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VCE Mathematical Methods ¾
AOS 2 Revision [2.0]
SAC 1

50 Marks. 15 Minutes Reading. 75 Minutes Writing.

Section A: SAC Questions (50 Marks)

Question 1 (12 marks)

One morning, when Gregor Samsa woke from troubled dreams, he found himself transformed in his bed into a horrible vermin. He lay on his armour-like back, and if he lifted his head a little he could see his brown belly, slightly domed and divided by arches into stiff sections. The bedding was hardly able to cover it and seemed ready to slide off any moment. His many legs, pitifully thin compared with the size of the rest of him, waved about helplessly as he looked.

We can model Gregor and his roach shell using the formula:

$$s: D \to R, s(x) = \frac{\sqrt{x} - x^2}{e^x}$$

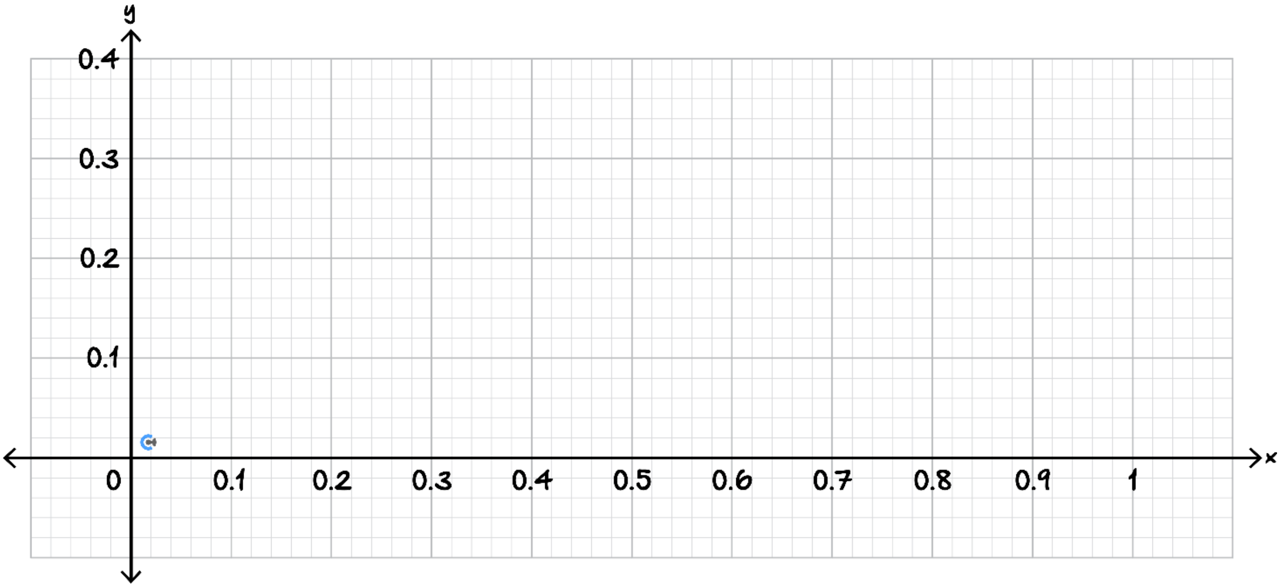
where $s(x)$ represents the height of the shell above the ground.

a. State a suitable domain D. Assume that Gregor is touching the ground. (1 mark)

b. Find $s'(x)$. (1 mark)

c. How tall is the height of the shell to two decimal places? (2 marks)

d. Draw the graph of $s(x)$, labelling the axes, intercept(s) and turning point(s) to two decimal places. (2 marks)



In disgust, Gregor's family try to trap him in a cage. This cage can be described with 2 straight lines, forming a triangle with the ground, which encloses $s(x)$.

One line is given by:

$$l_1: y - \frac{2}{5} = -\frac{1}{4}\left(x - \frac{1}{5}\right), \qquad \frac{1}{5} \le x \le \frac{9}{5}$$

If the lines meet at $\left(\frac{1}{5}, \frac{2}{5}\right)$ then:

e. Find a formula in terms of x, y and m that could describe the other line, l_2. (1 mark)

f. By considering the derivative of $s(x)$, show that the value of m that minimises the length of l_2 is $m = 1.21$. Give your answer to two decimal places. (3 marks)

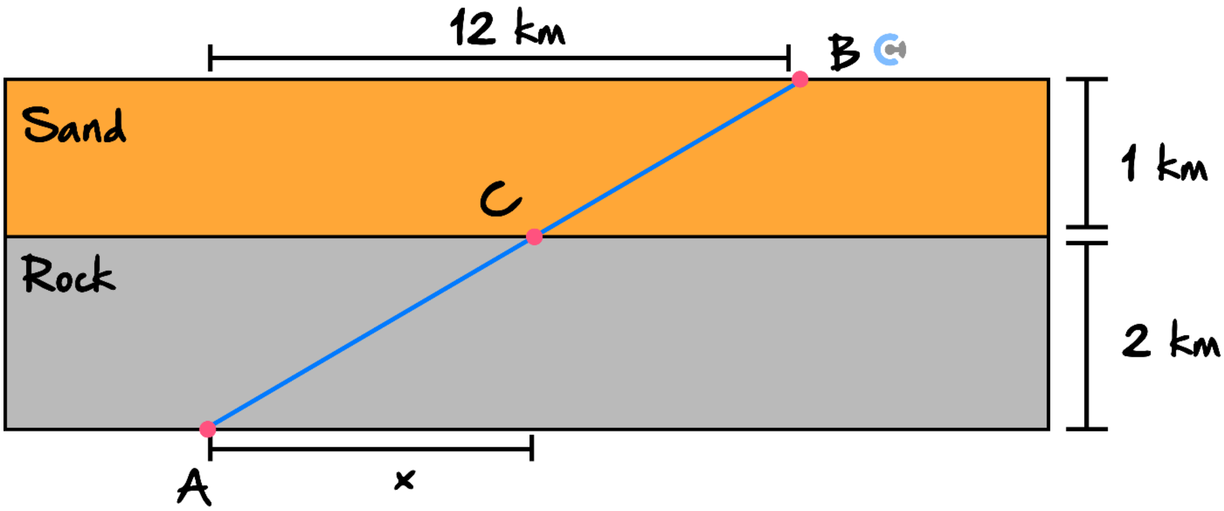
g. Hence, find the line l_2 in terms of x, specifying the domain with all answers to two decimal places. (1 mark)

h. Graph l_1 and l_2 on the graph above, labelling the points of intersection to two decimal places. (1 mark)

Space for Personal Notes

Question 2 (9 marks)

Gregor is late to work. His journey involves travelling across sand and rock. While Gregor can travel at 3 $km/hour$ over sand, he can only travel at 2 $km/hour$ over the rock due to his short beetle legs.



Let the horizontal distance between A and B be 12 m. Let x be the horizontal distance between A and C.

If Gregor travels to work along the path ACB.

a. Write an expression for the distance travelled between A and B. (1 mark)

b. Show that, $t(x)$, Gregor's time of travel is given by: (1 mark)

$$t(x) = \frac{\sqrt{x^2 - 24x + 145}}{3} + \frac{\sqrt{x^2 + 4}}{2}$$

c.

i. State $t'(x)$. (1 mark)

ii. Hence, find Gregor's minimum travel time to two decimal places. (2 marks)

d. What angle in degrees does Gregor make to the horizontal at point A to one decimal place? (2 marks)

e. By what percentage is this time faster than if Gregor travelled in a straight line to one decimal place? (2 marks)

Space for Personal Notes

Question 3 (16 marks)

After enough time living as a bug, Gregor has finally learnt how to fly. He can now commute to work a lot faster.

Gregor works in the n^{th} floor of an office building ($n \in N$), 400 metres above the ground.

We can model his flight path between home and work with the following hybrid function, $h(x)$, where h is Gregor's height above the ground and x is his horizontal displacement. Assume his flight is smooth.

$$h(x) = \begin{cases} ax^2 + bx + c, & if\ 0 \leq x \leq 600 \\ \frac{x}{2} - 200, & if\ 600 \leq x \leq 1200 \end{cases}$$

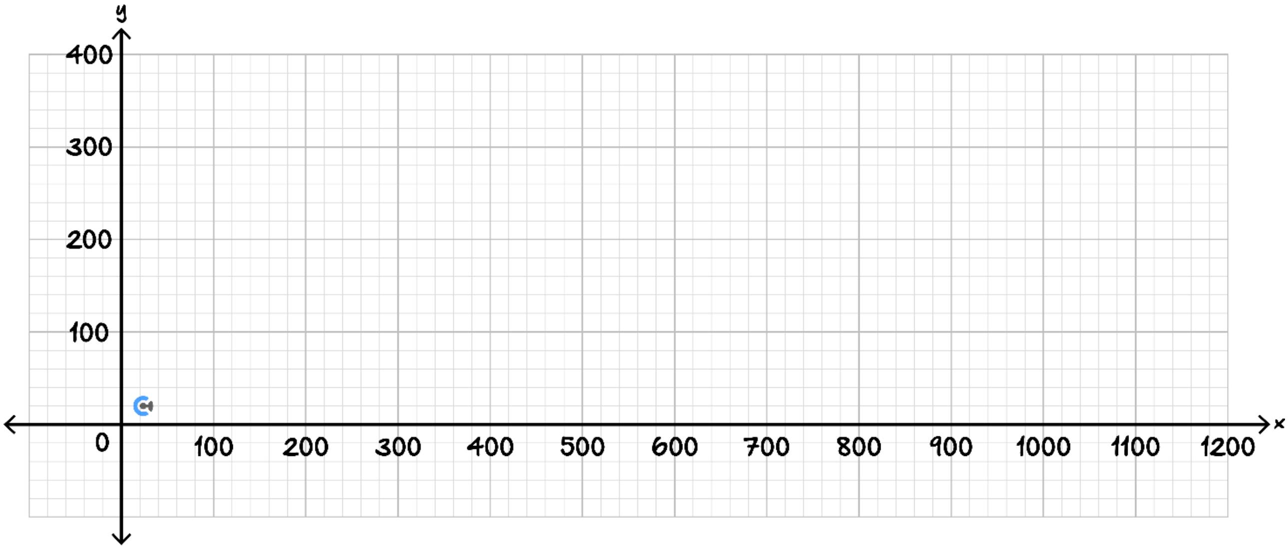
a. If Gregor departs his apartment from his apartment floor, 200 m from ground level.

i. State the values of $h(x)$ at $x = 0$ and $x = 600$. (1 mark)

ii. State the value of $h'(x)$ at $x = 600$. (1 mark)

iii. Hence, find the value of a, b and c. (2 marks)

iv. Plot the graph of $y = h(x)$, labelling intercepts, turning points, axes, endpoints as necessary. (3 marks)



b. Gregor is tired from flying and can barely overcome gravity. Now we can model the second half of his flight over $[600, 1200]$ with a logarithm, $g(x)$.

Assume this new flight path can be modelled by $g(x) = 20(\log_e(x - h)) + k$.

i. Describe how $y = g(x)$ can be transformed into the graph of $y = \log_e(x)$ in terms of h and k. (3 marks)

ii. State the value of $g(x)$ at $x = 600$. (1 mark)

iii. Find the value of h and k. (2 marks)

iv. Plot $g(x)$ on the same axes above. (1 mark)

v. Does Gregor fly high enough to reach his office? If not, what is the height difference between him and his office when he reaches the building? Give your answer correct to two decimal places. (2 marks)

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Question 4 (13 marks)

a. Consider the functions:

$$f(x) = e^{x^2}, x \in R$$

$$g(x) = \ln(x), x > 0$$

i. Does $g(f(x))$ exist? Explain why. (1 mark)

ii. What is the implied domain of $g(f(x))$? (1 mark)

When Gregor is flying, we can model the height in metres of his wingtips relative to his body over 1 flap using the following function:

$$w: [a, 2] \to R, w(t) = 4\,t^2(2-t)^2$$

b. State the range of this function's rule over its implied domain. (1 mark)

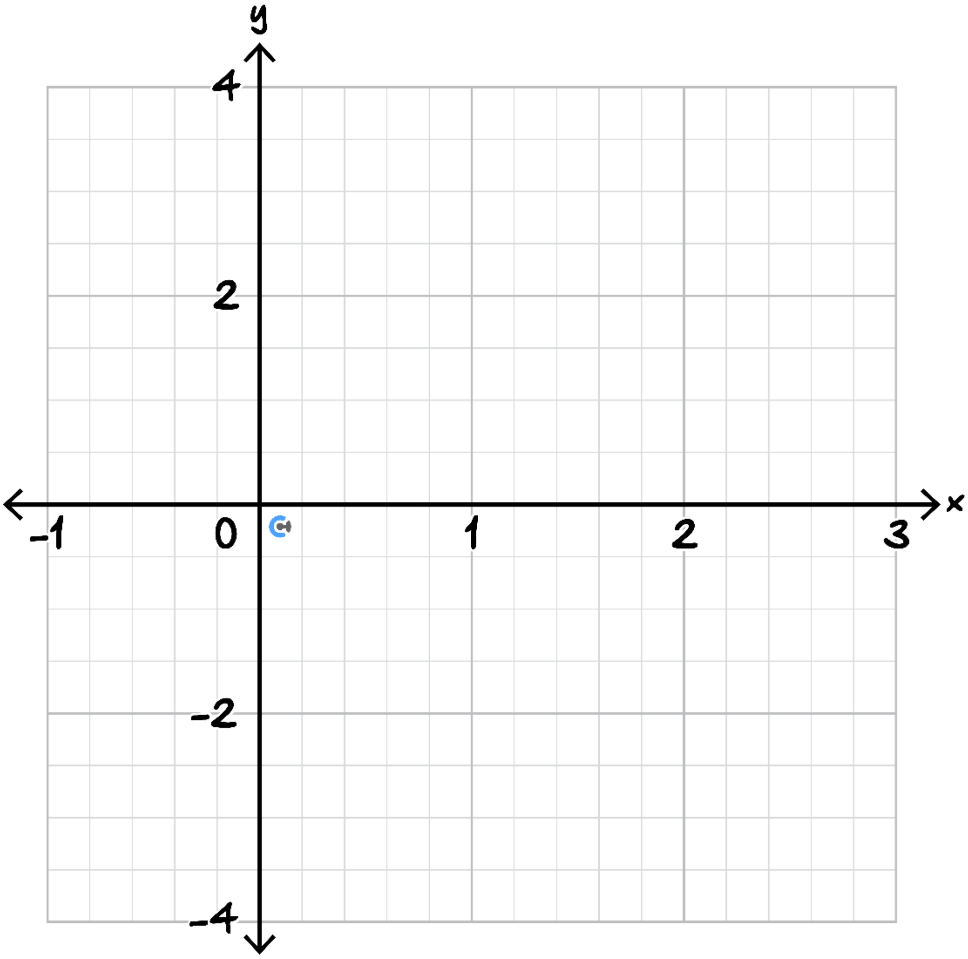
c.

i. Find the value(s) of t when Gregor's wings are at body level, correct to 4 decimal places. (3 marks)

ii. If Gregor's wings are below his body, above his body, then back below his body over 1 flap, find a. (2 marks)

iii. What is the fastest rate and the time when his wings **rise** during 1 flap? Give your answers correct to two decimal places. (3 marks)

d. Graph $y = w(t)$, labelling intercepts, axes and turning points. Give your answers correct to four decimal places. (2 marks)



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Section B: TI Solutions

Question Number	Solutions
1(a)	Define $s(x)=\frac{\sqrt{x}-x^2}{e^x}$ *Done* $\text{solve}(s(x)\geq 0,x)$ $0\leq x\leq 1$
1(b)	Define $ds(x)=\frac{d}{dx}(s(x))$ *Done* $ds(x)$ $\frac{\left(2\cdot x^{\frac{5}{2}}-4\cdot x^{\frac{3}{2}}-2\cdot x+1\right)\cdot e^{-x}}{2\cdot\sqrt{x}}$
1(c)	*methods_func\analysed*$(s(x),x,0,1)$ ▸ Start Point: $[0 \quad 0]$ ▸ End Point: $[1 \quad 0]$ ▸ Maximal Domain: $0\leq x\leq 1$ ▸ Asymptote: $y=0$ (Horizontal) ▸ x –Intercepts: (2) $[0 \quad 0],[1 \quad 0]$ ▸ Vertical Intercept: $[0 \quad 0]$ ▸ Derivative: $\frac{\left(2\cdot x^{\frac{5}{2}}-4\cdot x^{\frac{3}{2}}-2\cdot x+1\right)\cdot e^{-x}}{2\cdot\sqrt{x}}$ ▸ Inflection Point: $[0.803018 \quad 0.112565]$ (Decreasing) ▸ Stationary Point: $[0.264236 \quad 0.341067]$ (Local max.)

1(e)	Define $l1(x)=\frac{-1}{4}\cdot\left(x-\frac{1}{5}\right)+\frac{2}{5}$ *Done* Define $l2(x)=m\cdot\left(x-\frac{1}{5}\right)+\frac{2}{5}$ *Done*
1(f)	*methods_diffcalc\solve_touch*$(l2(x),s(x),x,m)$ ▸ Derivative 1: m ▸ Derivative 2: $\frac{\left(2\cdot x^{\frac{5}{2}}-4\cdot x^{\frac{3}{2}}-2\cdot x+1\right)\cdot e^{-x}}{2\cdot\sqrt{x}}$ ▸ Equating functions and derivatives. ▸ Solutions: $x=1.14217$ and $m=-0.50443$ $x=0.456388$ and $m=-0.405438$ $x=0.081532$ and $m=1.20662$ Only look at $0 \leq x \leq 1$ and positive gradient m.
1(g)	$l2(x)\|m=1.21$ $\quad 1.21\cdot x+0.158$
2(b)	Define $t(x)=\frac{\sqrt{x^2-24\cdot x+145}}{3}+\frac{\sqrt{x^2+4}}{2}$ *Done*
2(c)	$y=4-\frac{5\cdot x}{6}$ (Oblique) $y=\frac{5\cdot x}{6}-4$ (Oblique) ▸ No x –Intercepts Found ▸ Vertical Intercept: $\left[0 \quad \frac{\sqrt{145}}{3}+1\right]$ ▸ Derivative:

	$y=4-\frac{5\cdot x}{6}$ (Oblique) $y=\frac{5\cdot x}{6}-4$ (Oblique) ▸ No x –Intercepts Found ▸ Vertical Intercept: $\left[0 \quad \frac{\sqrt{145}}{3}+1\right]$ ▸ Derivative: $\frac{x-12}{3\cdot\sqrt{x^2-24\cdot x+145}}+\frac{x}{2\cdot\sqrt{x^2+4}}$ ▸ No Inflection Points Found ▸ Stationary Point: $[1.77366 \quad 4.76163]$ (Local min.)
2(d)	Define $m=\frac{2}{1.77366}$ *Done* $\frac{\tan^{-1}(m)\cdot 180}{\pi}$ 48.4324
2(e)	$1-\frac{4.76163}{t(8)}$ 0.133851
3(a)(i)	Define $h1(x)=a\cdot x^2+b\cdot x+c$ *Done* Define $h2(x)=\frac{x}{2}-200$ *Done* Define $dh1(x)=\frac{d}{dx}(h1(x))$ *Done* Define $dh2(x)=\frac{d}{dx}(h2(x))$ *Done*

3(a)(ii)	$dh2(600)$ $\quad \frac{1}{2}$
3(a)(iii)	Note: solve_smooth supports up to 2 parameters, so we need to know one of the parameters in $h1(x)$. We know that the vertical intercept is 200 so $c = 200$. $methods\_diffcalc\backslash solve\_smooth(h1(x),h2(x),x,\{a,b\},600)$ ▸ Left Derivative: $2\cdot a\cdot x+b$ ▸ Right Derivative: $\frac{1}{2}$ $\begin{bmatrix} \text{"At x=600:"} & \text{"Left Func."} & \text{"Right Func."} \\ \text{"Value:"} & 360000\cdot a+200\cdot(3\cdot b+1) & 100 \\ \text{"Gradient:"} & 1200\cdot a+b & \frac{1}{2} \end{bmatrix}$ ▸ Solutions: $a=\frac{1}{900}$ and $b=\frac{-5}{6}$
3(b)(i)	Note: We can use the transform program to check our transformations give the desired result. $methods\_func\backslash transform\left(g(x),x,\left\{y-k,x-h,\frac{y}{20}\right\}\right)$ ▸ Translation $-k$ units along the y−dir. $20\cdot\ln(x-h)$ ▸ Translation $-h$ units along the x−dir. $20\cdot\ln(x)$ ▸ Dilation by factor of $\frac{1}{20}$ from the x−axis $\ln(x)$
3(b)(iii)	Remember to define parameters for h1(x)

	Define $a=\frac{1}{900}$ *Done* Define $b=\frac{-5}{6}$ *Done* methods_diffcalc\solve_smooth$(h1(x),g(x),x,\{h,k\},600)$ ▸ Left Derivative: $\frac{x}{450}-\frac{5}{6}$ ▸ Right Derivative: $\frac{20}{x-h}$ $\begin{bmatrix} \text{"At x=600:"} & \text{"Left Func."} & \text{"Right Func."} \\ \text{"Value:"} & 100 & 20\cdot\ln(-(h-600))+k \\ \text{"Gradient:"} & \frac{1}{2} & \frac{-20}{h-600} \end{bmatrix}$ ▸ Solutions: $h=560$ and $k=-20\cdot(\ln(40)-5)$
3(b)(v)	Define $h=560$ *Done* Define $k=-20\cdot(\ln(40)-5)$ *Done* $g(1200)$ 155.452 $400-155.45177444479$ 244.548
4(a)	Define $f(x)=e^{x^2}$ *Done* Define $g(x)=\ln(x)$ *Done* domain$(g(f(x)),x)$ $-\infty<x<\infty$
4(b), (c) (i), (c) (iii)	Check y-value of endpoints and stationary points

Define $w(t)=4\cdot t^2\cdot(2-t)^2-2$ *Done*

methods_func\analysed$(w(t),t,0,2)$

- Start Point: $[0. \quad -2.]$
- End Point: $[2. \quad -2.]$
- Maximal Domain: $0.\le t\le 2.$
- t −Intercepts: (2.)
  $[0.458804 \quad 0.],[1.5412 \quad 0.]$
- Vertical Intercept: $[0. \quad -2.]$
- Derivative:
  $16.\cdot t^3-48.\cdot t^2+32.\cdot t$
- Inflection Points: (2.)
  $[0.42265 \quad -0.222222]$ (Increasing)
  $[1.57735 \quad -0.222222]$ (Decreasing)
- Stationary Points: (3.)
  $[0. \quad -2.]$ (Local min.)
  $[1. \quad 2.]$ (Local max.)
  $[2. \quad -2.]$ (Local min.)

Part c.iii. continued:
Fasting increase occurs at increasing inflection point.

Define $dw(t)=\frac{d}{dt}(w(t))$ *Done*

$dw(0.42)$ 6.15821

Show second derivative and set equal to 0 in working for full 3 marks.

$\frac{d}{dt}(dw(t))$ $48\cdot t^2-96\cdot t+32$

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