Website: contoureducation.com.au | Phone: 1800 888 300
Email: hello@contoureducation.com.au

VCE Mathematical Methods ¾
AOS 2 Revision [2.0]
Contour Check (Part 3) Solutions

Contour Check

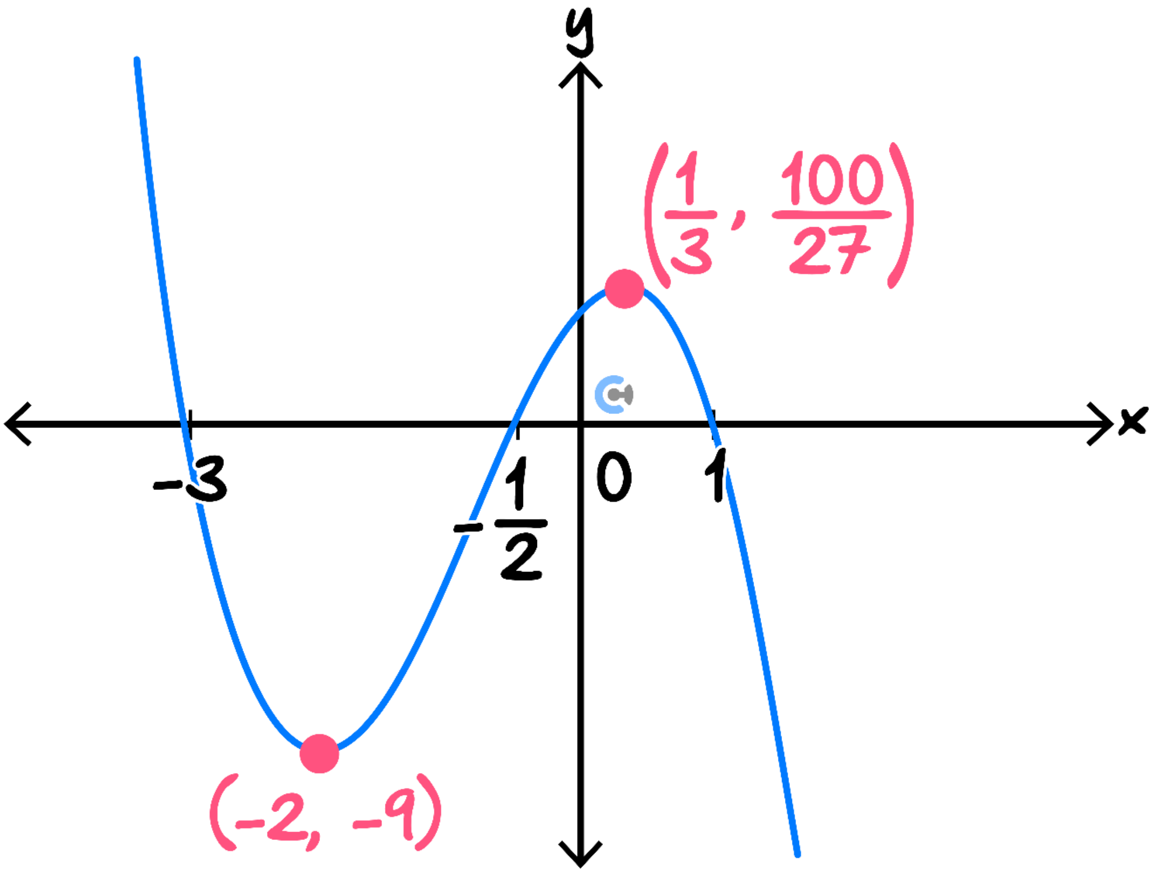
[2.1 - 2.7] - Exam 2 Overall (VCAA Qs) Pg 111-207

Section I: [2.1 - 2.7] - Exam 2 Overall (Checkpoints) (338 Marks)

Question 96 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2016
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2016/2016MM2-w.pdf#page=3

Part of the graph $y = f(x)$ of the polynomial function f is shown below.



$f'(x) < 0$ for:

A. $x \in (-2,0) \cup \left(\frac{1}{3}, \infty\right)$

B. $x \in \left(-9, \frac{100}{27}\right)$

C. $x \in (-\infty, -2) \cup \left(\frac{1}{3}, \infty\right)$

D. $x \in \left(-2, \frac{1}{3}\right)$

E. $x \in (-\infty, -2) \cup (1, \infty)$

Question 97 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2016
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2016/2016MM2-w.pdf#page=3

The average rate of change of the function f with rule, $f(x) = 3x^2 - 2\sqrt{x+1}$, between $x = 0$ and $x = 3$ is:

A. 8

B. 25

C. $\frac{53}{9}$

D. $\frac{25}{3}$

E. $\frac{13}{9}$

Question 98 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2016
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2016/2016MM2-w.pdf#page=6

For the curve $y = x^2 - 5$, the tangent to the curve will be parallel to the line connecting the positive x-intercept and the y-intercept when x is equal to:

A. $\sqrt{5}$

B. 5

C. -5

D. $\frac{\sqrt{5}}{2}$

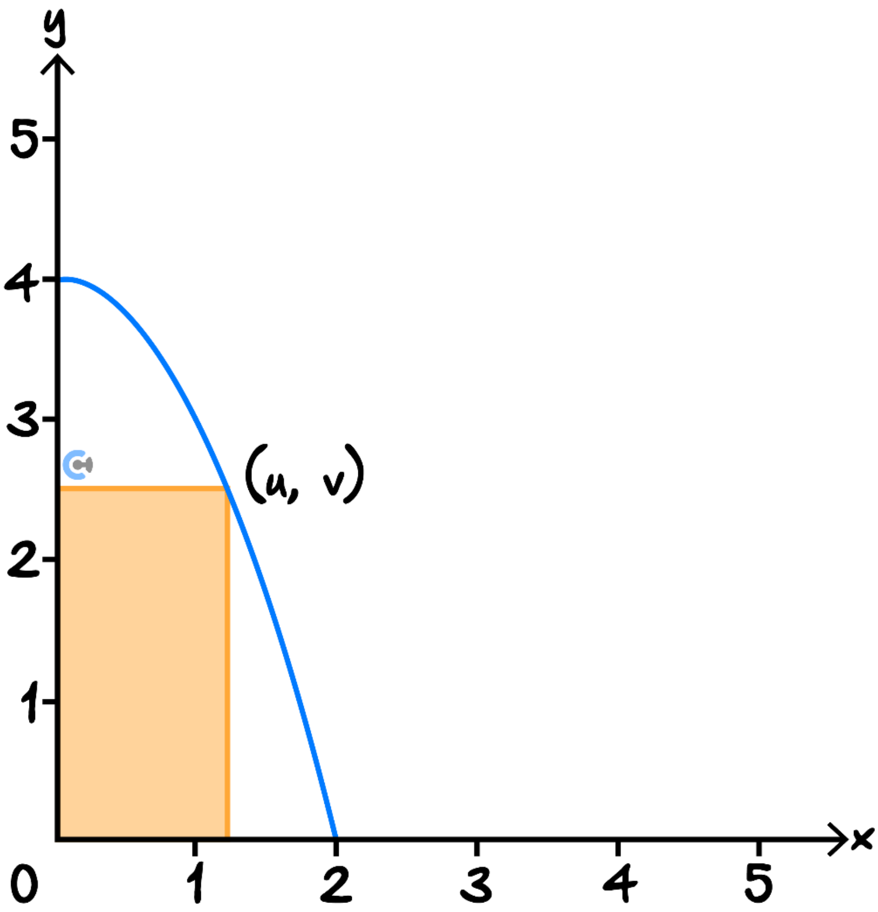
E. $\frac{1}{\sqrt{5}}$

Space for Personal Notes

Question 99 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2016
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2016/2016MM2-w.pdf#page=8

A rectangle is formed by using part of the coordinate axes and a point (u, v), where $u > 0$ on the parabola $y = 4 - x^2$.



Which one of the following is the maximum area of the rectangle?

A. 4

B. $\frac{2\sqrt{3}}{3}$

C. $\frac{8\sqrt{3}-4}{3}$

D. $\frac{8}{3}$

E. $\frac{16\sqrt{3}}{9}$

Area of the rectangle = $x(4-x^2)$

Let $A(x) = x(4-x^2)$

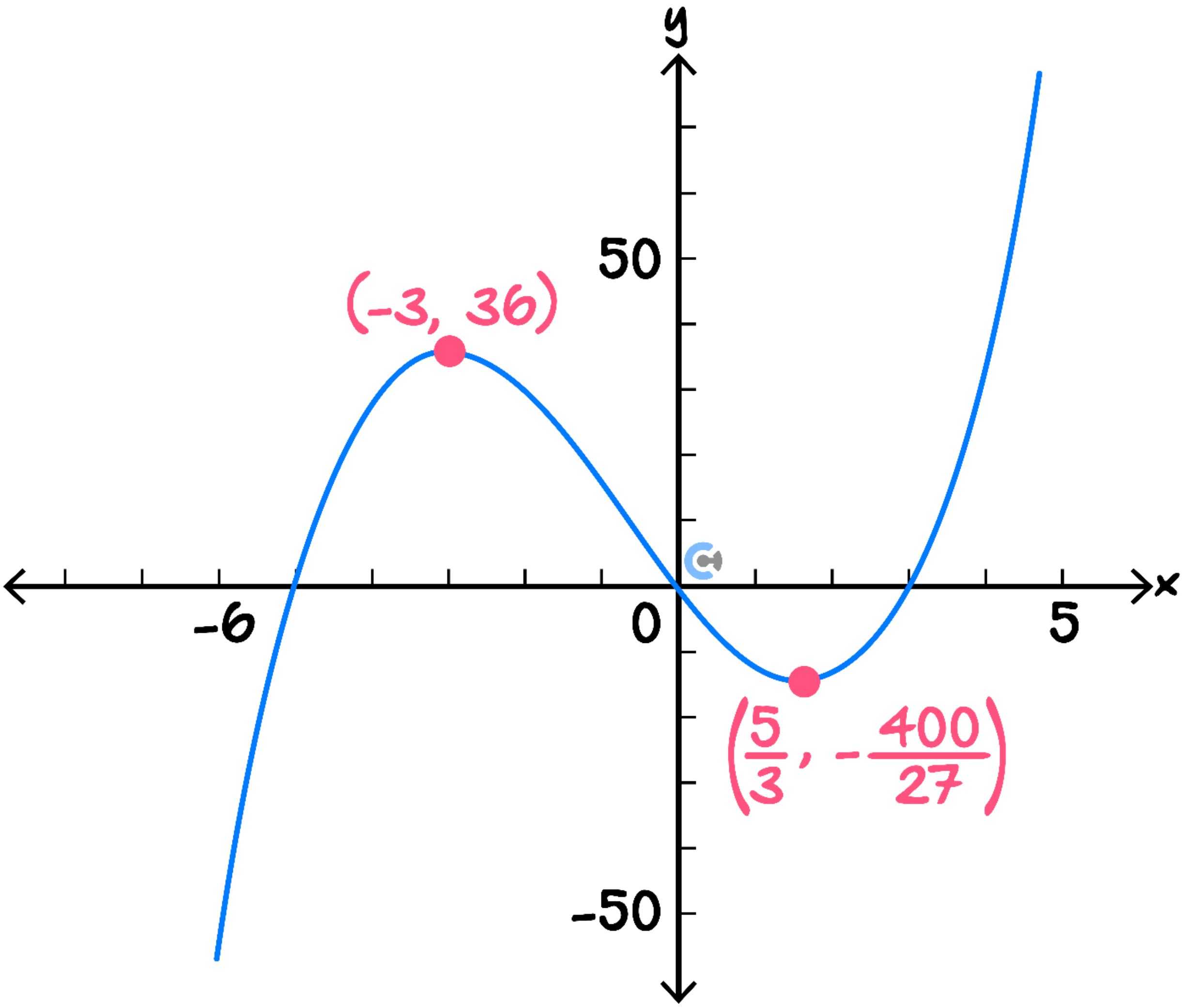
Solve $A'(x) = -3x^2 + 4 = 0$, $x = \frac{2\sqrt{3}}{3}$

$A\left(\frac{2\sqrt{3}}{3}\right) = \frac{16\sqrt{3}}{9}$

Question 100 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2017
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2017/2017MM2-w.pdf#page=2

Part of the graph of a cubic polynomial function f and the coordinates of its stationary points are shown below.



$f'(x) < 0$ for the interval:

A. $(0,3)$

B. $(-\infty,-5) \cup (0,3)$

C. $(-\infty,-3) \cup \left(\frac{5}{3},\infty\right)$

D. $\left(-3,\frac{5}{3}\right)$

E. $\left(\frac{-400}{27},36\right)$

Question 101 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2017
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2017/2017MM2-w.pdf#page=5

The average rate of change of the function with the rule $f(x) = x^2 - 2x$ over the interval $[1, a]$, where $a > 1$, is 8.

The value of a is:

A. 9

B. 8

C. 7

D. 4

E. $1 + \sqrt{2}$

Question 102 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2017
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2017/2017MM2-w.pdf#page=6

The function $f: R \to R, f(x) = x^3 + ax^2 + bx$ has a local maximum at $x = -1$ and a local minimum at $x = 3$.

The values of a and b are respectively:

A. -2 and -3.

B. 2 and 1.

C. 3 and -9.

D. -3 and -9.

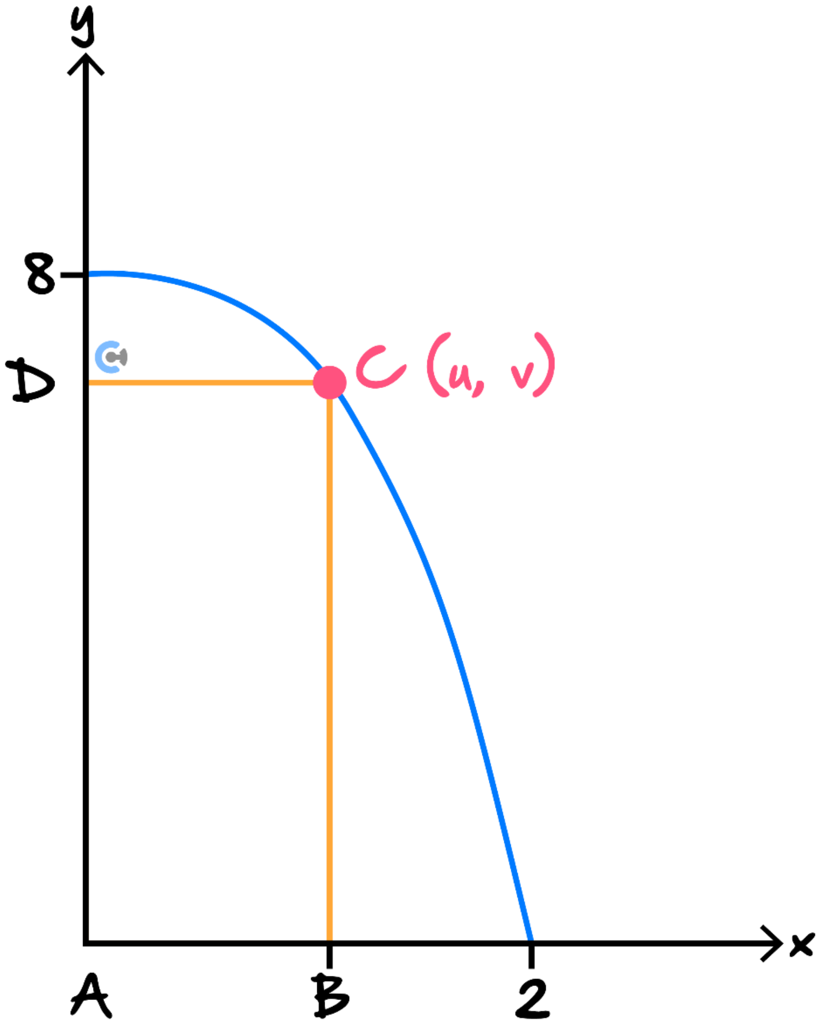
E. -6 and -15.

Space for Personal Notes

Question 103 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2017
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2017/2017MM2-w.pdf#page=7

A rectangle $ABCD$ has vertices $A(0,0), B(u,0), C(u,v)$, and $D(0,v)$, where (u,v) lies on the graph of $y = -x^3 + 8$, as shown below.



The maximum area of the rectangle is:

A. $\sqrt[3]{2}$

B. $6\sqrt[3]{2}$

C. 16

D. 8

E. $3\sqrt[3]{2}$

Question 104 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2018
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2018/2018MM2-w.pdf#page=3

Consider $f(x) = x^2 + \frac{p}{x}, x \neq 0, p \in R$.

There is a stationary point on the graph of f when $x = -2$.

The value of p is:

A. -16

B. -8

C. 2

D. 8

E. 16

Question 105 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2018
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2018/2018MM2-w.pdf#page=4

A tangent to the graph of $y = \log_e(2x)$ has a gradient of 2.

This tangent will cross the y-axis at:

A. 0

B. -0.5

C. -1

D. $-1 - \log_e(2)$

E. $-2\log_e(2)$

Space for Personal Notes

Question 106 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2018
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2018/2018MM2-w.pdf#page=8

Consider the functions, $f: R^+ \to R, f(x) = x^{\frac{p}{q}}$ and $g: R^+ \to R, g(x) = x^{\frac{m}{n}}$, where p, q, m and n are positive integers and $\frac{p}{q}$ and $\frac{m}{n}$ are fractions in simplest form.

If $\{x: f(x) > g(x)\} = (0,1)$ and $\{x: g(x) > f(x)\} = (1, \infty)$, which of the following must be **false**?

A. $q > n$ and $p = m$.

B. $m > p$ and $q = n$.

C. $pn < qm$.

D. $f'(c) = g'(c)$ for some $c \in (0,1)$.

E. $f'(d) = g'(d)$ for some $d \in (1, \infty)$.

$f'(d) = g'(d)$ for some $d \in (1, \infty)$ is false.

Options A to D could be seen to be true by substituting in values.

Question 107 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2019
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2019/2019MM2-w.pdf#page=3

Let $f: R\backslash\{4\} \to R, f(x) = \frac{a}{x-4}$, where $a > 0$.

The average rate of change of f from $x = 6$ to $x = 8$ is:

A. $a \log_e(2)$

B. $\frac{a}{2}\log_e(2)$

C. $2a$

D. $-\frac{a}{4}$

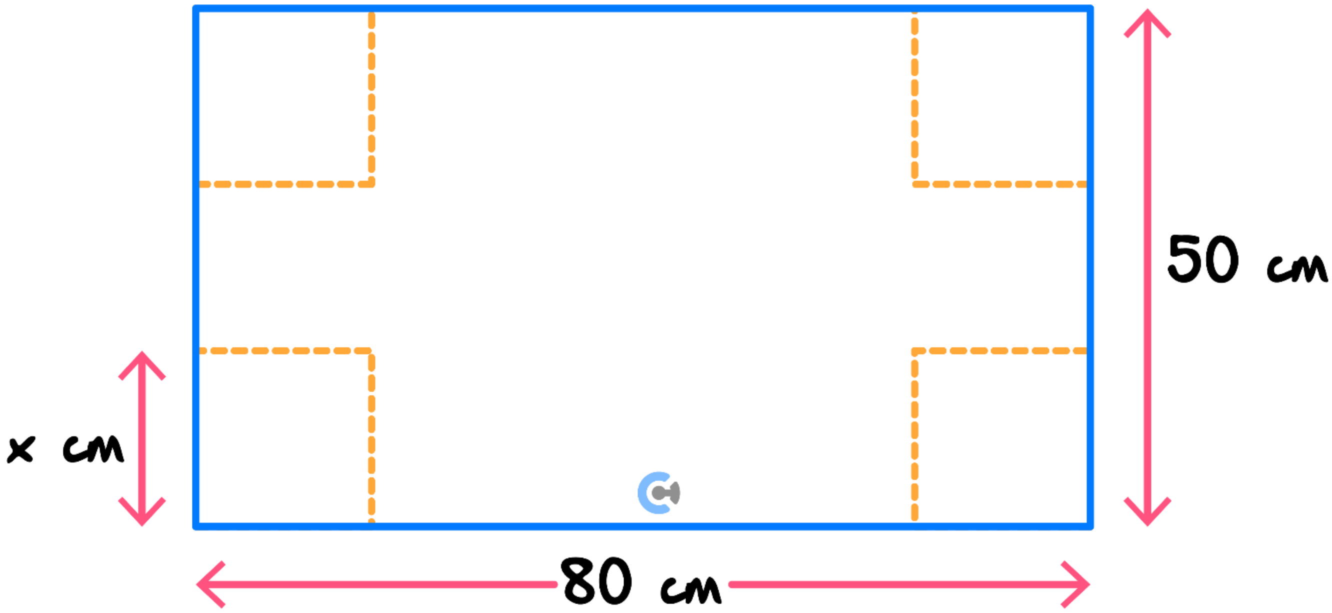
E. $-\frac{a}{8}$

Space for Personal Notes

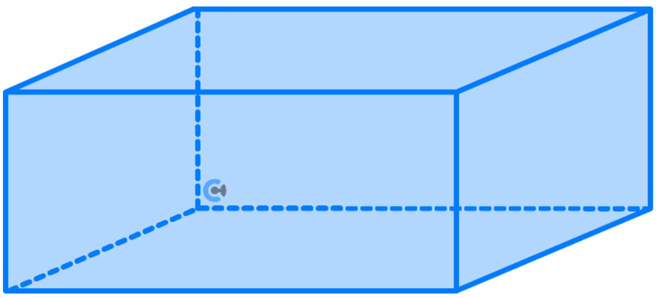
Question 108 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2019
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2019/2019MM2-w.pdf#page=4

A rectangular sheet of cardboard has a length of 80 cm and a width of 50 cm. Squares, of side length x centimetres, are cut from each of the corners, as shown in the diagram below.



A rectangular box with an open top is then constructed, as shown in the diagram below.



The volume of the box is maximum when x is equal to:

A. 10

B. 20

C. 25

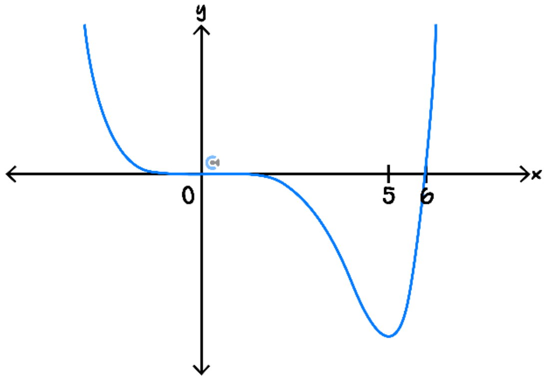
D. $\frac{100}{3}$

E. $\frac{200}{3}$

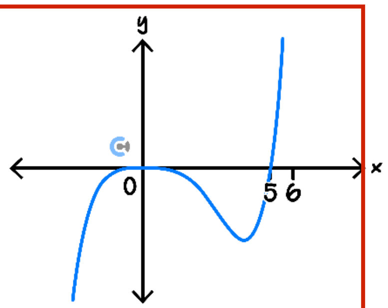
Question 109 (1 mark)

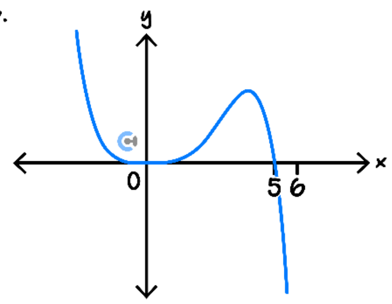
Inspired from VCAA Mathematical Methods ¾ Exam 2019
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2019/2019MM2-w.pdf#page=8

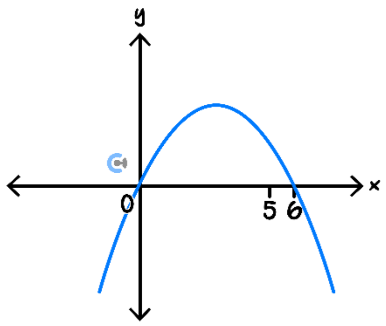
Part of the graph of $y = f(x)$ is shown below.

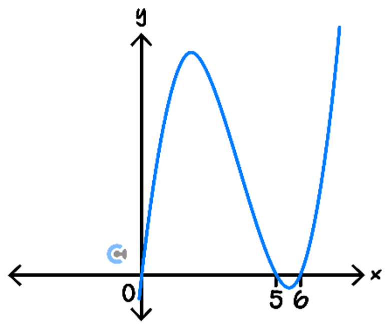


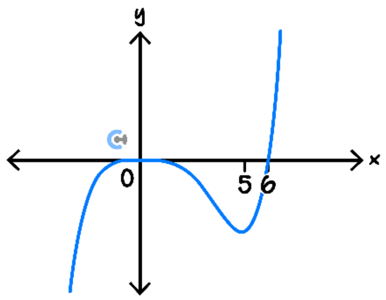
The corresponding part of the graph of $y = f'(x)$ is best represented by:

A.


B.


C.


D.


E.


Question 110 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2020
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2020/2020MM2-w.pdf#page=5

If $f(x) = e^{g(x^2)}$, where g is a differentiable function, then $f'(x)$ is equal to:

A. $2xe^{g(x^2)}$

B. $2xg(x^2)e^{g(x^2)}$

C. $2xg'(x^2)e^{g(x^2)}$

D. $2xg'(2x)e^{g(x^2)}$

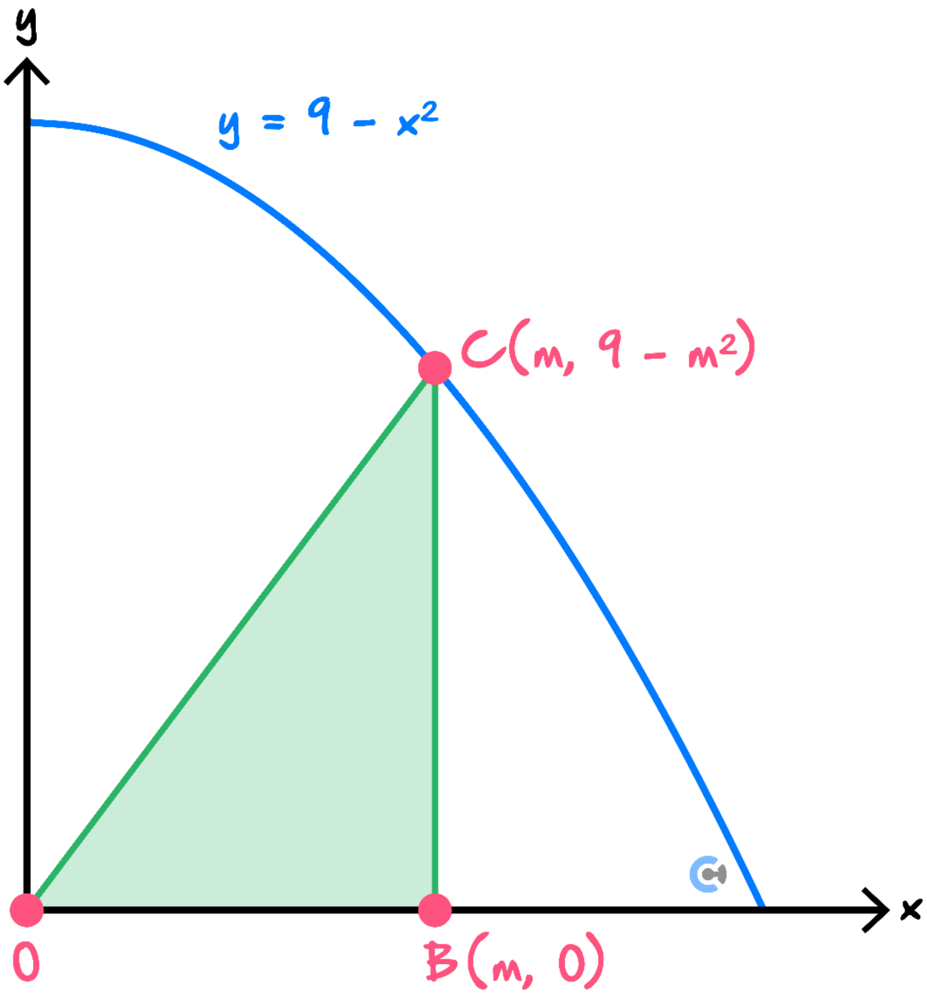
E. $2xg'(x^2)e^{g(2x)}$

Space for Personal Notes

Question 111 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2020
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2020/2020MM2-w.pdf#page=9

A right-angled triangle, OBC, is formed using the horizontal axis and the point $C(m, 9 - m^2)$, where $m \in (0, 3)$, on the parabola, $y = 9 - x^2$, as shown below.



The maximum area of the triangle OBC is:

A. $\frac{\sqrt{3}}{3}$

B. $\frac{2\sqrt{3}}{3}$

C. $\sqrt{3}$

D. $3\sqrt{3}$

E. $9\sqrt{3}$

Question 112 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2020
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2020/2020MM2-w.pdf#page=9

Let $f(x) = -\log_e(x + 2)$.

A tangent to the graph of f has a vertical axis intercept at $(0, c)$. The maximum value of c is:

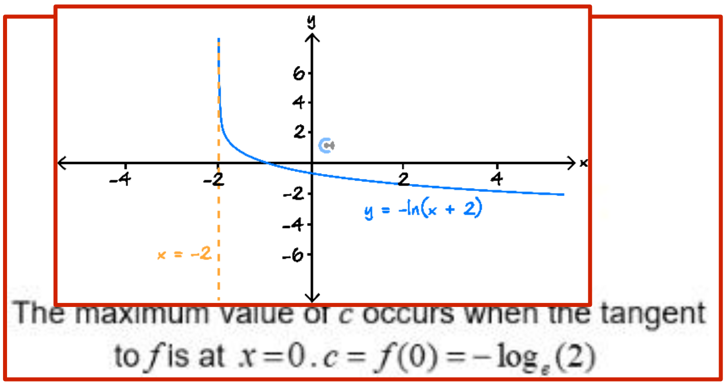
A. -1

B. $-1 + \log_e(2)$

C. $-\log_e(2)$

D. $-1 - \log_e(2)$

E. $\log_e(2)$



Question 113 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2021
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2021/2021MM2-w.pdf#page=3

The maximum value of the function $h: [0,2] \to R, h(x) = (x - 2)e^x$ is:

A. $-e$

B. 0

C. 1

D. 2

E. e

Space for Personal Notes

Question 114 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2021
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2021/2021MM2-w.pdf#page=4

The tangent to the graph of $y = x^3 - ax^2 + 1$ at $x = 1$ passes through the origin.

The value of a is:

A. $\frac{1}{2}$

B. 1

C. $\frac{3}{2}$

D. 2

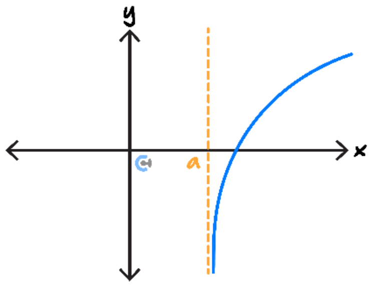
E. $\frac{5}{2}$

Space for Personal Notes

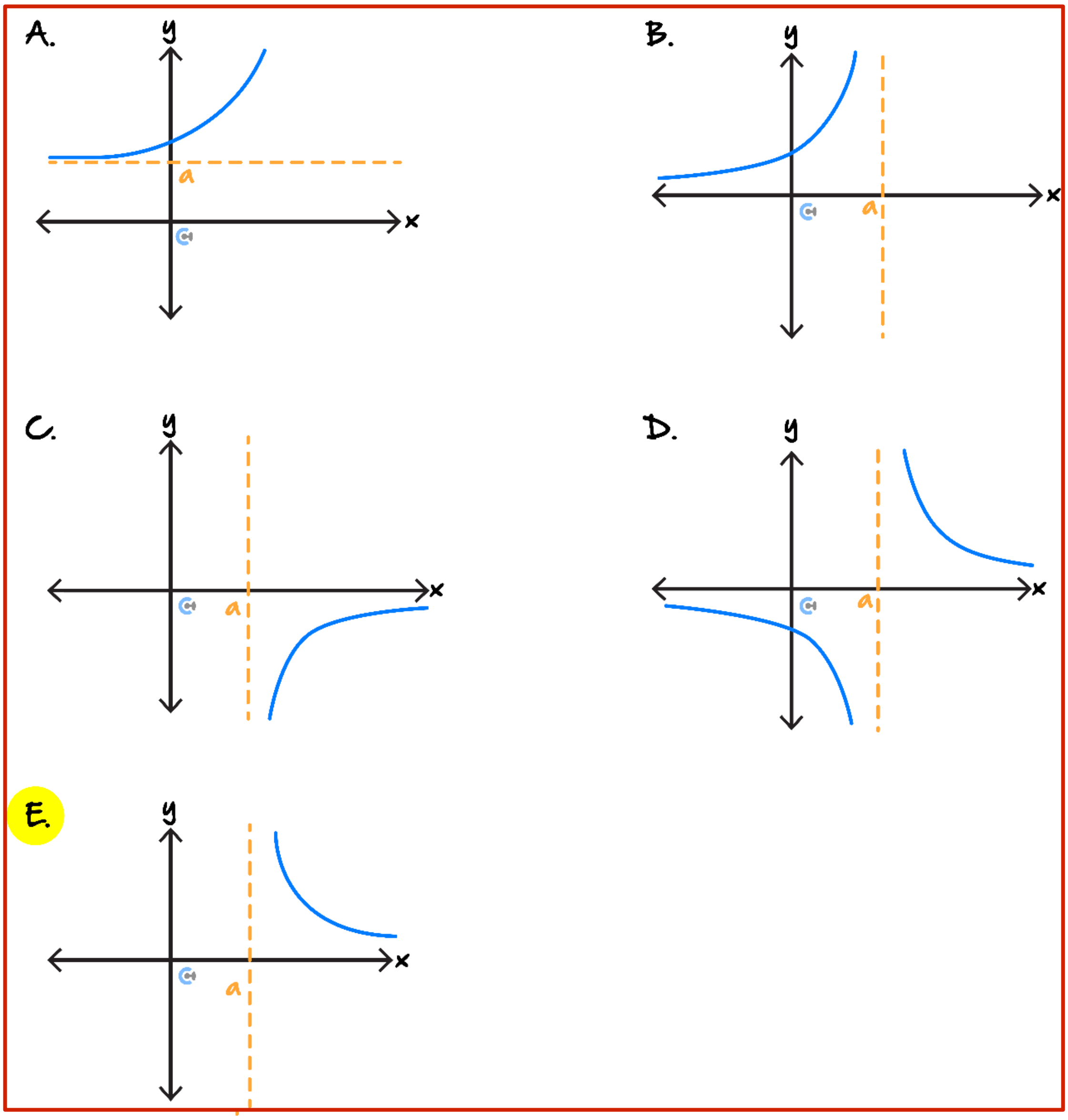
Question 115 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2021
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2021/2021MM2-w.pdf#page=5

The graph of the function f is shown below.



The graph corresponding to f' is:



Question 116 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2021
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2021/2021MM2-w.pdf#page=7

The value of an investment, in dollars, after n months can be modelled by the function:

$$f(n) = 2500 \times (1.004)^n$$

Where, $n \in \{0,1,2,\ldots\}$.

The average rate of change of the value of the investment over the first 12 months is closest to:

A. $10.00 per month

B. $10.20 per month.

C. $10.50 per month.

D. $125.00 per month.

E. $127.00 per month.

Question 117 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2021
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2021/2021MM2-w.pdf#page=7

A value of k for which, the average value of $y = \cos\left(kx - \frac{\pi}{2}\right)$ over the interval $[0, \pi]$ is equal to the average value of $y = \sin(x)$ over the same interval is:

A. $\frac{1}{6}$

B. $\frac{1}{5}$

C. $\frac{1}{4}$

D. $\frac{1}{3}$

E. $\frac{1}{2}$

Space for Personal Notes

Question 118 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2021
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2021/2021MM2-w.pdf#page=9

Which one of the following functions is differentiable for all real values of x?

A. $f(x) = \begin{cases} x & x < 0 \\ -x & x \geq 0 \end{cases}$

B. $f(x) = \begin{cases} x & x < 0 \\ -x & x > 0 \end{cases}$

C. $f(x) = \begin{cases} 8x+4 & x < 0 \\ (2x+1)^2 & x \geq 0 \end{cases}$

D. $f(x) = \begin{cases} 2x+1 & x < 0 \\ (2x+1)^2 & x \geq 0 \end{cases}$

E. $f(x) = \begin{cases} 4x+1 & x < 0 \\ (2x+1)^2 & x \geq 0 \end{cases}$

$f(x) = \begin{cases} 4x+1 & x < 0 \\ (2x+1)^2 & x \geq 0 \end{cases}$

$\lim\limits_{x \to 0^-} (f(x)) = \lim\limits_{x \to 0^+} (f(x)) = 1$

The graph of f is continuous over the interval $(-\infty, \infty)$.

$f'(x) = \begin{cases} 4 & x < 0 \\ 8x+4 & x \geq 0 \end{cases}$

$\lim\limits_{x \to 0^-} (f'(x)) = \lim\limits_{x \to 0^+} (f'(x)) = 4$

The graph of f is smooth at $x = 0$.

The function f where $f(x) = \begin{cases} 4x+1 & x < 0 \\ (2x+1)^2 & x \geq 0 \end{cases}$ is differentiable for all real values of x.

Question 119 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2022
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2022/2022MM2-w.pdf#page=2

The gradient of the graph of $y = e^{3x}$ at the point where the graph crosses the vertical axis is equal to:

A. 0

B. $\frac{1}{e}$

C. 1

D. e

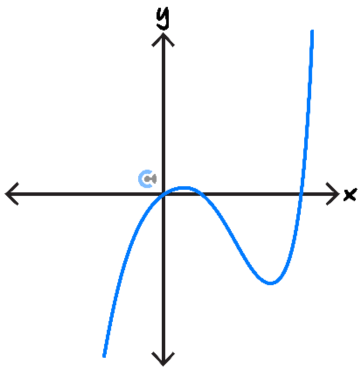
E. 3

Space for Personal Notes

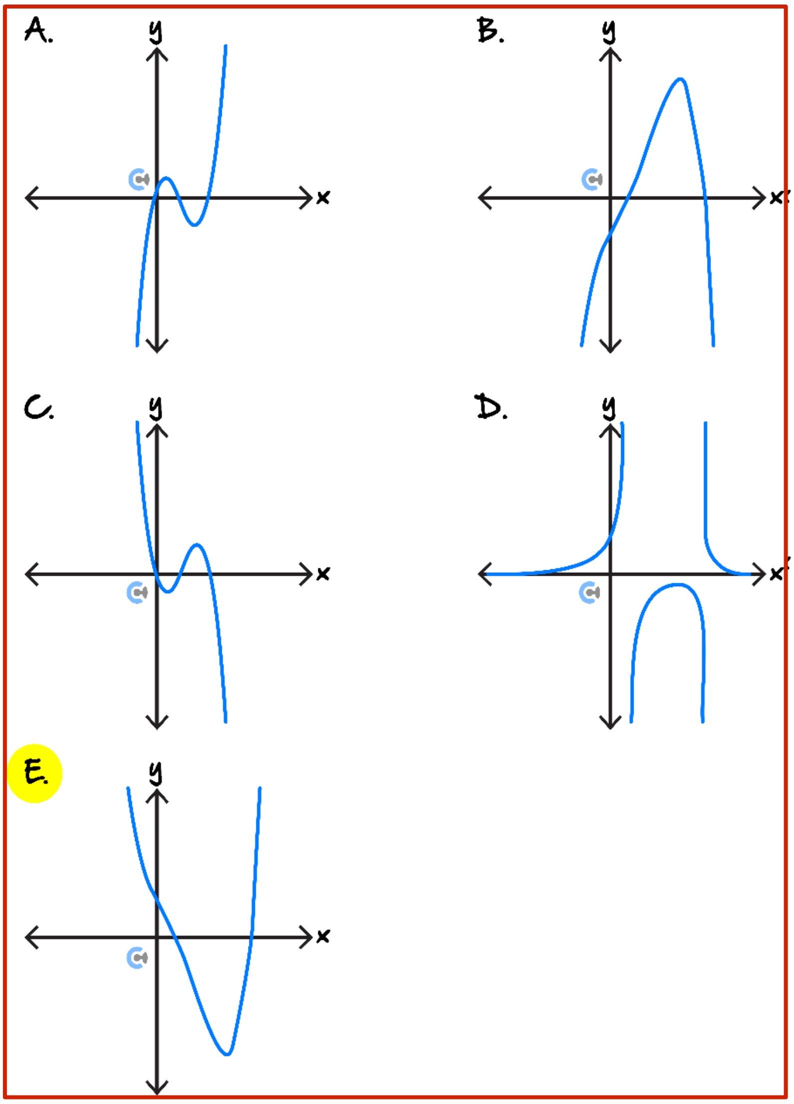
Question 120 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2022
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2022/2022MM2-w.pdf#page=4

The graph of $y = f(x)$ is shown below.



The graph of $y = f'(x)$, the first derivative of $f(x)$ with respect to x, could be:



Question 121 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2022
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2022/2022MM2-w.pdf#page=2

The function $f(x) = \frac{1}{3}x^3 + mx^2 + nx + p$, for $m, n, p \in R$, has turning points at $x = -3$ and $x = 1$ and passes through the point $(3,4)$.

The values of m, n, and p respectively are:

A. $m = 0, \quad n = -\frac{7}{3}, \quad p = 2$

B. $m = 1, \quad n = -3, \quad p = -5$

C. $m = -1, \quad n = -3, \quad p = 13$

D. $m = \frac{5}{4}, \quad n = \frac{3}{2}, \quad p = -\frac{83}{4}$

E. $m = \frac{5}{2}, \quad n = 6, \quad p = -\frac{91}{2}$

Question 122 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2022
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2022/2022MM2-w.pdf#page=8

A function g is continuous on the domain $x \in [a, b]$ and has the following properties:

- The average rate of change of g between $x = a$ and $x = b$ is positive.
- The instantaneous rate of change of g at $x = \frac{a+b}{2}$ is negative.

Therefore, on the interval $x \in [a, b]$, the function must be:

A. Many-to-one.

B. One-to-many.

C. One-to-one.

D. Strictly decreasing.

E. Strictly increasing.

$\frac{g(b) - g(a)}{b - a} > 0, \; g(b) > g(a)$

$g'(x) < 0$ at $x = \frac{a+b}{2}$

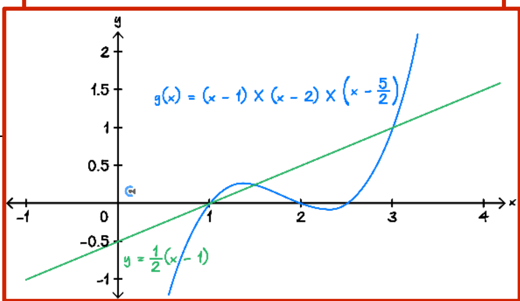
g is a many-to-one function.

An example is shown below using $g(x) = (x-1)(x-2)(x-2.5)$.

Let $a = 1$ and $b = 3$. The average rate of change is 0.5.

The gradient at $x = 2$ is negative.

So, g is a many-to-one function.



Question 123 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2022
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2022/2022MM2-w.pdf#page=8

A box is formed from a rectangular sheet of cardboard, which has a width of a units and a length of b units, by first cutting out squares of side length x units from each corner and then folding upwards to form a container with an open top.

The maximum volume of the box occurs when x is equal to:

A. $\frac{a-b+\sqrt{a^2-ab+b^2}}{6}$

B. $\frac{a+b+\sqrt{a^2-ab+b^2}}{6}$

C. $\frac{a-b-\sqrt{a^2-ab+b^2}}{6}$

D. $\frac{a+b-\sqrt{a^2-ab+b^2}}{6}$

E. $\frac{a+b-\sqrt{a^2-2ab+b^2}}{6}$

$V(x)=x(b-2x)(a-2x)$

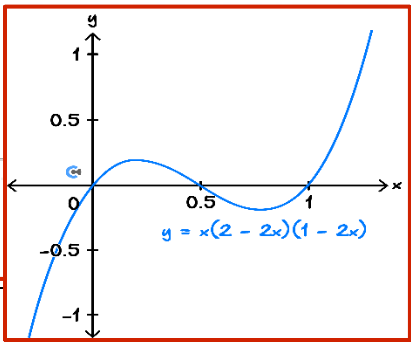
Solve $V'(x)=0$ for x.

$x=\frac{a+b\pm\sqrt{a^2-ab+b^2}}{6}$

$x=\frac{a+b-\sqrt{a^2-ab+b^2}}{6}$

The maximum occurs at the smaller x value.

An example is shown below for a general cubic function using $b=2$ and $a=1$.



Question 124 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2023
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2023/2023MM2-w.pdf#page=3

Which one of the following functions has a horizontal tangent at $(0,0)$?

A. $y=x^{-\frac{1}{3}}$

B. $y=x^{\frac{1}{3}}$

C. $y=x^{\frac{2}{3}}$

D. $y=x^{\frac{4}{3}}$

E. $y=x^{\frac{3}{4}}$

Space for Personal Notes

Question 125 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2023
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2023/2023MM2-w.pdf#page=4

Let $f(x) = \log_e x$, where $x > 0$ and $g(x) = \sqrt{1-x}$, where $x < 1$.

The domain of the derivative of $(f \circ g)(x)$ is:

A. $x \in R$

B. $x \in (-\infty, 1]$

C. $x \in (-\infty, 1)$

D. $x \in (0, \infty)$

E. $x \in (0,1)$

Question 126 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2023
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2023/2023MM2-w.pdf#page=5

The function f is given by:

$$f(x) = \begin{cases} \tan\left(\frac{x}{2}\right) & 4 \leq x < 2\pi \\ \sin(ax) & 2\pi \leq x \leq 8 \end{cases}$$

The value of a for which, f is continuous and smooth at $x = 2\pi$ is:

A. -2

B. $-\frac{\pi}{2}$

C. $-\frac{1}{2}$

D. $\frac{1}{2}$

E. 2

$$f(x) = \begin{cases} \tan\left(\frac{x}{2}\right) & 4 \leq x < 2\pi \\ \sin(ax) & 2\pi \leq x \leq 8 \end{cases}$$

For f to be continuous at $x = 2\pi$,

$$\tan\left(\frac{x}{2}\right) = \sin(ax)$$

For f to be smooth at $x = 2\pi$,

$$\frac{d}{dx}\tan\left(\frac{x}{2}\right) = \frac{d}{dx}\sin(ax)$$

So, solving $\tan\left(\frac{x}{2}\right) = \sin(ax)$ and

$\frac{d}{dx}\tan\left(\frac{x}{2}\right) = \frac{d}{dx}\sin(ax)$ for a,

$$a = -\frac{1}{2}$$

Space for Personal Notes

Question 127 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2023
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2023/2023MM2-w.pdf#page=5

Two functions, f and g are continuous and differentiable for all $x \in R$. It is given that $f(-2) = -7, g(-2) = 8$ and $f'(-2) = 3, g'(-2) = 2$.

The gradient of the graph $y = f(x) \times g(x)$ at the point where $x = -2$ is:

A. -10

B. -6

C. 0

D. 6

E. 10

Space for Personal Notes

Question 128 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2023
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2023/2023MM2-w.pdf#page=6

The following algorithm applies Newton's method for using a For loop with 3 iterations:

Inputs: $f(x), a\ function\ of\ x$
$df(x), the\ derivative\ of\ f(x)$
$x0, an\ initial\ estimate$

Define newton $(f(x), df(x), x0)$
For i **from** 1 **to** 3
If $df(x0) = 0$ **Then**
Return "Error: Division by zero"
Else
$x0 \leftarrow x0 - f(x0) \div df(x0)$
EndFor
Return $x0$

The **Return** value of the function newton $(x^3 + 3x - 3, 3x^2 + 3, 1)$ is closest to:

A. 0.83333

B. 0.81785

C. 0.81773

D. 1

E. 3

Space for Personal Notes

Question 129 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2023
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2023/2023MM2-w.pdf#page=7

A polynomial has the equation $y = x(3x - 1)(x + 3)(x + 1)$.

The number of tangents to this curve that pass through the positive x-intercept is:

A. 0

B. 1

C. 2

D. 3

E. 4

$y = x(3x-1)(x+3)(x+1)$

The positive x-intercept is $\frac{1}{3}$.

Find the tangent line at $x = a$.

$y_T = (12a^3 + 33a^2 + 10a - 3)x - a^2(9a^2 + 22a + 5)$

Solve $y_T\left(\frac{1}{3}\right) = 0$ for a.

$a = \frac{-\sqrt{7}-4}{3}$, $a = \frac{\sqrt{7}-4}{3}$ or $a = \frac{1}{3}$

Hence there are three solutions.

Alternatively, a graphical approach could be taken.

Question 130 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/2024MM2-w.pdf#page=2

A function $g: R \to R$ has the derivative $g'(x) = x^3 - x$.

Given that $g(0) = 5$, the value of $g(2)$ is:

A. 2

B. 3

C. 5

D. 7

Space for Personal Notes

Question 131 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/2024MM2-w.pdf#page=4

Suppose a function $f: [0,5] \to R$ and its derivative $f': [0,5] \to R$ are defined and continuous on their domains. If $f'(2) < 0$ and $f'(4) > 0$, which one of these statements must be true?

A. f is strictly decreasing on $[0,2]$.

B. f does not have an inverse function.

C. f is positive on $[4,5]$.

D. f has a local minimum at $x = 3$.

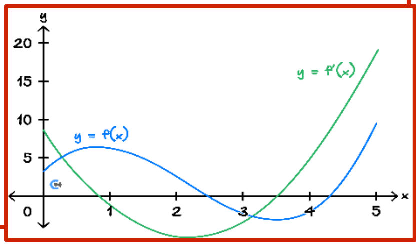
Possible graphs of f and f' are shown below.

f does not have to be strictly decreasing on $[0,2]$.

f does not have to be positive on $[4,5]$.

f does not have to have a local minimum at $x = 3$.

f is many-to-one on $[2,4]$, since f' changes sign. So f does not have an inverse function.



Question 132 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/2024MM2-w.pdf#page=8

The points of inflection of the graph of $y = 2 - \tan\left(\pi\left(x - \frac{1}{4}\right)\right)$ are:

A. $\left(k + \frac{1}{4}, 2\right), k \in Z$

B. $\left(k - \frac{1}{4}, 2\right), k \in Z$

C. $\left(k + \frac{1}{4}, -2\right), k \in Z$

D. $\left(k - \frac{3}{4}, -2\right), k \in Z$

Space for Personal Notes

Question 133 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/2024MM2-w.pdf#page=8

Suppose that a differentiable function $f: R \to R$ and its derivative $f': R \to R$ satisfy $f(4) = 25$ and $f'(4) = 15$.

Determine the gradient of the tangent line to the graph of $y = \sqrt{f(x)}$ at $x = 4$.

A. $\sqrt{15}$

B. $\frac{1}{10}$

C. $\frac{15}{2}$

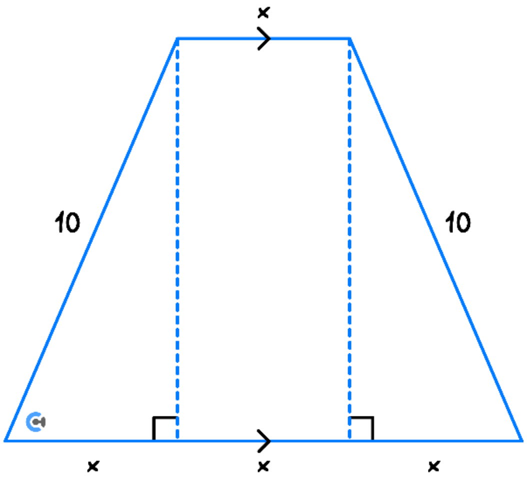
D. $\frac{3}{2}$

$$\frac{dy}{dx} = \frac{1}{2}(f(x))^{-\frac{1}{2}} \times f'(x)$$
$$= \frac{1}{2}(f(4))^{-\frac{1}{2}} \times f'(4)$$
$$= \frac{15}{2 \times \sqrt{25}}$$
$$= \frac{3}{2}$$

Question 134 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/2024MM2-w.pdf#page=10

Find the value of x which maximises the area of the trapezium below.



A. 10

B. $5\sqrt{2}$

C. 7

D. $\sqrt{10}$

Question 135 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2017
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2017/nht/2017MM2-nht-w.pdf#page=2

The gradient of a line perpendicular to the line that passes through $(3, 0)$ and $(0, -6)$ is:

A. $-\frac{1}{2}$

B. -2

C. $\frac{1}{2}$

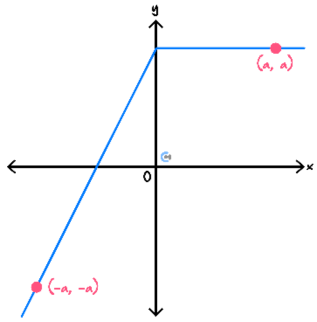
D. 4

E. 2

Question 136 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2017
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2017/nht/2017MM2-nht-w.pdf#page=4

Part of the graph of a function f is shown below.



Which one of the following is the average value of the function f over the interval $[-a, a]$?

A. 0

B. $\frac{3a}{4}$

C. $\frac{3a}{8}$

D. $\frac{a}{2}$

E. $\frac{a}{4}$

Question 137 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2017
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2017/nht/2017MM2-nht-w.pdf#page=5

The tangent to the graph of $y = 3\sin(2x) - 1$ is parallel to the line with equation $y = 3x + 1$ at the points where x is equal to:

A. $n\pi \pm \frac{\pi}{6}, n \in Z$

B. $-\frac{\pi}{3}, \frac{\pi}{3}$ only.

C. $\frac{\pi}{6}, \frac{5\pi}{6}$ only.

D. $n\pi \pm \frac{\pi}{3}, n \in Z$

E. $nx, n \in Z$

Question 138 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2017
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2017/nht/2017MM2-nht-w.pdf#page=2

Let $f(x) = x^m e^{ax}$, where a and m are non-zero real constants. If $(x + 2)$ is a factor of $f'(x)$, then which one of the following must be true?

A. $m = 2$

B. $m = -2$

C. $m = 2 - a$

D. $m = 2a$

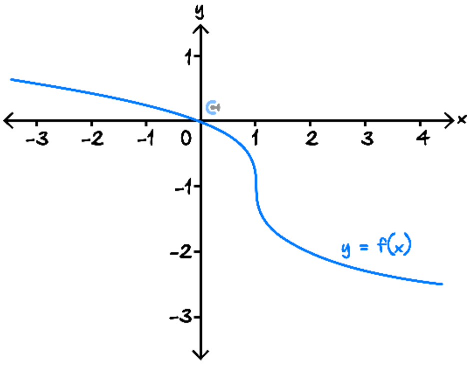
E. $m = -2a$

Space for Personal Notes

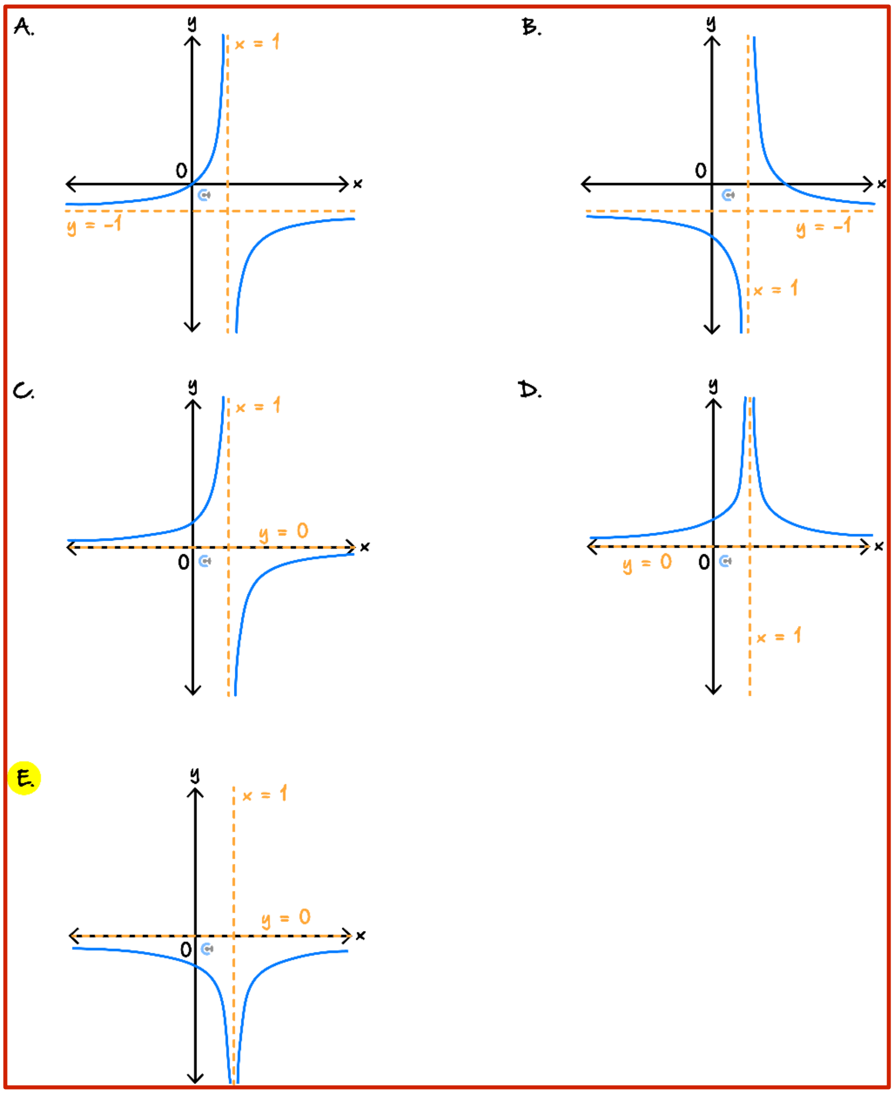
Question 139 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2018
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2018/nht/2018MM2-nht-w.pdf#page=5

Part of the graph of $y = f(x)$ is shown below.



The graph of $y = f'(x)$ is best represented by:



Question 140 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2018
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2018/nht/2018MM2-nht-w.pdf#page=20

Let f be a one-to-one differentiable function such that, $f(3) = 7, f(7) = 8, f'(3) = 2$ and $f'(7) = 3$.

The function g is differentiable and $g(x) = f^{-1}(x)$ for all x.

$g'(7)$ is equal to:

A. $\frac{1}{2}$

B. 2

C. $\frac{1}{6}$

D. $\frac{1}{8}$

E. $\frac{1}{3}$

$f(3) = 7,\ f'(3) = 2,\ g(x) = f^{-1}(x),\ g'(7) = \frac{1}{2}$ since

$f'(x) \times f'(y) = 1, g(x) = f'(y) = \frac{1}{f'(x)}$

Question 141 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2019
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2019/NHT/2019MM2-nht-w.pdf#page=6

The graph of $f(x) = x^3 - 6(b - 2)x^2 + 18x + 6$ has exactly two stationary points for:

A. $1 < b < 2$

B. $b = 1$

C. $b = \frac{4 \pm \sqrt{6}}{2}$

D. $\frac{4 - \sqrt{6}}{2} \leq b \leq \frac{4 + \sqrt{6}}{2}$

E. $b < \frac{4 - \sqrt{6}}{2}$ or $b > \frac{4 + \sqrt{6}}{2}$

Space for Personal Notes

Question 142 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2019
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2019/NHT/2019MM2-nht-w.pdf#page=10

Let $f(x) = (ax + b)^5$ and let g be the inverse function of f.

Given that $f(0) = 1$, what is the value of $g'(1)$?

A. $\frac{5}{a}$

B. 1

$$g(x)=f^{-1}(x)=\frac{x^{\frac{1}{5}}-b}{a},\ g'(x)=\frac{x^{-\frac{4}{5}}}{5a},\ g'(1)=\frac{1}{5a}$$

C. $\frac{1}{5a}$

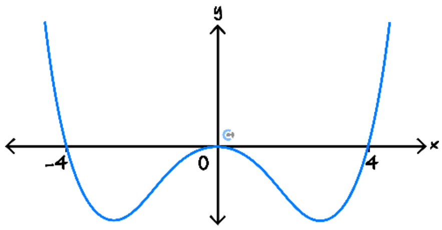
D. $5a(a+1)^4$

E. 0

Question 143 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2021
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2021/NHT/2021MM2-nht-w.pdf#page=5

Part of the graph of a polynomial function f is shown below. This graph has turning points at $(-2\sqrt{2}, -1)$, and $(2\sqrt{2}, -1)$.



$f(x)$ is strictly decreasing for:

A. $x \in (-\infty, -4] \cup [4, \infty)$

Solution Pending

B. $x \in [-4, 4]$

C. $x \in [-2\sqrt{2}, 2\sqrt{2}]$

D. $x \in (-\infty, -2\sqrt{2}] \cup [0, 2\sqrt{2}]$

E. $x \in [-2\sqrt{2}, 0] \cup [2\sqrt{2}, \infty)$

Question 144 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2021
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2021/NHT/2021MM2-nht-w.pdf#page=5

Consider the graph of $f : R \to R, f(x) = -x^2 - 4x + 5$.

The tangent to the graph of f is parallel to the line connecting the negative x-intercept and the y-intercept of f when x is equal to:

A. -3

B. $-\frac{5}{2}$

C. $-\frac{3}{2}$

D. -1

E. $-\frac{1}{2}$

Question 145 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2022
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2022/NHT/2022mm2-nht-w.pdf#page=6

Let $f(x) = g(x) \cdot \sqrt{1 - x^2}$, where g is a function that is continuous and differentiable for all $x \in R$. The gradient of the tangent to the graph of f at the point where f crosses the vertical axis is equal to:

A. 0

B. 1

C. $g(0)$

D. $g'(0)$

E. $g'(0) - g(0)$

Space for Personal Notes

Question 146 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2022
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2022/NHT/2022mm2-nht-w.pdf#page=8

At the point where $x = k$, the tangent to the circle given by the equation $x^2 + (y - 1)^2 = 1$ meets the positive direction of the x-axis at an angle of 135°.

The value of k could be:

A. $-\sqrt{3}$

B. -1

C. $-\frac{1}{\sqrt{2}}$

D. $-\frac{1}{\sqrt{3}}$

E. 0

$x^2+(y-1)^2=1$, $y=-\sqrt{1-x^2}+1$, $\tan(135°)=-1$, solve $\frac{d}{dx}\left(-\sqrt{1-x^2}+1\right)=-1$ at $x=k$, $k=-\frac{\sqrt{2}}{2}$

Question 147 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2023
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2023/NHT/2023MM2-nht-w.pdf#page=2

Let $f(t) = -\frac{1}{3}t^3 + \frac{1}{2}t^2 + 50t$.

The instantaneous rate of change of f when $t = 1$ is:

A. 247.5

B. 50.2

C. 50.0

D. −13.8

E. −22.0

Space for Personal Notes

Question 148 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2023
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2023/NHT/2023MM2-nht-w.pdf#page=2

If $u = g(x)$ and $v = e^{g(2x)}$, where g is a differentiable function, then $\frac{d}{dx}(uv)$ is equal to:

A. $3g(x)e^{g(2x)}$

B. $e^{g(2x)}\left(2g(x) + g'(x)\right)$

C. $e^{g(2x)}\left(g(x)g'(2x) + g'(x)\right)$

D. $e^{g(2x)}\left(2g(x)g'(2x) + g'(x)\right)$

E. $2g(x)g'(2x)e^{g'(2x)} + e^{g(2x)}g'(x)$

$u = g(x)$, $v = e^{g(2x)}$

$\frac{d}{dx}(uv)$

$= \frac{d}{dx}\left[g(x)e^{g(2x)}\right]$

$= g'(x)e^{g(2x)} + g(x) \times 2g'(2x)e^{g(2x)}$ using the product and chain rules

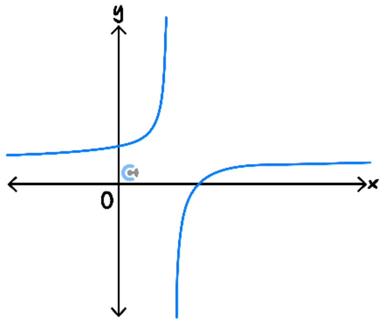
$= e^{g(2x)}\left(2g(x)g'(2x) + g'(x)\right)$

Space for Personal Notes

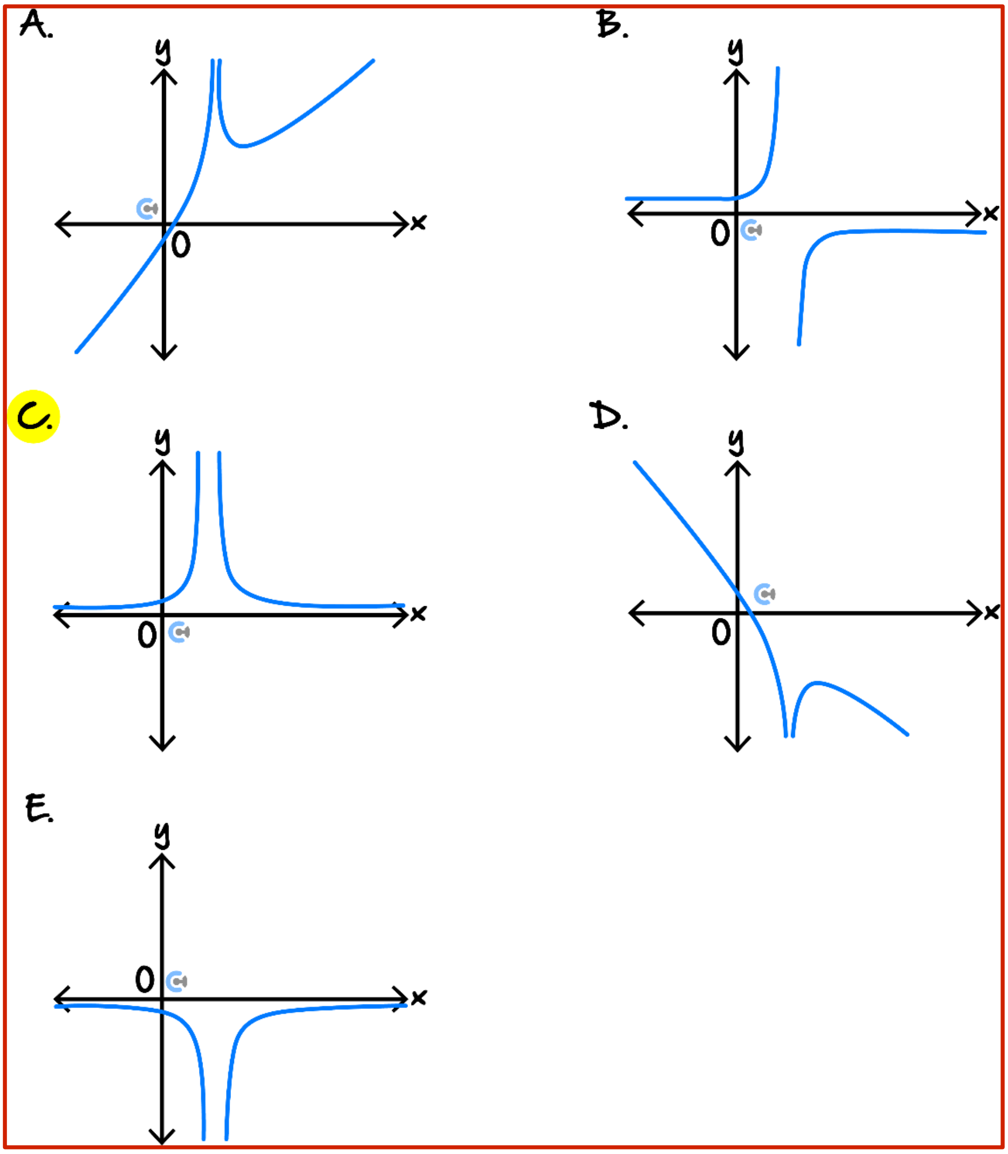
Question 149 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2023
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2023/NHT/2023MM2-nht-w.pdf#page=5

The graph of $y = f(x)$ is shown below.



The graph of $y = f'(x)$ is best represented by:



Question 150 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2023
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2023/NHT/2023MM2-nht-w.pdf#page=6

A tangent line to the graph of $y = \log_e(2x) + \log_e(x - 2)$ passes through the origin. The x-coordinate, correct to two decimal places, where the tangent line touches the graph is closest to:

A. -0.66

B. 1.25

C. 2.91

D. 4.19

E. 6.33

Question 151 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2023
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2023/NHT/2023MM2-nht-w.pdf#page=7

The graph of $y = \log_e(x - k)$, for $k \in R$, has a tangent with a maximum horizontal axis intercept of:

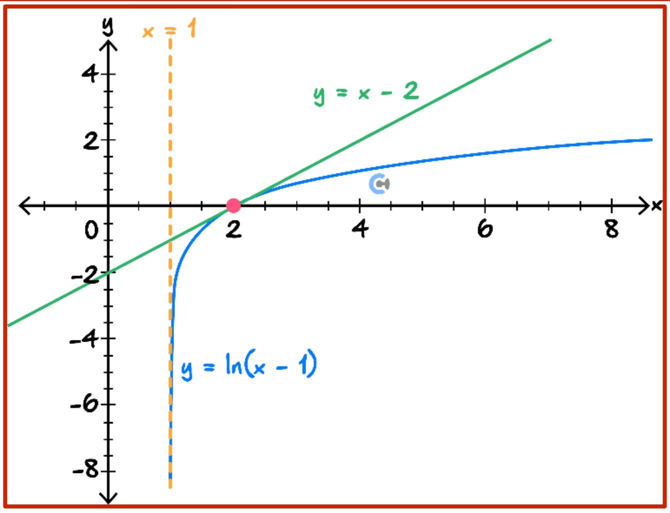
A. $x = 1$

B. $x = k$

C. $x = e$

D. $x = 1 + k$

E. $x = e + k$



The graph of $y = \log_e(x - k)$ has a tangent with a maximum horizontal intercept when $\log_e(x - k) = 0$.

$x - k = 1$

$x = 1 + k$

An example is shown above for $k = 1$.

Space for Personal Notes

Question 152 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/NHT/2024MM2-nht-w.pdf#page=3

A straight line passes through the positive x-intercept of the curve of the cubic $y = x^3 - x^2 - 2x$ and also through its point of inflection.

The gradient of this line is:

A. $\frac{4}{9}$

B. $\frac{2}{3}$

C. $\frac{1}{2}$

D. $-\frac{15}{7}$

E. $-\frac{20}{9}$

Question 153 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/NHT/2024MM2-nht-w.pdf#page=7

Newton's method is used to estimate the x-intercept of the function:

$$f: [0, \infty) \to R, f(x) = \log_e(2x + 1) - \left(4 - x^{\frac{5}{2}}\right)$$

With an initial estimate of $x_0 = 0$, the estimate for x_3 is closest to:

A. 1.4717

B. 1.4718

C. 1.4752

D. 1.5628

E. 2.0000

Question 154 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/NHT/2024MM2-nht-w.pdf#page=9

The finance team at a small technology company estimates that the production cost per item is given by the rule $C(n) = 0.5n^2 - 37.5n + 1900 + \frac{1250}{n}$, where $n \in Z^+$ and n is the number of items produced.

The minimum cost per item is closest to:

A. \$38.34

B. \$38.35

C. \$1229.83

D. \$1229.89

E. \$1230.05

$C(n) = 0.5n^2 - 37.5n + 1900 + \frac{1250}{n}$, $n \in Z^+$

Solve $C'(n) = 0$, $n = 38.34...$, but n is discrete.

$C(38) = 1229.89...$, $C(39) = 1230.05...$

The minimum cost is closest to \$1229.89.

Question 155 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/NHT/2024MM2-nht-w.pdf#page=9

The values for two continuous functions, f and g, and their derivatives are given in the tables below.

	$x = 0$	$x = 2$
$f(x)$	2	-1
$f'(x)$	-1	2

	$x = 0$	$x = 2$
$g(x)$	1	-1
$g'(x)$	0	3

What is the value of $\frac{d}{dx}\left((g \circ f)(x)\right)$ at $x = 0$?

A. -3

B. -1

C. 0

D. 1

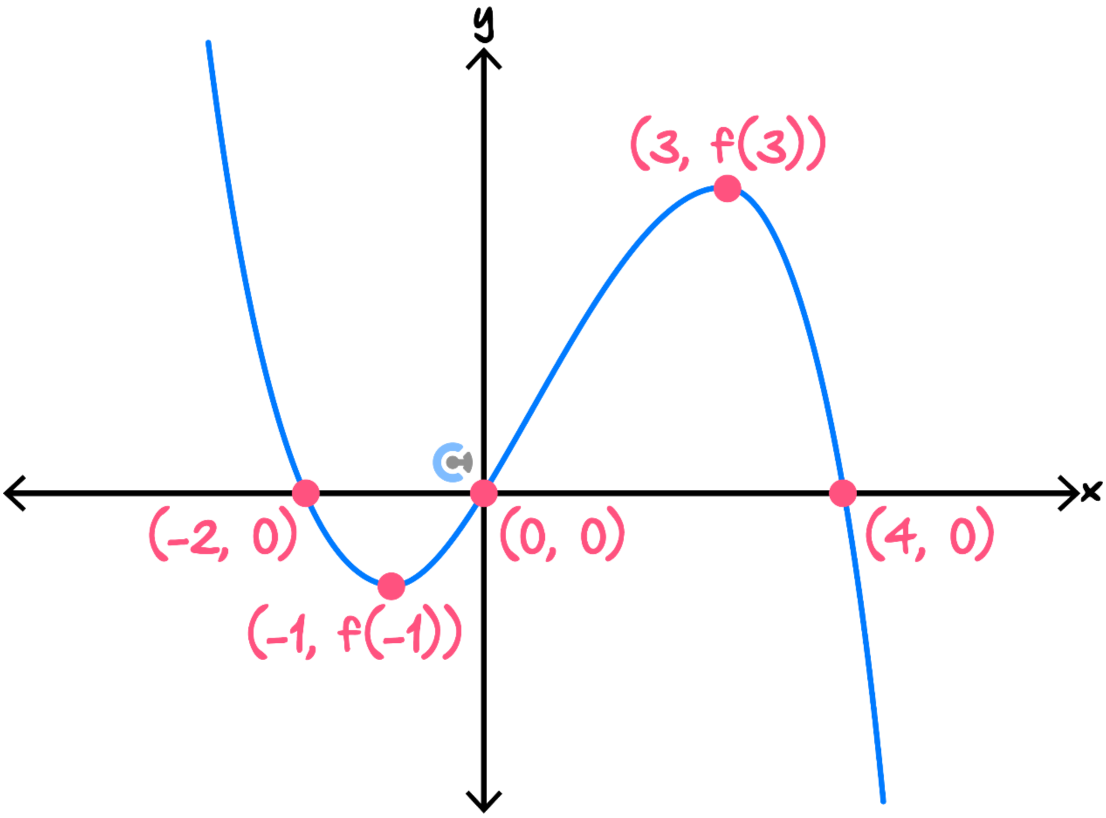
E. 2

$$\frac{d}{dx} g(f(x))$$
$$= g'(f(x)) \times f'(x)$$
$$= g'(f(0)) \times f'(0)$$
$$= g'(2) \times f'(0)$$
$$= 3 \times -1$$
$$= -3$$

Question 156 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/NHT/2024MM2-nht-w.pdf#page=10

Let $f : R \to R$ be a continuous and differentiable function. Part of the graph of f is shown below. The stationary points of f are at $(-1, f(-1))$ and $(3, f(3))$.



The solution to the inequality $(x^2 - x - 2)\, f'(x) > 0$ is:

A. $-1 < x < 2$

B. $-1 < x < 3$

C. $2 < x < 3$

D. $x < -1$ or $x > 2$.

E. $x < -1$ or $x > 3$.

$(x^2 - x - 2) f'(x) > 0$

$f'(x) > 0$ when $\{x : -1 < x < 3\}$, $x^2 - x - 2 > 0$ when $\{x : x < -1\} \cup \{x : x > 2\}$

So $(x^2 - x - 2) f'(x) > 0$, when $\{x : 2 < x < 3\}$.

$f'(x) < 0$ when $\{x : x < -1\} \cup \{x : x > 3\}$, $x^2 - x - 2 < 0$ when $\{x : -1 < x < 2\}$

So $(x^2 - x - 2) f'(x) > 0$, when $\{x : 2 < x < 3\}$ only.

Space for Personal Notes

Question 157 (11 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2016
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2016/2016MM2-w.pdf

Let $f: [0,8\pi] \to R, f(x) = 2\cos\left(\frac{x}{2}\right) + \pi$.

a. Find the period and range of f. (2 marks)

Marks	0	1	2	Average
%	10	33	57	1.5

$f:[0,8\pi] \to R, f(x) = 2\cos\left(\frac{x}{2}\right) + \pi$, Period = $\frac{2\pi}{\frac{1}{2}} = 4\pi$, Range = $[-2+\pi, 2+\pi]$

This question was answered well. However, some students included round brackets instead of square brackets for the range. Range = $[2+\pi, -2+\pi]$ was occasionally seen. Some students gave approximate answers instead of exact answers.

b. State the rule for the derivative function f'. (1 mark)

Marks	0	1	Average
%	13	87	0.9

$f'(x) = -\sin\left(\frac{x}{2}\right)$

This question was answered well. Some students did not write an equation, leaving their answer as $-\sin\left(\frac{x}{2}\right)$. Others made errors when using the chain rule. Some had their technology in degree mode rather than radian mode.

c. Find the equation of the tangent to the graph of f at $x = \pi$. (1 mark)

Marks	0	1	Average
%	33	67	0.7

$f'(\pi) = -1, f(\pi) = \pi, y - \pi = -(x - \pi), y = -x + 2\pi$

This question was answered well. Students were not required to show any working. The answer could be obtained directly using technology. Some left their answer as $-x + 2\pi$.

d. Find the equations of the tangents to the graph of $f: [0, 8\pi] \to R, f(x) = 2\cos\left(\frac{x}{2}\right) + \pi$ that have a gradient of 1. (2 marks)

Marks	0	1	2	Average
%	48	5	47	1.0

$f'(x) = 1, x = 3\pi$ or $x = 7\pi$, $f(3\pi) = \pi, f(7\pi) = \pi, y - \pi = 1(x - 3\pi), y - \pi = 1(x - 7\pi), y = x - 2\pi, y = x - 6\pi$

Once students found $x = 3\pi$ or $x = 7\pi$ the rest of the question could be completed using technology. Some students gave only one of the equations of the tangents.

e. The rule of f' can be obtained from the rule of f under a transformation T, such that:

$$T: R^2 \to R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & a \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -\pi \\ b \end{bmatrix}$$

Find the value of a and the value of b. (3 marks)

Marks	0	1	2	3	Average
%	48	34	9	10	0.8

$T: R^2 \to R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & a \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -\pi \\ b \end{bmatrix}, x' = x - \pi, x = x' + \pi, y' = ay + b, y = \frac{y' - b}{a}$

$y = 2\cos\left(\frac{x}{2}\right) + \pi, \frac{y' - b}{a} = 2\cos\left(\frac{x' + \pi}{2}\right) + \pi, y' = 2a\cos\left(\frac{x' + \pi}{2}\right) + a\pi + b, y' = -2a\sin\left(\frac{x'}{2}\right) + a\pi + b,$

$-2a = -1, a = \frac{1}{2}, a\pi + b = 0, b = -\frac{\pi}{2}$

This question was not answered well. Many students were able to take the equations out of the matrices and rearrange them. Some attempted to draw the graphs but were unable to describe the transformations.

f. Find the values of $x, 0 \leq x \leq 8\pi$, such that $f(x) = 2f'(x) + \pi$. (2 marks)

Marks	0	1	2	Average
%	49	4	47	1.0

Solve $f(x) = 2f'(x) + \pi$ for x, $2\cos\left(\frac{x}{2}\right) + \pi = -2\sin\left(\frac{x}{2}\right) + \pi, \tan\left(\frac{x}{2}\right) = -1, \frac{x}{2} = \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}, \frac{15\pi}{4}$

$x = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \frac{15\pi}{2}$

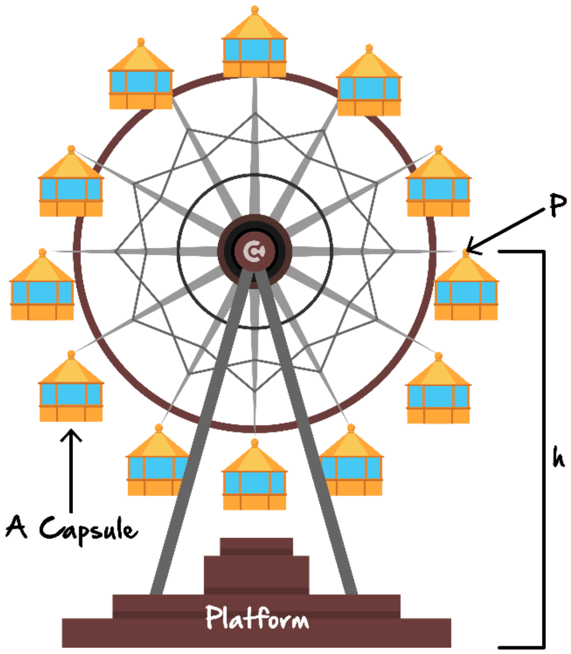
Some students gave $x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}$ as the answer. Others tried solving $2f'(x) + \pi = 0$ instead of $2f'(x) + \pi = f(x)$.

Space for Personal Notes

Question 158 (12 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2017
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2017/2017MM2-w.pdf

Sammy visits a giant Ferris wheel. Sammy enters a capsule on the Ferris wheel from a platform above the ground. The Ferris wheel is rotating anti-clockwise. The capsule is attached to the Ferris wheel at point P. The height of P above the ground, h, is modelled by $h(t) = 65 - 55\cos\left(\frac{\pi t}{15}\right)$ where t is the time in minutes after Sammy enters the capsule and h is measured in metres. Sammy exits the capsule after one complete rotation of the Ferris wheel.



a. State the minimum and maximum heights of P above the ground. (1 mark)

Marks	0	1	Average
%	11	89	0.9

$h(t) = 65 - 55\cos\left(\frac{\pi t}{15}\right)$, range $[-55+65, 55+65] = [10, 120]$, minimum height is 10 m and maximum height is 120 m

This question was well answered.

b. For how much time, is Sammy in the capsule? (1

Marks	0	1	Average
%	16	84	0.9

Period $= \frac{2\pi}{\left(\frac{\pi}{15}\right)} = 30$. He was in the capsule for 30 minutes.

A common incorrect answer was 15 minutes.

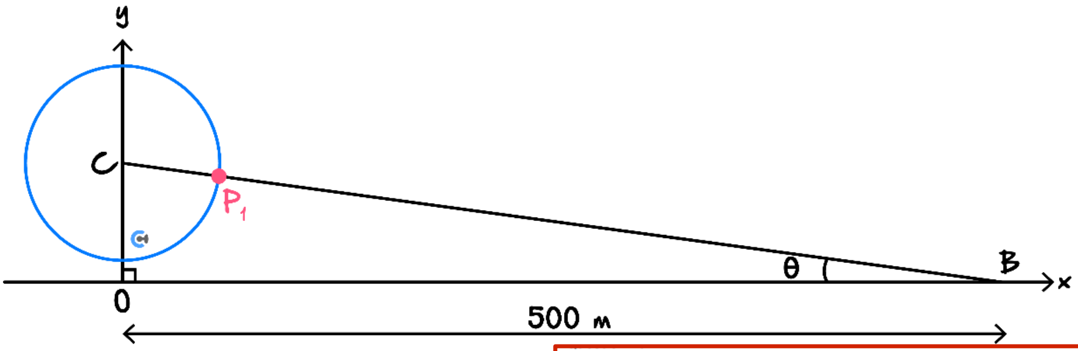
c. Find the rate of change of h with respect to t and, hence, state the value of t at which the rate of change of h is at its maximum. (2 marks)

Marks	0	1	2	Average
%	29	42	29	1

$h'(t) = \frac{11\pi}{3}\sin\left(\frac{\pi t}{15}\right)$, solve $h'(t) = \frac{11\pi}{3}$ for t, $t = 7.5$ minutes

Most students were able to find the derivative. There were occasions when $\frac{y_2 - y_1}{x_2 - x_1}$ was attempted (average rate of change). Some students had their technology in degree instead of radian mode, giving $h'(t) = \frac{11\pi^2 \sin\left(\frac{\pi t}{15}\right)}{540}$. Many could not find the maximum rate of change. A common incorrect answer was 15 minutes. Many found the value of t for the maximum value of h. Others gave a general solution or two t values.

As the Ferris wheel rotates, a stationary boat at B, on a nearby river, first becomes visible at point P_1. B is 500 m horizontally from the vertical axis through the centre C of the Ferris wheel and angle $CBO = \theta$, as shown below.



d. Find θ in degrees, correct to two decimal places. (1 mark)

Marks	0	1	Average
%	64	36	0.4

$\tan(\theta) = \frac{65}{500}$, $\theta = 7.41°$, correct to two decimal places

Many students knew to get $\tan^{-1}\left(\frac{65}{500}\right)$ but they did not specify 'degree' for their technology. A common incorrect answer was $\theta = \tan^{-1}\left(\frac{55}{500}\right) = 6.28°$. Some used $\theta = \tan\left(\frac{65}{500}\right)$ instead of $\theta = \tan^{-1}\left(\frac{65}{500}\right)$. Others used $\theta = \sin^{-1}\left(\frac{65}{500}\right)$.

Students should be familiar with the relevant functionality for the context and select it appropriately.

Part of the path of P is given by, $y = \sqrt{3025 - x^2} + 65, x \in [-55, 55]$, where x and y are in metres.

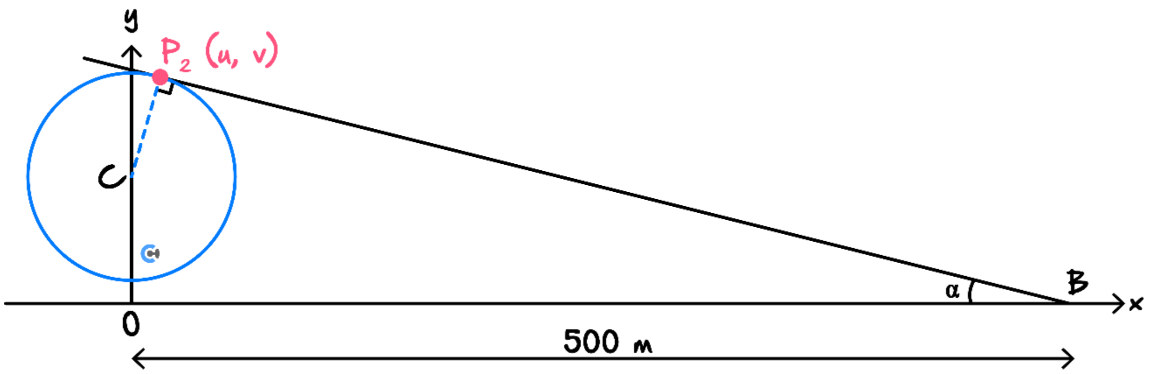
e. Find $\frac{dy}{dx}$. (1 mark)

Marks	0	1	Average
%	10	91	0.9

$\frac{dy}{dx} = \frac{-x}{\sqrt{3025 - x^2}}$

This question was answered well. Some students wrote $\frac{dy}{dx} = \frac{x}{\sqrt{3025 - x^2}}$. There was no need to rationalise the denominator.

As the Ferris wheel continues to rotate, the boat at B is no longer visible from the point $P_2(u, v)$ onwards. The line through B and P_2 is tangent to the path of P, where angle $OBP_2 = \alpha$.



f. Find the gradient of the line segment P_2B in terms of u and, hence, find the coordinates of P_2, correct to two decimal places. (3 marks)

Marks	0	1	2	3	Average
%	67	21	4	8	0.5

$m_{P_2B} = \frac{-u}{\sqrt{3025-u^2}}$ or $\frac{-\sqrt{3025-u^2}-65}{500-u}$, solve $\frac{-u}{\sqrt{3025-u^2}} = \frac{-\sqrt{3025-u^2}-65}{500-u}$ for u

$u = 12.9975... = 13.00$, $v = 118.4421... = 118.44$, correct to two decimal places

Many students were able to find the gradient of the line segment in terms of u, using their answer from Question 2e. Others used $h(t)$ or $y = \sqrt{3025-x^2}$ instead of $y = \sqrt{3025-x^2} + 65$. Many students were unable to find the second gradient expression where they were required to use rise over run for the line segment P_2B.

g. Find α in degrees, correct to two decimal places. (1 mark)

Marks	0	1	Average
%	93	7	0.1

$\alpha = \tan^{-1}\left(\frac{12.9975...}{\sqrt{3025-(12.9975...)^2}}\right) = 13.67°$, correct to two decimal places

Some students used radians instead of degrees.

h. Hence or otherwise, find the length of time, to the nearest minute, during which the boat at B is visible. (2 marks)

Marks	0	1	2	Average
%	94	5	2	0.1

Angle difference = $\theta = 90 - (13.669... - 7.406...) = 83.737...°$, $\frac{83.737...}{360} \times 30 = 6.978... = 7$ minutes, to the nearest minute

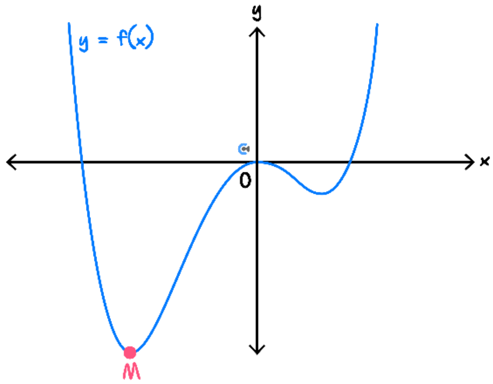
This question was not answered well. Some students wrote 7 minutes without showing any working. As indicated in the instructions on the examination, for questions worth more than one mark, appropriate working must be shown.

Space for Personal Notes

Question 159 (13 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2018
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2018/2018MM2-w.pdf

Consider the quartic $f: R \to R, f(x) = 3x^4 + 4x^3 - 12x^2$ and part of the graph of $y = f(x)$ below.



a. Find the coordinates of the point M, at whic

Marks	0	1	Average
%	5	95	1

$f: R \to R, f(x) = 3x^4 + 4x^3 - 12x^2$, $(-2, -32)$

This question was answered well. Some students only gave the x value when coordinates were required.

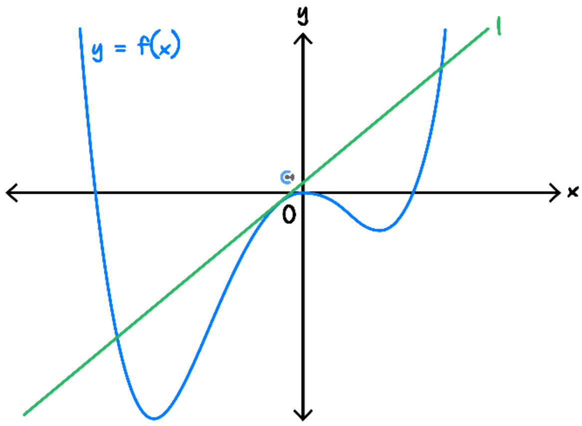
b. State the values of $b \in R$ for which the graph of $y = f(x) + b$ has no x-intercepts. (1 mark)

Marks	0	1	Average
%	35	65	0.7

$b > 32$

This question was answered well. Common incorrect answers were $(-\infty, 32)$, $b = 32$, $b \geq 32$, $[33, \infty)$ and $b = 33$. Others used the x-coordinate and gave $x > 2$ as their answer.

Part of the tangent, l, to $y = f(x)$ at $x = -\frac{1}{3}$ is shown below.



c. Find the equation of the tangent l. (1 mark)

Marks	0	1	Average
%	27	73	0.8

$l(x) = \frac{80}{9}x + \frac{41}{27}$

An equation and exact values were required.

d. The tangent l intersects $y = f(x)$ at $x = -\frac{1}{3}$ and at two other points.

State the x-values of the two other points of intersection. Express your answers in the form $\frac{a \pm \sqrt{b}}{c}$, where a, b and c are integers. (2 marks)

Marks	0	1	2	Average
%	20	13	67	1.5

Solve $l(x) = f(x)$ for x, $x = \frac{-1 \pm \sqrt{42}}{3}$

Exact values were required. There were many sign errors, for example $x = \frac{1 \pm \sqrt{42}}{3}$. Some students found the values of x where the gradient of l was equal to the gradient of f.

e. Find the total area of the regions bounded by the tangent l and $y = f(x)$. Express your answer in the form $\frac{a\sqrt{b}}{c}$, where a, b and c are positive integers. (2 marks)

Marks	0	1	2	Average
%	38	13	49	1.1

Area $= \int_{\frac{-1-\sqrt{42}}{3}}^{\frac{-1+\sqrt{42}}{3}} (l(x) - f(x))dx = \frac{784\sqrt{42}}{135}$

Students who answered Question 1d. correctly were generally able to answer this question correctly. Some students split the integral, which was unnecessary. Others put a negative sign in front of the integral for the bounded area below the x-axis. Some had their terminals or expressions the reverse of what was required.

Let $p: R \to R, p(x) = 3x^4 + 4x^3 + 6(a-2)x^2 - 12ax + a^2, a \in R$.

f. State the value of a for which $f(x) =$

Marks	0	1	Average
%	51	49	0.5

$a = 0$

Some students gave an additional expression $a = -6x(x-2)$, which was obtained if technology was used rather than equating coefficients.

g. Find all solutions to $p'(x) = 0$, in terms of a where appropriate. (1 mark)

Marks	0	1	Average
%	43	57	0.6

$x = 1,\ x = -1 \pm \sqrt{1-a}$

A common error was $x = 1 \pm \sqrt{1-a}$. $x = \frac{-1 \pm \sqrt{9-4a}}{2}$, $x = 0$ was often given. This comes from forgetting to differentiate $-12ax$ when differentiating $p(x)$.

Question 1hi.

h.

i. Find the values of a for which p has only one stationary point. (1 mark)

Marks	0	1	Average
%	82	18	0.2

$1-a<0,\ a>1$

This question was not answered well. Common incorrect answers were $a=1$, $a=0$ or $a>0$.

ii. Find the minimum value of p when $a = 2$. (1 mark)

Marks	0	1	Average
%	47	53	0.6

$a=2$, $p(x)=3x^4+4x^3-24x+4$, $p(1)=-13$, the minimum value is -13.

This question was answered well. The minimum value needed to be stated, not just the coordinates of the turning point.

iii. If p has only one stationary point, find the values of a for which $p(x) = 0$ has no solutions. (2 marks)

Marks	0	1	2	Average
%	92	4	4	0.2

Solve $p(x)>0$ for a when $x=1$, $a>\sqrt{14}+3$ as $a>1$

This question was not answered well. Many students did not attempt this question. Some students solved $p(x)=0$ or $p'(x)=0$ for x. Others tried to apply the discriminant to a cubic equation. Others, who used a correct method, sometimes gave an incorrect inequality, for example $a<\sqrt{14}+3$.

Question 160 (10 marks)

A drug, X, comes in 500 milligrams (mg) tablets.
The amount, b, of drug X in the bloodstream, in milligrams, t hours after one tablet is consumed is given by the function:

$$b(t)=\frac{4500}{7}\left(e^{\left(-\frac{t}{5}\right)}-e^{\left(-\frac{9t}{10}\right)}\right)$$

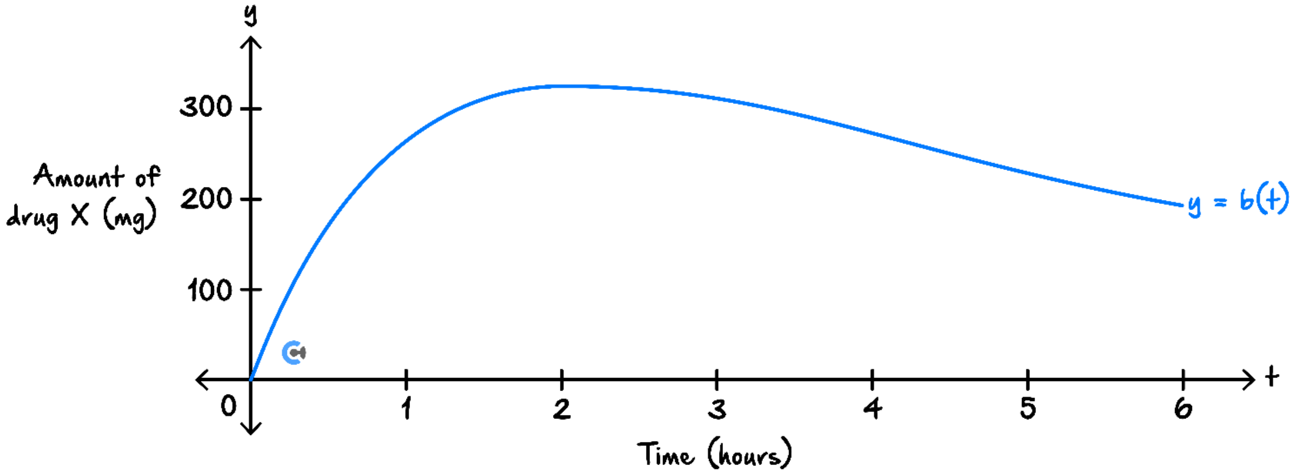
a. Find the time, in hours, it takes for drug X to reach a maximum amount in the bloodstream after one tablet is consumed. Express your answer in the form $a\log_e(c)$, where $a, c \in R$. (2 marks)

Marks	0	1	2	Average
%	19	8	73	1.6

$b'(t)=0$, $t=\frac{10\log_e(4.5)}{7}$ hours

This question was answered well. An exact answer was required. Some students converted $t = 2.148\ldots$ to 2 hours and 15 minutes.

The graph of $y = b(t)$ is shown below for $0 \leq t \leq 6$.



b. Find the average rate of change of the amount of drug X in the bloodstream, in milligrams per hour, over the interval $[2, 6]$. Give your answer correct to one decimal place. (2 marks)

Marks	0	1	2	Average
%	16	15	70	1.6

Average rate of change = $\frac{b(6)-b(2)}{6-2} = -33.5$ mg/h, correct to one decimal place

Some students used the average value of the function. Others made substitution errors. A common incorrect answer was 33.5. Several students used the graph to approximate values rather than find $b(6)$ and $b(2)$. Some students found the average of the gradient at $b = 2$ and $b = 6$.

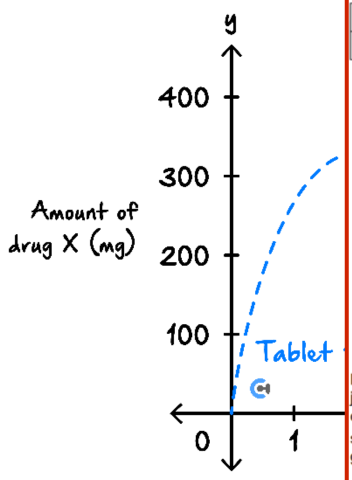
c. Find the average amount of drug X in the bloodstream, in milligrams, during the first six hours after one tablet is consumed. Give your answer correct to the nearest milligram. (2 marks)

Marks	0	1	2	Average
%	39	6	56	1.2

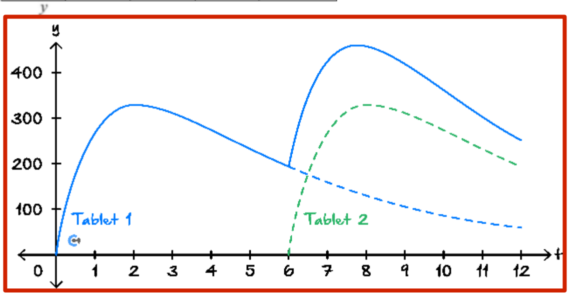
Average amount of drug = $\frac{1}{6}\int_0^6 (b(t))\,dt = 256$ mg to the nearest integer

Some students used the interval [2, 6] from Question 2c., instead of [0, 6]. Others did not divide by 6, which gave 1535.1… mg. $\frac{b(0)+b(1)+b(2)+b(3)+b(4)+b(5)+b(6)}{6}$ was often given. Some thought that the first six hours meant $t = 1$ to $t = 6$ instead of $t = 0$ to $t = 6$. Others evaluated $\int_0^6 t \times b(t)\,dt$ or found the average rate of change.

d. Six hours after one 500 milligram tablet of drug X is consumed (Tablet 1), a second identical tablet is consumed (Tablet 2). The amount of drug X in the bloodstream from each tablet consumed independently is shown in the graph below.



Marks	0	1	2	Average
%	35	29	35	1



Many students were able to trace over the first part of the graph from $t = 0$ to $t = 6$. Some did not join the two sections at $x = 6$, with some starting at the intersection of the two graphs. Students could use addition of ordinates or define the function $b_2(t) = b(t) + b(t-6)$ and use technology to sketch the graph and find the position of turning point. Some students shaded the area under the graph.

007A

i. On the graph above, sketch the total amount of drug X in the bloodstream during the first 12 hours after Tablet 1 is consumed. (2 marks)

ii. Find the maximum amount of drug X in the bloodstream in the first 12 hours and the time at which this maximum occurs. Give your answers correct to two decimal places. (2 marks)

Marks	0	1	2	Average
%	74	6	19	0.5

Total amount of drug = $b(t) + b(t-6)$, maximum amount of drug is 455.82 mg, correct to two decimal places, $t = 7.78$ h correct to two decimal places

This question was not answered well. Many students were unable to find the new rule and solved $b'(t) = 0$ for t, getting 324.34 mg for the maximum amount of drug. Some then added six to this answer, 324.34 + 6 = 330.34 mg. Some assumed that $t = 8$. Answers were required to two decimal places. Other students gave only one answer.

Question 161 (11 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2019
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2019/2019MM2-w.pdf

Let $f: R \to R, f(x) = x^2 e^{-x^2}$.

a. Find $f'(x)$. (1 mark)

Marks	0	1	Average
%	6	94	1.0

$f: R \to R, f(x) = x^2 e^{-x^2}$, $f'(x) = 2xe^{-x^2} - 2x^3 e^{-x^2}$

Other equivalent forms are acceptable.

This question was answered well. Some students appeared to transcribe the output from technology incorrectly: $f'(x) = 2x^3 e^{-x^2} - 2xe^{-x^2}$ and $f'(x) = 2e^{-x^2} - 2x^3 e^{-x^2}$ were occasionally seen. Others tried to find the derivative by hand or further engaged with the output from technology and made errors.

b.

i. State the nature of the stationary point on the graph of f at the origin. (1 mark)

Marks	0	1	Average
%	28	72	0.7

Minimum

This question was answered well. Some students did not understand what the term 'nature of the stationary point' meant. Common incorrect answers were point of inflection, stationary points and turning points. Some gave the coordinates of the turning point, (0, 0).

ii. Find the maximum value of the function f and the values of x for which the maximum occurs. (2 marks)

Marks	0	1	2	Average
%	10	22	67	1.6

Solve $f'(x) = 0$ for x, $x = -1$ or $x = 1$, Maximum $f(1) = \frac{1}{e}$

Some students included $x = 0$ or only gave one answer for x. Others did not find the maximum value. Some gave the approximate answer for the maximum value. An exact answer was required.

iii. Find the values of $d \in$

Marks	0	1	Average
%	65	35	0.4

$d < -\frac{1}{e}$

This question was not answered well. Common incorrect answers were $d = -\frac{1}{e}$, $d \le -\frac{1}{e}$, $d > \frac{1}{e}$, $d > -\frac{1}{e}$, $d < \frac{1}{e}$ or $d < -x^2e^{-x^2}$. Some students wrote $\left(-\frac{1}{e}, -\infty\right)$. Others did not attempt the question.

c.

i. Find the equation of the tangent to the graph of f at $x = -1$. (1 mark)

Marks	0	1	Average
%	21	79	0.8

$y = \frac{1}{e}$

This question was answered well. An equation was required.

ii. Find the area enclosed by the graph of f and the tangent to the graph of f at $x = -1$, correct to four decimal places. (2 marks)

Marks	0	1	2	Average
%	32	12	56	1.3

Area of rectangle – integral of function $= \frac{2}{e} - \int_{-1}^{1}\left(x^2e^{-x^2}\right)dx$ OR $\int_{-1}^{1}\left(\frac{1}{e} - x^2e^{-x^2}\right)dx$

Area = 0.3568 correct to four decimal places

Most students were able to subtract *f* from their tangent. Common incorrect methods were $\int_{0}^{1}\left(\frac{1}{e} - x^2e^{-x^2}\right)dx$ and $\int_{-1}^{1}\left(x^2e^{-x^2}\right)dx$ and $\int_{-1}^{1}\left(x^2e^{-x^2} - \frac{1}{e}\right)dx$.

d. Let $M(m,n)$ be a point on the graph of f, where $m \in [0,1]$.

Find the minimum distance between M and the point $(0,e)$, and the value of m for which this occurs, correct to three decimal places. (3 marks)

Marks	0	1	2	3	Average
%	46	23	7	24	1.1

$d=\sqrt{(0-m)^2+(e-f(m))^2}$, Solve $d'(m)=0$ for m, or, $m_{\text{Tangent}}=-(m^3-2m)e^{-m^2}$,

$m_{\text{Perpendicular}}=\frac{-1}{-(m^3-2m)e^{-m^2}}=\frac{1}{(m^3-2m)e^{-m^2}}$, $m_{\text{Perpendicular}}=\frac{y_2-y_1}{x_2-x_1}$, $\frac{1}{(m^3-2m)e^{-m^2}}=\frac{m^2e^{-m^2}-e}{m-0}$,

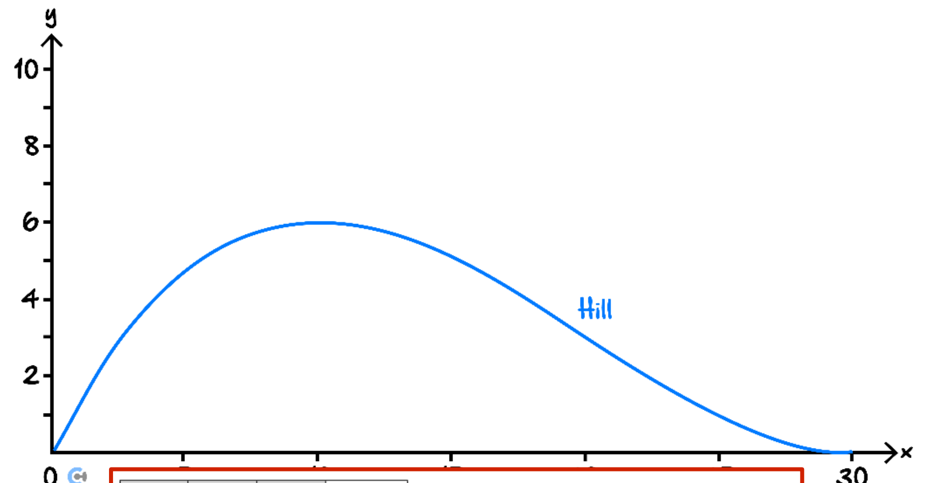
$m=0.783$ correct to three decimal places, $d(0.738...)=2.511$ correct to three decimal places

Many students were able to use the distance formula. Others found m but not the distance. Some gave their answers correct to two decimal places.

Question 162 (11 marks)

An amusement park is planning to build a zip-line above a hill on its property.

The hill is modelled by $y=\frac{3x(x-30)^2}{2000}$, $x \in [0,30]$, where x is the horizontal distance, in metres, from an origin and y is the height, in metres, above this origin, as shown in the graph below.



a. Find $\frac{dy}{dx}$. (1 mark)

Marks	0	1	Average
%	7	93	1.0

Let $f(x)=\frac{3x(x-30)^2}{2000}$, $f'(x)=\frac{3(-30+x)^2}{2000}+\frac{3x(-30+x)}{1000}=\frac{9(x-30)(x-10)}{2000}$

Other equivalent forms were acceptable.

This question was answered well. Common incorrect answers were $\frac{9x(x-30)(x-15)}{500}$ and $\frac{9(x^2-40x+30)}{2000}$.

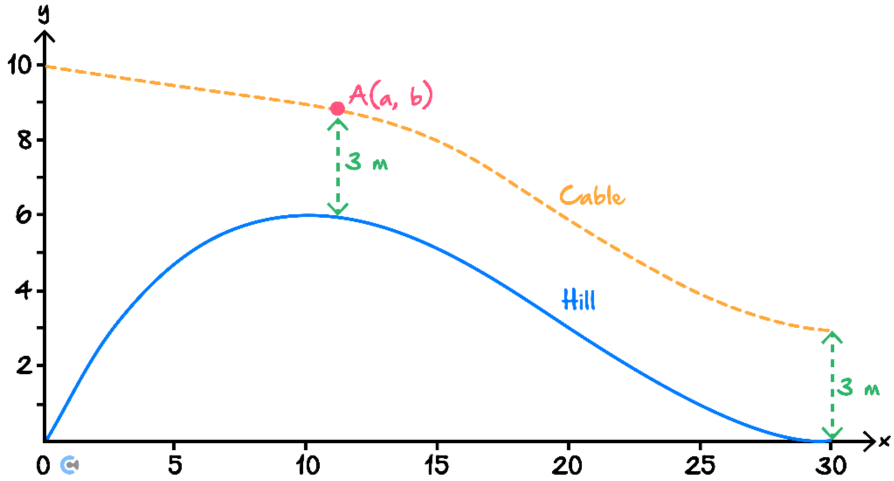
b. State the set of values for which the gradient of the hill is strictly decreasing. (1 mark)

Marks	0	1	Average
%	97	3	0.1

$x \in (0, 20]$

This question was not done well. Most students interpreted the question as asking where the function modelling the hill was strictly decreasing, rather than the gradient of the hill and so the most common incorrect response was $[10, 30]$ or a combination of round and square brackets with those two values.

The cable for the zip-l … $0 \leq x \leq a$, where $10 \leq a \leq 20$ … xactly 3 m vertically above the hi…



c. State the rule, in terms of x, for the height of the cable above the horizontal axis for $x \in [a, 30]$. (1 mark)

Marks	0	1	Average
%	38	62	0.6

$$h(x) = \frac{3x(x-30)^2}{2000} + 3 = \frac{3x^3}{2000} - \frac{9x^2}{100} + \frac{27x}{20} + 3$$

This question was generally well done. Some students did not give an equation. Others added 10, instead of 3, to f.

d. Find the values of x for which the gradient of the cable is equal to the average gradient of the hill for $x \in [10, 30]$. (3 marks)

Marks	0	1	2	3	Average
%	41	13	9	37	1.4

$$\text{Average Gradient} = \frac{f(30) - f(10)}{30 - 10} = \frac{1}{30-10}\int_{10}^{30} h'(x)\,dx = -\frac{3}{10},\quad f'(x) = \frac{9(x-30)(x-10)}{2000} = -\frac{3}{10},$$

$$x = \frac{10}{3}\left(6 \pm \sqrt{3}\right) = 20 \pm \frac{10}{\sqrt{3}} = 20 \pm \frac{10\sqrt{3}}{3} = \frac{60 \pm 10\sqrt{3}}{3}$$

A common incorrect answer for the average gradient was $\frac{3}{10}$.

Some students used $\frac{1}{30-10}\int_{10}^{30} h(x)\,dx$ instead of $\frac{1}{30-10}\int_{10}^{30} h'(x)\,dx$.

Some students gave approximate answers for the x values, 14.23 and 25.77.

Other students did not use brackets correctly, giving $x = \frac{\pm 10\left(\sqrt{3}+6\right)}{3}$ as their answer. Another common incorrect answer was $\frac{6 \pm 10\sqrt{3}}{3}$.

The gradients of the straight and curved sections of the cable approach the same value at $x = a$, so there is a continuous and smooth join at A.

e.

i. State the gradient of the cable at A, in terms of a. (1 mark)

Marks	0	1	Average
%	48	52	0.5

$\frac{9a^2}{2000} - \frac{9a}{50} + \frac{27}{20}$ or $\frac{h(a)-10}{a} = \frac{\frac{3a(a-30)^2}{2000}+3-10}{a} = \frac{3a^2}{2000} - \frac{9a}{100} - \frac{7}{a} + \frac{27}{20}$

Common incorrect answers were $\frac{9a^2}{2000} - \frac{9a}{50} + \frac{27}{50}$, $\frac{3}{2000}a^2 - \frac{9}{100}a - \frac{10}{a} + \frac{27}{20}$ and $\frac{3a^3 - 180a^2 + 2700a - 20000}{2000a}$.

Some students used $\frac{f(a)-10}{a}$ instead of $\frac{h(a)-10}{a}$. Other students wrote $\frac{b-10}{a}$.

The answer had to be given in terms of *a*.

ii. Find the coordinates of A, with each value correct to two decimal places. (3 marks)

Marks	0	1	2	3	Average
%	69	9	6	15	0.7

$\frac{3a^2}{2000} - \frac{9a}{100} - \frac{7}{a} + \frac{27}{20} = \frac{9(a-30)(a-10)}{2000}$ or $\frac{9(a-30)(a-10)}{2000}a + 10 = \frac{3a(a-30)^2}{2000} + 3$,

$(11.12,\ 8.95)$ correct to two decimal places

Many students did not equate the correct expressions. Some students found the value of *a* but not the value of *b*. Other students rounded their answers incorrectly, giving (11.11, 8.94).

iii. Find the value of the gradient at A, correct to one decimal place. (1 mark)

Marks	0	1	Average
%	80	20	0.2

$h'(11.1...) = \frac{9(a-30)(a-10)}{2000} = -0.1$ correct to one decimal place

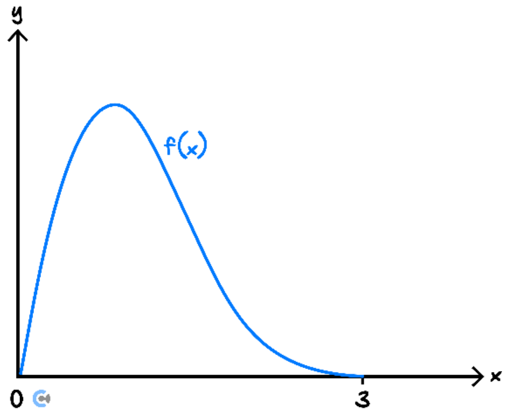
Students who obtained the correct value for *a* in Question 2eii. were generally successful with this question.

Space for Personal Notes

Question 163 (13 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2020
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2020/2020MM2-w.pdf

The graph of the function $f(x) = 2xe^{(1-x^2)}$, where $0 \leq x \leq 3$, is shown below.



a. Find the slope of the tangent to f at $x = 1$. (1 mark)

Marks	0	1	Average
%	23	77	0.8

$f(x) = 2xe^{(1-x^2)}$, $f'(1) = -2$

Some students wrote the equation of the tangent instead of its gradient.

b. Find the obtuse angle that the tangent to f at $x = 1$ makes with the positive direction of the horizontal axis. Give your answer correct to the nearest degree. (1 mark)

Marks	0	1	Average
%	63	37	0.4

$180 + \tan^{-1}(-2) = 117°$ to the nearest degree

63° and -63° were common incorrect answers.

c. Find the slope of the tangent to f at a point $x = p$. Give your answer in terms of p. (1 mark)

Marks	0	1	Average
%	33	67	0.7

$2(1-2p^2)e^{1-p^2}$ or $(2e-4p^2e)e^{-p^2}$

Some responses contained transcription errors.

Instead of writing $2(1-2p^2)e^{-p^2+1}$, some wrote $2(1-2p^2)e^{-p^2}+1$.

Brackets were not used well, and some students wrote the equation of the tangent instead of its gradient.

d.

i. Find the value of p for which the tangent to f at $x = 1$ and the tangent to f at $x = p$ are perpendicular to each other. Give your answer correct to three decimal places. (2 marks)

Marks	0	1	2	Average
%	38	8	54	1.2

Solve $2\left(1-2p^2\right)e^{1-p^2} = \frac{1}{2}$ for p, $p = 0.655$ correct to three decimal places

Some students solved $2\left(1-2p^2\right)e^{1-p^2} = 2$. Others knew that $m_1 m_2 = -1$ but were unable to connect this information to their previous answers. $p = 0.656$ was often seen.

ii. Hence, find the coordinates of the point where the tangents to the graph of f at $x = 1$ and $x = p$ intersect when they are perpendicular. Give your answer correct to two decimal places. (3 marks)

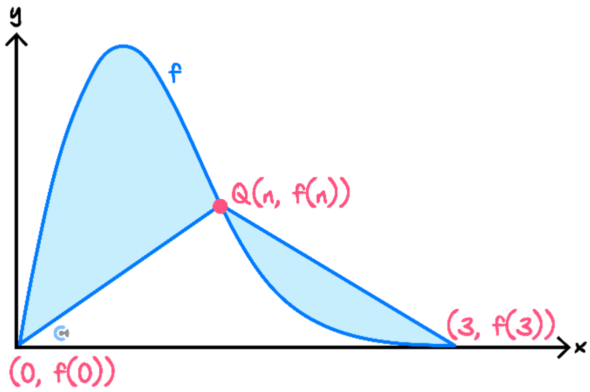
Marks	0	1	2	3	Average
%	46	11	17	26	1.2

$y = \frac{1}{2}(x-p) + f(p)$, $p = 0.65525\ldots$ or $y = 0.50\ldots x + 1.99\ldots$, $y = -2(x-1) + 2 = -2x + 4$,

$(0.80, 2.39)$

Many students successfully found that the point of intersection of the two tangents occurred at $x = 0.80$ but then substituted this into $f(x)$, getting the value 2.29 instead of substituting it into one of the two tangent equations. Some students managed to find the equation of the tangent at $x = 1$ but did not know what to do with this equation. Others rounded too early.

Two line segments connect the points $(0, f(0))$ and $(3, f(3))$ to a single point $Q(n, f(n))$, where $1 < n < 3$, as shown in the graph below.



e.

i. The first line segment connects the point $(0, f(0))$ and the point $Q(n, f(n))$, where $1 < n < 3$. Find the equation of this line segment in terms of n. (1 mark)

Marks	0	1	Average
%	56	44	0.4

$y_1 = 2e^{1-n^2}x$

Some students did not write a rule. Others left out x, giving the gradient as the final answer: $y = 2e^{1-n^2}$

A number of students wrote the rule in terms of $f(n)$ and not n. Other common incorrect answers were: $y = 2xe^{1-x^2}$, $y = 2ne^{1-n^2}$ and $y = 2e^{1-x^2}$.

ii. The second line segment connects the point $Q(n, f(n))$ and the point $(3, f(3))$, where $1 < n < 3$. Find the equation of this line segment in terms of n. (1 mark)

Marks	0	1	Average
%	72	28	0.3

$y_2 = \frac{f(3)-f(n)}{3-n}(x-n)+f(n)$, $y_2 = \frac{2ne^{1-n^2}-6e^{-8}}{n-3}(x-3)+6e^{-8}$ (many forms)

A rule was required. Some students only wrote down the gradient. Others assumed $f(3) = 0$.

There were a lot of transcription errors: e^{n^2-8} was often written as $e^{n^2}-8$.

The variable x sometimes looked like n and vice versa. Brackets were used poorly. Some students only wrote down part of the equation. Students need to make sure they scroll across the screen to ensure they

iii. Find the value of n, where $1 < n < 3$, if there are equal areas between the function f and each line segment. Give your answer correct to three decimal places. (3 marks)

Marks	0	1	2	3	Average
%	60	7	22	11	0.8

Solve $\int_0^n (f(x)-y_1)dx = \int_n^3 (y_2 - f(x))dx$ for n, $n = 1.088$ correct to three decimal places.

The majority of students who attempted this question were able to correctly set up the integrals. However, some were then unable to arrive at the final response. There was no need to write out entire expressions. This often led to transcription errors and misuse of brackets. Others used areas of triangles:

$\int_0^n f(x)dx - \frac{1}{2}nf(n) = \frac{1}{2}(3-n)(f(n)-f(3)) - \int_n^3 f(x)dx$, which gave $n = 1.087$.

The area from $x = n$ to $x = 3$ is a trapezium, not a triangle. So, the correct formulation is

$\int_0^n f(x)dx - \frac{1}{2}nf(n) = \frac{1}{2}(3-n)(f(n)+f(3)) - \int_n^3 f(x)dx$.

Question 164 (13 marks)

Let $f: R \to R, f(x) = x^3 - x$.

Let $g_a : R \to R$ be the function representing the tangent to the graph of f at $x = a$, where $a \in R$.

Let $(b, 0)$ be the x-intercept of the graph of g_a.

a. Show that $b = \frac{2a^3}{3a^2-1}$. (3 marks)

Marks	0	1	2	3	Average
%	36	7	9	49	1.7

$f'(a) = 3a^2 - 1$, equation of the tangent, $y - 0 = (3a^2-1)(x-b)$, substitute $(a, a^3 - a)$,

$a^3 - a = (3a^2-1)(a-b)$, $a^3 - a = 3a^3 - 3a^2b - a + b$, $-2a^3 = b(1-3a^2)$, $b = \frac{2a^3}{3a^2-1}$

Most students were able to find the equation of the tangent. When finding the equation of the tangent, many students left out brackets when multiplying $(x-a)$ by the gradient $(3a^2-1)$, writing $3a^2 - 1(x-a)$ instead of $(3a^2-1)(x-a)$. Some students did not show suitable steps.

b. State the values of a for which b does not exist. (1 mark)

Marks	0	1	Average
%	54	46	0.5

$3a^2 - 1 = 0$, $a = \pm\frac{1}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3}$

Some students wrote down only one solution, generally $a = \frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$. Other incorrect answers were $a = R \setminus \left\{\pm\frac{1}{\sqrt{3}}\right\}$, $\left[-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right]$ and $a < -\frac{\sqrt{3}}{3}$, $a > \frac{\sqrt{3}}{3}$. Some gave approximate answers.

c. State the nature of the graph of g_a, when b does not exist. (1 mark)

Marks	0	1	Average
%	77	23	0.2

Horizontal line

The concept of the 'nature of a tangent line' was not obvious for many students. Common incorrect answers were undefined, asymptote, increasing, decreasing, inflection, maximum and minimum. Many described the curve of f and not the tangent.

d.

i. State all values of a for which $b = 1.1$. Give your answers correct to four decimal places. (1 mark)

Marks	0	1	Average
%	40	60	0.6

Solve $\frac{2a^3}{3a^2-1} = 1.1$ for a, $a = -0.5052$ or $a = 0.8084$ or $a = 1.3468$ correct to four decimal places.

There were some decimal place errors such as $a = -0.5051$, $a = 1.3467$ or $a = 1.347$.

Sometimes $a = -0.5052$ was written as $a = 0.5052$

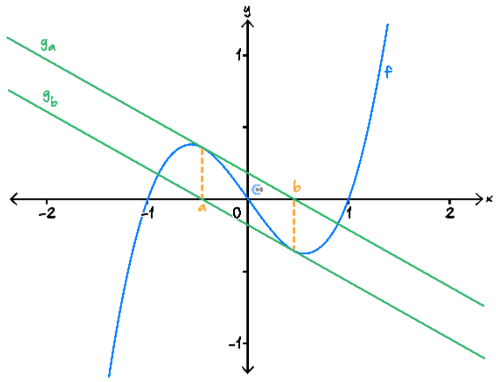
ii. The graph of f has an x-intercept at $(1, 0)$.

State the values of a for which $1 \leq b < 1.1$. Give your answers correct to three decimal places. (1 mark)

Marks	0	1	Average
%	87	13	0.1

Solve $1 \leq \frac{2a^3}{3a^2-1} < 1.1$ for a, $(-0.505, -0.500] \cup (0.808, 1.347)$ correct to three decimal places.

The coordinate $(b, 0)$ is the horizontal axis intercept of g_a.
Let g_b be the function representing the tangent to the graph of f at $x = b$, as shown in the graph below.



e. Find the values of a for which the graphs of g_a and g_b, where b exists, are parallel and where $b \neq a$. (3 marks)

Marks	0	1	2	3	Average
%	82	3	8	7	0.4

$f'(b) = 3b^2 - 1 = 3\left(\frac{2a^3}{3a^2-1}\right)^2 - 1$, $3\left(\frac{2a^3}{3a^2-1}\right)^2 - 1 = 3a^2 - 1$, $a = \left\{-1, -\frac{\sqrt{5}}{5}, 0, \frac{\sqrt{5}}{5}, 1\right\}$, checking the condition $a \neq b$ using the formula $b = \frac{2a^3}{3a^2-1}$ gives $a = \pm\frac{\sqrt{5}}{5}$.

Many students did not use $b = \frac{2a^3}{3a^2-1}$.

Others did not eliminate the values where $a = b$ and included $a = -1, 0$ and 1.

Let $p : R \to R, p(x) = x^3 + wx$, where $w \in R$.

f. Show that $p(-x) = -p(x)$ for all $w \in R$. (1 mark)

Marks	0	1	Average
%	43	57	0.6

$p(-x) = (-x)^3 - wx = -x^3 - wx = -(x^3 + wx) = -p(x)$

Some students did not expand the expression in brackets correctly. Others tried to show the required result by substitution.

A property of the graphs of p is that two distinct parallel tangents will always occur at $(t, p(t))$ and $(-t, p(-t))$ for all $t \neq 0$.

g. Find all values of w such that a tangent to the graph of p at $(t, p(t))$, for some $t > 0$, will have an x-intercept at $(-t, 0)$. (1 mark)

Marks	0	1	Average
%	97	3	0.03

$y - t^3 - wt = (3t^2 + w)(x - t)$, $0 = (3t^2 + w)(-2t) + t^3 + wt$, $w = -5t^2$, $t > 0, w < 0$

This question was attempted by a small number of students. Some students found $w = -5t^2$ but were unable to write down the values of w.

h. Let $T: R^2 \to R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} m & 0 \\ 0 & n \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} h \\ k \end{bmatrix}$, where $m, n \in R\backslash[0]$ and $h, k \in R$.

State any restrictions on the values of m, n, h and k, given that the image of p under the transformation T always has the property that parallel tangents occur at $x = -t$ and $x = t$ for **all** $t \neq 0$. (1 mark)

Marks	0	1	Average
%	98	2	0.02

$h = 0$ (odd function)

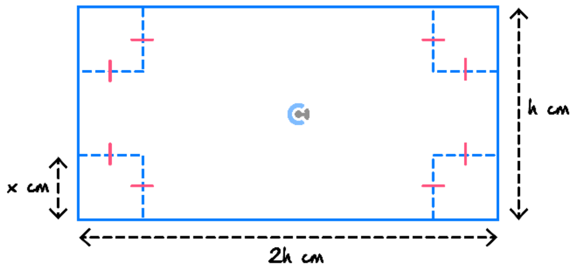
This question was attempted by only a small number of students. When the correct answer was given, it was sometimes accompanied with incorrect values of m and n: for example, $m, n \in R$. The key word in this part is **restrictions**. There were no restrictions on m, n or k.

Space for Personal Notes

Question 165 (14 marks)

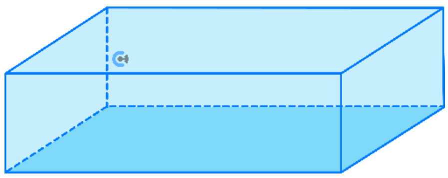
Inspired from VCAA Mathematical Methods ¾ Exam 2021
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2021/2021MM2-w.pdf

A rectangular sheet of cardboard has a width of h centimetres. Its length is twice its width.
Squares of side length x centimetres, where $x > 0$, are cut from each of the corners, as shown in the diagram below.



The sides of this sheet of cardboard are then folded up to make a rectangular box with an open top, as shown in the diagram below.

Assume that the thickness of the cardboard is negligible and that $V_{box} > 0$.



A box is to be made from a sheet of cardboard with $h = 25\ cm$.

a. Show that the volume, V_{box}, in cubic centimetres, is given by $V_{box}(x) = 2x(25 - 2x)(25 - x)$. (1 mark)

Marks	0	1	Average
%	29	71	0.7

$V = x(h-2x)(2h-2x) = x(25-2x)(50-2x) = 2x(25-2x)(25-x)$

This question was answered well.

Some students could not identify the dimensions correctly. Others used brackets incorrectly or omitted brackets, for example: $x \times 25 - 2x \times 50 - 2x$

b. State the domain of V_{box}. (1 mark)

Marks	0	1	Average
%	58	42	0.4

$(0, 12.5)$

$(0, 25)$ was often seen. Some students had incorrect brackets, for example, $(0, 12.5]$.

c. Find the derivative of V_{box} with respect to x. (1 mark)

Marks	0	1	Average
%	10	90	0.9

$12x^2 - 300x + 1250$

This question was well done. Some incorrectly wrote $12x^2 - 300x + 12500$.

d. Calculate the maximum possible volume of the box and for which value of x this occurs. (3 marks)

Marks	0	1	2	3	Average
%	17	13	21	49	2.0

Solve $V'(x) = 0$, $x = \frac{-25\left(\sqrt{3}-3\right)}{6} = \frac{-25\sqrt{3}}{6} + \frac{25}{2}$, $V = \frac{15625\sqrt{3}}{9}$ or $\frac{15625}{3\sqrt{3}}$

Exact values were required. Some students found the x-value but did not find the maximum volume. Others chose the incorrect x-value, $\frac{25\sqrt{3}}{6} + \frac{25}{2}$, to find the volume.

There were some transcription errors: $x = \frac{-25\left(\sqrt{3}+3\right)}{6}$ was often seen.

e. Waste minimisation is a goal when making cardboard boxes.
The percentage wasted is based on the area of the sheet of cardboard that is cut out before the box is made.
Find the percentage of the sheet of cardboard that is wasted when $x = 5$. (2 marks)

Marks	0	1	2	Average
%	35	19	46	1.1

$\frac{4 \times 5^2}{25 \times 50} \times 100\% = 8\%$

Some students used volume and not area in the denominator. Many were able to work out that 100 cm² was cut out. Some were not able to convert their fraction to a percentage: 0.08% was often seen.

Now, consider a box made from a rectangular sheet of cardboard where $h > 0$ and the box's length is still twice its width.

f.

i. Let V_{box} be the function that gives the volume of the box.
State the domain of V_{box} in terms of h. (1 mark)

Marks	0	1	Average
%	67	33	0.4

$\left(0, \frac{h}{2}\right)$

Some students gave the equation, $V = x(h-2x)(2h-2x)$, and not the domain. Others had incorrect brackets. $(0, 25h)$ was sometimes seen.

ii. Find the maximum volume for any such rectangular box, V_{box}, in terms of h. (3 marks)

Marks	0	1	2	3	Average
%	42	13	12	33	1.4

$V = x(h-2x)(2h-2x)$, $V'(x) = 0$, $x = \frac{-h\left(\sqrt{3}-3\right)}{6}$, $V = \frac{\sqrt{3}h^3}{9}$

Many students were able to find the formula, $V = x(h-2x)(2h-2x)$. Some chose the incorrect x-value, $x = \frac{h\left(\sqrt{3}+3\right)}{6}$ and then gave a negative volume.

g. Now, consider making a box from a square sheet of cardboard with side lengths of h centimetres.

Show that the maximum volume of the box occurs when $x = \frac{h}{6}$. (2 marks)

Marks	0	1	2	Average
%	55	10	35	0.8

$V = x(h-2x)^2$, $V'(x) = 0$, $x = \frac{h}{2}$ or $x = \frac{h}{6}$, $x = \frac{h}{6}$ as the domain is $\left(0, \frac{h}{2}\right)$

Some students were able to find the correct formula, $V = x(h-2x)^2$. Some did not show adequate working for a 'show that' question.

Question 166 (12 marks)

Let $q(x) = \log_e (x^2 - 1) - \log_e (1 - x)$.

a. State the maximal domain and the range of q. (2 marks)

Marks	0	1	2	Average
%	27	37	36	1.1

Domain $(-\infty, -1)$, range R

Some students only gave the domain and not the range. Others gave the domain as $(-\infty, 1)$ or $(-\infty, 1]$ or $(-1, -\infty)$.

b.

i. Find the equation of the tangent to the graph of q when $x = -2$. (1 mark)

Marks	0	1	Average
%	26	74	0.8

$y = -x - 2$

An equation was required. Many students worked out the equation without using technology. This would have been time consuming.

ii. Find the equation of the line that is perpendicular to the graph of q when $x = -2$ and passes through the point $(-2, 0)$. (1 mark)

Marks	0	1	Average
%	35	65	0.7

$y = x + 2$

Once again, an equation was required and it could be found easily using technology.

Let $p(x) = e^{-2x} - 2e^{-x} + 1$.

c. Explain why p is not a one-to-one function. (1 mark)

Marks	0	1	Average
%	34	66	0.7

'Fails the horizontal line test' or 'many-to-one function' or 'there exist two x-values for some y-values'.

Some students wrote that there exists two x-values for every y-value, which is not the case, or p fails the vertical line test. Others gave the meaning of a one-to-one function without relating it to the question.

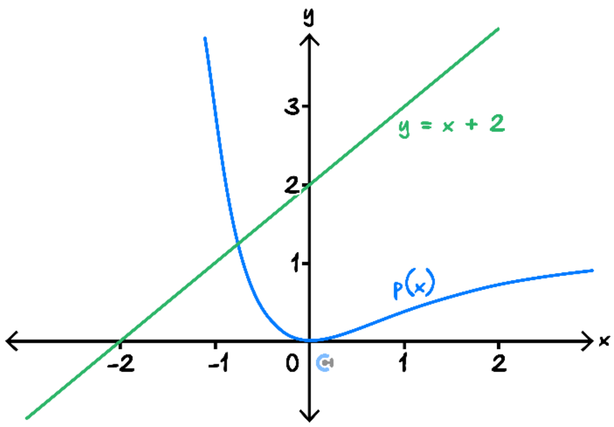
d. Find the gradient of the tangent to the graph of p at $x = a$. (1 mark)

Marks	0	1	Average
%	33	67	0.7

$2(e^{a}-1)e^{-2a}$

Some students gave their answer in terms of x and not a. There were some transcription errors and brackets were used poorly. Others wrote the equation of the tangent.

The diagram below shows parts of the graph of p and the line $y = x + 2$.



The line $y = x + 2$ and the tangent to the graph of p at $x = a$ intersect with an acute angle of θ between them.

e. Find the value(s) of a for which $\theta = 60°$. Give your answer(s) correct to two decimal places. (3 marks)

Marks	0	1	2	3	Average
%	83	4	9	4	0.4

$p'(a) = -\tan(75°) = \tan(105°)$, $a = -0.67$, $p'(a) = -\tan(15°) = \tan(165°)$, $a = -0.11$

This question was not answered well. Some students were able to find either $a = -0.67$ or $a = -0.11$ but not both.

f. Find the x-coordinate of the point of intersection between the line $y = x + 2$ and the graph of p, and hence, find the area bounded by $y = x + 2$, the graph of p and the x-axis, both correct to three decimal places. (3 marks)

Marks	0	1	2	3	Average
%	41	23	7	29	1.3

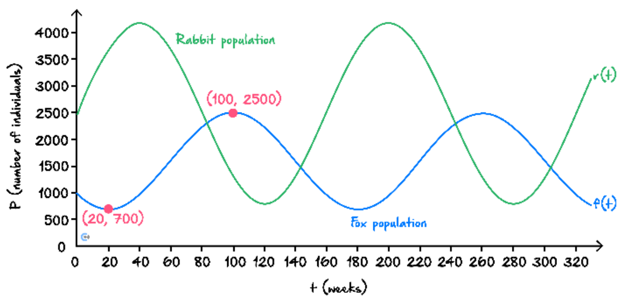
$x = -0.750$, $\int_{-2}^{-0.750...} (x+2)\,dx + \int_{-0.750...}^{0} p(x)\,dx = 1.038$

Many students were able to find $x = -0.750$. Some wrote their answer as $x = -0.75$, but three decimal places were required. Others were unable to set up the definite integrals correctly.

Question 167 (16 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2022
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2022/2022MM2-w.pdf

On a remote island, there are only two species of animals: Foxes and rabbits. The foxes are the predators and the rabbits are their prey.
The populations of foxes and rabbits increase and decrease in a periodic pattern, with the period of both populations being the same, as shown in the graph below, for all $t \geq 0$, where time t is measured in weeks. One point of minimum fox population, $(20, 700)$, and one point of maximum fox population, $(100, 2500)$, are also shown on the graph.
The graph has been drawn to scale.



The population of rabbits can be modelled by the rule $r(t) = 1700 \sin\left(\frac{\pi t}{80}\right) + 2500$.

a.

i. State the initial population of rabbits. (1 mark)

Question 2ai.

Marks	0	1	Average
%	3	97	1.0

2500

This question was done very well.

ii. State the minimum and maximum population of rabbits. (1 mark)

Marks	0	1	Average
%	11	89	0.9

800, 4200

Some students gave the coordinates (120, 800) and (40, 4200), without stating the minimum and maximum values. Others had the minimum as 700 and the maximum as 4000.

iii. State the number of weeks between maximum populations of rabbits. (1 mark)

Marks	0	1	Average
%	16	84	0.9

160

A common incorrect answer was 80.

The population of foxes can be modelled by the rule $f(t) = a\sin(b(t-60)) + 1600$.

b. Show that $a = 900$ and $b = \frac{\pi}{80}$. (2 marks)

Marks	0	1	2	Average
%	19	13	68	1.5

$a = \frac{2500-700}{2} = 900$, $\frac{2\pi}{b} = 160$, $b = \frac{\pi}{80}$

As this was a 'show that' question, appropriate working needed to be shown. Students used a range of techniques to find the correct values of *a* and *b*. Many students substituted in points from the graphs and solved simultaneous equations. Other students took a similar approach using the derivative.

c. Find the maximum combined population of foxes and rabbits. Give your answer correct to the nearest whole number. (1 mark)

Marks	0	1	Average
%	62	38	0.4

5339

This question was not answered well. There were rounding errors, with 5340 as a common incorrect answer. A common incorrect approach was to add the maximum value of rabbits to the maximum value of foxes, without recognising that the maximum values for each animal occurred at different times.

d. What is the number of weeks between the periods when the combined population of foxes and rabbits is a maximum? (1 mark)

Marks	0	1	Average
%	43	57	0.6

160

An exact answer was required. A common incorrect answer was 160.1.

The population of foxes is better modelled by the transformation of $y = \sin(t)$ under Q given by:

$$Q: R^2 \to R^2, Q\left(\begin{bmatrix} t \\ y \end{bmatrix}\right) = \begin{bmatrix} \frac{90}{\pi} & 0 \\ 0 & 900 \end{bmatrix}\begin{bmatrix} t \\ y \end{bmatrix} + \begin{bmatrix} 60 \\ 1600 \end{bmatrix}$$

e. Find the average population during the first 300 weeks for the combined population of foxes and rabbits, where the population of foxes is modelled by the transformation of $y = \sin(t)$ under the transformation Q. Give your answer correct to the nearest whole number. (4 marks)

Marks	0	1	2	3	4	Average
%	40	12	22	4	22	1.6

$t' = \frac{90}{\pi}t + 60,\ t = \frac{\pi(t'-60)}{90},\ y' = 900y + 1600,\ y = \frac{y'-1600}{900},\ y' = 900\sin\left(\frac{\pi(t'-60)}{90}\right) + 1600,$

$\frac{1}{300}\int_0^{300}(y' + r(t))\,dt = 4142$

Many students were able to do the transformation. Some did not add $r(t)$ in the integral. 1600 was a common incorrect answer. Others subtracted $r(t)$. Some students used average rate of change instead of average value.

Over a longer period of time, it is found that the increase and decrease in the population of rabbits gets smaller and smaller.

The population of rabbits over a longer period of time can be modelled by the rule:

$$s(t) = 1700\,.\,e^{-0.003t}.\sin\left(\frac{\pi t}{80}\right) + 2500, \quad \text{for all } t \geq 0$$

f. Find the average rate of change between the first two times when the population of rabbits is at a maximum. Give your answer correct to one decimal place. (2 marks)

Marks	0	1	2	Average
%	54	5	41	0.9

$\frac{s(198.058...) - s(38.058...)}{198.058... - 38.058...} = -3.6$

Some students rounded their values too early. $\frac{s(200) - s(40)}{200 - 40}$ was often seen. Some students found the average rate of change between the maximum and the minimum populations. Others used $r(t)$ instead of

g. Find the time, where $t > 40$, in weeks, when the rate of change of the rabbit population is at its greatest positive value. Give your answer correct to the nearest whole number. (2 marks)

Marks	0	1	2	Average
%	69	12	19	0.5

$\frac{d^2s}{dt^2} = 0,\ t = 156$

Many students solved $\frac{ds}{dt} = 0$. A common incorrect answer was 41.8, $s(156.11...) = 41.79...$ Another common incorrect answer was 76 weeks. This is when the rate of change of the rabbit population is at its greatest negative value.

h. Over time, the rabbit population approaches a particular value.

State this value. (1 mark)

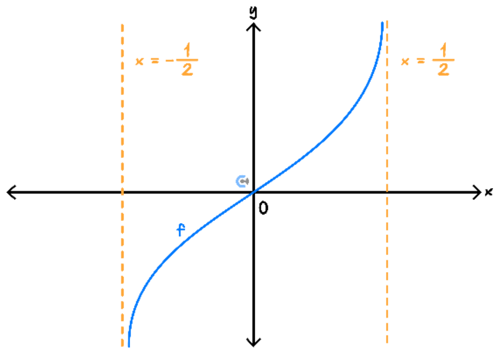
Marks	0	1	Average
%	44	56	0.6

2500

This question was reasonably well done. A common incorrect answer was 0.

Question 168 (9 marks)

Consider the function f, where $f:\left(-\frac{1}{2},\frac{1}{2}\right)\to R, f(x)=\log_e\left(x+\frac{1}{2}\right)-\log_e\left(\frac{1}{2}-x\right)$.
Part of the graph of $y = f(x)$ is shown below.



a. State the range of $f(x)$. (1 mark)

Marks	0	1	Average
%	18	82	0.8

R

This question was answered well. Some students gave the domain rather than the range. A common error was $(-26.2, 26.2)$.

b.

i. Find $f'(0)$. (2 marks)

Marks	0	1	2	Average
%	9	6	85	1.8

$f'(x)=\frac{1}{x+\frac{1}{2}}+\frac{1}{\frac{1}{2}-x}=\frac{2}{2x+1}-\frac{2}{2x-1}=-\frac{4}{(2x-1)(2x+1)}=-\frac{1}{x^2-0.25}$, $f'(0)=4$

Many students were able to find $f'(x)$. Some did not substitute $x=0$ into the derivative. A common incorrect answer was $f'(0)=0$.

ii. State the maximal dom

Marks	0	1	Average
%	43	57	0.6

$\left(-\frac{1}{2},\frac{1}{2}\right)$

Common incorrect answers were $\left(-\frac{1}{2},0\right)\cup\left(0,\frac{1}{2}\right)$, $R\backslash\left(-\frac{1}{2},\frac{1}{2}\right)$, $\left[-\frac{1}{2},\frac{1}{2}\right]$, $\left[0,\frac{1}{2}\right]$, $(-\infty,\infty)$ and $\left(0,\frac{1}{2}\right)$

c. Show that $f(x) + f(-x)$

Marks	0	1	Average
%	28	72	0.7

$f(x)+f(-x)=\log_e\left(x+\frac{1}{2}\right)-\log_e\left(\frac{1}{2}-x\right)+\log_e\left(-x+\frac{1}{2}\right)-\log_e\left(\frac{1}{2}+x\right)=0$

Some students substituted $-x$ incorrectly. Others substituted a value for x.

$\frac{x+\frac{1}{2}}{1}\times\frac{-x+\frac{1}{2}}{1}=0$ was occasionally seen.

d. Find the domain and the rule of f^{-1}, the inverse of f. (3 marks)

Marks	0	1	2	3	Average
%	12	14	23	51	2.2

Let $y=f(x)$, inverse swap x and y, solve $x=\log_e\left(y+\frac{1}{2}\right)-\log_e\left(\frac{1}{2}-y\right)$ for y,

$f^{-1}(x)=\frac{1}{2}-\frac{1}{e^x+1}=\frac{e^x-1}{2(e^x+1)}$, the domain is R

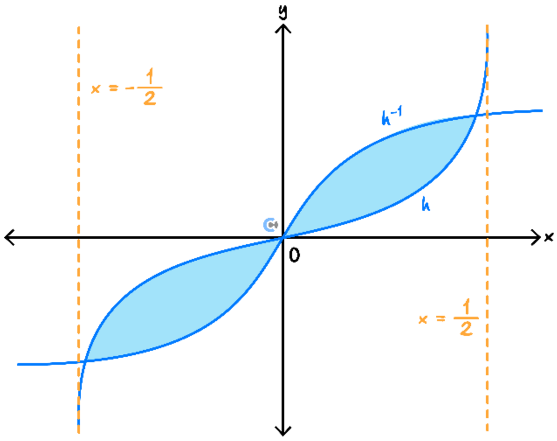
Many students were able to swap x and y. Some wrote $f^{-1}(x)=\frac{1}{2}\tan\left(\frac{x}{2}\right)$ instead of $f^{-1}(x)=\frac{1}{2}\tanh\left(\frac{x}{2}\right)$. The tanh function is not part of the study design but the output on some students' technology gave this function and it is correct. Other students did not find the domain. Some found $\frac{1}{f(x)}$. Some students did not use their technology and tried to find the inverse function by hand. This would have been time consuming.

Let h be the function $h:\left(-\frac{1}{2},\frac{1}{2}\right)\to R, h(x)=\frac{1}{k}\left(\log_e\left(x+\frac{1}{2}\right)-\log_e\left(\frac{1}{2}-x\right)\right)$, where $k\in R$ and $k>0$.

The inverse function of h is defined by $h^{-1}: R\to R, h^{-1}(x)=\frac{e^{kx}-1}{2(e^{kx}+1)}$.

The area of the regions bound by the functions h and h^{-1} can be expressed as a function, $A(k)$.

The graph below shows the relevant area shaded.



You are not required to find or define $A(k)$.

e. Determine the range of values of k such that $A(k) > 0$. (1 mark)

Marks	0	1	Average
%	94	6	0.1

$k>4$

This question was not answered well. Some incorrect responses were $k>0$ and $4<k<33$.

Question 169 (12 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2023
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2023/2023MM2-w.pdf

Consider the function $g : R \to R, g(x) = 2^x + 5$.

a. State the value of $\lim_{x \to -\infty} g(x)$. (1 mark)

Marks	0	1	Average
%	25	75	0.7

5

This question was done well. Some students did not attempt the question and appear not to have recognised the notation $\lim_{x \to -\infty} g(x)$. A common incorrect answer was 6.

b. The derivative, $g'(x)$, can be expressed in the form $g'(x) = k \times 2^x$.
Find the real number k. (1 mark)

Marks	0	1	Average
%	15	85	0.9

$\log_e(2)$ or $\ln(2)$

This question was done well. Some students did not include the base.

c.

i. Let a be a real number. Find, in terms of a, the equation of the tangent to g at the point $(a, g(a))$. (1 mark)

Marks	0	1	Average
%	48	52	0.5

$y = 2^a \log_e(2)x - (a\log_e(2) - 1) \times 2^a + 5$ or $y = 2^a \log_e(2)x - a2^a \log_e(2) + 2^a + 5$

An equation was required. There were many transcription errors such as $y = 2^a \ln(2) - 2^a a \ln(2) + 2a - 5$ and $y = 2^a \ln(2) - 2^a \ln(2) + 2^a + 5$. Some students attempted to find the equation by hand, making algebraic errors.

ii. Hence, or otherwise, find the equation of the tangent to g that passes through the origin, correct to three decimal places. (2 marks)

Marks	0	1	2	Average
%	52	34	15	0.6

$0 = -a2^a \log_e(2) + 2^a + 5$, $a = 2.617\,84\ldots$, $y = 4.255x$

Some students did not substitute $(0, 0)$ into the correct equation. Many misread the question and found the equation of the tangent line at $x = 0$, giving $y = 0.693x + 6$ as the answer. Some substituted $a = 0$ rather than $x = 0$ into their equation. $y = 4.255x + 8.14\text{E}-10$ was often seen.

Let $h: R \to R, h(x) = 2^x - x^2$.

d. Find the coordinates of the point of inflection for h, correct to two decimal places. (1 mark)

Marks	0	1	Average
%	42	58	0.6

$(2.06, -0.07)$

Many students gave the coordinates of the stationary points $(0.49, 1.16)$ and $(3.21, -1.05)$ rather than the coordinates of the point of inflection. There were some rounding errors.

e. Find the largest interval of x-values for which h is strictly decreasing.
Give your answer correct to two decimal places. (1 mark)

Marks	0	1	Average
%	65	35	0.4

$[0.49, 3.21]$

Round brackets were often seen; these were incorrect as the largest interval of x values was required, which included the interval endpoints. In some cases, it was impossible to determine whether the student meant round or square brackets. Another incorrect response was $(-\infty, 0.49] \cup [3.21, \infty)$. These students have incorrectly interpreted the question requirements as asking for intervals where the function is strictly increasing.

f. Apply Newton's method, with an initial estimate of $x_0 = 0$, to find an approximate x-intercept of h.

Write the estimates x_1, x_2 and x_3 in the table below, correct to three decimal places. (2 marks)

Marks	0	1	2	Average
%	36	10	54	1.2

$x_1 = -1.443$, $x_2 = -0.897$, $x_3 = -0.773$

Many students were familiar with Newton's method. Answers were required to three decimal places. Some students only had one correct answer. Others had rounding errors.

x_3	

g. For the function h, explain why a solution to the equation $\log_e(2) \times (2^x) - 2x = 0$ should not be used as an initial estimate x_0 in Newton's method. (1 mark)

Marks	0	1	Average
%	79	21	0.2

The solutions to $\log_e(2) \times 2^x - 2x = 0$ will give the x values of the turning points of the graph. The tangents to the graph will be horizontal lines and $h'(x) = 0$. Hence, $x_{n+1} = x_n - \dfrac{h(x)}{h'(x)}$ will be undefined.

h. There is a positive real number n for which the function $f(x) = n^x - x^n$ has a local minimum on the x-axis.

Find this value of n. (2 marks)

Marks	0	1	2	Average
%	86	12	3	0.2

$f(x)=0$ and $f'(x)=0$, $n=e$

This question was not done well. Many students indicated that $f'(x)=0$ but did not combine it with $f(x)=0$. Some formulated the question correctly but did not provide an answer. Others found an approximate value for the answer such as $n=2.7$. An exact answer was required.

Question 170 (12 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/2024MM2-w.pdf

Consider the function $f: R \to R, f(x) = (x+1)(x+a)(x-2)(x-2a)$ where $a \in R$.

a. State, in terms of a where required, the values of x for which $f(x) = 0$. (1 mark)

Marks	0	1	Average
%	15	85	0.9

$-1, -a, 2, 2a$

This question was answered well.

b. Find the values of a for which the graph of $y = f(x)$ has:

i. Exactly three x-intercepts. (2 marks)

Marks	0	1	2	Average
%	29	61	10	0.8

$-2, -\frac{1}{2}, 0$

Some of the values were often missing.

ii. Exactly four x-intercepts. (1 mark)

	0	1	Average
%	69	31	0.3

$R \setminus \left\{-2, -\frac{1}{2}, 0, 1\right\}$

This question was not answered well. Some students did not exclude 1.

c. Let g be the function $g: R \to R, g(x) = (x+1)^2(x-2)^2$, which is the function f where $a = 1$.

i. Find $g'(x)$. (1 mark)

Marks	0	1	Average
%	5	95	1.0

$g'(x) = 2(x-2)(x+1)(2x-1) = 4x^3 - 6x^2 - 6x + 4$

ii. Find the coordinates of the local maximum of g. (1 mark)

Marks	0	1	Average
%	24	76	0.8

$\left(\frac{1}{2}, \frac{81}{16}\right)$ or $(0.5, 5.0625)$

Exact answers were required. $(0.5, 5.06)$ was sometimes seen.

iii. Find the values of x for which $g'(x) > 0$. (1 mark)

Marks	0	1	Average
%	29	71	0.7

$\left(-1, \frac{1}{2}\right) \cup (2, \infty)$

This question was answered well. Some students used square brackets instead of round brackets. Others put $\cap$ instead of $\cup$.

iv. Consider the two tangent lines to the graph of $y = g(x)$ at the points where:

$$x = \frac{-\sqrt{3}+1}{2} \text{ and } x = \frac{\sqrt{3}+1}{2}.$$

Determine the coordinates of the point of intersection of these two tangent lines. (2 marks)

Marks	0	1	2	Average
%	32	17	51	1.2

$y_1 = 3\sqrt{3}x - \frac{3(2\sqrt{3}-9)}{4}$, $y_2 = -3\sqrt{3}x + \frac{3(2\sqrt{3}+9)}{4}$ or $y_1 = 3\sqrt{3}x - \frac{3\sqrt{3}}{2} + \frac{27}{4}$, $y_2 = -3\sqrt{3}x + \frac{3\sqrt{3}}{2} + \frac{27}{4}$

or $y_1 = \frac{3(\sqrt{3}(4x-2)+9)}{4}$, $y_2 = \frac{-3(\sqrt{3}(4x-2)-9)}{4}$, $\left(\frac{1}{2}, \frac{27}{4}\right)$ or $(0.5, 6.75)$

This question was answered reasonably well. A common incorrect answer was $\left(\frac{1}{2}, \frac{81}{16}\right)$.

d. Let g remain as the function $g: R \to R, g(x) = (x+1)^2(x-2)^2$, which is the function f where $a = 1$. Let h be the function $h: R \to R, h(x) = (x+1)(x-1)(x+2)(x-2)$, which is the function f where $a = -1$.

i. Using translations only, describe a sequence of transformations of h, for which its image would have a local maximum at the same coordinates as that of g. (1 mark)

Marks	0	1	Average
%	63	37	0.4

Translate $\frac{1}{2}$ unit to the right and $\frac{17}{16} = 1.0625$ units up.

Some students incorrectly translated to the left and down. Others had an incorrect value for the vertical translation such as $\frac{81}{16}$. Exact answers were required. 1.06 was sometimes seen.

ii. Using dilation and translations, describe a different sequence of transformations of h, for which its image would have both local minimums at the same coordinates as that of g. (2 marks)

Marks	0	1	2	Average
%	76	19	5	0.3

Dilate by a factor of $\frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$ from the vertical axis, translate $\frac{1}{2}$ unit to the right, translate $\frac{9}{4}$ units up

OR translate $\frac{\sqrt{10}}{6}$ units to the right, dilate by a factor of $\frac{3}{\sqrt{10}}$ from the vertical axis, translate $\frac{9}{4}$ units up.

This question was not done well. The vertical translation could be completed at any stage in the sequence. The other transformations had to be in the correct order.

Question 171 (11 marks)

A model for the temperature in a room, in degrees Celsius, is given by:

$$f(t) = \begin{cases} 12 + 30t & 0 \le t \le \frac{1}{3} \\ 22 & t > \frac{1}{3} \end{cases}$$

Where t represents the time in hours after a heater is switched on.

a. Express the derivative $f'(t)$ as a hybrid function. (2 marks)

Marks	0	1	2	Average
%	11	41	48	1.4

$f'(t) = \begin{cases} 30 & 0 \le t < \frac{1}{3} \\ 0 & t > \frac{1}{3} \end{cases}$ or $f'(t) = \begin{cases} 30 & 0 < t < \frac{1}{3} \\ 0 & t > \frac{1}{3} \end{cases}$

Many students included $\frac{1}{3}$ in the domain.

b. Find the average rate of change in temperature predicted by the model between $t = 0$ and $t = \frac{1}{2}$.

Give your answer in degrees Celsius per hour. (1 mark)

Marks	0	1	Average
%	35	65	0.7

Average rate of change $= 20$

A common incorrect answer was $30°\text{C/h}$ where $\frac{1}{2}$ was substituted into $12 + 30t$, giving $\frac{27-12}{\frac{1}{2}} = 30$.

Some students incorrectly found the average value of the temperature.

c. Another model for the temperature in the room is given by $g(t) = 22 - 10e^{-6t}, t \ge 0$.

i. Find the derivative $g'(t)$. (1 mark)

Marks	0	1	Average
%	5	95	1.0

$g'(t) = 60e^{-6t}$

This question was answered well.

ii. Find the value of t for which $g'(t) = 10$.

Give your answer correct to three decimal places. (1 mark)

Marks	0	1	Average
%	15	85	0.9

$g'(t) = 10$, $t = 0.299$

Some students gave the exact answer. There were some rounding errors: $t = 0.298$ and $t = 0.300$ were occasionally seen.

d. Find the time $t \in (0, 1)$ when the temperatures predicted by the models f and g are equal.

Give your answer correct to two decimal places. (1 mark)

Marks	0	1	Average
%	37	63	0.7

$f(t) = g(t)$, $t = 0.27$

Some students incorrectly included $t = 0$ but $t \in (0,1)$.

e. Find the time $t \in (0, 1)$ when the difference between the temperatures predicted by the two models is the greatest.

Give your answer correct to two decimal places. (1 mark)

Marks	0	1	Average
%	71	29	0.3

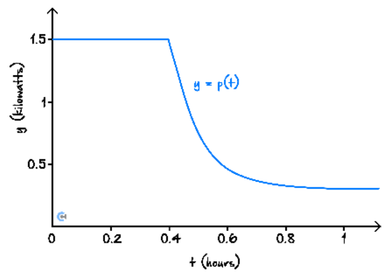
$t = 0.12$

A common incorrect answer was $\frac{1}{3}$, which is the time when $f(t) - g(t)$ is a maximum. The maximum difference occurs when $g(t) - f(t)$ is a maximum or $|f(t) - g(t)| = |g(t) - f(t)|$ is a maximum.

f. The amount of power, in kilowatts, used by the heater t hours after it is switched on, can be modelled by the continuous function p, whose graph is shown below.

$$p(t) = \begin{cases} 1.5 & 0 \le t \le 0.4 \\ 0.3 + Ae^{-10t} & t > 0.4 \end{cases}$$

The amount of energy used by the heater, in kilowatt-hours, can be estimated by evaluating the area between the graph of $y = p(t)$ and the t-axis.



i. Given that $p(t)$ is continuous for $t \ge 0$, show that $A = 1.2e^4$. (1 mark)

Marks	0	1	Average
%	45	55	0.6

$0.3 + Ae^{-10\times0.4} = 1.5$, $0.3 + Ae^{-4} = 1.5$, $Ae^{-4} = 1.2$, $A = 1.2e^4$

Most students substituted the correct values into the equation. Students must make sure they show adequate working for 'show that' questions.

ii. Find how long it takes after the heater is switched on until the heater has used 0.5-kilowatt hours of energy.
Give your answer in hours. (1 mark)

Marks	0	1	Average
%	63	37	0.4

$1.5t = 0.5$, $t = \frac{1}{3}$

An exact answer was required. Some students solved $p(t) = 0.5$ for t or found $p(0.5)$.

iii. Find how long it takes after the heater is switched on until the heater has used 1-kilowatt hour of energy.
Give your answer in hours, correct to two decimal places. (2 marks)

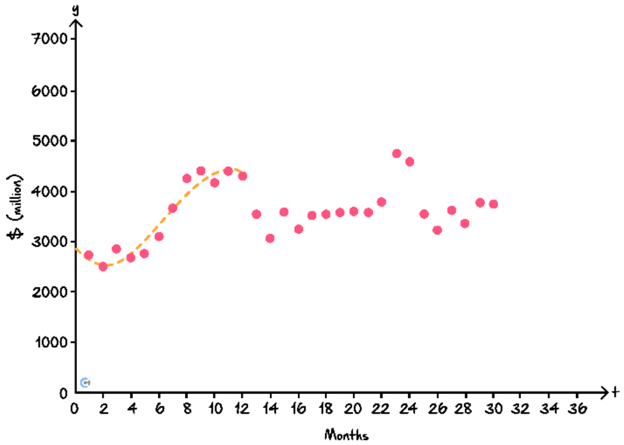
Marks	0	1	2	Average
%	61	11	27	0.7

$\int_{0.4}^{a} \left(0.3 + Ae^{-10t}\right)dt = 0.4$, $a = 1.33$

Some students solved $p(t) = 1$ for t or found $p(1)$. Others just gave the answer. For questions worth more than 1 mark, appropriate working must be shown. Some students transcribed A incorrectly into the function.

Question 172 (11 marks)

The points shown in the chart below represent monthly online sales in Australia. The variable y represents sales in millions of dollars. The variable t represents the month when the sales were made, where $t = 1$ corresponds to January 2021, $t = 2$ corresponds to February 2021 and so on.



Source : Australian Bureau of Statistics, Retail Trade, Australia, December 2023

a. A cubic polynomial $p : (0, 12] \to R, p(t) = at^3 + bt^2 + ct + d$ can be used to model monthly online sales in 2021.
The graph of $y = p(t)$ is shown as a dashed curve on the set of axes above.
It has a local minimum at $(2, 2500)$ and a local maximum at $(11, 4400)$.

i. Find, correct to two decimal places, the values of a, b, c and d. (3 marks)

Marks	0	1	2	3	Average
%	30	16	12	42	1.7

$p(2) = 2500$, $p(11) = 4400$, $p'(2) = 0$, $p'(11) = 0$

$8a + 4b + 2c + d = 2500$, $12a + 4b + c = 0$, $1331a + 121b + 11c + d = 4400$, $363a + 22b + c = 0$

$a = -5.21$, $b = 101.65$, $c = -344.03$, $d = 2823.18$

Some students only wrote the answers without showing adequate working. Others had only two correct equations.

Some had $p'(2) = 2500$ and $p'(11) = 4400$. Others rounded 101.646... to 101.64. The value of d was sometimes missing.

ii. Let $q: (12,24] \to R, q(t) = p(t - h) + k$ be a cubic function obtained by translating p, which can be used to model monthly online sales in 2022. Find the values of h and k such that the graph of $y = q(t)$ has a local maximum at $(23, 4750)$. (2 marks)

Marks	0	1	2	Average
%	65	9	26	0.6

$h = 12$, $k = 350$

Many students did not realise they only needed to translate the point $(11, 4400)$ to the point $(23, 4750)$.

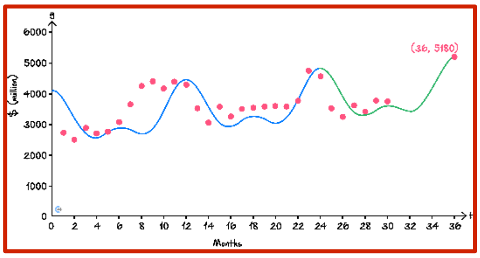
Some gave solutions outside the domain. Others translated the local minimum, giving $h = 21$, $k = 2250$ as their answers. $h = -12$, $k = 350$ was sometimes seen.

b. Another function f can be used to model monthly online sales, where:

$$f:(0,36] \to R, f(t) = 3000 + 30t + 700\cos\left(\frac{\pi t}{6}\right) + 400\cos\left(\frac{\pi t}{3}\right)$$

Part of the graph of f is shown on the axes below.

Marks	0	1	2	Average
%	19	18	63	1.5



i. Complete the graph of f on t
Label the endpoint at $t = 3$

More care needs to be taken when sketching graphs. The turning points and endpoint needed to be in the correct positions. Some students labelled the endpoint incorrectly. Round brackets are required around the coordinates. Others made the graph discontinuous at $t = 24$.

ii. The function f predicts that every 12 months, monthly online sales increase by n million dollars. Find the value of n. (1 mark)

Marks	0	1	Average
%	41	59	0.6

360

A common incorrect answer was 30 which is found by calculating $f'(12)$.

iii. Find the derivative $f'(t)$. (1 mark)

Marks	0	1	Average
%	23	77	0.8

$$f'(t) = -\frac{400\pi\sin\left(\frac{\pi t}{3}\right)}{3} - \frac{350\pi\sin\left(\frac{\pi t}{6}\right)}{3} + 30$$

This question was answered well. There were some transcription errors: π was often omitted and brackets were not used well.

iv. Hence, find the maximum instantaneous rate of change for the function f, correct to the nearest million dollars per month, and the values of t in the interval $(0, 36]$ when this maximum rate occurs, correct to one decimal place. (2 marks)

Marks	0	1	2	Average
%	65	14	21	0.6

Maximum instantaneous rate of change $= 725$

$t = 10.2$, $t = 22.2$, $t = 34.2$

Many students gave extra t values or only one t value. Others did not give the maximum instantaneous rate of change or found the minimum instantaneous rate of change.

Question 173 (11 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2017
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2017/nht/2017MM2-nht-w.pdf

The temperature, T°C, in an office, is controlled. For a particular weekday, the temperature at time t, where t is the number of hours after midnight, is given by the function:

$$T(t) = 19 + 6\sin\left(\frac{\pi}{12}(t-8)\right), 0 \leq t \leq 24$$

a. What are the maximum and minimum temperatures in the office? (2 marks)

$T(t) = 19 + 6\sin\left(\frac{\pi}{12}(t-8)\right)$, range of function is $[-6+19,\ 6+19]$ = $[13,\ 25]$, minimum temperature is 13 °C, maximum temperature is 25 °C

b. What is the temperature in the office at 6.00 AM? (1 mark)

$T(6) = 16$ °C

c. Most of the people working in the office arrive at 8.00 AM.

What is the temperature in the office when they arrive? (1 mark)

$T(8) = 19$ °C

d. For how many hours of the day is the temperature greater than or equal to 19°C? (2 marks)

Solve $T(t) \geq 19$ °C, $8 \leq t \leq 20$, $20 - 8 = 12$ hours

e. What is the average rate of change of the temperature in the office between 8.00 AM and noon? (2 marks)

Average rate of change = $\frac{T(12) - T(8)}{12 - 8} = \frac{3\sqrt{3}}{4}$ °C/hr

f.

i. Find $T'(t)$. (1 mark)

$T'(t) = \frac{\pi}{2}\cos\left(\frac{\pi}{12}(t-8)\right)$ or $T'(t) = -\frac{\pi}{2}\cos\left(\frac{\pi}{12}t + \frac{\pi}{3}\right)$, or equivalent

ii. At what time of the day is the temperature in the office decreasing most rapidly? (2 marks)

Find the minimum of the derivative, decreasing most rapidly at 8.00 pm or 20 hours.

Space for Personal Notes

Question 174 (13 marks)

Let $f: R \to R$, where $f(x) = (x-2)^2(x-5)$.

a. Find $f'(x)$. (1 mark)

$f: R \to R$, where $f(x) = (x-2)^2(x-5)$, $f'(x) = 3(x-4)(x-2)$, or equivalent

b. For what values of x is $f'(x) < 0$? (1 mark)

Solve $f'(x) < 0$, $2 < x < 4$

c.

i. Find the gradient of the line segment joining the points on the graph of $y = f(x)$ where $x = 1$ and $x = 5$. (1 mark)

$f(1) = -4$, $f(5) = 0$, $\dfrac{f(5) - f(1)}{5 - 1} = 1$

ii. Show that the midpoint of the line segment in **part c.i.** also lies on the graph of $y = f(x)$. (2 marks)

Midpoint $\left(\frac{5+1}{2}, \frac{-4+0}{2}\right) = (3, -2)$, $f(3) = -2$ hence midpoint lies on the graph of $y = f(x)$

iii. Find the values of x for which the gradient of the tangent to the graph of $y = f(x)$ is equal to the gradient of the line segment joining the points on the graph where $x = 1$ and $x = 5$. (2 marks)

Solve $f'(x) = 1$, $x = \frac{9+2\sqrt{3}}{3}$ or $x = \frac{9-2\sqrt{3}}{3}$

Let $g: R \to R$, where $g(x) = (x-2)^2(x-a)$, where $a \in R$.

d. The coordinates of the stationary points of g are $P(2,0)$ and $Q(p(a+1), q(a-2)^3)$, where p and q are rational numbers.

Find the values of p and q. (2 marks)

$g: R \to R$, where $g(x) = (x-2)^2(x-a)$, $g'(x) = 0$, $x = 2$ or $x = \frac{2(a+1)}{3}$,

$p = \frac{2}{3}$, $g\left(\frac{2(a+1)}{3}\right) = -\frac{4}{27}(a-2)^3$, $q = -\frac{4}{27}$

e. Show that the gradient of the tangent to the graph of $y = g(x)$ at the point $(a, 0)$ is positive for $a \in R\backslash\{2\}$. (1 mark)

$g'(a) = (a-2)^2$, $(a-2)^2 \geq 0$, when $a = 2$, $g'(x) = 0$, gradient of the tangent is positive for $a \in R\backslash\{2\}$

f.

i. Find the coordinates of another point where the tangent to the graph of $y = g(x)$ is parallel to the tangent at $x = a$. (2 marks)

$$g'(x) = (a-2)^2, \left(\frac{8-a}{3}, -\frac{4}{27}(a-2)^3\right)$$

ii. Hence, find the distance between this point and point Q when $a > 2$. (1 mark)

$Q\left(\frac{2(a+1)}{3}, -\frac{4}{27}(a-2)^3\right)$ and $\left(\frac{8-a}{3}, -\frac{4}{27}(a-2)^3\right)$,

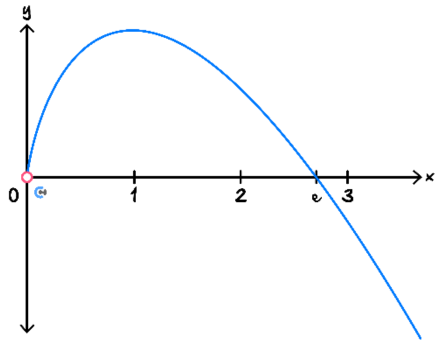
distance = $\sqrt{\left(-\frac{4}{27}(a-2)^3 - -\frac{4}{27}(a-2)^3\right)^2 + \left(\frac{8-a}{3} - \frac{2(a+1)}{3}\right)^2} = a-2$

Space for Personal Notes

Question 175 (12 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2018
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2018/nht/2018MM2-nht-w.pdf

Let $f: (0, \infty) \to R, f(x) = x - x\log_e(x)$. Part of the graph of f is shown below.



a. Find the values of x for which:

i. $-1 < f'(x) < -\frac{1}{2}$. (2 marks)

$f'(x) = -\log_e(x)$, $\sqrt{e} < x < e$

ii. $\frac{1}{2} < f'(x) < 1$. (1 mark)

$\frac{1}{e} < x < \frac{1}{\sqrt{e}}$

b.

i. Find the equation of the tangent to the graph of f at the point $(a, f(a))$ in the form $y = mx + c$. (1 mark)

$y = -\log_e(a)x + a$

ii. Find the coordinates of the point of intersection of the tangent to the graph of f at $x = a$ and the tangent to the graph of f at $x = \frac{1}{a}$. (2 marks)

Tangent at $x = \frac{1}{a}$, $y = -\log_e\left(\frac{1}{a}\right)x + \frac{1}{a}$, $-\log_e(a)x + a = -\log_e\left(\frac{1}{a}\right)x + \frac{1}{a}$, $\left(\frac{a^2 - 1}{2a\log_e(a)}, \frac{a^2 + 1}{2a}\right)$

iii. Hence, find the coordinates of the point of intersection of the tangents to the graph of f at $x = e$ and $x = \frac{1}{e}$. Express each coordinate in terms of e. (1 mark)

$$\left(\frac{e^2-1}{2e}, \frac{e^2+1}{2e}\right)$$

c.

i. For a value of $b > e$, the tangent to f at the point $(b, f(b))$ and the tangent to f at the point $(2, f(2))$ intersect the x-axis at the same point.

Find the value of b. (2 marks)

$$\frac{2}{\log_e(2)} = \frac{b}{\log_e(b)}, \ b = 2 \text{ or } 4, \text{ since } b > e, \ b = 4$$

ii. If the tangent to f at the point $(p, f(p))$, where $1 < p < e$, and the tangent to f at the point $(q, f(q))$, where $q > e$, intersect on the x-axis, show that $p^q = q^P$. (2 marks)

$$\frac{p}{\log_e(p)} = \frac{q}{\log_e(q)}, \ p\log_e(q) = q\log_e(p), \ \log_e\left(q^p\right) = \log_e\left(p^q\right), \ q^p = p^q$$

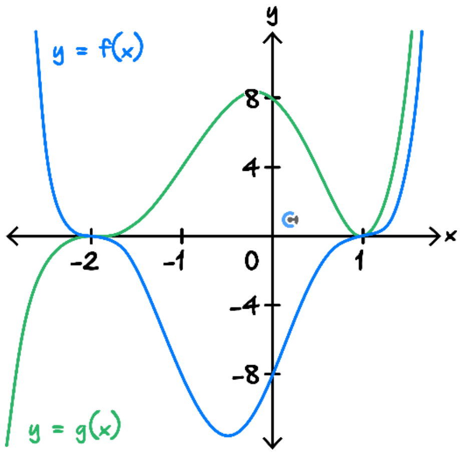
d. Find the equation of the tangent to the graph of f at the point where $x = e^{\frac{1}{2}}$. (1 mark)

$$y = -\frac{x}{2} + e^{\frac{1}{2}}$$

Question 176 (9 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2019
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2019/NHT/2019MM2-nht-w.pdf

Parts of the graphs of $f(x) = (x-1)^3(x+2)^3$ and $g(x) = (x-1)^2(x+2)^3$ are shown on the axes below.



The two graphs intersect at three points, $(-2, 0)$, $(1, 0)$ and (c, d). The point (c, d) is not shown in the diagram above.

a. Find the values of c and d. (2 marks)

$c = 2,\ d = 64$

b. Find the values of x such that $f(x) > g(x)$. (1 mark)

$(-\infty, -2) \cup (2, \infty)$

c. State the values of x for which:

i. $f'(x) > 0$. (1 mark)

$\left(-\frac{1}{2}, 1\right) \cup (1, \infty)$

ii. $g'(x) > 0$. (1 mark)

$(-\infty, -2) \cup \left(-2, -\frac{1}{5}\right) \cup (1, \infty)$

d. Show that $f(1+m) = f(-2-m)$ for all m. (1 mark)

$f(1+m) = m^3(m+3)^3$, $f(-2-m) = (-m-3)^3(-m)^3 = m^3(m+3)^3$, so $f(1+m) = f(-2-m)$

e. Find the values of h such that $g(x+h) = 0$ has exactly one negative solution. (2 marks)

$-2 < h \leq 1$

f. Find the values of k such that $f(x) + k = 0$ has no solutions. (1 mark)

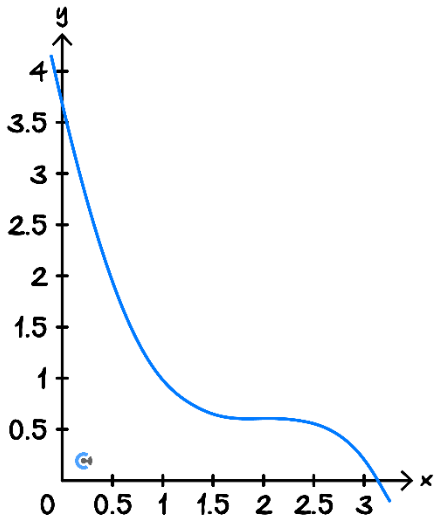
$k > \frac{729}{64}$

Question 177 (10 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2022
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2022/NHT/2022mm2-nht-w.pdf

Let $f: R \to R, f(x) = -\frac{2}{5}(x-2)^3 + \frac{3}{5}$.

Part of the graph of f is shown below.



a. Find $f'(x)$, the derivative of f, with respect to x. (1 mark)

$f'(x) = -\frac{6}{5}(x-2)^2$ or $f'(x) = -\frac{6}{5}x^2 + \frac{24x}{5} - \frac{24}{5}$

b. Give the coordinates of the stationary point of f. (1 mark)

$\left(2, \frac{3}{5}\right)$

c. The graph of f has a tangent with a gradient of $-\frac{6}{5}$ when $x = 1$.

The graph of f also has a tangent with a gradient of $-\frac{6}{5}$ at another point, D.

i. Show that the x-coordinate of D is 3. (1 mark)

$f'(x) = -\frac{6}{5}(x-2)^2 = -\frac{6}{5}$,

$(x-2)^2 = 1$,

$x - 2 = \pm 1$,

$x = 1$ or $x = 3$, since $x = 1$ is given, the other point D is at $x = 3$.

ii. Determine the equation of the tangent that touches the graph of f at point D. (1 mark)

$$y = \frac{19}{5} - \frac{6x}{5}$$

iii. The tangent to f at point D intersects the graph of f at another point, M.
Give the coordinates of point M. (2 marks)

Solving $f(x) = \frac{19}{5} - \frac{6x}{5}$

$\left(0, \frac{19}{5}\right)$ or $(0, 0.38)$

iv. Find the obtuse angle, in degrees, that the tangent to f at point D makes with the positive direction of the horizontal axis. Give your answer correct to one decimal place. (1 mark)

$$129.8°$$

v. The graph has two regions:

The first region is bounded by the graph of f and the tangent to f at point D.

The second region is bounded by the graph of f, the tangent to f at point D and the horizontal axis.

Find the total area of the two regions. Give your answer correct to four decimal places. (3 marks)

$$A_{Total} = \int_0^3 \left(\frac{19}{5} - \frac{6x}{5} - f(x)\right)dx + \int_3^{\frac{19}{6}} \left(\frac{19}{5} - \frac{6x}{5}\right)dx - \int_3^{\frac{3^{\frac{1}{3}} \times 2^{\frac{2}{3}} + 4}{2}} f(x)\,dx = 2.7015,$$

or

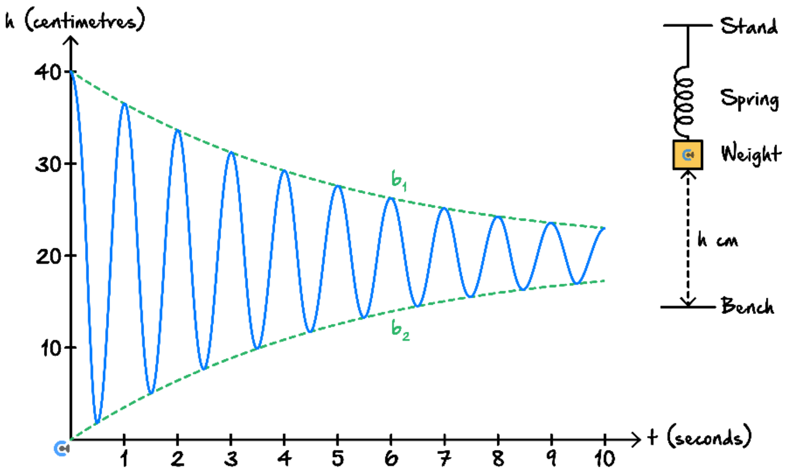
$$A_{Total} = \int_0^{\frac{3^{\frac{1}{3}} \times 2^{\frac{2}{3}} + 4}{2}} \left(\frac{19}{5} - \frac{6x}{5} - f(x)\right)dx + \int_{\frac{3^{\frac{1}{3}} \times 2^{\frac{2}{3}} + 4}{2}}^{\frac{19}{6}} \left(\frac{19}{5} - \frac{6x}{5}\right)dx = 2.7015$$

Question 178 (9 marks)

A spring with a weight attached is suspended from a stand. The base of the weight is 40 cm above a bench.
The spring is released and moves vertically up and down above the surface of the bench, such that the height of the base of the weight above the bench over the next 10 seconds is given by the function:

$$h(t) = 20e^{-\frac{1}{5}}\cos(2\pi t) + 20, \quad 0 \le t \le 10$$

Where t is the time, measured in seconds. A graph of the function h over the first 10 seconds is shown below.



The dashed curve b_1 lies above the graph of h and the dashed curve b_2 lies below the graph of h. Both b_1 and b_2 bound the graph of h.

The dashed curve b_1, has the equation $b_1(t) = 20e^{-\frac{t}{5}} + 20$.

a. State the equation of the dashed curve b_2. (1 mark)

$$b_2(t) = -20e^{-\frac{t}{5}} + 20$$

b. Find the average value of the height, in centimetres, of the base of the weight above the bench over the first 10 seconds. Give your answer correct to two decimal places. (2 marks)

$$\frac{1}{10-0}\int_0^{10} h(t)\,dt$$

$= 20.01$

c.

i. Write down the rule for the derivative of h. (1 mark)

$$h'(t) = -4e^{-\frac{t}{5}}\left(\cos(2\pi t) + 10\pi\sin(2\pi t)\right)$$

ii. Find the time, in seconds, and the height above the surface of the bench, in centimetres, of the point of maximum positive rate of change in h over the first 10 seconds. Write your answer as a coordinate pair, correct to one decimal place. (3 marks)

Max of $h'(t)$, (0.7, 18.9)

d. Determine the total distance travelled by the base of the weight over the first 2 seconds of its motion. Give your answer correct to the nearest centimetre. (2 marks)

Method 1:

$(40-1.894...)+(36.382...-1.894...)+(36.382...-5.176...)+(33.413...-5.176...)+(33.413...-33.406...)=$ 132

Method 2:

$\int_0^2 |h'(t)|\,dt = 132$

Space for Personal Notes

Question 179 (13 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2023
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2023/NHT/2023MM2-nht-w.pdf

The amount of caffeine present in Kim's body after they drink espresso coffee can be modelled mathematically. Students suggest that the amount of caffeine, C in milligrams, in Kim's body t hours after consuming an espresso coffee can be modelled by the function $C(t) = 65e^{-\frac{1}{8}}$.

a. How much caffeine will be present in Kim's body 2 hours after they consume an espresso? Give your answer in milligrams, correct to one decimal place. (1 mark)

50.6

b. How long will it take for the amount of caffeine in Kim's body to reach 10 milligrams after drinking an espresso? Give your answer in hours and minutes, correct to the nearest minute. (2 marks)

Solving $C(t) = 10$ for t

898 minutes which is 14 hours 58 minutes

c. At what rate is the amount of caffeine in Kim's body decaying 4 hours after they drink an espresso? Give your answer in milligrams per hour in the form $\frac{a}{b\sqrt{e}}$, where a and b are positive integers. (2 marks)

$$C'(4) = -\frac{65}{8}e^{-\frac{1}{2}}$$

$$\frac{a}{b\sqrt{e}} = \frac{65}{8\sqrt{e}}$$

Kim consumes another espresso coffee 4 hours after consuming the first.
The students then suggest that a more appropriate model for the absorption of caffeine, in milligrams, in Kim's body t hours after they consume the first espresso is:

$$C_2(t) = \begin{cases} 65e^{-\frac{1}{8}} & 0 \le t < 4 \\ 65\left(\frac{1-e}{1-\sqrt{e}}\right)e^{-\frac{1}{8}} & t \ge 4 \end{cases}$$

d. When $t \geq 4$, is the function C_2 strictly increasing, strictly decreasing or neither? (1 mark)

Strictly decreasing

e. Show that the function C_2 is not continuous for $t > 0$. (1 mark)

C_2 is not continuous at $t=4$ and so is not continuous for $t>0$

$65e^{-\frac{1}{2}} \neq 65\left(\frac{1-e}{1-\sqrt{e}}\right)e^{-\frac{1}{2}}$, $104.4 \neq 39.4$

f. Using C_2, find the maximum amount of active caffeine in Kim's body and the time at which this level was reached. Give the maximum amount of caffeine, in milligrams, correct to one decimal place. (2 marks)

$t=4$

$C_2(4) = 104.4$

g. Find the derivative $C_2'(t)$, giving your answer as a hybrid function that includes the relevant domains. (2 marks)

$$C_2'(t) = \begin{cases} -\frac{65}{8}e^{-\frac{t}{8}} & 0 < t < 4 \\ -\frac{65}{8}\left(\frac{1-e}{1-\sqrt{e}}\right)e^{-\frac{t}{8}} & t > 4 \end{cases}$$

h. Use the derivative $C_2'(t)$ to find the times during which the amount of active caffeine is decreasing by at least 8 milligrams per hour. Express your answer in interval notation, correct to one decimal place. (2 marks)

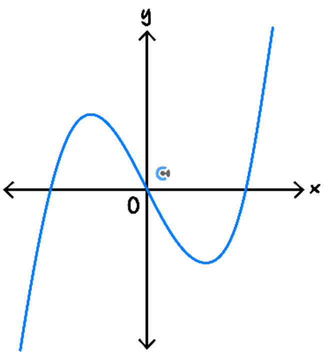
Solving $C_2'(t) \le -8$

$(0, 0.1] \cup (4, 7.9]$

Question 180 (10 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/NHT/2024MM2-nht-w.pdf

Consider the function $f: R \to R, f(x) = x^3 - px$, where $p \in R$.
Part of the graph of f is shown below when $p = 3$.



a. Find the values of the x-intercepts of f, when $p = 3$. (1 mark)

$x = 0, x = \pm\sqrt{3}$

b. Use the derivative f' to find the coordinates of the turning points of f, when $p = 3$. (2 marks)

Solving $f'(x) = 0$ or $3x^2 - 3 = 0$

$(-1, 2)$ and $(1, -2)$

c.

i. Find the value of p for which f would have exactly one stationary point. (1 mark)

$p = 0$

ii. Find the values of p for which f would not have any stationary points. (1 mark)

$p < 0$

d. The graph of f passes through the origin for all values of p.

i. Use calculus to show that the tangent line to f at the origin has the equation $y = -px$. (2 marks)

$f'(x) = 3x^2 - p$ gives $f'(0) = -p$

The tangent line is given by $y = -px + c$, where $c = 0$ as it goes through the origin

Therefore $y = -px$

ii. Find, in terms of p, the area of the region bounded by the function f, the line $y = -px$ and the line $x = p$, where $p > 0$. (2 marks)

$$A = \int_0^p (f(x) + px)\,dx$$

$$= \frac{p^4}{4}$$

iii. The expression for the area found in **part d.ii.** also gives the area bounded by a cubic function $y = kx^3$, the x-axis and the line $x = p$, where $p > 0$.

Find all possible values of k. (1 mark)

$k = \pm 1$

VCE Mathematical Methods ¾

Free 1-on-1 Support

Be Sure to Make The Most of These (Free) Services!

- Experienced Contour tutors (45+ raw scores, 99+ ATARs).
- For fully enrolled Contour students with up-to-date fees.
- After school weekdays and all-day weekends.

1-on-1 Video Consults	Text-Based Support
➤ Book via bit.ly/contour-methods-consult-2025 (or QR code below). ➤ One active booking at a time (must attend before booking the next).	➤ Message +61 440 138 726 with questions. ➤ Save the contact as "Contour Methods".

Booking Link for Consults
bit.ly/contour-methods-consult-2025

Number for Text-Based Support
+61 440 138 726