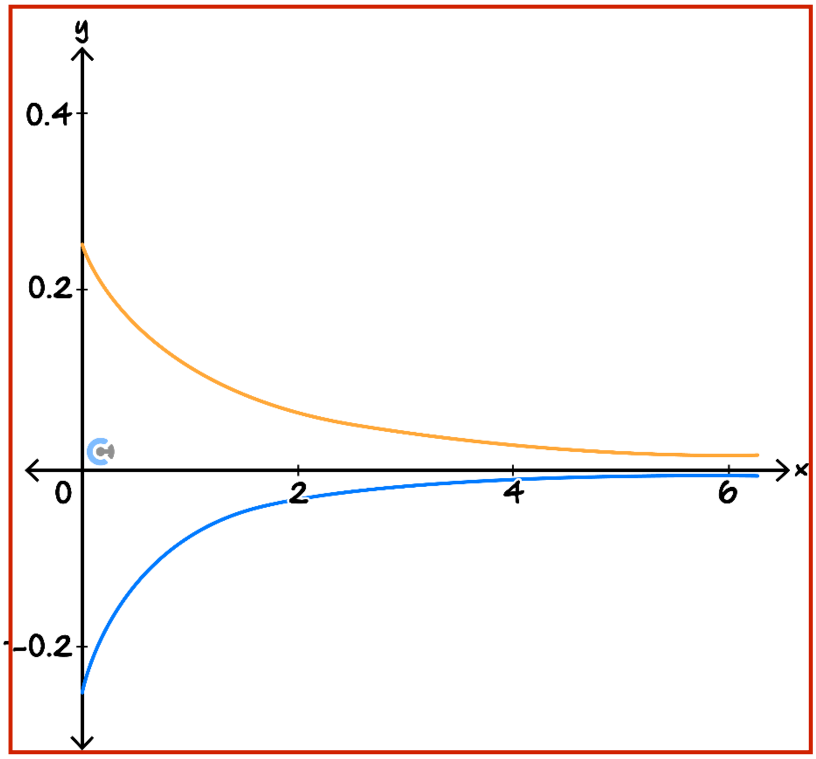
a. Define the derivative function $f'(x)$. (2 marks)

$$f' : (0, \infty) \to R, f'(x) = \frac{-2}{(x+2)^3}$$

b. State the value of the gradient $f(x)$ approaches as x approaches 0. Hence, sketch the graph of $f'(x)$ on the axes below. Label endpoints and intercepts (if any) with their coordinates. (3 marks)

$$-\frac{1}{4}$$

Note: Left endpoint should be open circle.



c.

i. Find the tangent line to $f(x)$ at $x = 0.25$. (2 marks)

$$y = \frac{176}{729} - \frac{128x}{729}$$

ii. Find the area of the triangle that is formed by this tangent line, the x-axis and the y-axis. Give your answer correct to 2 decimal places. (2 marks)

0.17 sq. units

There exists another tangent to $f(x)$ at $x = a$.

d.

i. Find the equation of the tangent line at $x = a$. (1 mark)

$$y = \frac{3a+2}{(a+2)^3} - \frac{2x}{(a+2)^3}$$

ii. Hence, find the value of a that maximises the area of the triangle enclosed by the tangent line, the x-axis and the y-axis. Hence, find this area. (3 marks)

$a = 2$, Area $= \frac{1}{4}$ sq. units

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Question 5 (8 marks)

The polynomial $h(x)$ is given by the equation $h(x) = 5x^3 + 60x^2 + 180x$.

a. State the coordinates of the x-intercepts of $h(x)$. (2 marks)

$(-6, 0), (0, 0)$

b. Show that $h(x)$ can be expressed in the form $h(x) = c(x - a)(x - b)^2$, where a, b, and c are integers. (2 marks)

$$h(x) = 5x(x^2 + 12x + 36)$$
$$h(x) = 5x(x + 6)^2$$
$$h(x) = 5(x - 0)(x - (-6))^2$$

Let another function $g(x) = h(x) + k$.

c. Find the possible value(s) of k such that $g(x) = 0$ has 3 solutions. (2 marks)

$0 < k < 160$

d. Find the coordinates of the inflection point of $g(x)$ in terms of k. (2 marks)

$(-4, k - 80)$

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Section B: TI Solutions

Question Number	Solutions
1(a), (b), (c), (d)	$methods\_func \backslash analyse(f(x),x)$ ▸ Start Point: $[-\infty \quad 0]$ ▸ End Point: $[\infty \quad \infty]$ ▸ Maximal Domain: $-2<x<\infty$ ▸ Asymptotes: (2) $x=-2$ (Vertical) $y=0$ (Horizontal) ▸ x-Intercept: $[-1 \quad 0]$ ▸ Vertical Intercept: $[0 \quad e^{2}\cdot \ln(2)]$ ▸ Derivative: $e^{x+2}\cdot \ln(x+2)+\frac{e^{x+2}}{x+2}$ ▸ Inflection Point: $[-1.40814 \quad -0.94792]$ (Increasing) ▸ No Stationary Points Found
2(a), (b), (c)	$methods\_misc \backslash linear\_info(y=2\cdot x+1,[3 \quad -2])$ ▸ x-int: $\left[\frac{-1}{2} \quad 0\right]$ ▸ y-int: $[0 \quad 1]$ ▸ $[3 \quad -2]$ does not lie on $y=2\cdot x+1$ ▸ Perp. Line: $y=\frac{-x}{2}-\frac{1}{2}$ ▸ Intersection: $\left[\frac{-3}{5} \quad \frac{-1}{5}\right]$ ▸ Shortest Distance: $\frac{9\cdot \sqrt{5}}{5}$ Use the distance between two points formula in your working for part c!

2 (d)	Get intersection point with perpendicular: $\text{methods\_misc\linear\_info}\left(y=\frac{-x}{2}-\frac{1}{2}, y=2\cdot x-4\right)$ ▸ Intersection Point: $\begin{bmatrix}\frac{7}{5} & \frac{-6}{5}\end{bmatrix}$ ▸ Acute Angle: 90. ° Find distance between this point and Q. $\text{methods\_misc\linear\_info}(y=2\cdot x+1, y=2\cdot x-4)$ Lines are parallel ▸ Distance: $\sqrt{5}$
3 (a)	$\tan\left(\frac{59.7\cdot\pi}{180}\right)$ 1.71129
3 (b)	$\text{methods\_misc\linear\_info}([2\ \ 1], 1.71)$ ▸ Point: $[2\ \ 1]$ Gradient: 1.71 ▸ Linear Equation: $y=1.71\cdot x-2.42$ ▸ x−Intercept: $[1.4152\ \ 0]$ ▸ y−Intercept: $[0\ \ -2.42]$
3 (c)	Note: Press [ctrl][enter] to evaluate Define $s(x)=\frac{25\cdot x+41}{25\cdot(x-2)}$ *Done* $\text{expand}(s(x))$ $\frac{3.64}{x-2.}+1.$
3 (d) (i)	Define $ds(x)=\frac{d}{dx}(s(x))$ *Done* $ds(x)$ $\frac{-91}{25\cdot(x-2)^2}$

3 (d) (ii)	$\text{solve}(ds(x)<-1,x)\|x>2$ $\quad$ $2.<x<3.90788$
4 (a)	Define $f(x)=\dfrac{1}{(x+2)^2}$ $\quad$ *Done* Define $df(x)=\dfrac{d}{dx}(f(x))$ $\quad$ *Done* $df(x)$ $\quad$ $\dfrac{-2}{(x+2)^3}$
4 (b)	$df(0)$ $\quad$ $\dfrac{-1}{4}$
4 (c) (i)	$methods\_diffcalc\backslash tangent\_line\left(f(x),x,\dfrac{1}{4}\right)$ ▸ Derivative: $\dfrac{-2}{(x+2)^3}$ ▸ Gradient: $\dfrac{-128}{729}$ ▸ Passes Through: $\begin{bmatrix}\frac{1}{4} & \frac{16}{81}\end{bmatrix}$ ▸ x −Intercept: $\begin{bmatrix}\frac{11}{8} & 0\end{bmatrix}$ ▸ Vertical Intercept: $\begin{bmatrix}0 & \frac{176}{729}\end{bmatrix}$ ▸ Tangent Line: $\dfrac{176}{729}-\dfrac{128\cdot x}{729}$ Note: Input point as a fraction to obtain an exact answer for the tangent line.

4 (c) (ii)	$\frac{1}{2}\cdot\frac{11}{8}\cdot\frac{176}{729}$ 0.165981
4 (d) (i)	methods_diffcalc\tangent_line$(f(x),x,a)$ ▸ Derivative: $\frac{-2}{(x+2)^3}$ ▸ Gradient: $\frac{-2}{(a+2)^3}$ ▸ Passes Through: $\left[a \quad \frac{1}{(a+2)^2}\right]$ ▸ x –Intercept: $\left[\frac{3\cdot a+2}{2} \quad 0\right]$ ▸ Vertical Intercept: $\left[0 \quad \frac{3\cdot a+2}{(a+2)^3}\right]$ ▸ Tangent Line: $\frac{3\cdot a+2}{(a+2)^3}-\frac{2\cdot x}{(a+2)^3}$
4 (d) (ii)	Define $t(a)=\frac{1}{2}\cdot\frac{3\cdot a+2}{(a+2)^3}\cdot\frac{3\cdot a+2}{2}$ *Done* methods_func\analyse$(t(a),a)$ ▸ Start Point: $[-\infty \quad 0]$ ▸ End Point: $[\infty \quad 0]$ ▸ Maximal Domain: $a\neq-2$ and $-\infty<a<\infty$ ▸ Asymptotes: (2) $a=-2$ (Vertical) $y=0$ (Horizontal) ▸ a –Intercept: $\left[\frac{-2}{3} \quad 0\right]$ ▸ Vertical Intercept: $\left[0 \quad \frac{1}{8}\right]$ ▸ Derivative:

	$\frac{-3\cdot(a-2)\cdot(3\cdot a+2)}{4\cdot(a+2)^4}$ ▶ Inflection Points: (2) $\left[\frac{-2\cdot(2\cdot\sqrt{3}-3)}{3} \quad \frac{-3\cdot(\sqrt{3}-3)}{64}\right]$ (Increasing) $\frac{-3\cdot(a-2)\cdot(3\cdot a+2)}{4\cdot(a+2)^4}$ ▶ Inflection Points: (2) $\left[\frac{-2\cdot(2\cdot\sqrt{3}-3)}{3} \quad \frac{-3\cdot(\sqrt{3}-3)}{64}\right]$ (Increasing) $\left[\frac{2\cdot(2\cdot\sqrt{3}+3)}{3} \quad \frac{3\cdot(\sqrt{3}+3)}{64}\right]$ (Decreasing) ▶ Stationary Points: (2) $\left[\frac{-2}{3} \quad 0\right]$ (Local min.) $\left[2 \quad \frac{1}{4}\right]$ (Local max.)
5 (a)	Define $h(x)=5\cdot x^3+60\cdot x^2+180\cdot x$ *Done* zeros$(h(x),x)$ $\{-6,0\}$
5 (b)	factor$(h(x),x)$ $5\cdot x\cdot(x+6)^2$

5 (c)	$methods\_func \backslash analyse(h(x), x)$ ▶ Start Point: $[-\infty \quad -\infty]$ ▶ End Point: $[\infty \quad \infty]$ ▶ Maximal Domain: $-\infty<x<\infty$ ▶ x –Intercepts: (2) $[-6 \quad 0], [0 \quad 0]$ ▶ Vertical Intercept: $[0 \quad 0]$ ▶ Derivative: $15 \cdot x^2+120 \cdot x+180$ ▶ Inflection Point: $[-4 \quad -80]$ (Decreasing) ▶ Stationary Points: (2) $[-6 \quad 0]$ (Local max.) $[-2 \quad -160]$ (Local min.) Local min cannot be translated up to 160 units or more, and local max cannot be translated down 0 units or more. Therefore, the vertical translation is bounded strictly between 0 and 160 units in the positive y-direction.

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