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VCE Mathematical Methods ¾
AOS 2 Revision [2.0]
Contour Check (Part 3)

Contour Check

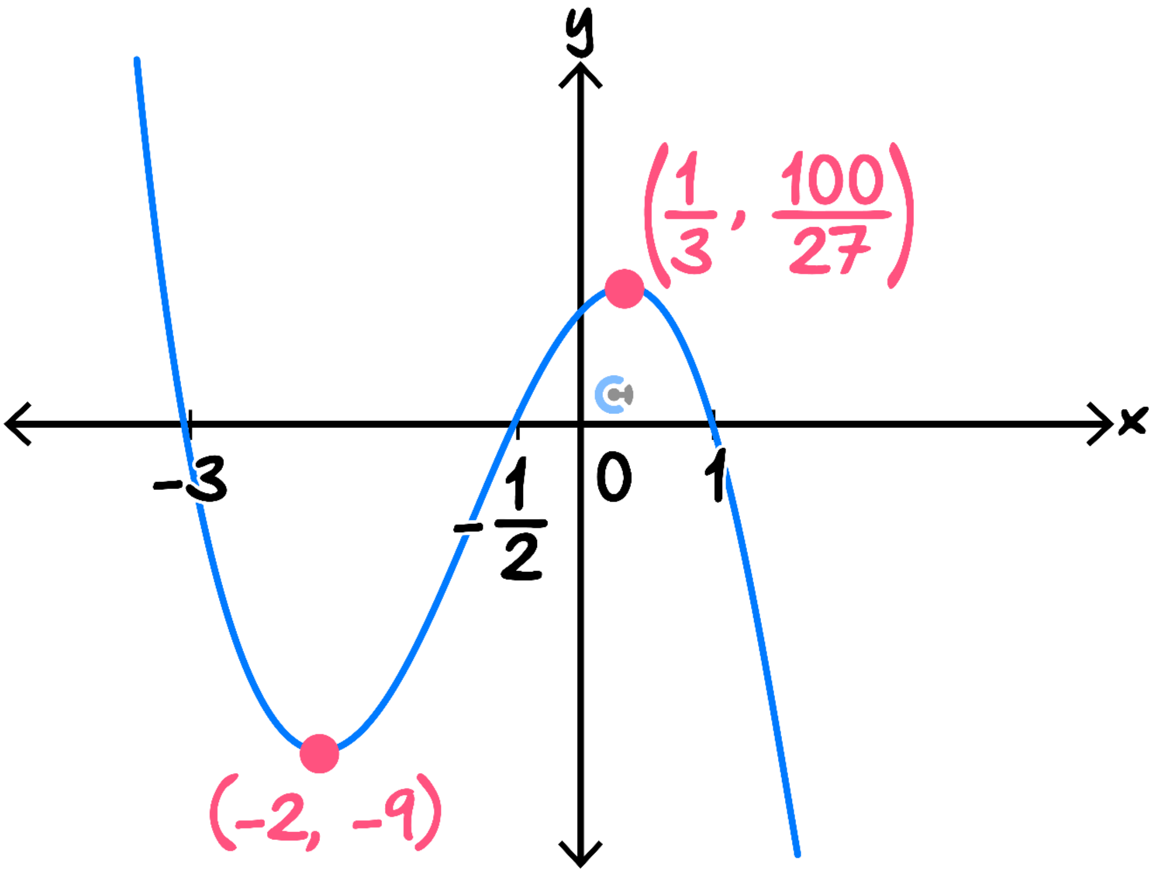
[2.1 - 2.7] - Exam 2 Overall (VCAA Qs) Pg 111-207	

Section I: [2.1 - 2.7] - Exam 2 Overall (Checkpoints) (338 Marks)

Question 96 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2016
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2016/2016MM2-w.pdf#page=3

Part of the graph $y = f(x)$ of the polynomial function f is shown below.



$f'(x) < 0$ for:

A. $x \in (-2,0) \cup \left(\frac{1}{3}, \infty\right)$

B. $x \in \left(-9, \frac{100}{27}\right)$

C. $x \in (-\infty, -2) \cup \left(\frac{1}{3}, \infty\right)$

D. $x \in \left(-2, \frac{1}{3}\right)$

E. $x \in (-\infty, -2) \cup (1, \infty)$

Question 97 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2016
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2016/2016MM2-w.pdf#page=3

The average rate of change of the function f with rule, $f(x) = 3x^2 - 2\sqrt{x+1}$, between $x = 0$ and $x = 3$ is:

A. 8

B. 25

C. $\frac{53}{9}$

D. $\frac{25}{3}$

E. $\frac{13}{9}$

Question 98 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2016
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2016/2016MM2-w.pdf#page=6

For the curve $y = x^2 - 5$, the tangent to the curve will be parallel to the line connecting the positive x-intercept and the y-intercept when x is equal to:

A. $\sqrt{5}$

B. 5

C. -5

D. $\frac{\sqrt{5}}{2}$

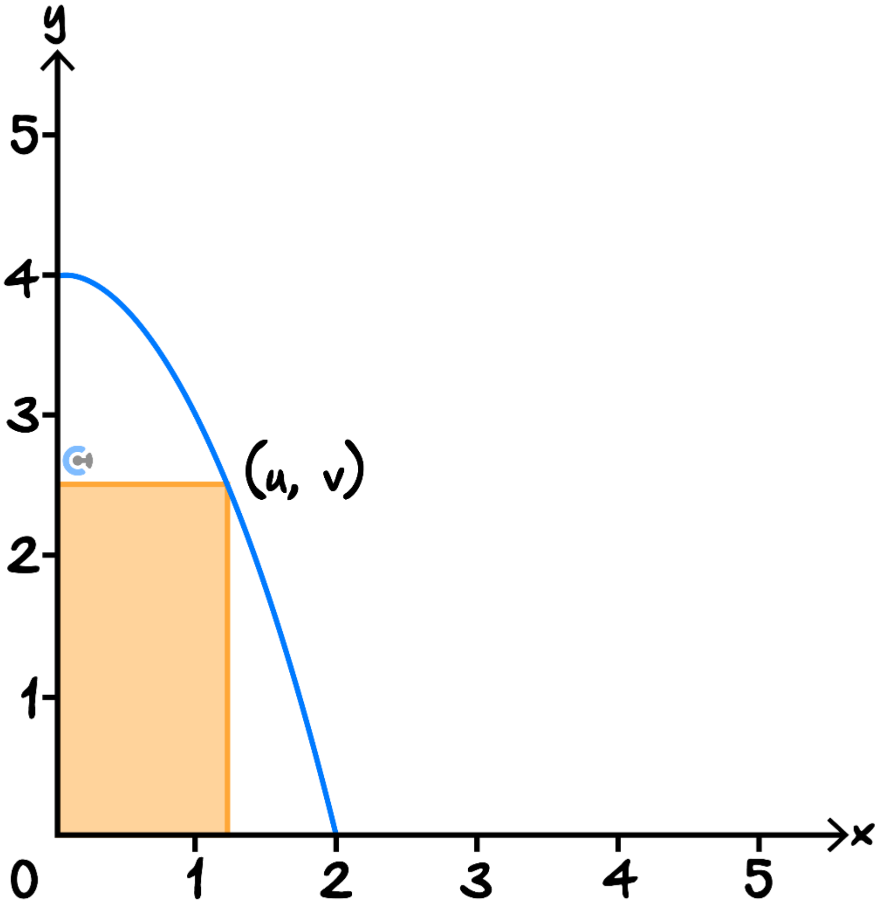
E. $\frac{1}{\sqrt{5}}$

Space for Personal Notes

Question 99 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2016
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2016/2016MM2-w.pdf#page=8

A rectangle is formed by using part of the coordinate axes and a point (u, v), where $u > 0$ on the parabola $y = 4 - x^2$.



Which one of the following is the maximum area of the rectangle?

A. 4

B. $\frac{2\sqrt{3}}{3}$

C. $\frac{8\sqrt{3}-4}{3}$

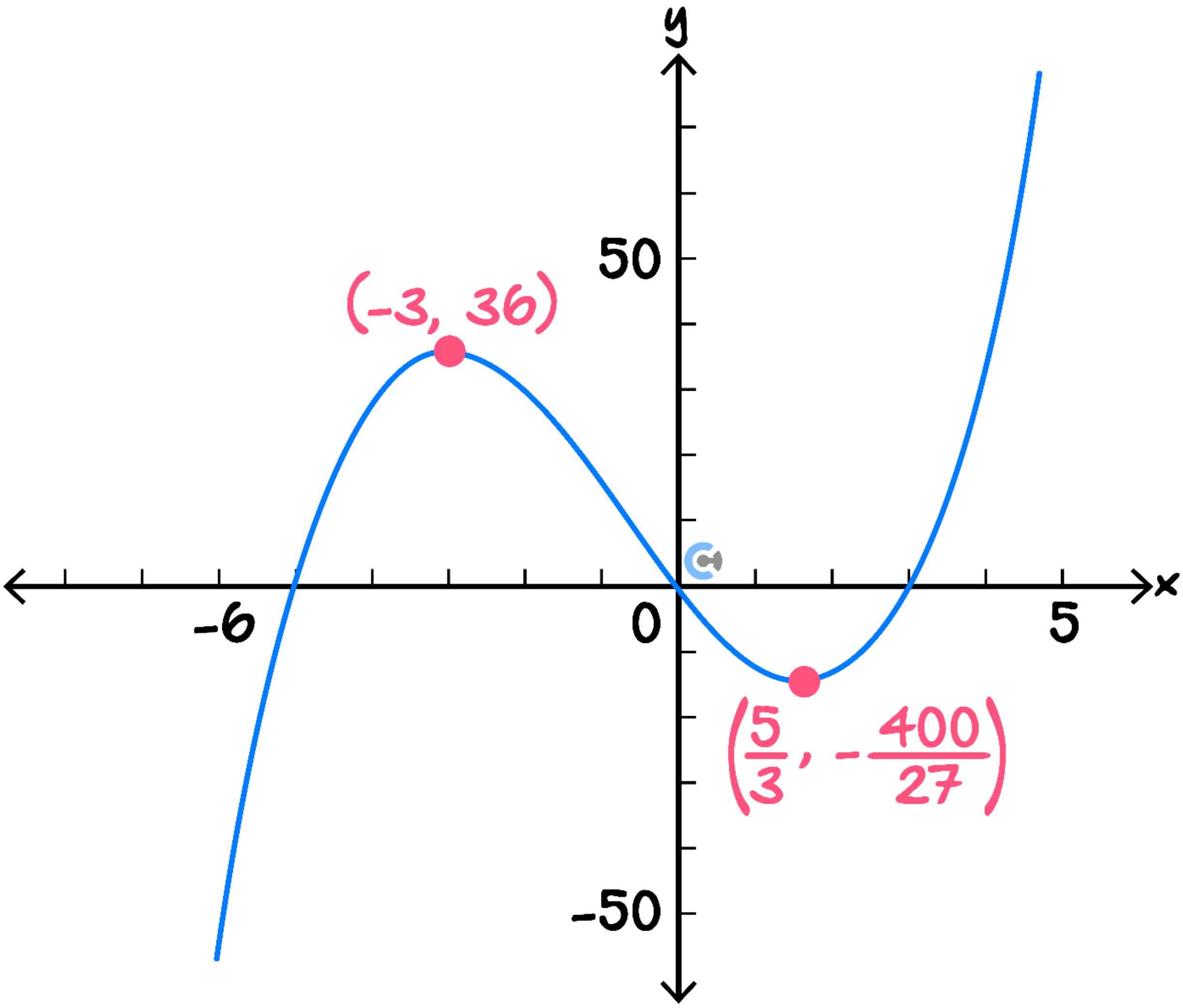
D. $\frac{8}{3}$

E. $\frac{16\sqrt{3}}{9}$

Question 100 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2017
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2017/2017MM2-w.pdf#page=2

Part of the graph of a cubic polynomial function f and the coordinates of its stationary points are shown below.



$f'(x) < 0$ for the interval:

A. $(0,3)$

B. $(-\infty,-5) \cup (0,3)$

C. $(-\infty,-3) \cup \left(\frac{5}{3},\infty\right)$

D. $\left(-3,\frac{5}{3}\right)$

E. $\left(\frac{-400}{27},36\right)$

Question 101 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2017
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2017/2017MM2-w.pdf#page=5

The average rate of change of the function with the rule $f(x) = x^2 - 2x$ over the interval $[1, a]$, where $a > 1$, is 8.

The value of a is:

A. 9

B. 8

C. 7

D. 4

E. $1 + \sqrt{2}$

Question 102 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2017
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2017/2017MM2-w.pdf#page=6

The function $f: R \rightarrow R, f(x) = x^3 + ax^2 + bx$ has a local maximum at $x = -1$ and a local minimum at $x = 3$.

The values of a and b are respectively:

A. -2 and -3.

B. 2 and 1.

C. 3 and -9.

D. -3 and -9.

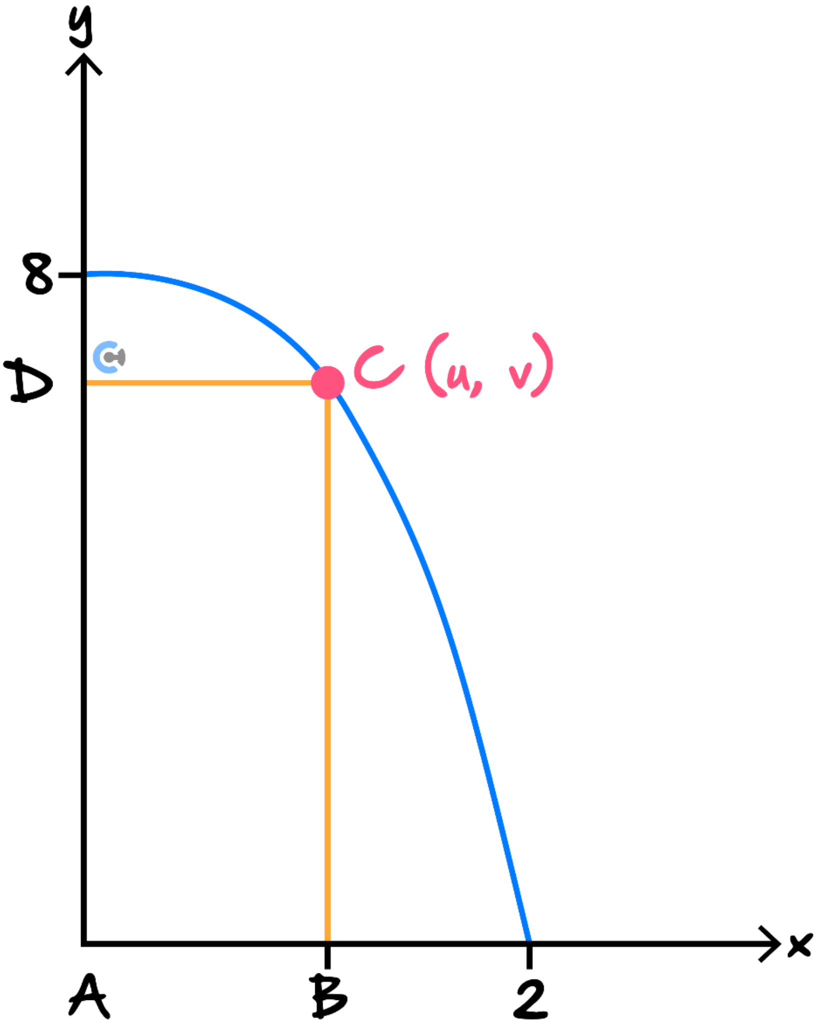
E. -6 and -15.

Space for Personal Notes

Question 103 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2017
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2017/2017MM2-w.pdf#page=7

A rectangle $ABCD$ has vertices $A(0, 0), B(u, 0), C(u, v)$, and $D(0, v)$, where (u, v) lies on the graph of $y = -x^3 + 8$, as shown below.



The maximum area of the rectangle is:

A. $\sqrt[3]{2}$

B. $6\sqrt[3]{2}$

C. 16

D. 8

E. $3\sqrt[3]{2}$

Question 104 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2018
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2018/2018MM2-w.pdf#page=3

Consider $f(x) = x^2 + \frac{p}{x}, x \neq 0, p \in R$.

There is a stationary point on the graph of f when $x = -2$.

The value of p is:

A. -16

B. -8

C. 2

D. 8

E. 16

Question 105 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2018
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2018/2018MM2-w.pdf#page=4

A tangent to the graph of $y = \log_e(2x)$ has a gradient of 2.

This tangent will cross the y-axis at:

A. 0

B. -0.5

C. -1

D. $-1 - \log_e(2)$

E. $-2\log_e(2)$

Space for Personal Notes

Question 106 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2018
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2018/2018MM2-w.pdf#page=8

Consider the functions, $f: R^+ \to R, f(x) = x^{\frac{p}{q}}$ and $g: R^+ \to R, g(x) = x^{\frac{m}{n}}$, where p, q, m and n are positive integers and $\frac{p}{q}$ and $\frac{m}{n}$ are fractions in simplest form.

If $\{x: f(x) > g(x)\} = (0,1)$ and $\{x: g(x) > f(x)\} = (1, \infty)$, which of the following must be **false**?

A. $q > n$ and $p = m$.

B. $m > p$ and $q = n$.

C. $pn < qm$.

D. $f'(c) = g'(c)$ for some $c \in (0,1)$.

E. $f'(d) = g'(d)$ for some $d \in (1, \infty)$.

Question 107 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2019
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2019/2019MM2-w.pdf#page=3

Let $f: R\backslash\{4\} \to R, f(x) = \frac{a}{x-4}$, where $a > 0$.

The average rate of change of f from $x = 6$ to $x = 8$ is:

A. $a \log_e(2)$

B. $\frac{a}{2}\log_e(2)$

C. $2a$

D. $-\frac{a}{4}$

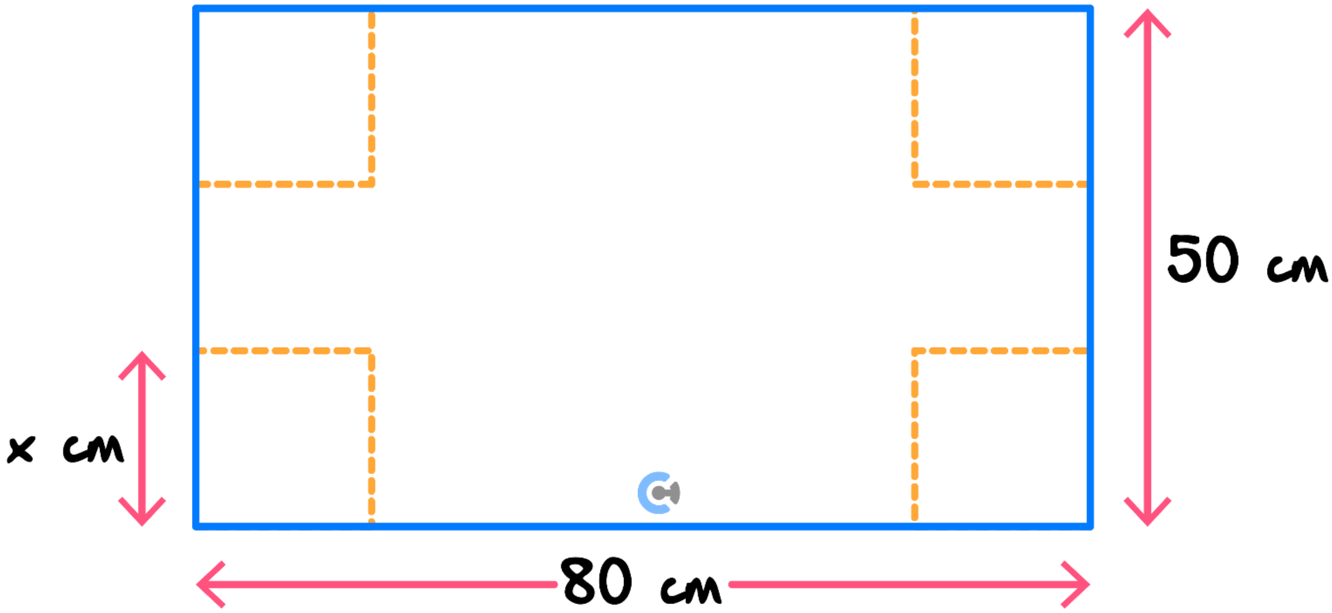
E. $-\frac{a}{8}$

Space for Personal Notes

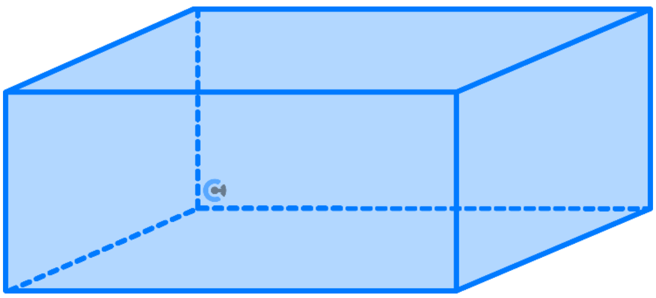
Question 108 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2019
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2019/2019MM2-w.pdf#page=4

A rectangular sheet of cardboard has a length of 80 cm and a width of 50 cm. Squares, of side length x centimetres, are cut from each of the corners, as shown in the diagram below.



A rectangular box with an open top is then constructed, as shown in the diagram below.



The volume of the box is maximum when x is equal to:

A. 10

B. 20

C. 25

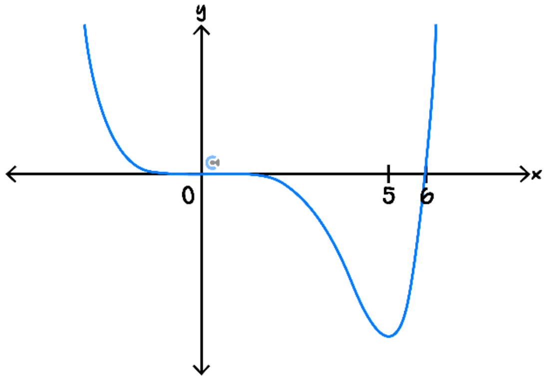
D. $\frac{100}{3}$

E. $\frac{200}{3}$

Question 109 (1 mark)

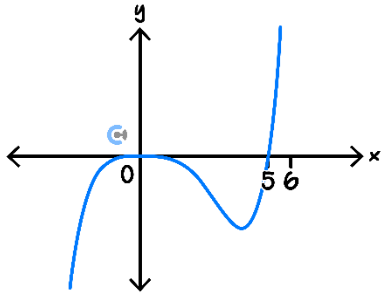
Inspired from VCAA Mathematical Methods ¾ Exam 2019
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2019/2019MM2-w.pdf#page=8

Part of the graph of $y = f(x)$ is shown below.

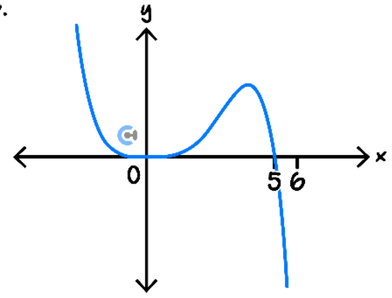


The corresponding part of the graph of $y = f'(x)$ is best represented by:

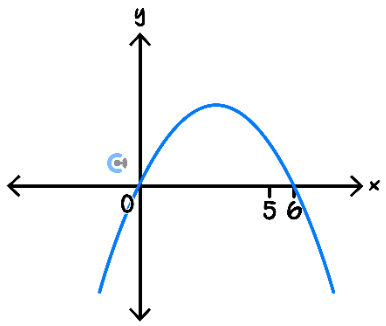
A.



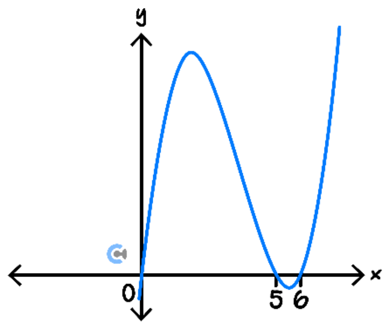
B.



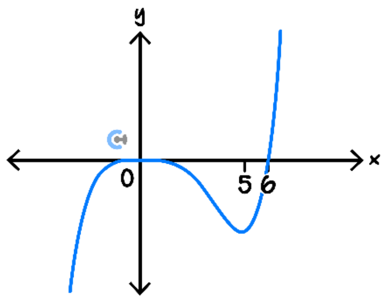
C.



D.



E.



Question 110 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2020
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2020/2020MM2-w.pdf#page=5

If $f(x) = e^{g(x^2)}$, where g is a differentiable function, then $f'(x)$ is equal to:

A. $2xe^{g(x^2)}$

B. $2xg(x^2)e^{g(x^2)}$

C. $2xg'(x^2)e^{g(x^2)}$

D. $2xg'(2x)e^{g(x^2)}$

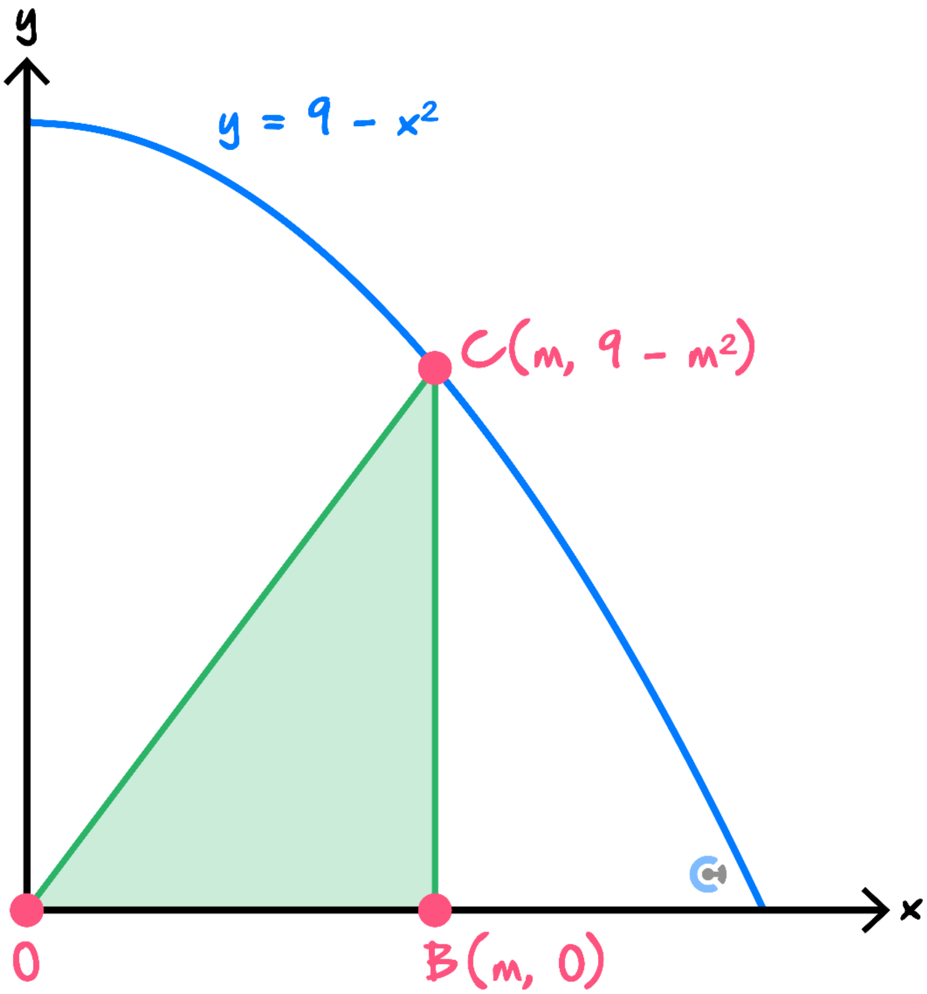
E. $2xg'(x^2)e^{g(2x)}$

Space for Personal Notes

Question 111 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2020
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2020/2020MM2-w.pdf#page=9

A right-angled triangle, OBC, is formed using the horizontal axis and the point $C(m, 9 - m^2)$, where $m \in (0, 3)$, on the parabola, $y = 9 - x^2$, as shown below.



The maximum area of the triangle OBC is:

A. $\frac{\sqrt{3}}{3}$

B. $\frac{2\sqrt{3}}{3}$

C. $\sqrt{3}$

D. $3\sqrt{3}$

E. $9\sqrt{3}$

Question 112 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2020
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2020/2020MM2-w.pdf#page=9

Let $f(x) = -\log_e(x+2)$.

A tangent to the graph of f has a vertical axis intercept at $(0, c)$. The maximum value of c is:

A. -1

B. $-1 + \log_e(2)$

C. $-\log_e(2)$

D. $-1 - \log_e(2)$

E. $\log_e(2)$

Question 113 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2021
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2021/2021MM2-w.pdf#page=3

The maximum value of the function $h: [0,2] \to R, h(x) = (x-2)e^x$ is:

A. $-e$

B. 0

C. 1

D. 2

E. e

Space for Personal Notes

Question 114 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2021
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2021/2021MM2-w.pdf#page=4

The tangent to the graph of $y = x^3 - ax^2 + 1$ at $x = 1$ passes through the origin.

The value of a is:

A. $\frac{1}{2}$

B. 1

C. $\frac{3}{2}$

D. 2

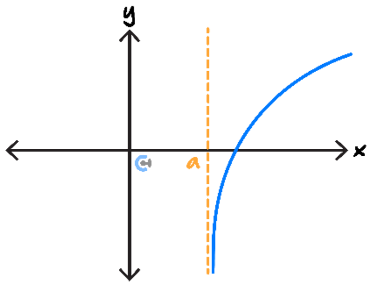
E. $\frac{5}{2}$

Space for Personal Notes

Question 115 (1 mark)

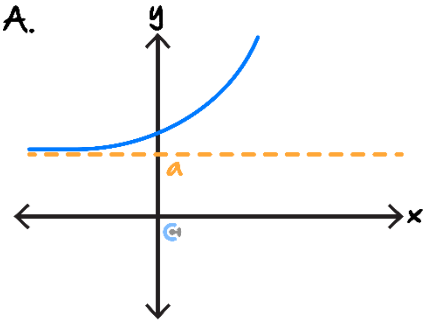
Inspired from VCAA Mathematical Methods ¾ Exam 2021
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2021/2021MM2-w.pdf#page=5

The graph of the function f is shown below.

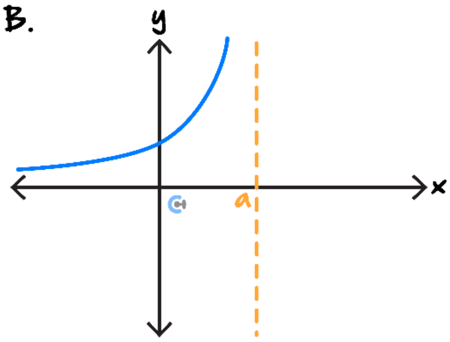


The graph corresponding to f' is:

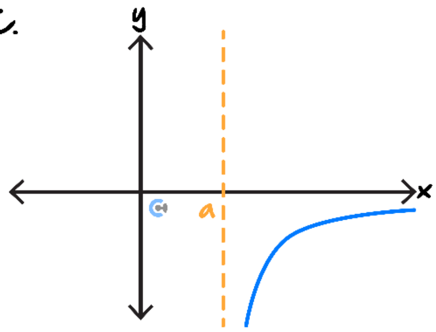
A.



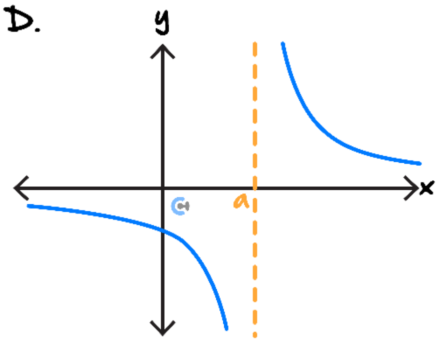
B.



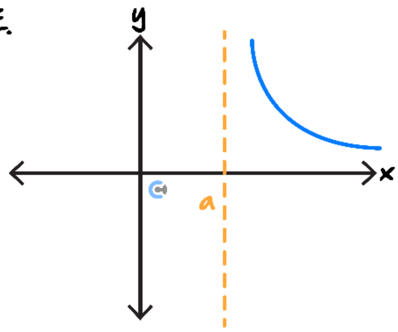
C.



D.



E.



Question 116 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2021
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2021/2021MM2-w.pdf#page=7

The value of an investment, in dollars, after n months can be modelled by the function:

$$f(n) = 2500 \times (1.004)^n$$

Where, $n \in \{0,1,2,\dots\}$.

The average rate of change of the value of the investment over the first 12 months is closest to:

A. \$10.00 per month

B. \$10.20 per month.

C. \$10.50 per month.

D. \$125.00 per month.

E. \$127.00 per month.

Question 117 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2021
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2021/2021MM2-w.pdf#page=7

A value of k for which, the average value of $y = \cos\left(kx - \frac{\pi}{2}\right)$ over the interval $[0, \pi]$ is equal to the average value of $y = \sin(x)$ over the same interval is:

A. $\frac{1}{6}$

B. $\frac{1}{5}$

C. $\frac{1}{4}$

D. $\frac{1}{3}$

E. $\frac{1}{2}$

Space for Personal Notes

Question 118 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2021
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2021/2021MM2-w.pdf#page=9

Which one of the following functions is differentiable for all real values of x?

A. $f(x) = \begin{cases} x & x < 0 \\ -x & x \geq 0 \end{cases}$

B. $f(x) = \begin{cases} x & x < 0 \\ -x & x > 0 \end{cases}$

C. $f(x) = \begin{cases} 8x + 4 & x < 0 \\ (2x + 1)^2 & x \geq 0 \end{cases}$

D. $f(x) = \begin{cases} 2x + 1 & x < 0 \\ (2x + 1)^2 & x \geq 0 \end{cases}$

E. $f(x) = \begin{cases} 4x + 1 & x < 0 \\ (2x + 1)^2 & x \geq 0 \end{cases}$

Question 119 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2022
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2022/2022MM2-w.pdf#page=2

The gradient of the graph of $y = e^{3x}$ at the point where the graph crosses the vertical axis is equal to:

A. 0

B. $\frac{1}{e}$

C. 1

D. e

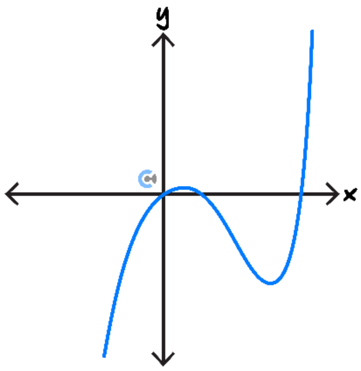
E. 3

Space for Personal Notes

Question 120 (1 mark)

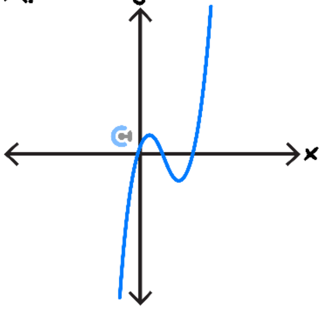
Inspired from VCAA Mathematical Methods ¾ Exam 2022
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2022/2022MM2-w.pdf#page=4

The graph of $y = f(x)$ is shown below.

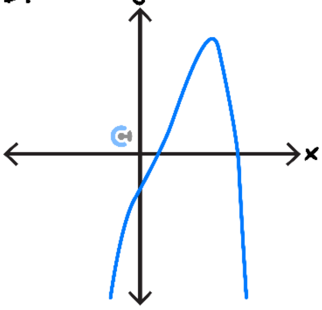


The graph of $y = f'(x)$, the first derivative of $f(x)$ with respect to x, could be:

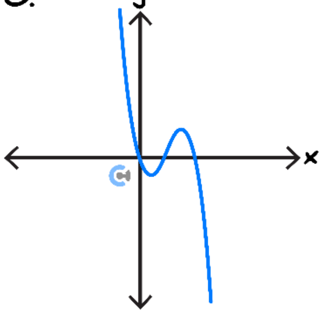
A.



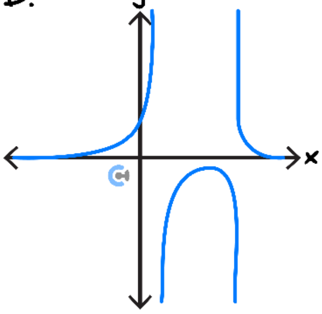
B.



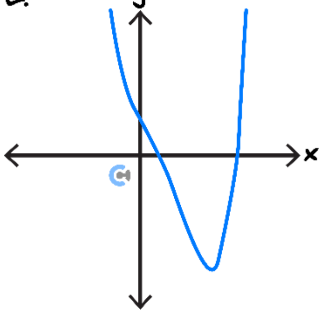
C.



D.



E.



Question 121 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2022
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2022/2022MM2-w.pdf#page=2

The function $f(x) = \frac{1}{3}x^3 + mx^2 + nx + p$, for $m, n, p \in R$, has turning points at $x = -3$ and $x = 1$ and passes through the point $(3,4)$.

The values of m, n, and p respectively are:

A. $m = 0, \quad n = -\frac{7}{3}, \quad p = 2$

B. $m = 1, \quad n = -3, \quad p = -5$

C. $m = -1, \quad n = -3, \quad p = 13$

D. $m = \frac{5}{4}, \quad n = \frac{3}{2}, \quad p = -\frac{83}{4}$

E. $m = \frac{5}{2}, \quad n = 6, \quad p = -\frac{91}{2}$

Question 122 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2022
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2022/2022MM2-w.pdf#page=8

A function g is continuous on the domain $x \in [a, b]$ and has the following properties:

- The average rate of change of g between $x = a$ and $x = b$ is positive.
- The instantaneous rate of change of g at $x = \frac{a+b}{2}$ is negative.

Therefore, on the interval $x \in [a, b]$, the function must be:

A. Many-to-one.

B. One-to-many.

C. One-to-one.

D. Strictly decreasing.

E. Strictly increasing.

Question 123 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2022
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2022/2022MM2-w.pdf#page=8

A box is formed from a rectangular sheet of cardboard, which has a width of a units and a length of b units, by first cutting out squares of side length x units from each corner and then folding upwards to form a container with an open top.

The maximum volume of the box occurs when x is equal to:

A. $\frac{a-b+\sqrt{a^2-ab+b^2}}{6}$

B. $\frac{a+b+\sqrt{a^2-ab+b^2}}{6}$

C. $\frac{a-b-\sqrt{a^2-ab+b^2}}{6}$

D. $\frac{a+b-\sqrt{a^2-ab+b^2}}{6}$

E. $\frac{a+b-\sqrt{a^2-2ab+b^2}}{6}$

Question 124 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2023
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2023/2023MM2-w.pdf#page=3

Which one of the following functions has a horizontal tangent at $(0, 0)$?

A. $y = x^{-\frac{1}{3}}$

B. $y = x^{\frac{1}{3}}$

C. $y = x^{\frac{2}{3}}$

D. $y = x^{\frac{4}{3}}$

E. $y = x^{\frac{3}{4}}$

Space for Personal Notes

Question 125 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2023
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2023/2023MM2-w.pdf#page=4

Let $f(x) = \log_e x$, where $x > 0$ and $g(x) = \sqrt{1-x}$, where $x < 1$.

The domain of the derivative of $(f \circ g)(x)$ is:

A. $x \in R$

B. $x \in (-\infty, 1]$

C. $x \in (-\infty, 1)$

D. $x \in (0, \infty)$

E. $x \in (0,1)$

Question 126 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2023
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2023/2023MM2-w.pdf#page=5

The function f is given by:

$$f(x) = \begin{cases} \tan\left(\frac{x}{2}\right) & 4 \le x < 2\pi \\ \sin(ax) & 2\pi \le x \le 8 \end{cases}$$

The value of a for which, f is continuous and smooth at $x = 2\pi$ is:

A. -2

B. $-\frac{\pi}{2}$

C. $-\frac{1}{2}$

D. $\frac{1}{2}$

E. 2

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Question 127 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2023
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2023/2023MM2-w.pdf#page=5

Two functions, f and g are continuous and differentiable for all $x \in R$. It is given that $f(-2) = -7, g(-2) = 8$ and $f'(-2) = 3, g'(-2) = 2$.

The gradient of the graph $y = f(x) \times g(x)$ at the point where $x = -2$ is:

A. -10

B. -6

C. 0

D. 6

E. 10

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Question 128 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2023
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2023/2023MM2-w.pdf#page=6

The following algorithm applies Newton's method for using a For loop with 3 iterations:

Inputs: $f(x), a\ function\ of\ x$
$df(x), the\ derivative\ of\ f(x)$
$x0, an\ initial\ estimate$

Define newton $(f(x), df(x), x0)$
For i **from** 1 **to** 3
If $df(x0) = 0$ **Then**
Return "Error: Division by zero"
Else
$x0 \leftarrow x0 - f(x0) \div df(x0)$
EndFor
Return $x0$

The **Return** value of the function newton $(x^3 + 3x - 3, 3x^2 + 3, 1)$ is closest to:

A. 0.83333

B. 0.81785

C. 0.81773

D. 1

E. 3

Space for Personal Notes

Question 129 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2023
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2023/2023MM2-w.pdf#page=7

A polynomial has the equation $y = x(3x - 1)(x + 3)(x + 1)$.

The number of tangents to this curve that pass through the positive x-intercept is:

A. 0

B. 1

C. 2

D. 3

E. 4

Question 130 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/2024MM2-w.pdf#page=2

A function $g: R \to R$ has the derivative $g'(x) = x^3 - x$.

Given that $g(0) = 5$, the value of $g(2)$ is:

A. 2

B. 3

C. 5

D. 7

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Question 131 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/2024MM2-w.pdf#page=4

Suppose a function $f: [0,5] \rightarrow R$ and its derivative $f': [0,5] \rightarrow R$ are defined and continuous on their domains. If $f'(2) < 0$ and $f'(4) > 0$, which one of these statements must be true?

A. f is strictly decreasing on $[0,2]$.

B. f does not have an inverse function.

C. f is positive on $[4,5]$.

D. f has a local minimum at $x = 3$.

Question 132 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/2024MM2-w.pdf#page=8

The points of inflection of the graph of $y = 2 - \tan\left(\pi\left(x - \frac{1}{4}\right)\right)$ are:

A. $\left(k + \frac{1}{4}, 2\right), k \in Z$

B. $\left(k - \frac{1}{4}, 2\right), k \in Z$

C. $\left(k + \frac{1}{4}, -2\right), k \in Z$

D. $\left(k - \frac{3}{4}, -2\right), k \in Z$

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Question 133 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/2024MM2-w.pdf#page=8

Suppose that a differentiable function $f: R \to R$ and its derivative $f': R \to R$ satisfy $f(4) = 25$ and $f'(4) = 15$.

Determine the gradient of the tangent line to the graph of $y = \sqrt{f(x)}$ at $x = 4$.

A. $\sqrt{15}$

B. $\frac{1}{10}$

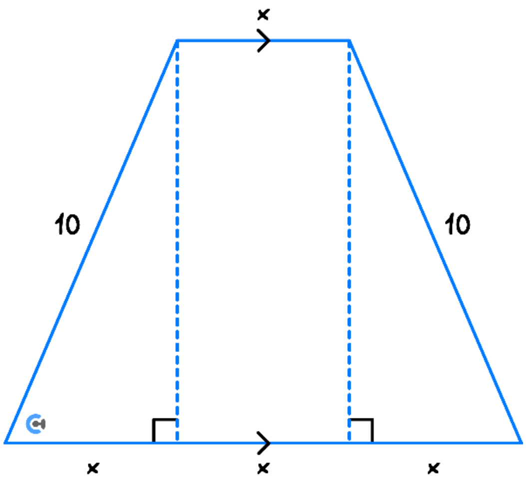
C. $\frac{15}{2}$

D. $\frac{3}{2}$

Question 134 (1 mark)

Inspired from VCAA Mathematical Methods ¾ Exam 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/2024MM2-w.pdf#page=10

Find the value of x which maximises the area of the trapezium below.



A. 10

B. $5\sqrt{2}$

C. 7

D. $\sqrt{10}$

Question 135 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2017
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2017/nht/2017MM2-nht-w.pdf#page=2

The gradient of a line perpendicular to the line that passes through $(3, 0)$ and $(0, -6)$ is:

A. $-\frac{1}{2}$

B. -2

C. $\frac{1}{2}$

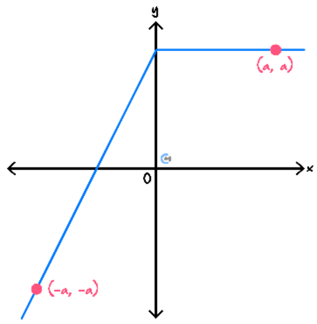
D. 4

E. 2

Question 136 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2017
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2017/nht/2017MM2-nht-w.pdf#page=4

Part of the graph of a function f is shown below.



Which one of the following is the average value of the function f over the interval $[-a, a]$?

A. 0

B. $\frac{3a}{4}$

C. $\frac{3a}{8}$

D. $\frac{a}{2}$

E. $\frac{a}{4}$

Question 137 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2017
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2017/nht/2017MM2-nht-w.pdf#page=5

The tangent to the graph of $y = 3\sin(2x) - 1$ is parallel to the line with equation $y = 3x + 1$ at the points where x is equal to:

A. $n\pi \pm \frac{\pi}{6}, n \in Z$

B. $-\frac{\pi}{3}, \frac{\pi}{3}$ only.

C. $\frac{\pi}{6}, \frac{5\pi}{6}$ only.

D. $n\pi \pm \frac{\pi}{3}, n \in Z$

E. $nx, n \in Z$

Question 138 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2017
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2017/nht/2017MM2-nht-w.pdf#page=2

Let $f(x) = x^m e^{ax}$, where a and m are non-zero real constants. If $(x + 2)$ is a factor of $f'(x)$, then which one of the following must be true?

A. $m = 2$

B. $m = -2$

C. $m = 2 - a$

D. $m = 2a$

E. $m = -2a$

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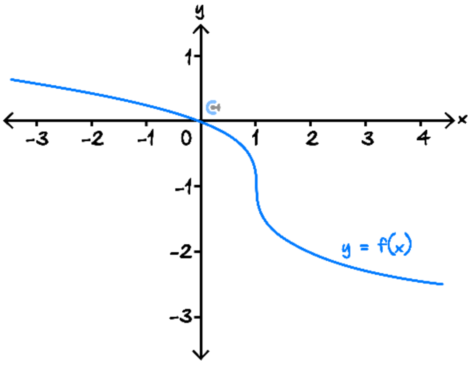
Question 139 (1 mark)

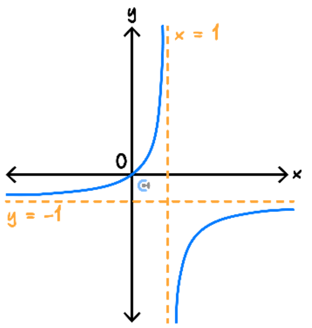
Inspired from VCAA Mathematical Methods ¾ NHT Exam 2018
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2018/nht/2018MM2-nht-w.pdf#page=5

Part of the graph of $y = f\ (x)$ is shown below.

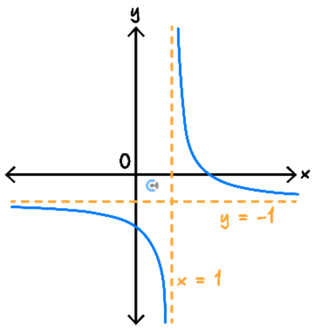


The graph of $y = f'(x)$ is best represented by:

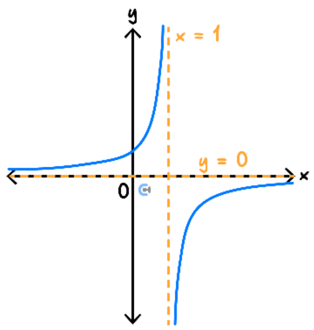
A.



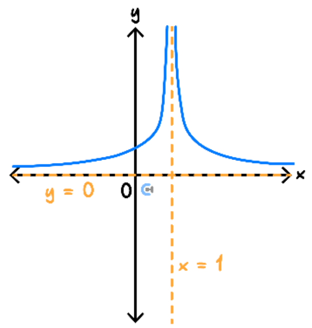
B.



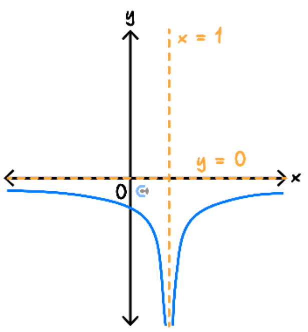
C.



D.



E.



Question 140 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2018
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2018/nht/2018MM2-nht-w.pdf#page=20

Let f be a one-to-one differentiable function such that, $f(3) = 7, f(7) = 8, f'(3) = 2$ and $f'(7) = 3$.

The function g is differentiable and $g(x) = f^{-1}(x)$ for all x.

$g'(7)$ is equal to:

A. $\frac{1}{2}$

B. 2

C. $\frac{1}{6}$

D. $\frac{1}{8}$

E. $\frac{1}{3}$

Question 141 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2019
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2019/NHT/2019MM2-nht-w.pdf#page=6

The graph of $f(x) = x^3 - 6(b - 2)x^2 + 18x + 6$ has exactly two stationary points for:

A. $1 < b < 2$

B. $b = 1$

C. $b = \frac{4 \pm \sqrt{6}}{2}$

D. $\frac{4-\sqrt{6}}{2} \leq b \leq \frac{4+\sqrt{6}}{2}$

E. $b < \frac{4-\sqrt{6}}{2}$ or $b > \frac{4+\sqrt{6}}{2}$

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Question 142 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2019
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2019/NHT/2019MM2-nht-w.pdf#page=10

Let $f(x) = (ax + b)^5$ and let g be the inverse function of f.

Given that $f(0) = 1$, what is the value of $g'(1)$?

A. $\frac{5}{a}$

B. 1

C. $\frac{1}{5a}$

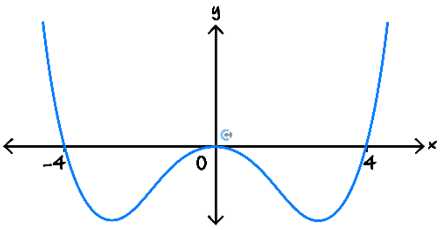
D. $5a(a + 1)^4$

E. 0

Question 143 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2021
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2021/NHT/2021MM2-nht-w.pdf#page=5

Part of the graph of a polynomial function f is shown below. This graph has turning points at $(-2\sqrt{2}, -1)$, and $(2\sqrt{2}, -1)$.



$f(x)$ is strictly decreasing for:

A. $x \in (-\infty, -4] \cup [4, \infty)$

B. $x \in [-4, 4]$

C. $x \in [-2\sqrt{2}, 2\sqrt{2}]$

D. $x \in (-\infty, -2\sqrt{2}] \cup [0, 2\sqrt{2}]$

E. $x \in [-2\sqrt{2}, 0] \cup [2\sqrt{2}, \infty)$

Question 144 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2021
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2021/NHT/2021MM2-nht-w.pdf#page=5

Consider the graph of $f : R \to R, f(x) = -x^2 - 4x + 5$.

The tangent to the graph of f is parallel to the line connecting the negative x-intercept and the y-intercept of f when x is equal to:

A. -3

B. $-\frac{5}{2}$

C. $-\frac{3}{2}$

D. -1

E. $-\frac{1}{2}$

Question 145 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2022
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2022/NHT/2022mm2-nht-w.pdf#page=6

Let $f(x) = g(x) \cdot \sqrt{1 - x^2}$, where g is a function that is continuous and differentiable for all $x \in R$. The gradient of the tangent to the graph of f at the point where f crosses the vertical axis is equal to:

A. 0

B. 1

C. $g(0)$

D. $g'(0)$

E. $g'(0) - g(0)$

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Question 146 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2022
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2022/NHT/2022mm2-nht-w.pdf#page=8

At the point where $x = k$, the tangent to the circle given by the equation $x^2 + (y - 1)^2 = 1$ meets the positive direction of the x-axis at an angle of $135°$.

The value of k could be:

A. $-\sqrt{3}$

B. -1

C. $-\frac{1}{\sqrt{2}}$

D. $-\frac{1}{\sqrt{3}}$

E. 0

Question 147 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2023
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2023/NHT/2023MM2-nht-w.pdf#page=2

Let $f(t) = -\frac{1}{3}t^3 + \frac{1}{2}t^2 + 50t$.

The instantaneous rate of change of f when $t = 1$ is:

A. 247.5

B. 50.2

C. 50.0

D. −13.8

E. −22.0

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Question 148 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2023
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2023/NHT/2023MM2-nht-w.pdf#page=2

If $u = g(x)$ and $v = e^{g(2x)}$, where g is a differentiable function, then $\frac{d}{dx}(uv)$ is equal to:

A. $3g(x)e^{g(2x)}$

B. $e^{g(2x)}\left(2g(x) + g'(x)\right)$

C. $e^{g(2x)}\left(g(x)g'(2x) + g'(x)\right)$

D. $e^{g(2x)}\left(2g(x)g'(2x) + g'(x)\right)$

E. $2g(x)g'(2x)e^{g'(2x)} + e^{g(2x)}g'(x)$

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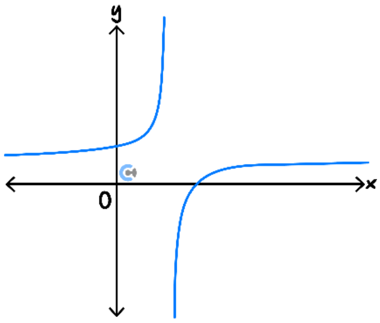
Question 149 (1 mark)

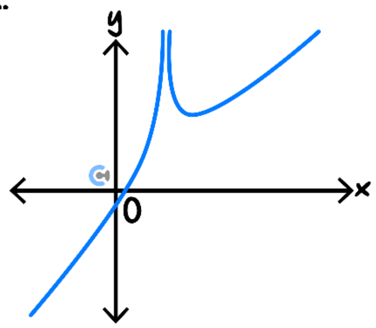
Inspired from VCAA Mathematical Methods ¾ NHT Exam 2023
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2023/NHT/2023MM2-nht-w.pdf#page=5

The graph of $y = f(x)$ is shown below.

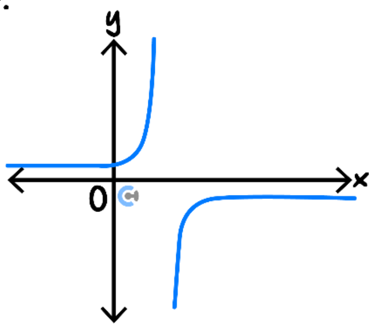


The graph of $y = f'(x)$ is best represented by:

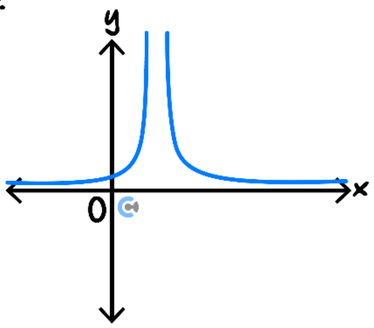
A.



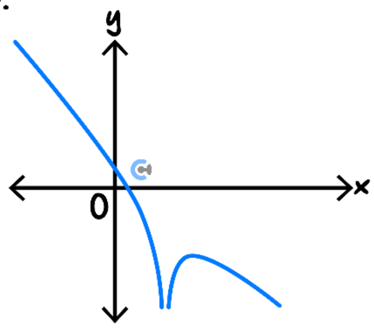
B.



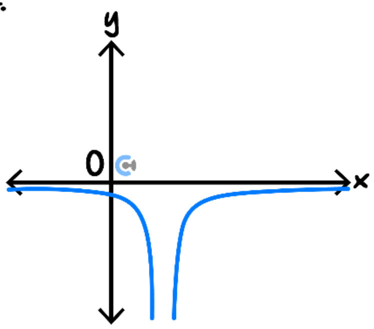
C.



D.



E.



Question 150 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2023
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2023/NHT/2023MM2-nht-w.pdf#page=6

A tangent line to the graph of $y = \log_e(2x) + \log_e(x - 2)$ passes through the origin. The x-coordinate, correct to two decimal places, where the tangent line touches the graph is closest to:

A. -0.66

B. 1.25

C. 2.91

D. 4.19

E. 6.33

Question 151 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2023
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2023/NHT/2023MM2-nht-w.pdf#page=7

The graph of $y = \log_e(x - k)$, for $k \in R$, has a tangent with a maximum horizontal axis intercept of:

A. $x = 1$

B. $x = k$

C. $x = e$

D. $x = 1 + k$

E. $x = e + k$

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Question 152 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/NHT/2024MM2-nht-w.pdf#page=3

A straight line passes through the positive x-intercept of the curve of the cubic $y = x^3 - x^2 - 2x$ and also through its point of inflection.

The gradient of this line is:

A. $\frac{4}{9}$

B. $\frac{2}{3}$

C. $\frac{1}{2}$

D. $-\frac{15}{7}$

E. $-\frac{20}{9}$

Question 153 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/NHT/2024MM2-nht-w.pdf#page=7

Newton's method is used to estimate the x-intercept of the function:

$$f: [0, \infty) \to R, f(x) = \log_e(2x + 1) - \left(4 - x^{\frac{5}{2}}\right)$$

With an initial estimate of $x_0 = 0$, the estimate for x_3 is closest to:

A. 1.4717

B. 1.4718

C. 1.4752

D. 1.5628

E. 2.0000

Question 154 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/NHT/2024MM2-nht-w.pdf#page=9

The finance team at a small technology company estimates that the production cost per item is given by the rule $C(n) = 0.5n^2 - 37.5n + 1900 + \frac{1250}{n}$, where $n \in Z^+$ and n is the number of items produced.

The minimum cost per item is closest to:

A. \$38.34

B. \$38.35

C. \$1229.83

D. \$1229.89

E. \$1230.05

Question 155 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/NHT/2024MM2-nht-w.pdf#page=9

The values for two continuous functions, f and g, and their derivatives are given in the tables below.

	$x = 0$	$x = 2$
$f(x)$	2	-1
$f'(x)$	-1	2

	$x = 0$	$x = 2$
$g(x)$	1	-1
$g'(x)$	0	3

What is the value of $\frac{d}{dx}\big((g \circ f)(x)\big)$ at $x = 0$?

A. -3

B. -1

C. 0

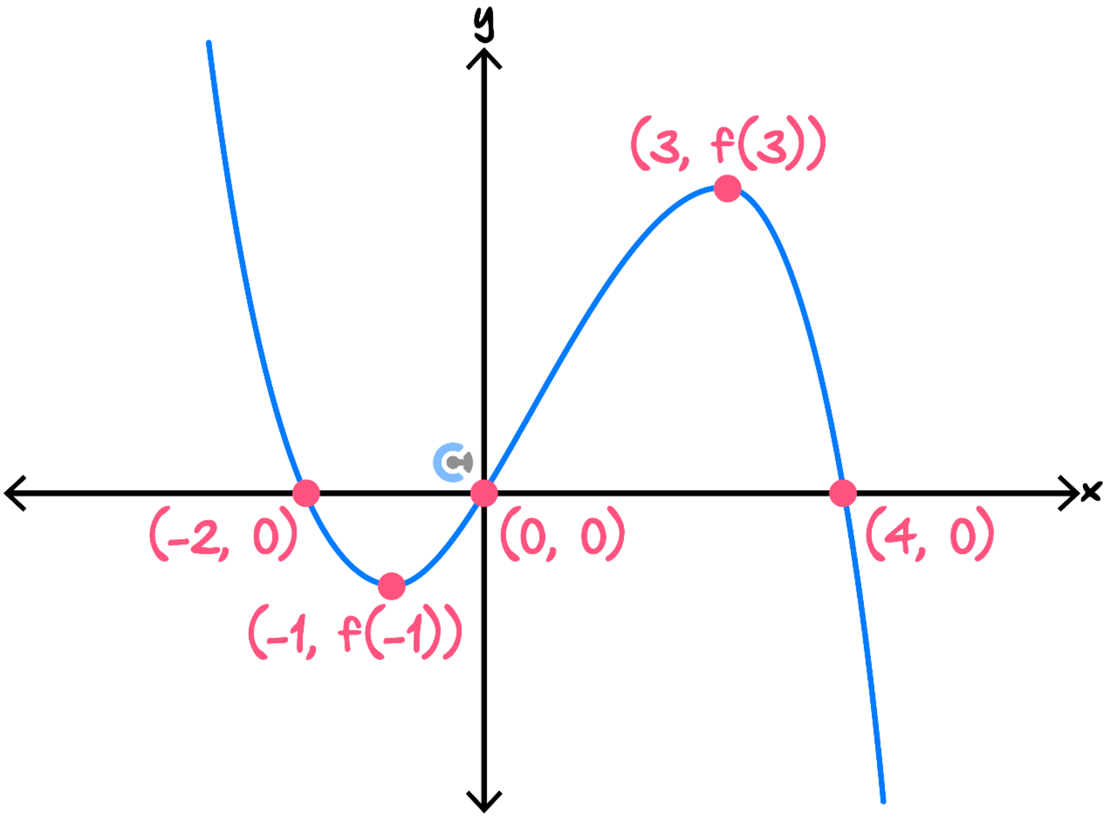
D. 1

E. 2

Question 156 (1 mark)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/NHT/2024MM2-nht-w.pdf#page=10

Let $f : R \to R$ be a continuous and differentiable function. Part of the graph of f is shown below. The stationary points of f are at $(-1, f(-1))$ and $(3, f(3))$.



The solution to the inequality $(x^2 - x - 2)\, f'(x) > 0$ is:

A. $-1 < x < 2$

B. $-1 < x < 3$

C. $2 < x < 3$

D. $x < -1$ or $x > 2$.

E. $x < -1$ or $x > 3$.

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Question 157 (11 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2016
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2016/2016MM2-w.pdf

Let $f: [0,8\pi] \to R, f(x) = 2\cos\left(\frac{x}{2}\right) + \pi$.

a. Find the period and range of f. (2 marks)

b. State the rule for the derivative function f'. (1 mark)

c. Find the equation of the tangent to the graph of f at $x = \pi$. (1 mark)

d. Find the equations of the tangents to the graph of $f: [0, 8\pi] \to R, f(x) = 2\cos\left(\frac{x}{2}\right) + \pi$ that have a gradient of 1. (2 marks)

e. The rule of f' can be obtained from the rule of f under a transformation T, such that:

$$T: R^2 \to R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & a \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -\pi \\ b \end{bmatrix}$$

Find the value of a and the value of b. (3 marks)

f. Find the values of $x, 0 \leq x \leq 8\pi$, such that $f(x) = 2f'(x) + \pi$. (2 marks)

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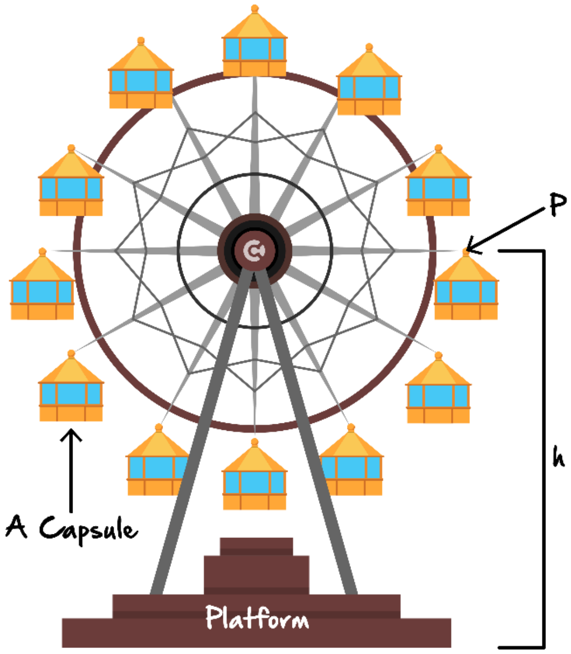
Question 158 (12 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2017
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2017/2017MM2-w.pdf

Sammy visits a giant Ferris wheel. Sammy enters a capsule on the Ferris wheel from a platform above the ground. The Ferris wheel is rotating anti-clockwise. The capsule is attached to the Ferris wheel at point P. The height of P above the ground, h, is modelled by $h(t) = 65 - 55\cos\left(\frac{\pi t}{15}\right)$ where t is the time in minutes after Sammy enters the capsule and h is measured in metres. Sammy exits the capsule after one complete rotation of the Ferris wheel.

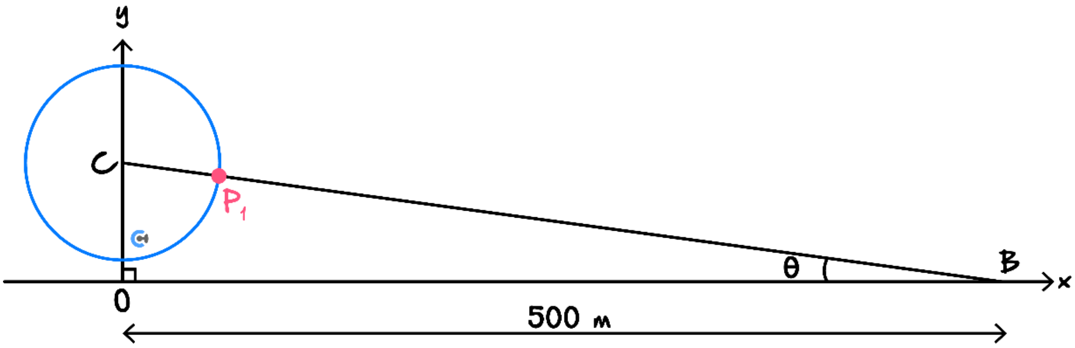


a. State the minimum and maximum heights of P above the ground. (1 mark)

b. For how much time, is Sammy in the capsule? (1 mark)

c. Find the rate of change of h with respect to t and, hence, state the value of t at which the rate of change of h is at its maximum. (2 marks)

As the Ferris wheel rotates, a stationary boat at B, on a nearby river, first becomes visible at point P_1. B is 500 m horizontally from the vertical axis through the centre C of the Ferris wheel and angle $CBO = \theta$, as shown below.

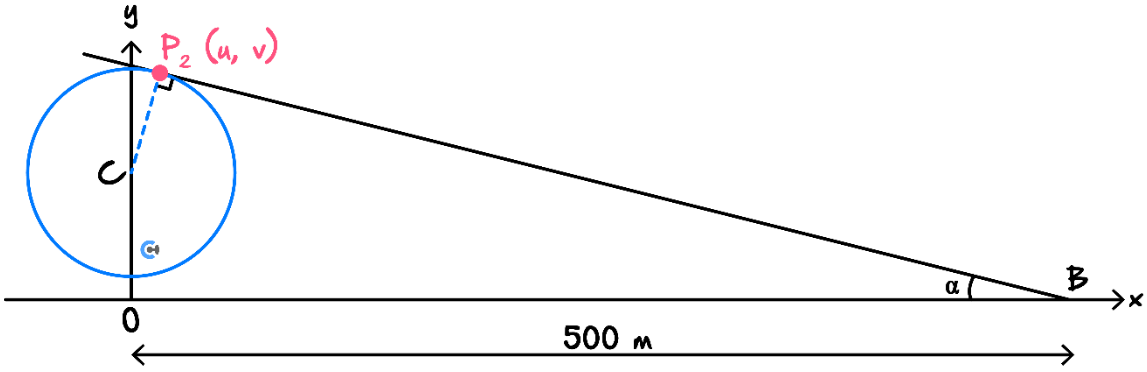


d. Find θ in degrees, correct to two decimal places. (1 mark)

Part of the path of P is given by, $y = \sqrt{3025 - x^2} + 65, x \in [-55, 55]$, where x and y are in metres.

e. Find $\frac{dy}{dx}$. (1 mark)

As the Ferris wheel continues to rotate, the boat at B is no longer visible from the point $P_2(u, v)$ onwards. The line through B and P_2 is tangent to the path of P, where angle $OBP_2 = \alpha$.



f. Find the gradient of the line segment P_2B in terms of u and, hence, find the coordinates of P_2, correct to two decimal places. (3 marks)

g. Find α in degrees, correct to two decimal places. (1 mark)

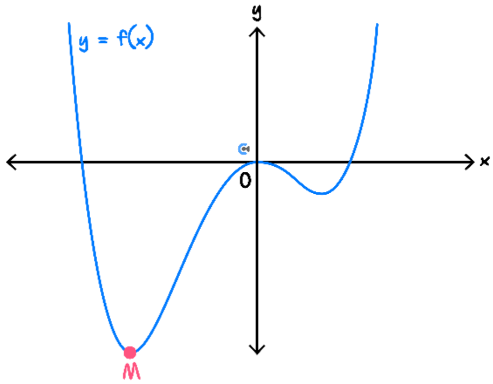
h. Hence or otherwise, find the length of time, to the nearest minute, during which the boat at B is visible. (2 marks)

Space for Personal Notes

Question 159 (13 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2018
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2018/2018MM2-w.pdf

Consider the quartic $f: R \to R, f(x) = 3x^4 + 4x^3 - 12x^2$ and part of the graph of $y = f(x)$ below.

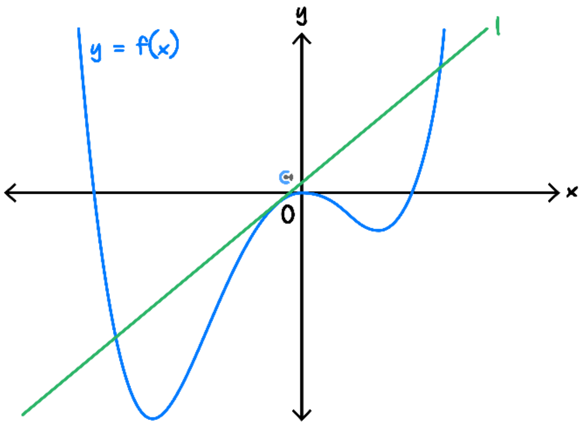


a. Find the coordinates of the point M, at which the minimum value of the function f occurs. (1 mark)

---

b. State the values of $b \in R$ for which the graph of $y = f(x) + b$ has no x-intercepts. (1 mark)

Part of the tangent, l, to $y = f(x)$ at $x = -\frac{1}{3}$ is shown below.



c. Find the equation of the tangent l. (1 mark)

---

d. The tangent l intersects $y = f(x)$ at $x = -\frac{1}{3}$ and at two other points.

State the x-values of the two other points of intersection. Express your answers in the form $\frac{a \pm \sqrt{b}}{c}$, where a, b and c are integers. (2 marks)

e. Find the total area of the regions bounded by the tangent l and $y = f(x)$. Express your answer in the form $\frac{a\sqrt{b}}{c}$, where a, b and c are positive integers. (2 marks)

Let $p: R \to R, p(x) = 3x^4 + 4x^3 + 6(a-2)x^2 - 12ax + a^2, a \in R$.

f. State the value of a for which $f(x) = p(x)$ for all x. (1 mark)

g. Find all solutions to $p'(x) = 0$, in terms of a where appropriate. (1 mark)

h.

i. Find the values of a for which p has only one stationary point. (1 mark)

ii. Find the minimum value of p when $a = 2$. (1 mark)

iii. If p has only one stationary point, find the values of a for which $p(x) = 0$ has no solutions. (2 marks)

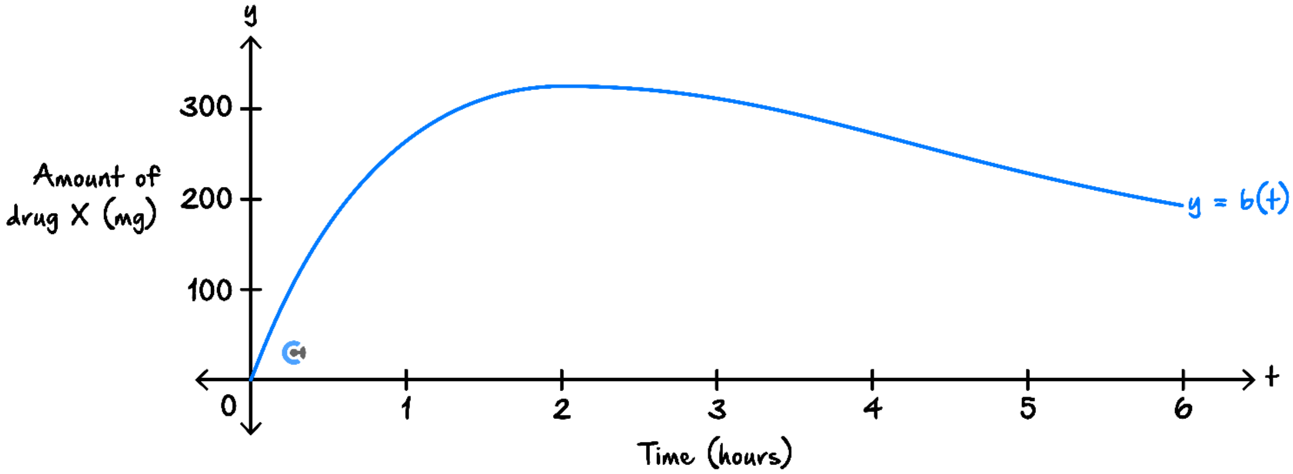
Question 160 (10 marks)

A drug, X, comes in 500 milligrams (mg) tablets.
The amount, b, of drug X in the bloodstream, in milligrams, t hours after one tablet is consumed is given by the function:

$$b(t) = \frac{4500}{7}\left(e^{\left(-\frac{t}{5}\right)} - e^{\left(-\frac{9t}{10}\right)}\right)$$

a. Find the time, in hours, it takes for drug X to reach a maximum amount in the bloodstream after one tablet is consumed. Express your answer in the form $a\log_e(c)$, where $a, c \in R$. (2 marks)

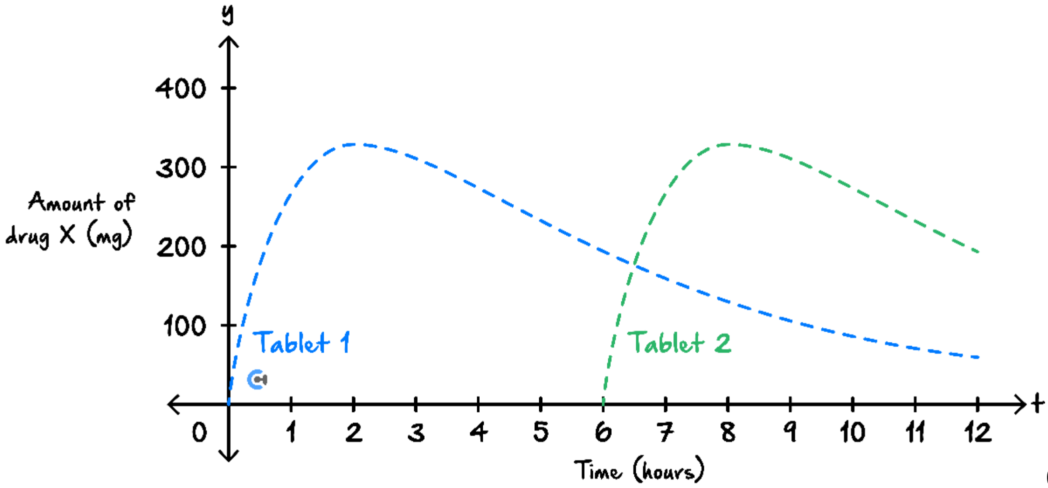
The graph of $y = b(t)$ is shown below for $0 \leq t \leq 6$.



b. Find the average rate of change of the amount of drug X in the bloodstream, in milligrams per hour, over the interval $[2, 6]$. Give your answer correct to one decimal place. (2 marks)

c. Find the average amount of drug X in the bloodstream, in milligrams, during the first six hours after one tablet is consumed. Give your answer correct to the nearest milligram. (2 marks)

d. Six hours after one 500 milligram tablet of drug X is consumed (Tablet 1), a second identical tablet is consumed (Tablet 2). The amount of drug X in the bloodstream from each tablet consumed independently is shown in the graph below.



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i. On the graph above, sketch the total amount of drug X in the bloodstream during the first 12 hours after Tablet 1 is consumed. (2 marks)

ii. Find the maximum amount of drug X in the bloodstream in the first 12 hours and the time at which this maximum occurs. Give your answers correct to two decimal places. (2 marks)

Question 161 (11 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2019
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2019/2019MM2-w.pdf

Let $f: R \to R, f(x) = x^2 e^{-x^2}$.

a. Find $f'(x)$. (1 mark)

b.

i. State the nature of the stationary point on the graph of f at the origin. (1 mark)

ii. Find the maximum value of the function f and the values of x for which the maximum occurs. (2 marks)

iii. Find the values of $d \in R$ for which $f(x) + d$ is always negative. (1 mark)

c.

i. Find the equation of the tangent to the graph of f at $x = -1$. (1 mark)

ii. Find the area enclosed by the graph of f and the tangent to the graph of f at $x = -1$, correct to four decimal places. (2 marks)

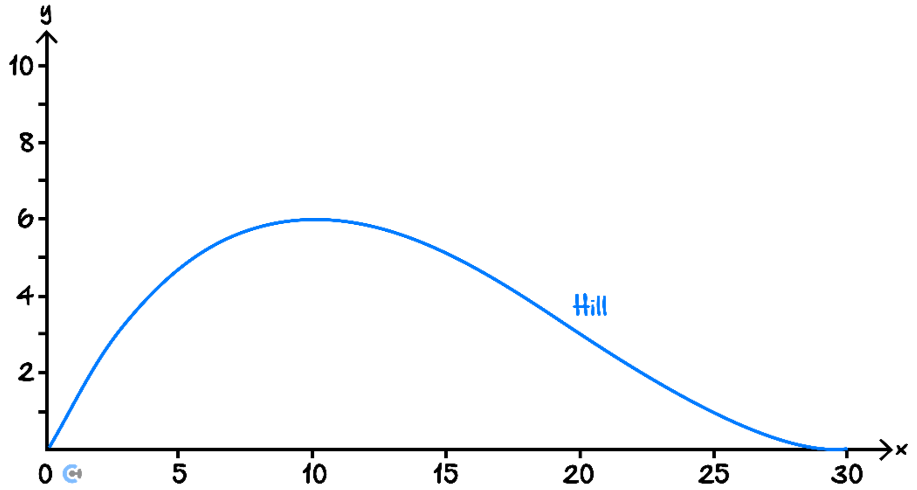
d. Let $M(m, n)$ be a point on the graph of f, where $m \in [0, 1]$.

Find the minimum distance between M and the point $(0, e)$, and the value of m for which this occurs, correct to three decimal places. (3 marks)

Question 162 (11 marks)

An amusement park is planning to build a zip-line above a hill on its property.

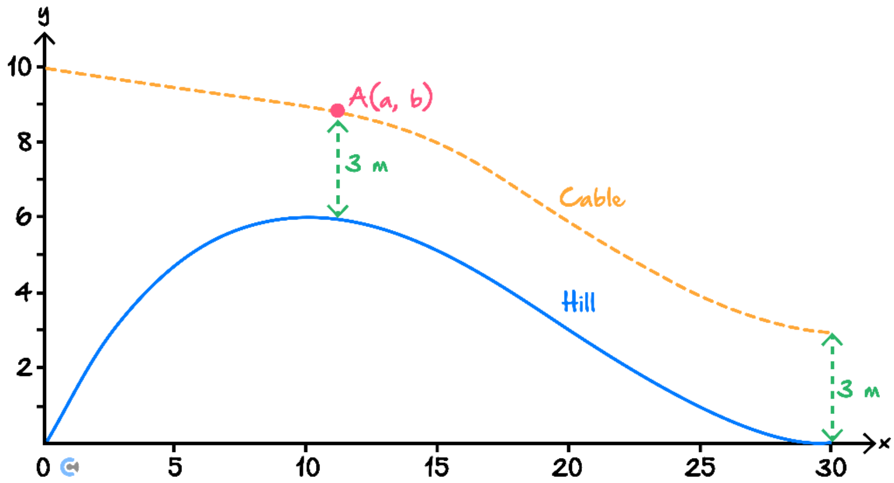
The hill is modelled by $y = \frac{3x(x-30)^2}{2000}, x \in [0,30]$, where x is the horizontal distance, in metres, from an origin and y is the height, in metres, above this origin, as shown in the graph below.



a. Find $\frac{dy}{dx}$. (1 mark)

b. State the set of values for which the gradient of the hill is strictly decreasing. (1 mark)

The cable for the zip-line is connected to a pole at the origin at a height of 10 m and is straight for $0 \leq x \leq a$, where $10 \leq a \leq 20$. The straight section joins the curved section at $A(a, b)$. The cable is then exactly 3 m vertically above the hill from $a \leq x \leq 30$, as shown in the graph below.



c. State the rule, in terms of x, for the height of the cable above the horizontal axis for $x \in [a, 30]$. (1 mark)

---

d. Find the values of x for which the gradient of the cable is equal to the average gradient of the hill for $x \in [10, 30]$. (3 marks)

The gradients of the straight and curved sections of the cable approach the same value at $x = a$, so there is a continuous and smooth join at A.

e.

i. State the gradient of the cable at A, in terms of a. (1 mark)

ii. Find the coordinates of A, with each value correct to two decimal places. (3 marks)

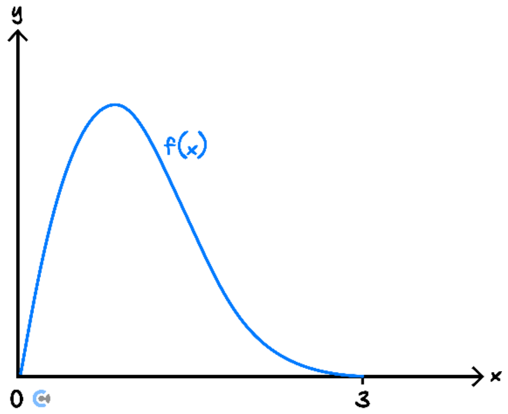
iii. Find the value of the gradient at A, correct to one decimal place. (1 mark)

Space for Personal Notes

Question 163 (13 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2020
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2020/2020MM2-w.pdf

The graph of the function $f(x) = 2xe^{(1-x^2)}$, where $0 \leq x \leq 3$, is shown below.



a. Find the slope of the tangent to f at $x = 1$. (1 mark)

b. Find the obtuse angle that the tangent to f at $x = 1$ makes with the positive direction of the horizontal axis. Give your answer correct to the nearest degree. (1 mark)

c. Find the slope of the tangent to f at a point $x = p$. Give your answer in terms of p. (1 mark)

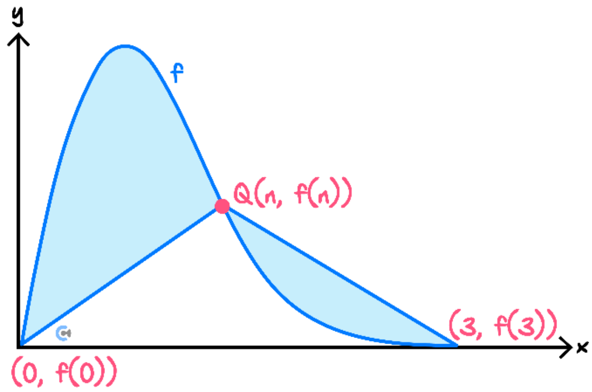
d.

i. Find the value of p for which the tangent to f at $x = 1$ and the tangent to f at $x = p$ are perpendicular to each other. Give your answer correct to three decimal places. (2 marks)

---

ii. Hence, find the coordinates of the point where the tangents to the graph of f at $x = 1$ and $x = p$ intersect when they are perpendicular. Give your answer correct to two decimal places. (3 marks)

Two line segments connect the points $(0, f(0))$ and $(3, f(3))$ to a single point $Q(n, f(n))$, where $1 < n < 3$, as shown in the graph below.



e.

i. The first line segment connects the point $(0, f(0))$ and the point $Q(n, f(n))$, where $1 < n < 3$. Find the equation of this line segment in terms of n. (1 mark)

---

ii. The second line segment connects the point $Q(n, f(n))$ and the point $(3, f(3))$, where $1 < n < 3$. Find the equation of this line segment in terms of n. (1 mark)

---

iii. Find the value of n, where $1 < n < 3$, if there are equal areas between the function f and each line segment. Give your answer correct to three decimal places. (3 marks)

Question 164 (13 marks)

Let $f: R \to R, f(x) = x^3 - x$.

Let $g_a : R \to R$ be the function representing the tangent to the graph of f at $x = a$, where $a \in R$.

Let $(b, 0)$ be the x-intercept of the graph of g_a.

a. Show that $b = \frac{2a^3}{3a^2-1}$. (3 marks)

b. State the values of a for which b does not exist. (1 mark)

c. State the nature of the graph of g_a, when b does not exist. (1 mark)

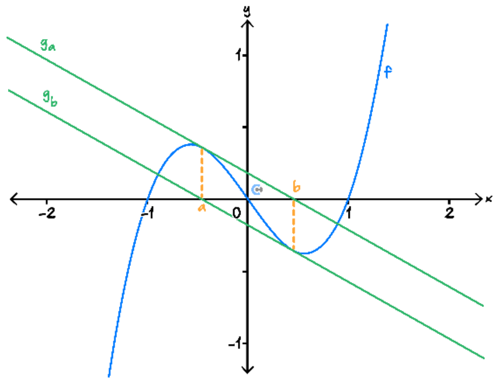
d.

i. State all values of a for which $b = 1.1$. Give your answers correct to four decimal places. (1 mark)

ii. The graph of f has an x-intercept at (1,0).

State the values of a for which $1 \leq b < 1.1$. Give your answers correct to three decimal places. (1 mark)

The coordinate $(b, 0)$ is the horizontal axis intercept of g_a.
Let g_b be the function representing the tangent to the graph of f at $x = b$, as shown in the graph below.



e. Find the values of a for which the graphs of g_a and g_b, where b exists, are parallel and where $b \neq a$. (3 marks)

Let $p : R \to R, p(x) = x^3 + wx$, where $w \in R$.

f. Show that $p(-x) = -p(x)$ for all $w \in R$. (1 mark)

A property of the graphs of p is that two distinct parallel tangents will always occur at $(t, p(t))$ and $(-t, p(-t))$ for all $t \neq 0$.

g. Find all values of w such that a tangent to the graph of p at $(t, p(t))$, for some $t > 0$, will have an x-intercept at $(-t, 0)$. (1 mark)

h. Let $T: R^2 \to R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} m & 0 \\ 0 & n \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} h \\ k \end{bmatrix}$, where $m, n \in R\backslash[0\}$ and $h, k \in R$.

State any restrictions on the values of m, n, h and k, given that the image of p under the transformation T always has the property that parallel tangents occur at $x = -t$ and $x = t$ for **all** $t \neq 0$. (1 mark)

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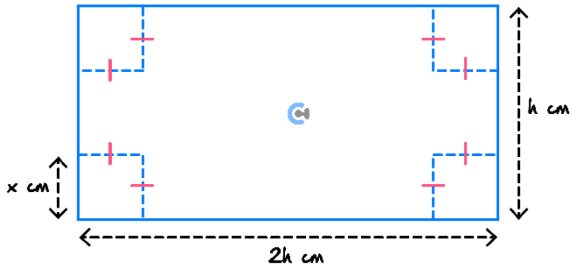
Question 165 (14 marks)

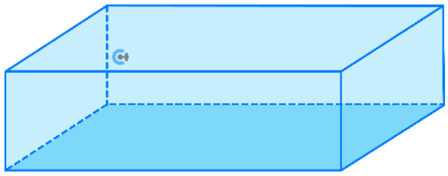
Inspired from VCAA Mathematical Methods ¾ Exam 2021
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2021/2021MM2-w.pdf

A rectangular sheet of cardboard has a width of h centimetres. Its length is twice its width.
Squares of side length x centimetres, where $x > 0$, are cut from each of the corners, as shown in the diagram below.



The sides of this sheet of cardboard are then folded up to make a rectangular box with an open top, as shown in the diagram below.

Assume that the thickness of the cardboard is negligible and that $V_{box} > 0$.



A box is to be made from a sheet of cardboard with $h = 25\ cm$.

a. Show that the volume, V_{box}, in cubic centimetres, is given by $V_{box}(x) = 2x(25 - 2x)(25 - x)$. (1 mark)

b. State the domain of V_{box}. (1 mark)

c. Find the derivative of V_{box} with respect to x. (1 mark)

d. Calculate the maximum possible volume of the box and for which value of x this occurs. (3 marks)

e. Waste minimisation is a goal when making cardboard boxes.
The percentage wasted is based on the area of the sheet of cardboard that is cut out before the box is made.
Find the percentage of the sheet of cardboard that is wasted when $x = 5$. (2 marks)

Now, consider a box made from a rectangular sheet of cardboard where $h > 0$ and the box's length is still twice its width.

f.

i. Let V_{box} be the function that gives the volume of the box.
State the domain of V_{box} in terms of h. (1 mark)

ii. Find the maximum volume for any such rectangular box, V_{box}, in terms of h. (3 marks)

g. Now, consider making a box from a square sheet of cardboard with side lengths of h centimetres.

Show that the maximum volume of the box occurs when $x = \frac{h}{6}$. (2 marks)

Question 166 (12 marks)

Let $q(x) = \log_e (x^2 - 1) - \log_e (1 - x)$.

a. State the maximal domain and the range of q. (2 marks)

b.

i. Find the equation of the tangent to the graph of q when $x = -2$. (1 mark)

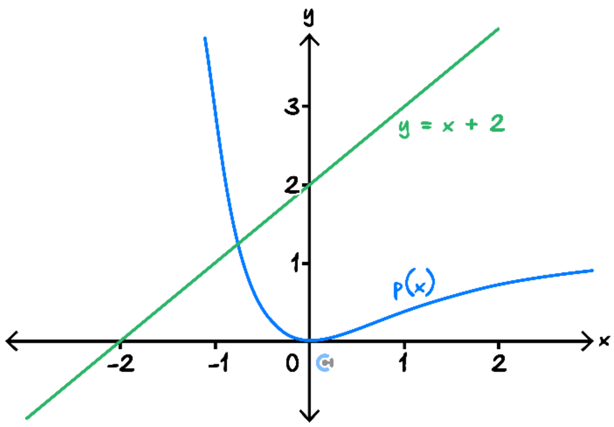
ii. Find the equation of the line that is perpendicular to the graph of q when $x = -2$ and passes through the point $(-2, 0)$. (1 mark)

Let $p(x) = e^{-2x} - 2e^{-x} + 1$.

c. Explain why p is not a one-to-one function. (1 mark)

d. Find the gradient of the tangent to the graph of p at $x = a$. (1 mark)

The diagram below shows parts of the graph of p and the line $y = x + 2$.



The line $y = x + 2$ and the tangent to the graph of p at $x = a$ intersect with an acute angle of θ between them.

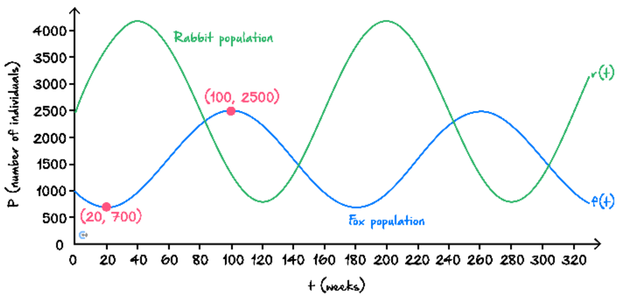
e. Find the value(s) of a for which $\theta = 60°$. Give your answer(s) correct to two decimal places. (3 marks)

f. Find the x-coordinate of the point of intersection between the line $y = x + 2$ and the graph of p, and hence, find the area bounded by $y = x + 2$, the graph of p and the x-axis, both correct to three decimal places. (3 marks)

Question 167 (16 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2022
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2022/2022MM2-w.pdf

On a remote island, there are only two species of animals: Foxes and rabbits. The foxes are the predators and the rabbits are their prey.
The populations of foxes and rabbits increase and decrease in a periodic pattern, with the period of both populations being the same, as shown in the graph below, for all $t \geq 0$, where time t is measured in weeks. One point of minimum fox population, $(20, 700)$, and one point of maximum fox population, $(100, 2500)$, are also shown on the graph.
The graph has been drawn to scale.



The population of rabbits can be modelled by the rule $r(t) = 1700 \sin\left(\frac{\pi t}{80}\right) + 2500$.

a.

i. State the initial population of rabbits. (1 mark)

ii. State the minimum and maximum population of rabbits. (1 mark)

iii. State the number of weeks between maximum populations of rabbits. (1 mark)

The population of foxes can be modelled by the rule $f(t) = a\sin(b(t-60)) + 1600$.

b. Show that $a = 900$ and $b = \frac{\pi}{80}$. (2 marks)

c. Find the maximum combined population of foxes and rabbits. Give your answer correct to the nearest whole number. (1 mark)

d. What is the number of weeks between the periods when the combined population of foxes and rabbits is a maximum? (1 mark)

The population of foxes is better modelled by the transformation of $y = \sin(t)$ under Q given by:

$$Q: R^2 \to R^2, Q\left(\begin{bmatrix} t \\ y \end{bmatrix}\right) = \begin{bmatrix} \frac{90}{\pi} & 0 \\ 0 & 900 \end{bmatrix}\begin{bmatrix} t \\ y \end{bmatrix} + \begin{bmatrix} 60 \\ 1600 \end{bmatrix}$$

e. Find the average population during the first 300 weeks for the combined population of foxes and rabbits, where the population of foxes is modelled by the transformation of $y = \sin(t)$ under the transformation Q. Give your answer correct to the nearest whole number. (4 marks)

Over a longer period of time, it is found that the increase and decrease in the population of rabbits gets smaller and smaller.
The population of rabbits over a longer period of time can be modelled by the rule:

$$s(t) = 1700 \, . \, e^{-0.003t} . \sin\left(\frac{\pi t}{80}\right) + 2500, \quad \text{for all } t \geq 0$$

f. Find the average rate of change between the first two times when the population of rabbits is at a maximum. Give your answer correct to one decimal place. (2 marks)

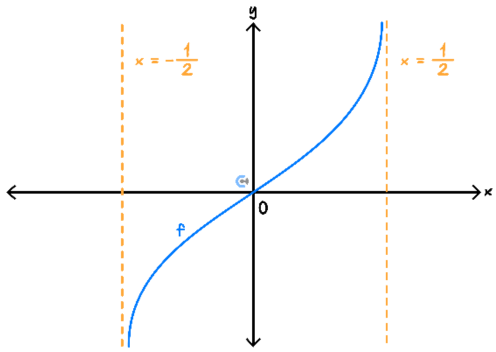
g. Find the time, where $t > 40$, in weeks, when the rate of change of the rabbit population is at its greatest positive value. Give your answer correct to the nearest whole number. (2 marks)

h. Over time, the rabbit population approaches a particular value.

State this value. (1 mark)

Question 168 (9 marks)

Consider the function f, where $f:\left(-\frac{1}{2},\frac{1}{2}\right)\rightarrow R, f(x)=\log_e\left(x+\frac{1}{2}\right)-\log_e\left(\frac{1}{2}-x\right)$.
Part of the graph of $y = f(x)$ is shown below.



a. State the range of $f(x)$. (1 mark)

b.

i. Find $f'(0)$. (2 marks)

ii. State the maximal domain over which f is strictly increasing. (1 mark)

c. Show that $f(x) + f(-x) = 0$. (1 mark)

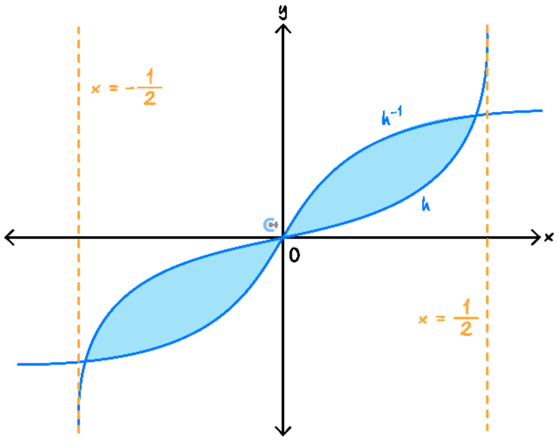
d. Find the domain and the rule of f^{-1}, the inverse of f. (3 marks)

Let h be the function $h\colon \left(-\frac{1}{2}, \frac{1}{2}\right) \to R, h(x) = \frac{1}{k}\left(\log_e\left(x + \frac{1}{2}\right) - \log_e\left(\frac{1}{2} - x\right)\right)$, where $k \in R$ and $k > 0$.

The inverse function of h is defined by $h^{-1}: R \to R, h^{-1}(x) = \frac{e^{kx}-1}{2(e^{kx}+1)}$.

The area of the regions bound by the functions h and h^{-1} can be expressed as a function, $A(k)$.

The graph below shows the relevant area shaded.



You are not required to find or define $A(k)$.

e. Determine the range of values of k such that $A(k) > 0$. (1 mark)

Question 169 (12 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2023
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2023/2023MM2-w.pdf

Consider the function $g : R \to R, g(x) = 2^x + 5$.

a. State the value of $\lim_{x \to -\infty} g(x)$. (1 mark)

b. The derivative, $g'(x)$, can be expressed in the form $g'(x) = k \times 2^x$.
Find the real number k. (1 mark)

c.

i. Let a be a real number. Find, in terms of a, the equation of the tangent to g at the point $(a, g(a))$. (1 mark)

ii. Hence, or otherwise, find the equation of the tangent to g that passes through the origin, correct to three decimal places. (2 marks)

Let $h : R \to R, h(x) = 2^x - x^2$.

d. Find the coordinates of the point of inflection for h, correct to two decimal places. (1 mark)

---

e. Find the largest interval of x-values for which h is strictly decreasing.
Give your answer correct to two decimal places. (1 mark)

---

f. Apply Newton's method, with an initial estimate of $x_0 = 0$, to find an approximate x-intercept of h.

Write the estimates x_1 , x_2 and x_3 in the table below, correct to three decimal places. (2 marks)

x_0	0
x_1	
x_2	
x_3	

g. For the function h, explain why a solution to the equation $\log_e(2) \times (2^x) - 2x = 0$ should not be used as an initial estimate x_0 in Newton's method. (1 mark)

---

h. There is a positive real number n for which the function $f(x) = n^x - x^n$ has a local minimum on the x-axis.

Find this value of n. (2 marks)

Question 170 (12 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/2024MM2-w.pdf

Consider the function $f: R \to R, f(x) = (x+1)(x+a)(x-2)(x-2a)$ where $a \in R$.

a. State, in terms of a where required, the values of x for which $f(x) = 0$. (1 mark)

b. Find the values of a for which the graph of $y = f(x)$ has:

i. Exactly three x-intercepts. (2 marks)

ii. Exactly four x-intercepts. (1 mark)

c. Let g be the function $g: R \to R, g(x) = (x+1)^2(x-2)^2$, which is the function f where $a = 1$.

i. Find $g'(x)$. (1 mark)

ii. Find the coordinates of the local maximum of g. (1 mark)

iii. Find the values of x for which $g'(x) > 0$. (1 mark)

iv. Consider the two tangent lines to the graph of $y = g(x)$ at the points where:

$$x = \frac{-\sqrt{3}+1}{2} \text{ and } x = \frac{\sqrt{3}+1}{2}.$$

Determine the coordinates of the point of intersection of these two tangent lines. (2 marks)

d. Let g remain as the function $g: R \to R, g(x) = (x+1)^2(x-2)^2$, which is the function f where $a = 1$. Let h be the function $h: R \to R, h(x) = (x+1)(x-1)(x+2)(x-2)$, which is the function f where $a = -1$.

i. Using translations only, describe a sequence of transformations of h, for which its image would have a local maximum at the same coordinates as that of g. (1 mark)

ii. Using dilation and translations, describe a different sequence of transformations of h, for which its image would have both local minimums at the same coordinates as that of g. (2 marks)

Question 171 (11 marks)

A model for the temperature in a room, in degrees Celsius, is given by:

$$f(t) = \begin{cases} 12 + 30t & 0 \leq t \leq \frac{1}{3} \\ 22 & t > \frac{1}{3} \end{cases}$$

Where t represents the time in hours after a heater is switched on.

a. Express the derivative $f'(t)$ as a hybrid function. (2 marks)

b. Find the average rate of change in temperature predicted by the model between $t = 0$ and $t = \frac{1}{2}$.

Give your answer in degrees Celsius per hour. (1 mark)

c. Another model for the temperature in the room is given by $g(t) = 22 - 10e^{-6t}, t \geq 0$.

i. Find the derivative $g'(t)$. (1 mark)

ii. Find the value of t for which $g'(t) = 10$.

Give your answer correct to three decimal places. (1 mark)

d. Find the time $t \in (0, 1)$ when the temperatures predicted by the models f and g are equal.

Give your answer correct to two decimal places. (1 mark)

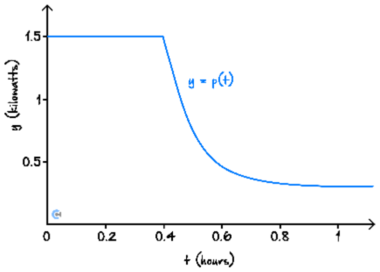
e. Find the time $t \in (0, 1)$ when the difference between the temperatures predicted by the two models is the greatest.

Give your answer correct to two decimal places. (1 mark)

f. The amount of power, in kilowatts, used by the heater t hours after it is switched on, can be modelled by the continuous function p, whose graph is shown below.

$$p(t) = \begin{cases} 1.5 & 0 \leq t \leq 0.4 \\ 0.3 + Ae^{-10t} & t > 0.4 \end{cases}$$

The amount of energy used by the heater, in kilowatt-hours, can be estimated by evaluating the area between the graph of $y = p(t)$ and the t-axis.



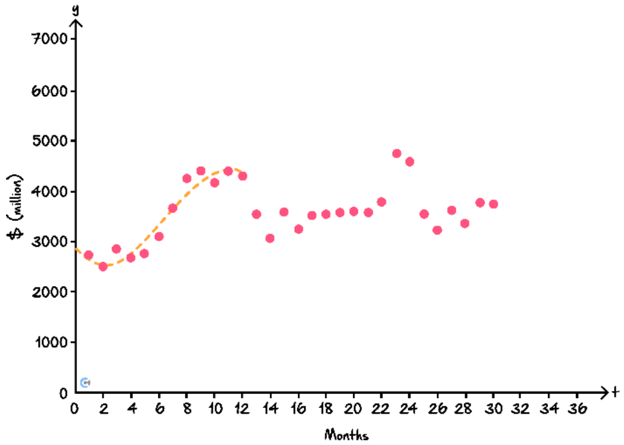
i. Given that $p(t)$ is continuous for $t \geq 0$, show that $A = 1.2e^4$. (1 mark)

ii. Find how long it takes after the heater is switched on until the heater has used 0.5-kilowatt hours of energy.
Give your answer in hours. (1 mark)

iii. Find how long it takes after the heater is switched on until the heater has used 1-kilowatt hour of energy.
Give your answer in hours, correct to two decimal places. (2 marks)

Question 172 (11 marks)

The points shown in the chart below represent monthly online sales in Australia. The variable y represents sales in millions of dollars. The variable t represents the month when the sales were made, where $t = 1$ corresponds to January 2021, $t = 2$ corresponds to February 2021 and so on.



Source : Australian Bureau of Statistics, Retail Trade, Australia, December 2023

a. A cubic polynomial $p : (0, 12] \to R, p(t) = at^3 + bt^2 + ct + d$ can be used to model monthly online sales in 2021.
The graph of $y = p(t)$ is shown as a dashed curve on the set of axes above.
It has a local minimum at $(2, 2500)$ and a local maximum at $(11, 4400)$.

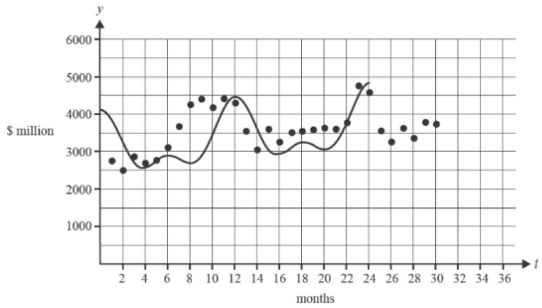
i. Find, correct to two decimal places, the values of a, b, c and d. (3 marks)

ii. Let $q: (12,24] \to R, q(t) = p(t - h) + k$ be a cubic function obtained by translating p, which can be used to model monthly online sales in 2022. Find the values of h and k such that the graph of $y = q(t)$ has a local maximum at $(23, 4750)$. (2 marks)

b. Another function f can be used to model monthly online sales, where:

$$f: (0,36] \rightarrow R, f(t) = 3000 + 30t + 700\cos\left(\frac{\pi t}{6}\right) + 400\cos\left(\frac{\pi t}{3}\right)$$

Part of the graph of f is shown on the axes below.



i. Complete the graph of f on the set of axes above until December 2023, that is, for $t \in (24, 36]$. Label the endpoint at $t = 36$ with its coordinates. (2 marks)

ii. The function f predicts that every 12 months, monthly online sales increase by n million dollars. Find the value of n. (1 mark)

iii. Find the derivative $f'(t)$. (1 mark)

iv. Hence, find the maximum instantaneous rate of change for the function f, correct to the nearest million dollars per month, and the values of t in the interval $(0, 36]$ when this maximum rate occurs, correct to one decimal place. (2 marks)

Question 173 (11 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2017
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2017/nht/2017MM2-nht-w.pdf

The temperature, T°C, in an office, is controlled. For a particular weekday, the temperature at time t, where t is the number of hours after midnight, is given by the function:

$$T(t) = 19 + 6\sin\left(\frac{\pi}{12}(t-8)\right), 0 \le t \le 24$$

a. What are the maximum and minimum temperatures in the office? (2 marks)

b. What is the temperature in the office at 6.00 AM? (1 mark)

c. Most of the people working in the office arrive at 8.00 AM.

What is the temperature in the office when they arrive? (1 mark)

d. For how many hours of the day is the temperature greater than or equal to 19°C? (2 marks)

e. What is the average rate of change of the temperature in the office between 8.00 AM and noon? (2 marks)

f.

i. Find $T'(t)$. (1 mark)

ii. At what time of the day is the temperature in the office decreasing most rapidly? (2 marks)

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Question 174 (13 marks)

Let $f: R \to R$, where $f(x) = (x-2)^2(x-5)$.

a. Find $f'(x)$. (1 mark)

b. For what values of x is $f'(x) < 0$? (1 mark)

c.

i. Find the gradient of the line segment joining the points on the graph of $y = f(x)$ where $x = 1$ and $x = 5$. (1 mark)

ii. Show that the midpoint of the line segment in **part c.i.** also lies on the graph of $y = f(x)$. (2 marks)

iii. Find the values of x for which the gradient of the tangent to the graph of $y = f(x)$ is equal to the gradient of the line segment joining the points on the graph where $x = 1$ and $x = 5$. (2 marks)

Let $g: R \to R$, where $g(x) = (x-2)^2 (x-a)$, where $a \in R$.

d. The coordinates of the stationary points of g are $P(2, 0)$ and $Q(p(a+1), q(a-2)^3)$, where p and q are rational numbers.

Find the values of p and q. (2 marks)

e. Show that the gradient of the tangent to the graph of $y = g(x)$ at the point $(a, 0)$ is positive for $a \in R \backslash \{2\}$. (1 mark)

f.

i. Find the coordinates of another point where the tangent to the graph of $y = g(x)$ is parallel to the tangent at $x = a$. (2 marks)

ii. Hence, find the distance between this point and point Q when $a > 2$. (1 mark)

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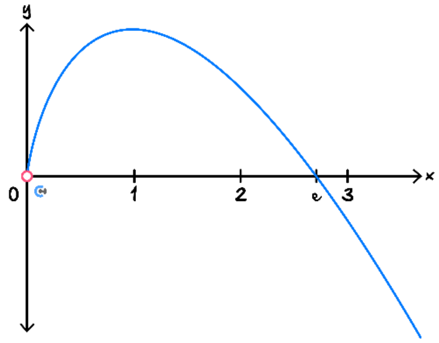
Question 175 (12 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2018
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2018/nht/2018MM2-nht-w.pdf

Let $f: (0, \infty) \to R, f(x) = x - x\log_e(x)$. Part of the graph of f is shown below.



a. Find the values of x for which:

i. $-1 < f'(x) < -\frac{1}{2}$. (2 marks)

ii. $\frac{1}{2} < f'(x) < 1$. (1 mark)

b.

i. Find the equation of the tangent to the graph of f at the point $(a, f(a))$ in the form $y = mx + c$. (1 mark)

ii. Find the coordinates of the point of intersection of the tangent to the graph of f at $x = a$ and the tangent to the graph of f at $x = \frac{1}{a}$. (2 marks)

iii. Hence, find the coordinates of the point of intersection of the tangents to the graph of f at $x = e$ and $x = \frac{1}{e}$. Express each coordinate in terms of e. (1 mark)

c.

i. For a value of $b > e$, the tangent to f at the point $(b, f(b))$ and the tangent to f at the point $(2, f(2))$ intersect the x-axis at the same point.

Find the value of b. (2 marks)

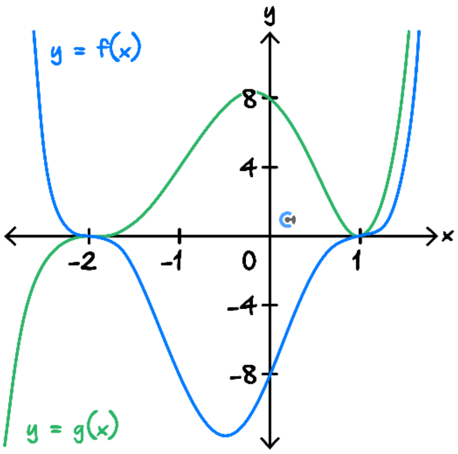
ii. If the tangent to f at the point $(p, f(p))$, where $1 < p < e$, and the tangent to f at the point $(q, f(q))$, where $q > e$, intersect on the x-axis, show that $p^q = q^P$. (2 marks)

d. Find the equation of the tangent to the graph of f at the point where $x = e^{\frac{1}{2}}$. (1 mark)

Question 176 (9 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2019
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2019/NHT/2019MM2-nht-w.pdf

Parts of the graphs of $f(x) = (x-1)^3(x+2)^3$ and $g(x) = (x-1)^2(x+2)^3$ are shown on the axes below.



The two graphs intersect at three points, $(-2, 0)$, $(1, 0)$ and (c, d). The point (c, d) is not shown in the diagram above.

a. Find the values of c and d. (2 marks)

b. Find the values of x such that $f(x) > g(x)$. (1 mark)

c. State the values of x for which:

i. $f'(x) > 0$. (1 mark)

ii. $g'(x) > 0$. (1 mark)

d. Show that $f(1 + m) = f(-2 - m)$ for all m. (1 mark)

e. Find the values of h such that $g(x + h) = 0$ has exactly one negative solution. (2 marks)

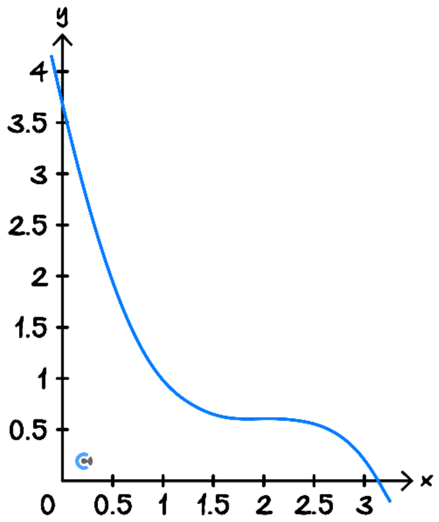
f. Find the values of k such that $f(x) + k = 0$ has no solutions. (1 mark)

Question 177 (10 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2022
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2022/NHT/2022mm2-nht-w.pdf

Let $f: R \to R, f(x) = -\frac{2}{5}(x-2)^3 + \frac{3}{5}$.

Part of the graph of f is shown below.



a. Find $f'(x)$, the derivative of f, with respect to x. (1 mark)

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b. Give the coordinates of the stationary point of f. (1 mark)

---

c. The graph of f has a tangent with a gradient of $-\frac{6}{5}$ when $x = 1$.

The graph of f also has a tangent with a gradient of $-\frac{6}{5}$ at another point, D.

i. Show that the x-coordinate of D is 3. (1 mark)

---

ii. Determine the equation of the tangent that touches the graph of f at point D. (1 mark)

iii. The tangent to f at point D intersects the graph of f at another point, M.
Give the coordinates of point M. (2 marks)

iv. Find the obtuse angle, in degrees, that the tangent to f at point D makes with the positive direction of the horizontal axis. Give your answer correct to one decimal place. (1 mark)

v. The graph has two regions:

The first region is bounded by the graph of f and the tangent to f at point D.

The second region is bounded by the graph of f, the tangent to f at point D and the horizontal axis.

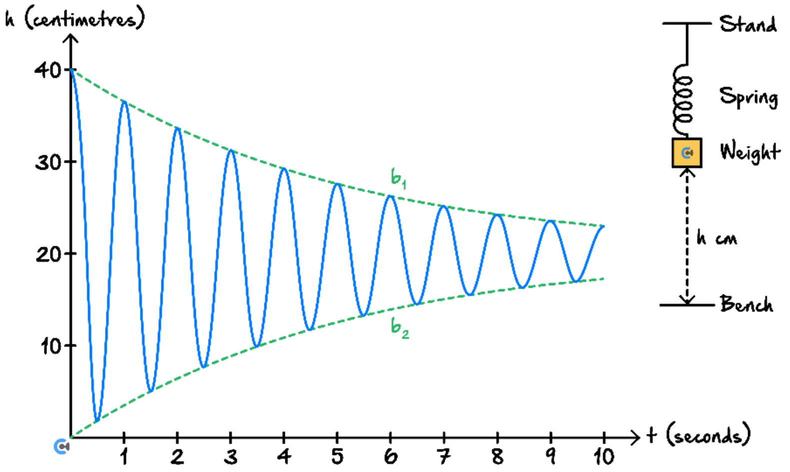
Find the total area of the two regions. Give your answer correct to four decimal places. (3 marks)

Question 178 (9 marks)

A spring with a weight attached is suspended from a stand. The base of the weight is 40 cm above a bench.
The spring is released and moves vertically up and down above the surface of the bench, such that the height of the base of the weight above the bench over the next 10 seconds is given by the function:

$$h(t) = 20e^{-\frac{1}{5}}\cos(2\pi t) + 20, \quad 0 \le t \le 10$$

Where t is the time, measured in seconds. A graph of the function h over the first 10 seconds is shown below.



The dashed curve b_1 lies above the graph of h and the dashed curve b_2 lies below the graph of h. Both b_1 and b_2 bound the graph of h.

The dashed curve b_1, has the equation $b_1(t) = 20e^{-\frac{t}{5}} + 20$.

a. State the equation of the dashed curve b_2. (1 mark)

b. Find the average value of the height, in centimetres, of the base of the weight above the bench over the first 10 seconds. Give your answer correct to two decimal places. (2 marks)

c.

i. Write down the rule for the derivative of h. (1 mark)

ii. Find the time, in seconds, and the height above the surface of the bench, in centimetres, of the point of maximum positive rate of change in h over the first 10 seconds. Write your answer as a coordinate pair, correct to one decimal place. (3 marks)

d. Determine the total distance travelled by the base of the weight over the first 2 seconds of its motion. Give your answer correct to the nearest centimetre. (2 marks)

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Question 179 (13 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2023
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2023/NHT/2023MM2-nht-w.pdf

The amount of caffeine present in Kim's body after they drink espresso coffee can be modelled mathematically. Students suggest that the amount of caffeine, C in milligrams, in Kim's body t hours after consuming an espresso coffee can be modelled by the function $C(t) = 65e^{-\frac{1}{8}}$.

a. How much caffeine will be present in Kim's body 2 hours after they consume an espresso? Give your answer in milligrams, correct to one decimal place. (1 mark)

b. How long will it take for the amount of caffeine in Kim's body to reach 10 milligrams after drinking an espresso? Give your answer in hours and minutes, correct to the nearest minute. (2 marks)

c. At what rate is the amount of caffeine in Kim's body decaying 4 hours after they drink an espresso? Give your answer in milligrams per hour in the form $\frac{a}{b\sqrt{e}}$, where a and b are positive integers. (2 marks)

Kim consumes another espresso coffee 4 hours after consuming the first.
The students then suggest that a more appropriate model for the absorption of caffeine, in milligrams, in Kim's body t hours after they consume the first espresso is:

$$C_2(t) = \begin{cases} 65e^{-\frac{1}{8}} & 0 \le t < 4 \\ 65\left(\dfrac{1-e}{1-\sqrt{e}}\right)e^{-\frac{1}{8}} & t \ge 4 \end{cases}$$

d. When $t \geq 4$, is the function C_2 strictly increasing, strictly decreasing or neither? (1 mark)

e. Show that the function C_2 is not continuous for $t > 0$. (1 mark)

f. Using C_2, find the maximum amount of active caffeine in Kim's body and the time at which this level was reached. Give the maximum amount of caffeine, in milligrams, correct to one decimal place. (2 marks)

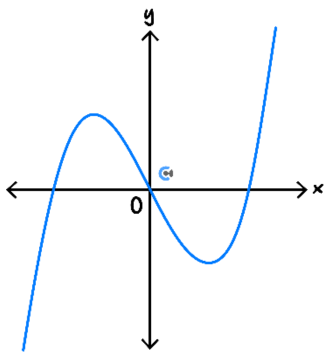
g. Find the derivative $C_2'(t)$, giving your answer as a hybrid function that includes the relevant domains. (2 marks)

h. Use the derivative $C_2{}'(t)$ to find the times during which the amount of active caffeine is decreasing by at least 8 milligrams per hour. Express your answer in interval notation, correct to one decimal place. (2 marks)

Question 180 (10 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/NHT/2024MM2-nht-w.pdf

Consider the function $f: R \to R, f(x) = x^3 - px$, where $p \in R$.
Part of the graph of f is shown below when $p = 3$.



a. Find the values of the x-intercepts of f, when $p = 3$. (1 mark)

b. Use the derivative f' to find the coordinates of the turning points of f, when $p = 3$. (2 marks)

c.

i. Find the value of p for which f would have exactly one stationary point. (1 mark)

ii. Find the values of p for which f would not have any stationary points. (1 mark)

d. The graph of f passes through the origin for all values of p.

i. Use calculus to show that the tangent line to f at the origin has the equation $y = -px$. (2 marks)

ii. Find, in terms of p, the area of the region bounded by the function f, the line $y = -px$ and the line $x = p$, where $p > 0$. (2 marks)

iii. The expression for the area found in **part d.ii.** also gives the area bounded by a cubic function $y = kx^3$, the x-axis and the line $x = p$, where $p > 0$.

Find all possible values of k. (1 mark)

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