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VCE Mathematical Methods ¾
AOS 2 Revision [2.0]
Contour Check (Part 2) Solutions

Contour Check

Section H: [2.1 - 2.7] – Exam 1 Overall (Checkpoints) (125 Marks)

Question 69 (4 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2016
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2016/2016MM1-w.pdf#page=3

a. Let $y = \frac{\cos(x)}{x^2+2}$.

Find $\frac{dy}{dx}$. (2 marks)

Marks	0	1	2	Average
%	11	33	56	1.5

$$\frac{dy}{dx} = \frac{-\sin(x)(x^2+2) - 2x\cos(x)}{(x^2+2)^2}$$

Most students were able to confidently apply the quotient rule. However, many students did not obtain full marks due to errors caused by, for example, a denominator of x^4+4 as the supposed expansion of $(x^2+2)^2$. Students should very carefully consider the placement and usage of brackets. For example, the expression $x^2+2\times-\sin(x)$ is not equivalent to $(x^2+2)\times-\sin(x)$.

b. Let $f(x) = x^2e^{5x}$.

Evaluate $f'(1)$. (2 marks)

Marks	0	1	2	Average
%	13	18	69	1.6

$f'(x) = 2xe^{5x} + 5x^2e^{5x}$

$f'(1) = 7e^5$

This question was well answered. Most students correctly identified the product rule but did not evaluate (as instructed) or their answers were incomplete. An incorrect combination of the product and chain rule resulted in an answer of $10xe^{5x}$ as a common error.

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Question 70 (3 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2016
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2016/2016MM1-w.pdf#page=4

Let $f: \left(-\infty, \frac{1}{2}\right] \to R$, where $f(x) = \sqrt{1-2x}$.

a. Find $f'(x)$. (1 mark)

Marks	0	1	Average
%	31	69	0.7

$f'(x) = -\frac{1}{\sqrt{1-2x}}$

Most students utilised the chain rule on an expression involving a fractional exponent. However, many students then missed the negative sign in the final answer, forgetting that the derivative of $(1-2x)$ is -2.

b. Find the angle θ from the positive direction of the x-axis to the tangent to the graph of f at $x = -1$, measured in the anticlockwise direction. (2 marks)

Marks	0	1	2	Average
%	42	39	20	0.8

$\tan\theta = f'(-1) = -\frac{1}{\sqrt{3}}$

$\theta = \frac{5\pi^c}{6}$ *or* 150°

This question was not answered well. Many students who knew the connection between $\tan(\theta)$ and $f'(-1)$ had difficulty in finding the required angle. Students should know the exact values of circular functions in all quadrants. Many students incorrectly assumed that $\text{gradient} = f(-1)$ or wasted time finding the equation of the tangent.

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Question 71 (2 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2016
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2016/2016MM1-w.pdf#page=10

Let $f: [-\pi, \pi] \to R$, where $f(x) = 2\sin(2x) - 1$.

Calculate the average rate of change of f between $x = -\frac{\pi}{3}$ and $x = \frac{\pi}{6}$.

Marks	0	1	2	Average
%	37	31	32	1

$f\left(-\frac{\pi}{3}\right) = 2\sin\left(-\frac{2\pi}{3}\right) - 1 = -\sqrt{3} - 1$

$f\left(\frac{\pi}{6}\right) = 2\sin\left(\frac{\pi}{3}\right) - 1 = \sqrt{3} - 1$

Average rate of change $= \dfrac{(\sqrt{3}-1)-(-\sqrt{3}-1)}{\frac{\pi}{6} - \left(-\frac{\pi}{3}\right)}$

$= \dfrac{4\sqrt{3}}{\pi}$

Most students used the correct gradient rule but erred when evaluating, particularly $f\left(-\frac{\pi}{3}\right)$ or in dealing with fractions in the denominator. A few students confused average rate of change with average value, and some incorrectly found the average of derivatives.

Question 72 (4 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2017
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2017/2017MM1-w.pdf#page=3

a. Let $f: (-2, \infty) \to R, f(x) = \frac{x}{x+2}$.

Differentiate f with respect to x. (2 marks)

Marks	0	1	2	Average
%	12	19	69	1.3

$f'(x) = \dfrac{(x+2)\times 1 - x \times 1}{(x+2)^2} = \dfrac{2}{(x+2)^2}$

This question was well handled. Students choosing to use the quotient rule tended to progress better than those using the product rule. Some very poor algebraic slips were made. The most common was 'cancelling' $x + 2$ in the numerator with $x + 2$ in the denominator. Others unnecessarily expanded $(x + 2)^2$ and did so incorrectly.

b. Let $g(x) = (2 - x^3)^3$.

Evaluate $g'(1)$. (2 marks)

Marks	0	1	2	Average
%	14	21	66	1.1

$g'(x) = 3(2 - x^3)^2 \times (-3x^2)$
$g'(1) = -9(2 - 1)^2 = -9$

Students competently applied the chain rule; however, some erred with the derivative of $(2 - x^3)^3$, especially with negatives. Some students opted unnecessarily to take the longer route by (often incorrectly) expanding the rule given by *g*. Others forgot to evaluate $g'(1)$.

Question 73 (3 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2018
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2018/2018MM1-w.pdf#page=3

a. If $y = (-3x^3 + x^2 - 64)^3$, find $\frac{dy}{dx}$. (1 mark)

Marks	0	1	Average
%	42	58	0.6

$\frac{dy}{dx} = 3(-9x^2 + 2x)(-3x^3 + x^2 - 64)^2$

Students generally recognised the need to deploy the chain rule; however, a significant number of students could not be awarded the mark. Poor use of brackets (or lack of brackets) resulted in an incorrect expression. For example, the expression $3(-3x^3 + x^2 - 64)^2(-9x^2 + 2x)$ is not equivalent to $3(-3x^3 + x^2 - 64)^2 - 9x^2 + 2x$. Transcription errors (especially with exponents) and arithmetic errors with unnecessary expansions were also observed.

b. Let $f(x) = \frac{e^x}{\cos(x)}$.

Evaluate $f'(\pi)$. (2 marks)

Marks	0	1	2	Average
%	10	39	51	1.4

$$f'(x) = \frac{e^x \cos(x) + e^x \sin(x)}{\cos^2(x)}$$

$$f'(\pi) = -e^\pi$$

Students competently applied the quotient rule; however, many were unable to carry out the required evaluation, often omitting it completely. Students who opted to use the product and chain rules tended to make little progress due to confusion with negative signs or negative exponents. Students should take care with legibility, for example, to distinguishing clearly the variable x and the constant π.

Question 74 (4 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2019
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2019/2019MM1-w.pdf#page=3

Let $f: \left(\frac{1}{3}, \infty\right) \to R, f(x) = \frac{1}{3x-1}$.

a.

i. Find $f'(x)$. (1 mark)

Marks	0	1	Average
%	33	67	0.7

$$f'(x) = \frac{-3}{(3x-1)^2}$$

The majority of students correctly applied the chain rule. Errors were generally arithmetic in nature or with the negative exponent.

ii. Find an antiderivative of $f(x)$. (1 mark)

Marks	0	1	Average
%	50	50	0.5

$$\left(\frac{1}{3}\right)\log_e(3x-1) \quad \text{or} \quad \left(\frac{1}{3}\right)\log_e\left(x-\frac{1}{3}\right)$$

There were various ways of expressing the anti-derivative, with the above being the most common. The most common error was placing a constant of 3 or 1 (rather than $\left(\frac{1}{3}\right)$) in front of the log expression. Students should note that they could easily verify their answer by using the chain rule to differentiate their answer, and checking whether or not this derivative was in fact the rule for f.

b. Let $g: R\backslash\{-1\} \to R, g(x) = \frac{\sin(\pi x)}{x+1}$.

Evaluate $g'(1)$. (2 marks)

Marks	0	1	2	Average
%	12	39	50	1.4

$$g'(x) = \frac{(x+1)(\pi\cos(\pi x)) - (\sin(\pi x)) \times 1}{(x+1)^2}$$

Hence $g'(1) = -\frac{\pi}{2}$

Though generally well handled, poor placement of, or lack of, brackets when using quotient rule (or the combination of product and chain rules) led to errors in evaluation. Other errors included the misconception that $cos(\pi) = 1$ or misquoting the relevant differentiation rule (which is listed on the formula sheet).

Some students did not answer the question in its entirety (i.e. completely forgetting to evaluate $g'(1)$).

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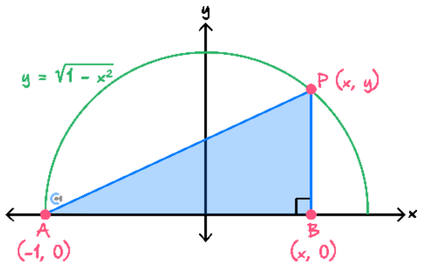
Question 75 (4 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2019
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2019/2019MM1-w.pdf#page=9

The graph of the relation $y = \sqrt{1 - x^2}$ is shown on the axes below. P is a point on the graph of this relation, A is the point $(-1, 0)$ and B is the point $(x, 0)$.



a. Find an expression for the length PB in terms of x only. (1 mark)

Marks	0	1	Average
%	42	58	0.6

$PB = \sqrt{1-x^2}$

Though mostly well done, some students left their answer as an incorrect and unsimplified form of the distance formula.

b. Find the maximum area of the triangle ABP. (3 marks)

Marks	0	1	2	3	Average
%	41	33	15	11	1

$A(x) = \frac{1}{2}(x+1)\left(\sqrt{1-x^2}\right)$

$A'(x) = \frac{1}{2}\left(\left(1 \times \sqrt{1-x^2}\right) + (x+1) \times (1-x^2)^{-\frac{1}{2}} \times (-2x)\right)$

$A'(x) = 0, \frac{1-x-2x^2}{2\sqrt{1-x^2}} = 0$ so solve $2x^2 + x - 1 = 0$

Maximum Area : $A\left(\frac{1}{2}\right) = \frac{3\sqrt{3}}{8}$

The majority of students used calculus to solve this problem. Some students used geometry and trigonometry to obtain a correct solution. Most students managed to find an expression for the area in terms of x. Many of those who used calculus found the differentiation of the expression difficult, generally as a result of poor setting out, particularly with lack of brackets, or dealing with negative terms. Students are encouraged to practice differentiations involving combinations of product and chain rules.

Question 76 (9 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2019
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2019/2019MM1-w.pdf#page=12

Consider the functions $f: R \to R, f(x) = 3 + 2x - x^2$ and $g: R \to R, g(x) = e^x$.

a. State the rule of $g(f(x))$. (1 mark)

Marks	0	1	Average
%	6	94	1

$g(f(x)) = e^{3+2x-x^2}$

This question was done well.

b. Find the values of x for which the derivative of $g(f(x))$ is negative. (2 marks)

Marks	0	1	2	Average
%	59	19	22	0.7

Let $h(x) = g(f(x))$

$h'(x) = (2-2x)e^{3+2x+x^2}$

$h'(x) > 0$ when $(2-2x) > 0$

The required set of values is $(1, \infty)$ or $x > 1$.

Students generally applied the chain rule to find the derivative; however, poor expression resulted in incorrect answers. The expression $(2-2x)e^{3+2x-x^2}$ is **not** equivalent to $2 - 2xe^{3+2x-x^2}$. Some students did find the correct answer; however, it was not supported by correct reasoning.

c. State the rule of $f(g(x))$. (1 mark)

Marks	0	1	Average
%	14	86	0.9

$f(g(x)) = 3 + 2e^x - (e^x)^2$

This question was done well. Some students incorrectly stated $f(g(x)) = 3 + 2e^x - e^{x^2}$.

d. Solve $f(g(x)) = 0$. (2 marks)

Marks	0	1	2	Average
%	34	19	48	1.2

$3 + 2e^x - e^{2x} = 0$

$(3 - e^x)(1 + e^x) = 0$

Hence $x = \log_e(3)$ since $e^x > 0$

Most students were able to form a quadratic equation. Some students faltered with the correct factorisation. The inclusion of $x = \log_e(-1)$ was a common error.

e. Find the coordinates of the stationary point of the graph of $f(g(x))$. (2 marks)

Marks	0	1	2	Average
%	42	28	30	0.9

$2e^x(1-e^x)=0$

$e^x=1 \Rightarrow x=0$

Thus the stationary point is at $(0,4)$

This question was well attempted but not so well done. Common errors included an incorrect derivative and omitting the y-coordinate of the stationary point.

f. State the number of solutions to $g(f(x)) + f(g(x)) = 0$. (1 mark)

Marks	0	1	Average
%	81	19	0.2

$g(f(x)) + f(g(x)) = 0$ has exactly one solution.

This question was not well done. Few students attempted to draw a rough sketch of each equation and use addition of ordinates.

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Question 77 (3 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2020
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2020/2020MM1-w.pdf#page=3

a. Let $y = x^2 \sin(x)$.

Find $\frac{dy}{dx}$. (1 mark)

Marks	0	1	Average
%	14	86	0.9

$$\frac{dy}{dx} = 2x\sin(x) + x^2\cos(x)$$

This question was well answered. Most students competently and confidently applied the product rule.

b. Evaluate $f'(1)$, where $f: R \to R, f(x) = e^{x^2-x+3}$. (2 marks)

Marks	0	1	2	Average
%	20	21	60	1.4

$$f'(x) = (2x-1)e^{x^2-x+3}$$

$$f'(1) = e^3$$

Students applied the chain rule; however, too often the lack of brackets resulted in an incorrect answer: for example, $(2x-1)e^{x^2-x+3} \neq 2x - 1e^{x^2-x+3}$

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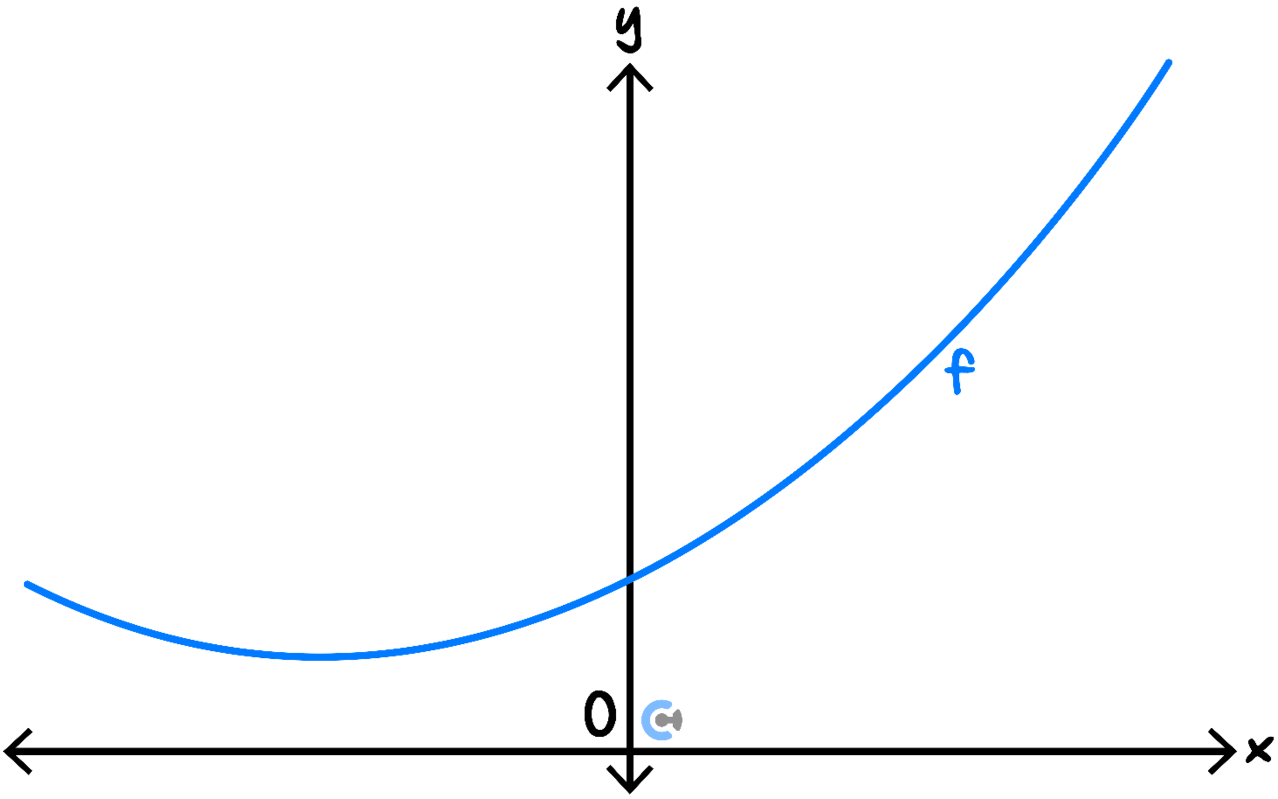
Question 78 (8 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2020
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2020/2020MM1-w.pdf#page=10

Consider the function $f(x) = x^2 + 3x + 5$ and the point $P(1, 0)$. Part of the graph of $y = f(x)$ is shown below.



a. Show that point P is not on the graph of $y = f(x)$. (1 mark)

Marks	0	1	Average
%	15	85	0.9

$f(1) = 1^2 + 3 \times 1 + 5 = 9 \neq 0$

Alternatively, $x = 1, y = 9 \neq 0$

There were several ways to complete this question. Most students chose to show that the point $(1, 0)$ was not on the graph through the use of substitution as indicated above. This was a 'show that' question, so those students who simply stated $f(1) = 9$ without explaining the relevance of this were not awarded the mark. Some students found the discriminant of the quadratic to be negative or simply stated it was negative without evidence but did not relate this to the question.

b. Consider a point $Q(a, f(a))$ to be a point on the graph of f.

i. Find the slope of the line connecting points P and Q in terms of a. (1 mark)

Marks	0	1	Average
%	48	52	0.5

$$\frac{a^2+3a+5}{a-1}$$

Many students wrote down an expression for gradient but went no further. Some students made algebraic errors, in particular cancellations of 'a' or dealing with negative coefficients.

ii. Find the slope of the tangent to the graph of f at point Q in terms of a. (1 mark)

Marks	0	1	Average
%	33	67	0.7

$f'(a) = 2a + 3$

Most students recognised that an evaluation of the derivative was required. Some students incorrectly assumed the question required the equation of the tangent at $x = a$.

iii. Let the tangent to the graph of f at $x = a$ pass through point P.

Find the values of a. (2 marks)

Marks	0	1	2	Average
%	56	14	31	0.8

$f'(a) = m$

$$(2a+3) = \frac{a^2+3a+5}{a-1}$$

$a^2 - 2a - 8 = 0$

$a = 4, -2$

Alternatively find the equation of the tangent:

$y - f(a) = (2a+3)(x-a)$

$y = (2a+3)(x-a) + (a^2+3a+5)$

Since tangent passes through (1,0)

$a^2 - 2a - 8 = 0$

$a = 4, -2$

Students who equated gradients tended to score more highly. Many of those who used the 'equation of the tangent' method could not form the correct quadratic equation.

iv. Give the equation of one of the lines passing through point P that is tangent to the graph of f. (1 mark)

Marks	0	1	Average
%	71	29	0.3

If $x = -2$, $f'(-2) = -1$ so $y = 1 - x$

If $x = 4$, $f'(4) = 11$ so $y = 11x - 11$

The most common error was students assuming that their value of 'a' was the gradient of the line instead of substituting into $f'(a)$.

c. Find the value, k, that gives the shortest possible distance between the graph of the function of $y = f(x - k)$ and point P. (2 marks)

Marks	0	1	2	Average
%	87	3	10	0.2

The turning point occurs at $x = -\frac{3}{2}$ and is a minimum.

Translate $f(x)$ 2.5 units in the positive x direction so that the turning point is directly above (1,0).

Thus $k = \frac{5}{2}$

Many students used the distance formula and then attempted to differentiate and equate to zero (often with limited success due to error in differentiation or algebra). Students who used a geometric approach tended to score more highly.

Question 79 (3 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2021
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2021/2021MM1-w.pdf#page=2

a. Differentiate $y = 2e^{-3x}$ with respect to x. (1 mark)

Marks	0	1	Average
%	13	87	0.9

$-6e^{-3x}$ or $-\frac{6}{e^{3x}}$

This question was well answered. Most students accurately and confidently applied the chain rule and knew how to differentiate the exponential.

b. Evaluate $f'(4)$, where $f(x) = x\sqrt{2x+1}$. (2 marks)

Marks	0	1	2	Average
%	30	28	43	1.2

$f'(x) = \frac{3x+1}{\sqrt{2x+1}}$, $f'(4) = 4\frac{1}{3} = \frac{13}{3}$

Students generally handled this question well, applying the product rule to give the derivative. A common error was not differentiating the $2x$ within the $\sqrt{2x+1}$. The evaluation of the derived function involved evaluating square roots and this caused problems for some.

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Question 80 (3 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2022
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2022/2022MM1-w.pdf#page=3

a. Let $y = 3xe^{2x}$.

Find $\frac{dy}{dx}$. (1 mark)

Marks	0	1	Average
%	35	65	0.7

$6xe^{2x} + 3e^{2x}$

This question was well attempted; however a number of students wrote $3x2e^{2x} + 3e^{2x}$ as their final answer. This was an incomplete answer, as the term $3x2e^{2x}$ needed to be written as $6xe^{2x}$. Some students did not use the product rule that was required. Many students chose to factorise their answer and in doing so factorised incorrectly. It is important to note that there was no requirement to express the answer in factorised form. If students further engage with their answer, and the final response is incorrect, even if a correct answer has been previously written, full marks cannot be awarded.

b. Find and simplify the rule of $f'(x)$, where $f: R \to R, f(x) = \frac{\cos(x)}{e^x}$. (2 marks)

Marks	0	1	2	Average
%	9	37	53	1.5

$$f'(x) = \frac{-e^x \cdot sin(x) - e^x \cdot cos(x)}{e^{2x}}$$

$$= -\frac{(sin(x) + cos(x))}{e^x} \text{ or } -e^{(-x)}(sin(x) + cos(x))$$

Students generally responded to this question well, applying either the product rule or quotient rule to obtain the derivative. Some students did not demonstrate an understanding of what was required to simplify the expression. Some students cancelled only one of the e^x terms.

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Question 81 (4 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2023
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2023/2023MM1-w.pdf#page=3

a. Let $y = \frac{x^2-x}{e^x}$.

Find and simplify $\frac{dy}{dx}$. (2 marks)

Marks	0	1	2	Average
%	8	50	42	1.3

$$\frac{dy}{dx} = \frac{e^x(2x-1)-e^x(x^2-x)}{e^{2x}}$$

$$\frac{dy}{dx} = \frac{-x^2+3x-1}{e^x} \text{ or } \frac{-(x^2-3x+1)}{e^x} \text{ or } (-x^2+3x-1)e^{-x}$$

This question was well attempted and required students to use either the product rule or quotient rule to find the derivative. The question required students to simplify their answers. Many students simplified the quadratic component but left the exponential terms unsimplified. Some students did not use brackets around terms and subsequently did not correctly develop the signs of terms, or collect 'like terms'. For example, $\frac{-x^2+x-1}{e^x}$ was a common incorrect response.

b. Let $f(x) = \sin(x)\,e^{2x}$.

Find $f'\left(\frac{\pi}{4}\right)$. (2 marks)

Marks	0	1	2	Average
%	9	26	65	1.6

$$f'(x) = 2\sin(x)e^{2x} + \cos(x)e^{2x}$$

$$f'\left(\frac{\pi}{4}\right) = 2\sin\left(\frac{\pi}{4}\right)e^{\frac{\pi}{2}} + \cos\left(\frac{\pi}{4}\right)e^{\frac{\pi}{2}}$$

$$f'\left(\frac{\pi}{4}\right) = \sqrt{2}e^{\frac{\pi}{2}} + \frac{\sqrt{2}e^{\frac{\pi}{2}}}{2} = \frac{3\sqrt{2}}{2}e^{\frac{\pi}{2}} \text{ or } \frac{3e^{\frac{\pi}{2}}}{\sqrt{2}}$$

Students generally responded to this question well. The question required students to use the product rule to differentiate and then evaluate the derivative at $x = \frac{\pi}{4}$. There was no requirement to give the answer in a particular form. Some students did not demonstrate an understanding of how to differentiate e^{2x} correctly and produced responses that had a combination of e^{2x} and e^x terms. Some students presented responses indicating that they did not know how to arithmetically engage with the surd terms in their answer.

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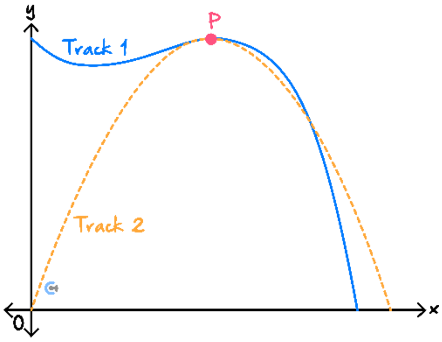
Question 82 (6 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2023
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2023/2023MM1-w.pdf#page=12

The shapes of two walking tracks are shown below.



Track 1 is described by the function $f(x) = a - x(x - 2)^2$.

Track 2 is defined by the function $g(x) = 12x + bx^2$.

The unit of length is kilometres.

a. Given that $f(0) = 12$ and $g(1) = 9$, verify that $a = 12$ and $b = -3$. (1 mark)

Marks	0	1	Average
%	10	90	0.9

$f(0) = a - 0(0-2)^2 = a - 0 = 12,\ f(0) = a$

$\therefore\ a = 12$

$g(1) = 12 \times 1 + b \times 1^2 = 12 + b = 9$

$\therefore\ b = -3$

This question was frequently attempted successfully. Most students knew that in order to 'verify' the values they needed to show working to support this.

b. Verify that $f(x)$ and $g(x)$ both have a turning point at P.

Give the co-ordinates of P.

Marks	0	1	2	Average
%	34	36	30	1.0

$f(x) = 12 - x(x-2)^2$

$= -x^3 + 4x^2 - 4x + 12$

$f'(x) = -(3x^2 - 8x + 4)$

$f'(2) = -(3(2)^2 - 8(2) + 4) = 0$

$g(x) = 12x - 3x^2$ OR $g(x) = 12x - 3x^2$

$g'(x) = 12 - 6x$ $= -3x^2 + 12x$

$g'(2) = 0$

Turning point at:

$$x = \frac{-b}{2a} = \frac{-12}{-6} = 2$$

Maxima of the graph; either use f or g:

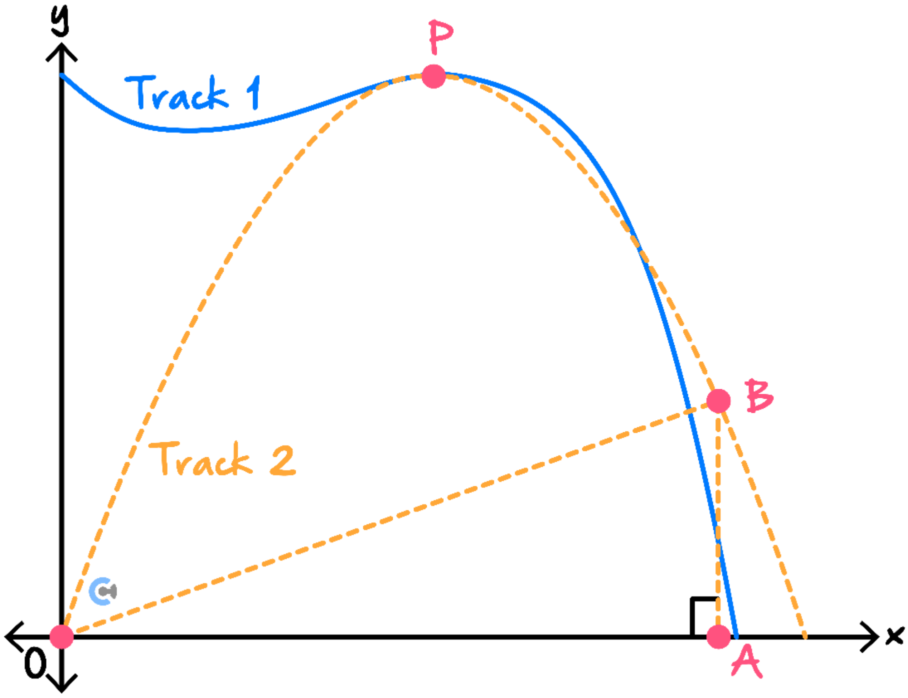
$f(2) = 12 - 0 = 12$

$(2,12)$

This question required students to verify that both $f(x)$ and $g(x)$ have a turning point at P. It was not sufficient to assume by inspection that the tracks met at P. Some students misinterpreted the question and solved $f(x) = g(x)$; they stopped short of showing that the point of intersection was a turning point for both curves. Errors were involved in expanding the brackets of $f(x)$ and finding $f'(x)$. Some students just showed that $g(x)$ had a turning point at $x = 2$, not addressing the turning points of $f(x)$. Some students incorrectly used the product rule to differentiate $f(x)$ and gave $f'(x) = x(x-2)^2$. Most students were successful in finding the coordinates of P at (2,12).

c. A theme park is planned whose boundaries will form the triangle ΔOAB where O is the origin, A is at $(k, 0)$ and B is at $(k, g(k))$, as shown below, where $k \in (0, 4)$.

Find the maximum possible area of the theme park, in km^2. (3 marks)



Marks	0	1	2	3	Average
%	65	5	17	13	0.8

Area of triangle:

$$A(x) = \frac{1}{2} \times x \times (12x - 3x^2)$$

$$= 6x^2 - \frac{3}{2}x^3$$

Maximum area at $A'(x) = 0$:

$$A'(x) = 12x - \frac{9}{2}x^2 = \frac{1}{2}(24x - 9x^2) = 0$$

$$3x\left(4 - \frac{3}{2}x\right) = 0$$

$$x = \frac{24}{9} = \frac{8}{3}$$

$$A\left(\frac{8}{3}\right) = \frac{1}{2} \times \frac{8}{3} \times \left(12\left(\frac{8}{3}\right) - 3\left(\frac{8}{3}\right)^2\right)$$

$$= \frac{4}{3}\left(32 - \frac{64}{3}\right)$$

$$= \frac{1152}{81} = \frac{128}{9}$$

This question was not well attempted. Those students who did complete the question generally were able to state the equation for the area of the triangle as either $A(k) = \frac{1}{2}k(12k - 3k^2)$ or $A(k) = 6k^2 - \frac{3}{2}k^3$ or an equivalent equation in terms of the variable x. Many students were able to differentiate to get $A'(k) = 0$, although some students incorrectly wrote this as $A'(x)$ when the variable they were using was k. Most students who were able to solve $A'(k) = 0$ found $k = \frac{8}{3}$ or its equivalent, $k = \frac{24}{9}$. Many arithmetic mistakes occurred when students tried to substitute the value of $k = \frac{8}{3}$ or $k = \frac{24}{9}$ into their expression of $A(k)$ to find the maximum area.

Question 83 (3 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/2024MM1-w.pdf#page=2

a. Let $y = e^x \cos(3x)$.

Find $\frac{dy}{dx}$. (1 mark)

Marks	0	1	Average
%	18	82	0.8

$$\frac{dy}{dx} = e^x\cos(3x) - 3e^x\sin(3x) = e^x(\cos(3x) - 3\sin(3x))$$

This question was well attempted and required students to use the product rule to find the derivative. Many students did not tidy up the negative signs in their answer and left their answer as $e^x\cos(3x) + -3e^x\sin(3x)$ or $e^x\cos(3x) + e^x - 3\sin(3x)$. Some students did not use brackets around terms and this had the potential to be misinterpreted. Some students altered the argument of the sine term and wrote $e^x\cos(3x) - 3e^x\sin(x)$.

b. Let $f(x) = \log_e(x^3 - 3x + 2)$.

Find $f'(3)$. (2 marks)

Marks	0	1	2	Average
%	31	15	54	1.2

$$f'(x) = \frac{1}{(x^3 - 3x + 2)} \times (3x^2 - 3) = \frac{3x^2 - 3}{x^3 - 3x + 2} \quad \left(or \quad \frac{1}{x+2} + \frac{2}{x-1} \right)$$

$$f'(3) = \frac{3(3)^2 - 3}{3^3 - 3(3) + 2}$$

$$= \frac{24}{20} = \frac{6}{5}$$

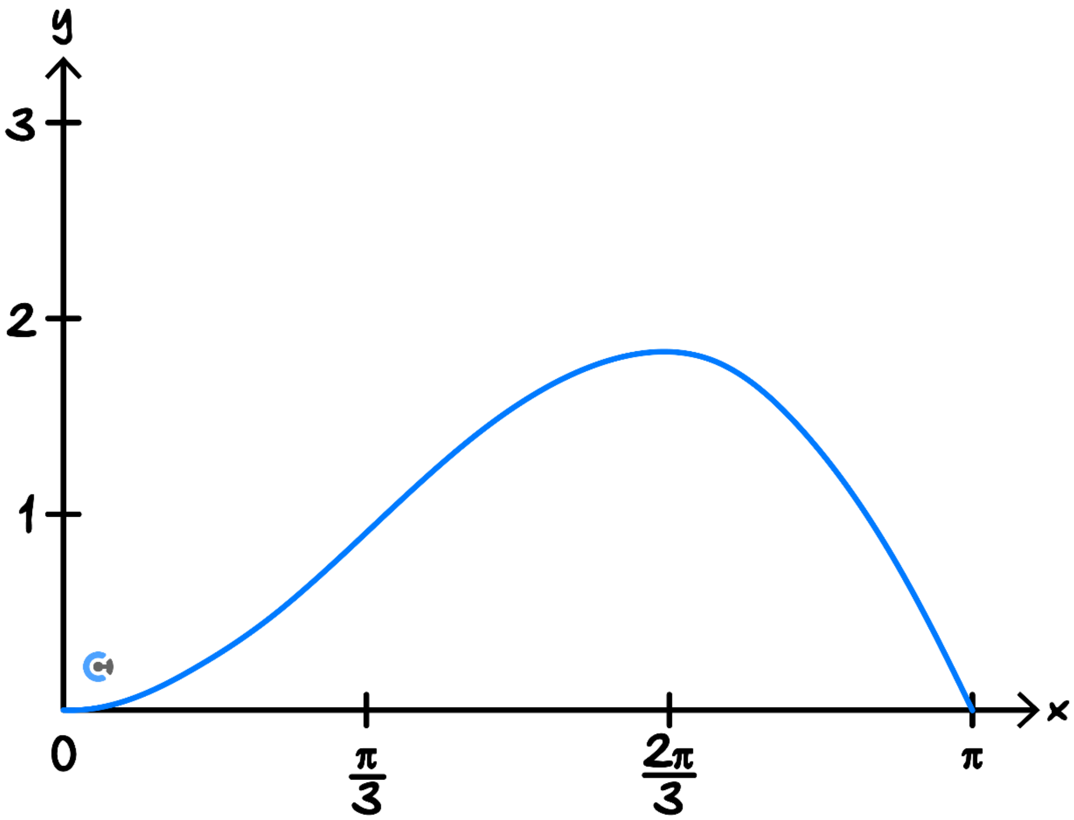
This question was well attempted and required students to use the chain rule to find the derivative then evaluate the derivative at $x = 3$. Some students did not correctly execute the chain rule and omitted the numerator. Some students did not put brackets around the quadratic term; this created ambiguity in their solution process with the evaluation of the derivative at $x = 3$. A correct answer must emerge from correct working.

Space for Personal Notes

Question 84 (9 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/2024MM1-w.pdf#page=8

Part of the graph of $f: [-\pi, \pi] \to R, f(x) = \sin(x)$ is shown below.



a. Use the trapezium rule with a step size of $\frac{\pi}{3}$ to determine an approximation of the total area between the graph of $y = f(x)$ and the x-axis over the interval $x \in [0, \pi]$. (3 marks)

Marks	0	1	2	3	Average
%	28	31	11	30	1.5

Key values for this question were:

x	$f(x)$
0	0
$\frac{\pi}{3}$	$\frac{\pi}{3} \times \frac{\sqrt{3}}{2} = \frac{\pi\sqrt{3}}{6}$
$\frac{2\pi}{3}$	$\frac{2\pi}{3} \times \frac{\sqrt{3}}{2} = \frac{2\pi\sqrt{3}}{6}$
π	0

Using the trapezium rule:

$$A_{trapezium} = \frac{\pi - 0}{2 \times 3}\left(f(0) + 2f\left(\frac{\pi}{3}\right) + 2f\left(\frac{2\pi}{3}\right) + f(\pi)\right)$$
$$= \frac{\pi}{6} \times \left(0 + 2 \times \frac{\pi\sqrt{3}}{6} + 2 \times \frac{2\pi\sqrt{3}}{6} + 0\right)$$
$$= \frac{\pi}{6} \times \frac{6\pi\sqrt{3}}{6}$$
$$= \frac{\pi^2\sqrt{3}}{6} \quad or \quad \frac{6\sqrt{3}\pi^2}{36} = \frac{3\sqrt{3}\pi^2}{18} = \frac{\pi^2}{2\sqrt{3}}$$

Or using a combination of triangles/trapeziums:

$$A_{triangle1} = \frac{1}{2} \times \frac{\pi}{3} \times f\left(\frac{\pi}{3}\right) = \frac{\pi}{6} \times \frac{\pi\sqrt{3}}{6} = \frac{\pi^2\sqrt{3}}{36}$$
$$A_{trapezium1} = \frac{1}{2} \times \frac{\pi}{3} \times \left(f\left(\frac{\pi}{3}\right) + f\left(\frac{2\pi}{3}\right)\right) = \frac{\pi}{6} \times \left(\frac{\pi\sqrt{3}}{6} + \frac{2\pi\sqrt{3}}{6}\right) = \frac{\pi^2\sqrt{3}}{12}$$
$$A_{triangle2} = \frac{1}{2} \times \frac{\pi}{3} \times f\left(\frac{2\pi}{3}\right) = \frac{\pi}{6} \times \frac{2\pi\sqrt{3}}{6} = \frac{2\pi^2\sqrt{3}}{36}$$
$$Total\ Area = \frac{\pi^2\sqrt{3}}{36} + \frac{\pi^2\sqrt{3}}{12} + \frac{2\pi^2\sqrt{3}}{36} = \frac{\pi^2\sqrt{3}}{6}$$

This question required that students use three trapeziums to approximate the area between the curve and the x-axis over the interval $[0, \pi]$, as per the trapezium rule. Therefore, any attempt to calculate this area using integral calculus was not acceptable. Although many students knew the trapezium rule, some students did not apply it correctly, often writing $\frac{\pi}{6}\left(f(0) + f\left(\frac{\pi}{3}\right) + f\left(\frac{2\pi}{3}\right) + f(\pi)\right)$, with the coefficient '2' missing from the middle two terms. Some students gave the formula as stated on the formula sheet with values relevant to the question, however, many did not proceed to calculate $f\left(\frac{\pi}{3}\right)$ and $f\left(\frac{2\pi}{3}\right)$ correctly. It is expected that students will have a way of remembering the exact values of $\sin\theta$, $\cos\theta$ and $\tan\theta$ for values of θ between 0 and $\frac{\pi}{2}$ inclusive, $\left(0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\right)$ as specified in the key knowledge of the study design. While some students were able to identify the exact value of $\sin\left(\frac{\pi}{3}\right)$ and $\sin\left(\frac{2\pi}{3}\right)$, many did not multiply these by $\frac{\pi}{3}$ and $\frac{2\pi}{3}$. The arithmetic manipulation of fractions and surds presented a challenge for some students, with some leaving their answer as the sum of two or three separate area parts, instead of combining them into a single term. Other errors included incorrectly evaluating $\sin\left(\frac{2\pi}{3}\right)$ as $-\frac{\sqrt{3}}{2}$.

b.

i. Find $f'(x)$. (1 mark)

Marks	0	1	Average
%	16	84	0.9

$\sin(x)+x\cos(x)$

Most students were able to apply the product rule appropriately. Students should be aware of the use of notation when naming their answer and use brackets to make their response clear.

ii. Determine the range of $f'(x)$ over the interval $\left[\frac{\pi}{2}, \frac{2\pi}{3}\right]$. (1 mark)

Marks	0	1	Average
%	80	20	0.2

Some students were able to substitute values effectively here to attain the correct endpoints of the interval. Some students were careless with notation and omitted the brackets or wrote curved parentheses. Some students, incorrectly, reversed the order of the interval. Students are encouraged to use the graph as a guide. A common incorrect answer was $[0,1]$.

iii. Hence, verify that $f(x)$ has a stationary point for $x \in \left[\frac{\pi}{2}, \frac{2\pi}{3}\right]$. (1 mark)

Marks	0	1	Average
%	88	12	0.1

Since $f'(x)$ is continuous and $\frac{\sqrt{3}}{2}-\frac{\pi}{3}<0$ and $1>0$ $\therefore f'(x)=0$ at some point in the interval $\left[\frac{\pi}{2}, \frac{2\pi}{3}\right]$

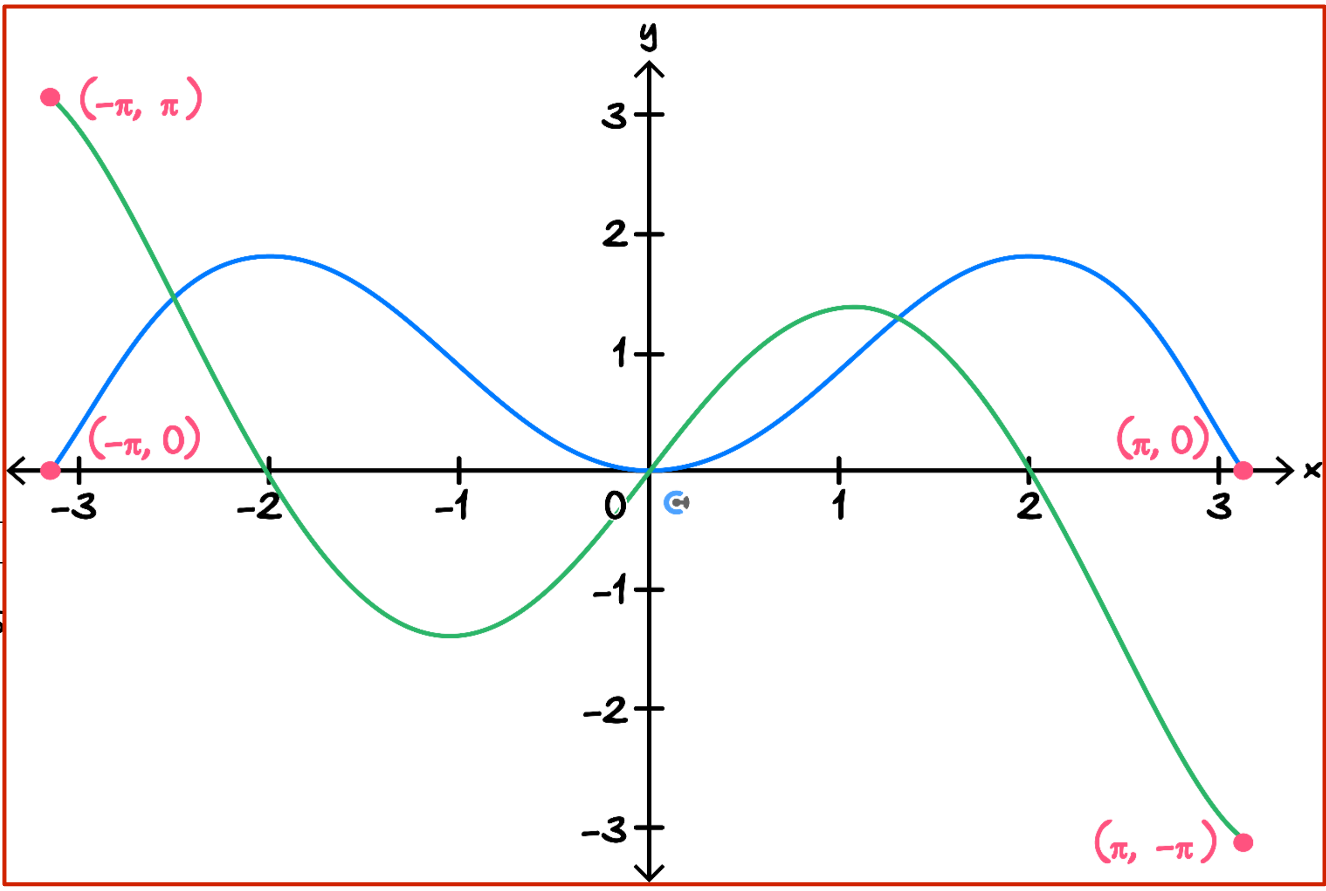
Or since $f'(x)$ is continuous and $\text{Range}(f'(x))=\left[\frac{\sqrt{3}}{2}-\frac{\pi}{3},\ 1\right]$ includes 0 $\therefore f'(x)=0$ at some point in the interval $\left[\frac{\pi}{2}, \frac{2\pi}{3}\right]$

Or since $f'(x)$ is continuous and $f'\left(\frac{\pi}{2}\right)\times f'\left(\frac{2\pi}{3}\right)=\left(\frac{\sqrt{3}}{2}-\frac{\pi}{3}\right)\times 1<0$ $\therefore f'(x)=0$ at some point in the interval $\left[\frac{\pi}{2}, \frac{2\pi}{3}\right]$

There were many ways students could use the values they found in Question 7b.ii to verify that $f(x)$ has a stationary point in the interval $\left[\frac{\pi}{2}, \frac{2\pi}{3}\right]$. This question required students to 'hence, verify […]' so it was not appropriate to attempt to use a calculus technique. Students are reminded that the word 'verify' means to demonstrate or check the truth of a statement, so it was not sufficient to merely discuss the interval in terms of general positive or negative tendencies without referring to specific values and showing the 'check' had been completed. Of the students who attained the correct interval in Question 7b.ii, many were able to provide a correct explanation. A common error was to not recognise $\frac{\sqrt{3}}{2}-\frac{\pi}{3}<0$.

c. On the set of axes below, sketch the graph of $y = f'(x)$ on the domain $[-\pi, \pi]$, labelling the endpoints with their coordinates.

You may use the fact that the graph of $y = f'(x)$ has a local minimum of approximately $(-1.1, -1.4)$ and a local maximum of approximately $(1.1, 1.4)$. (3 marks)



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Question 85 (7 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/2024MM1-w.pdf#page=10

Let $g: R \to R, g(x) = \sqrt[3]{x-k} + m$, where $k \in R\backslash\{0\}$ and $m \in R$.

Let the point P be the y-intercept of the graph of $y = g(x)$.

a. Find the coordinates of P, in terms of k and m. (1 mark)

Marks	0	1	Average
%	39	61	0.6

$\left(0, m - \sqrt[3]{k}\right)$ or $\left(0, -k^{\frac{1}{3}} + m\right)$ or $\left(0, \sqrt[3]{-k} + m\right)$

Many students attained the y value of the intercept, but did not correctly write the answer in coordinate form, with correct brackets. Some students demonstrated difficulties in dealing with a negative sign inside a cube-root, noting the odd nature of the function. Some students incorrectly wrote the radical sign as $3\sqrt{-k} + m$ and this often led to errors in later parts of Question 8.

b. Find the gradient of g at P, in terms of k. (2 marks)

Marks	0	1	2	Average
%	55	13	32	0.8

$g'(x) = \frac{1}{3}(x-k)^{-\frac{2}{3}} = \frac{1}{3(x-k)^{\frac{2}{3}}}$

$g'(0) = \frac{1}{3}(-k)^{-\frac{2}{3}}$

$= \frac{1}{3(-k)^{\frac{2}{3}}} = \frac{1}{3k^{\frac{2}{3}}}$ $\left(\text{or } \frac{1}{3}k^{-\frac{2}{3}}, \text{ or } \frac{k^{\frac{1}{3}}}{3k} \text{ or equivalent}\right)$

There were many ways the answer could be expressed in this question and students were not required to give their answer in a particular form. Some students confused the process involved in this question using integration instead of differentiation, leading to answers that involved an incorrect power of $\frac{4}{3}$. Many students calculated the $g'(0)$ but mistakenly took the negative sign out, leaving their final response as $-\frac{1}{3k^{\frac{2}{3}}}$. Some students did not manipulate or write the surds correctly.

c. Given that the graph of $y = g(x)$ passes through the origin, express k in terms of m. (1 mark)

Marks	0	1	Average
%	53	47	0.5

$k = m^3$

This question was well attempted.

d. Let the point Q be a point different from the point P, such that the gradient of g at points P and Q are equal.

Given that the graph of $y = g(x)$ passes through the origin, find the coordinates of Q in terms of m. (3 marks)

Marks	0	1	2	3	Average
%	74	15	2	9	0.5

There were many ways that a response to this question could be attempted.

Method 1: Using derivatives in terms of k

$g'(x) = g'(0)$

$\frac{1}{3}(x-k)^{-\frac{2}{3}} = \frac{1}{3}(-k)^{-\frac{2}{3}}$

$(x-k)^{-\frac{2}{3}} = (-k)^{-\frac{2}{3}}$

$(x-k)^{-2} = (-k)^{-2}$

$(x-k)^{2} = (-k)^{2}$

$x - k = \pm k$

$x = 0 \text{ or } x = 2k$

Then find y value as above

So $Q(2m^3, 2m)$

Or in terms of m

$$g'(x) = \frac{1}{3(x-k)^{\frac{2}{3}}} = \frac{1}{3\left(x-m^3\right)^{\frac{2}{3}}}$$

$$g'(0) = \frac{1}{3k^{\frac{2}{3}}} = \frac{1}{3m^2}$$

$$\Rightarrow \frac{1}{3\left(x-m^3\right)^{\frac{2}{3}}} = \frac{1}{3m^2}$$

$$\Rightarrow \left(x-m^3\right)^{\frac{1}{3}} = m$$

$$x - m^3 = m^3$$

$$x = 2m^3$$

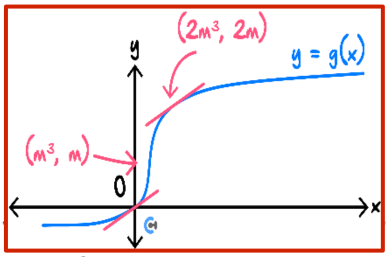
$$g\left(2m^3\right) = \left(2m^3 - m^3\right)^{\frac{1}{3}} + m$$

$$= \left(m^3\right)^{\frac{1}{3}} + m$$

$$= m + m = 2m$$

$\therefore$ the required coordinates of Q $\left(2m^3, 2m\right)$

Method 2: Using symmetry of graph of $g(x)$



So $Q(2m^3, 2m)$

Method 3: Using inverse

Inverse is $g^{-1}(x) = (x-m)^3 + k$

This has a point of inflection at (m, k).

The inverse must also pass through the origin.

By symmetry, the gradient at the origin is equal to the gradient at $x = 2m$

$g^{-1}(2m) = (2m-m)^3 + k$

$= m^3 + k$

$= 2m^3$

Then swap x and y values to get the coordinates of Q as $Q(2m^3, 2m)$

Sp

Question 86 (4 marks)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2017
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2017/nht/2017MM1-nht-w.pdf#page=3

a. Let $y = e^{2x} \cos\left(\frac{x}{2}\right)$.

Find $\frac{dy}{dx}$. (2 marks)

$$\frac{dy}{dx} = 2e^{2x}\cos\left(\frac{x}{2}\right) - \frac{1}{2}e^{2x}\sin\left(\frac{x}{2}\right)$$

Students are reminded to take care with notation, especially with the placement of negative signs and brackets.

b. Let $f: (0, \pi) \to R$, where $f(x) = \log_e(\sin(x))$.

Evaluate $f'\left(\frac{\pi}{3}\right)$. (2 marks)

$$f'(x) = \frac{\cos(x)}{\sin(x)}$$

$$f'\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$$

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Question 87 (4 marks)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2018
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2018/nht/2018MM1-nht-w.pdf#page=3

a. Let $f(x) = \frac{e^x}{(x^2-3)}$.

Find $f'(x)$. (2 marks)

$$f'(x) = \frac{(x^2-3)e^x - e^x(2x)}{(x^2-3)^2}$$

$$= \frac{e^x(x^2-2x-3)}{(x^2-3)^2}$$

Use of the quotient rule was the most straightforward method.

b. Let $y = (x+5)\log_e(x)$.

Find $\frac{dy}{dx}$, when $x = 5$. (2 marks)

$$\frac{dy}{dx} = \log_e(x) + \frac{x+5}{x}$$

At $x = 5$, $\frac{dy}{dx} = \log_e(5) + 2$

Students are reminded to take care with notation when dealing with logarithms.

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Question 88 (4 marks)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2019
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2019/NHT/2019MM1-nht-w.pdf#page=3

a. Let $y = \frac{2e^{2x}-1}{e^x}$.

Find $\frac{dy}{dx}$. (2 marks)

$y = 2e^x - e^{-x}$ so $\frac{dy}{dx} = 2e^x + e^{-x}$

Or $\frac{dy}{dx} = \frac{4e^{2x}e^x - (2e^{2x}-1)e^x}{(e^x)^2} = \frac{2e^{3x}+e^x}{e^{2x}}$ (quotient rule)

Some students used a combination of product and chain rules.

b. Let $f(x) = x^2\cos(3x)$.

Find $f'\left(\frac{\pi}{3}\right)$. (2 marks)

$f'(x) = 2x\cos(3x) - 3x^2\sin(3x)$

$f'\left(\frac{\pi}{3}\right) = -\frac{2\pi}{3}$

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Question 89 (8 marks)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2019
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2019/NHT/2019MM1-nht-w.pdf#page=6

A function g has rule $g(x) = \log_e(x - 3) + 2$.

a. State the maximal domain of g and the range of g over its maximal domain. (2 marks)

$g(x) = \log_e(x-3) + 2$

Domain: $x > 3$ or $(3, \infty)$
Range: R

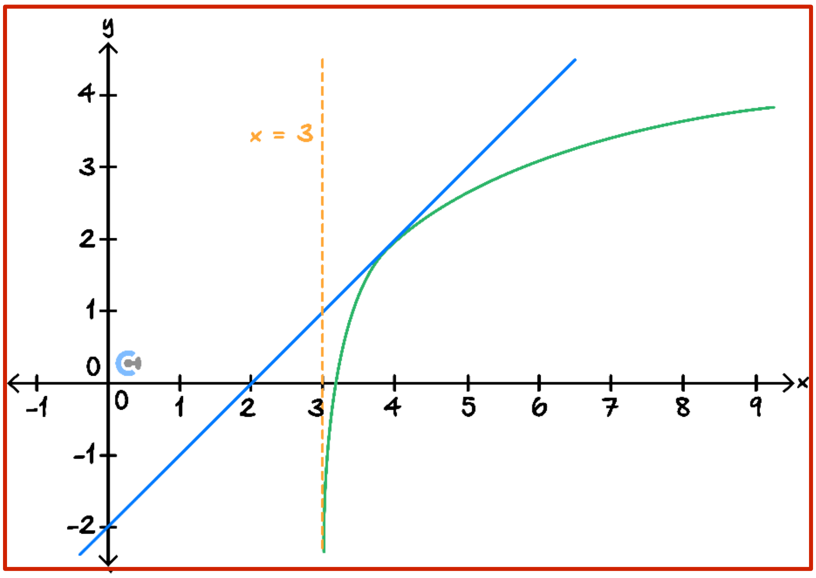
b.

i. Find the equation of the tangent to the graph of g at $(4, 2)$. (2 marks)

$$g'(x) = \frac{1}{x-3}$$

Using $g(4) = 2$ and $g'(4) = 1$ the tangent is $y = x - 2$

ii. On the axes on page 7, sketch the graph of the function g, labelling any asymptote with its equation. Also draw the tangent to the graph of g at $(4, 2)$. (4 marks)



Question 90 (4 marks)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2021
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2021/NHT/2021MM1-nht-w.pdf#page=4

a. Find the derivative of $\frac{e^{2x}}{2x+1}$. (2 marks)

The most efficient method is direct use of quotient rule.

$$\frac{2e^{2x}(2x+1)-2e^{2x}}{(2x+1)^2}=\frac{4xe^{2x}}{(2x+1)^2}$$

Alternatively, the combination of chain and product rule could be used.

b. Let $f: R \to R, f(x) = \sin^4(2x)$.

Evaluate $f'\left(\frac{\pi}{4}\right)$. (2 marks)

$$f'(x) = 4\sin^3(2x) \times 2\cos(2x) = 8\sin^3(2x)\cos(2x)$$

$$f'\left(\frac{\pi}{4}\right)=0$$

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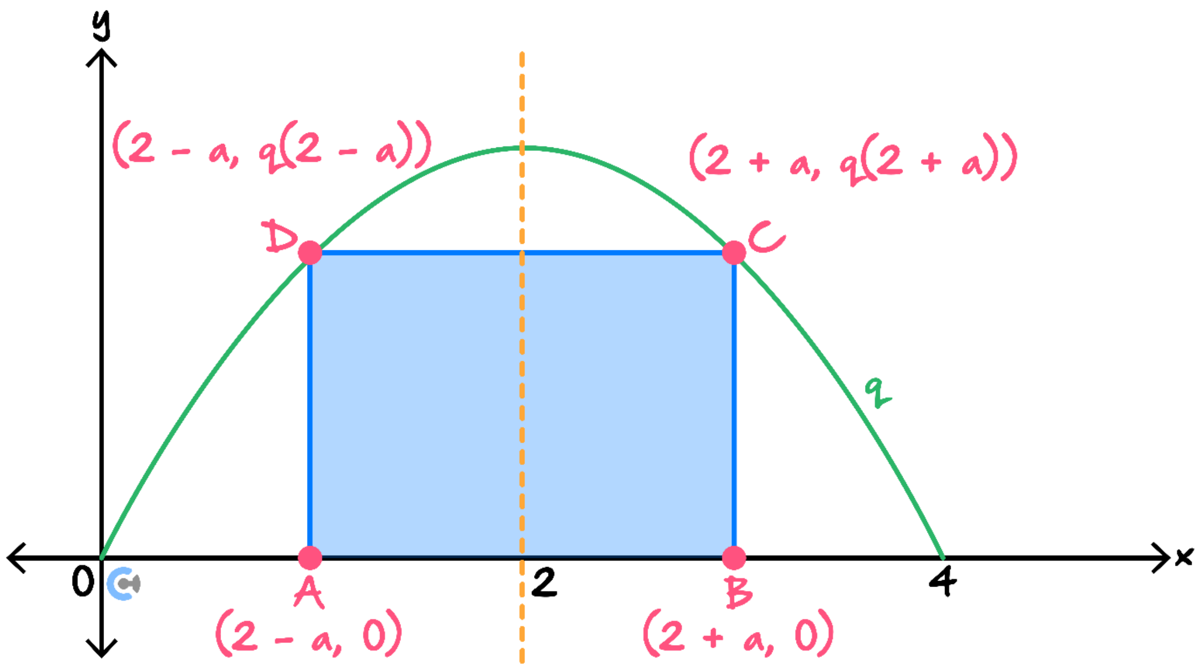
Question 91 (5 marks)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2021
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2021/NHT/2021MM1-nht-w.pdf#page=10

Let $q: [0,4] \to R, q(x) = x(4 - x)$.

A rectangle $ABCD$ is inscribed between the graph of the function q and the x-axis. Its vertices are a units, where $a > 0$, from the axis of symmetry, $x = 2$, as shown below.



a. Find the value of a when the rectangle is a square. Give your answer in the form $b + \sqrt{c}$, where b is an integer and c is a positive integer. (2 marks)

$$2a = q(2 + a)$$

$$a^2 + 2a - 4 = 0$$

$$a = -1 + \sqrt{5}$$

b. Find the maximum area of the rectangle $ABCD$. Give your answer in the form $\frac{m\sqrt{n}}{p}$, where m, n and p are positive integers. (3 marks)

$Area = A = 2a \times q(2+a) = 8a - 2a^3$

$\frac{dA}{da} = 8 - 6a^2 = 0$

Max when $a = \frac{2}{\sqrt{3}}$

$$A\left(\frac{2}{\sqrt{3}}\right) = \frac{4}{\sqrt{3}} \times \frac{8}{3} = \frac{32\sqrt{3}}{9}$$

Many students forgot to find the maximum area.

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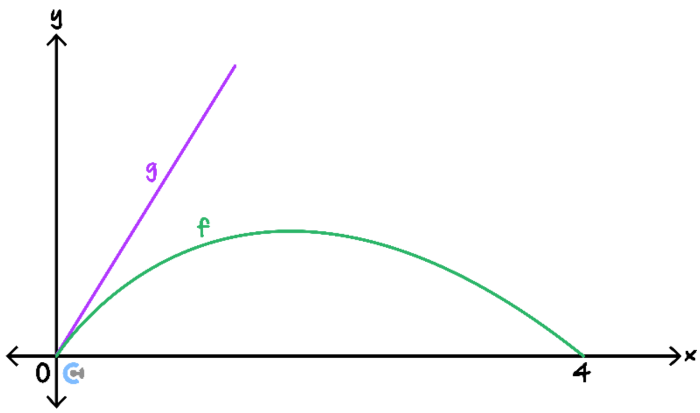
Question 92 (7 marks)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2021
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2021/NHT/2021MM1-nht-w.pdf#page=12

The graph of $f: [0,4] \to R, f(x) = x\left(2 - \sqrt{x}\right)$ and part of the graph of $g : [0, \infty) \to R, g(x) = 2x$ are shown below.



a. Find $f'(x)$. (1 mark)

$$f'(x) = 2 - \frac{3}{2}\sqrt{x} = \frac{4-3\sqrt{x}}{2} = 2 - \frac{3}{2}x^{\frac{1}{2}}$$

b. The tangent to the graph of f at the point $B\left(b, f(b)\right)$ is perpendicular to the graph of g.

Find the coordinates of B. (3 marks)

$$f'(x) = 2 - \frac{3}{2}\sqrt{x} = -\frac{1}{2}$$

$$\left(\frac{25}{9}, \frac{25}{27}\right)$$

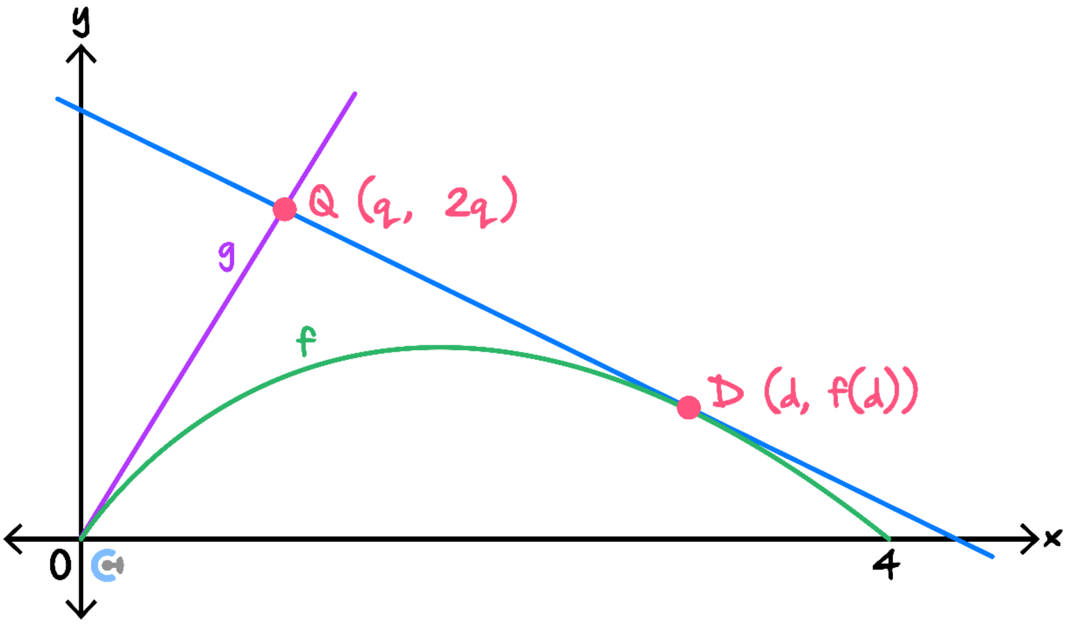
c. Show that the graphs of f and g intersect only at the origin. (1 mark)

$2x = 2x - x^{\frac{3}{2}} \Rightarrow x = 0$

Some students did not apply the null factor law appropriately.

d. Let $Q\ (q, 2q)$, where $q > 0$, be a point on the graph of g.

The tangent to the graph of f at the point $D(d, f(d))$ passes through Q, as shown below.



It can be shown that $d = 3q$.

Determine the values of q for which the tangent to the graph of f passes through Q and has an x-axis intercept greater than 4. (2 marks)

$f'(x) = 0$ when $x = \frac{16}{9}$

$q \in \left(\frac{16}{27}, \frac{4}{3}\right)$

Question 93 (3 marks)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2021
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2021/NHT/2021MM1-nht-w.pdf#page=14

A differentiable function $f : R \rightarrow R$ has the following two properties:

- $f'(x) = f(x)(4 - f(x))$.
- The range of f is $(0,4)$.

a. Find $f'(0)$ if $f(0) = 1$. (1 mark)

$f'(0) = f(0)(4 - f(0)) = 1(4 - 1) = 3$

b. Determine, with appropriate justification, the number of stationary points of the graph of f. (1 mark)

$g = f(x)$ has zero stationary points, because:

$f' \neq 0$ for all x since $f' = 0$ only when $f = 0$ or 4.

But Ran of f $\in (0,4)$

c. State the range of f'. (1 mark)

The correct answer is $(0,4]$.

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Question 94 (3 marks)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2022
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2022/NHT/2022mm1-nht-w.pdf#page=3

a. If $y = \sin(x^2 + 1)$, find $\frac{dy}{dx}$. (1 mark)

This question involved the chain rule for differentiation. A common error included writing a 2 rather than $2x$.

$2x\cos(x^2 + 1)$

b. If $f(x) = x^2 \log_e(x)$, find $f'(e)$. (2 marks)

This question involved the product rule for differentiation.

Students are reminded to use the notation given to name the derivative.

$f'(x) = 2x\log_e(x) + x$

$f'(e) = 2e\log_e e + e$

$f'(e) = 3e$

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Question 95 (4 marks)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/NHT/2024MM1-nht-w.pdf#page=2

a. Let $y = xe^{x^2+1}$.

Find and factorise $\frac{dy}{dx}$. (2 marks)

This question involved using the product and chain rules for differentiation. The answer was required to be presented in a factorised form.

$$\frac{dy}{dx} = (2x)xe^{(x^2+1)} + e^{(x^2+1)}$$

$$= (2x^2+1)e^{(x^2+1)} \quad or \quad 2(x^2+\frac{1}{2})e^{(x^2+1)}$$

b. Let $f(x) = \frac{x^3}{\log_e(x)}$.

Evaluate $f'(x)$ at $x = e$. (2 marks)

This question involved using either the quotient or product rule for differentiation. Some students did not evaluate the derivative at $x = e$

$$f'(x) = \frac{3x^2\log_e x - x^3\frac{1}{x}}{(\log_e x)^2}$$

$$= \frac{(3\log_e x - 1)x^2}{(\log_e x)^2} \text{ or } \frac{-x^2}{(\log_e x)^2} - \frac{3x^2}{\log_e x}$$

At $x = e$

$$f'(x) = \frac{(3\log_e e - 1)e^2}{(\log_e e)^2}$$

$$= \frac{2e^2}{1} = 2e^2$$

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