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VCE Mathematical Methods ¾
AOS 2 Revision [2.0]
Contour Check (Part 2)

Contour Check

[2.1 - 2.7] – Exam 1 Overall (VCAA Qs) Pg 74-108

Section H: [2.1 - 2.7] – Exam 1 Overall (Checkpoints) (125 Marks)

Question 69 (4 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2016
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2016/2016MM1-w.pdf#page=3

a. Let $y = \frac{\cos(x)}{x^2+2}$.

Find $\frac{dy}{dx}$. (2 marks)

b. Let $f(x) = x^2e^{5x}$.

Evaluate $f'(1)$. (2 marks)

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Question 70 (3 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2016
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2016/2016MM1-w.pdf#page=4

Let $f: \left(-\infty, \frac{1}{2}\right] \to R$, where $f(x) = \sqrt{1-2x}$.

a. Find $f'(x)$. (1 mark)

b. Find the angle θ from the positive direction of the x-axis to the tangent to the graph of f at $x = -1$, measured in the anticlockwise direction. (2 marks)

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Question 71 (2 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2016
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2016/2016MM1-w.pdf#page=10

Let $f: [-\pi, \pi] \to R$, where $f(x) = 2\sin(2x) - 1$.

Calculate the average rate of change of f between $x = -\frac{\pi}{3}$ and $x = \frac{\pi}{6}$.

Question 72 (4 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2017
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2017/2017MM1-w.pdf#page=3

a. Let $f: (-2, \infty) \to R, f(x) = \frac{x}{x+2}$.

Differentiate f with respect to x. (2 marks)

b. Let $g(x) = (2 - x^3)^3$.

Evaluate $g'(1)$. (2 marks)

Question 73 (3 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2018
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2018/2018MM1-w.pdf#page=3

a. If $y = (-3x^3 + x^2 - 64)^3$, find $\frac{dy}{dx}$. (1 mark)

b. Let $f(x) = \frac{e^x}{\cos(x)}$.

Evaluate $f'(\pi)$. (2 marks)

Question 74 (4 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2019
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2019/2019MM1-w.pdf#page=3

Let $f: \left(\frac{1}{3}, \infty\right) \to R, f(x) = \frac{1}{3x-1}$.

a.

i. Find $f'(x)$. (1 mark)

ii. Find an antiderivative of $f(x)$. (1 mark)

b. Let $g: R\backslash\{-1\} \to R, g(x) = \frac{\sin(\pi x)}{x+1}$.

Evaluate $g'(1)$. (2 marks)

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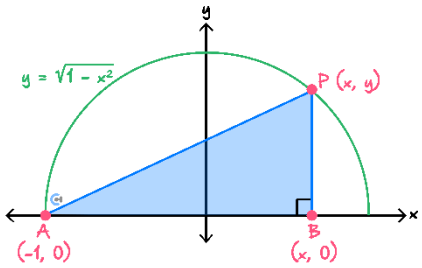
Question 75 (4 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2019
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2019/2019MM1-w.pdf#page=9

The graph of the relation $y = \sqrt{1 - x^2}$ is shown on the axes below. P is a point on the graph of this relation, A is the point $(-1, 0)$ and B is the point $(x, 0)$.



a. Find an expression for the length PB in terms of x only. (1 mark)

b. Find the maximum area of the triangle ABP. (3 marks)

Question 76 (9 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2019
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2019/2019MM1-w.pdf#page=12

Consider the functions $f: R \to R, f(x) = 3 + 2x - x^2$ and $g: R \to R, g(x) = e^x$.

a. State the rule of $g(f(x))$. (1 mark)

b. Find the values of x for which the derivative of $g(f(x))$ is negative. (2 marks)

c. State the rule of $f(g(x))$. (1 mark)

d. Solve $f(g(x)) = 0$. (2 marks)

e. Find the coordinates of the stationary point of the graph of $f\big(g(x)\big)$. (2 marks)

f. State the number of solutions to $g\big(f(x)\big) + f\big(g(x)\big) = 0$. (1 mark)

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Question 77 (3 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2020
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2020/2020MM1-w.pdf#page=3

a. Let $y = x^2 \sin(x)$.

Find $\frac{dy}{dx}$. (1 mark)

b. Evaluate $f'(1)$, where $f: R \to R, f(x) = e^{x^2-x+3}$. (2 marks)

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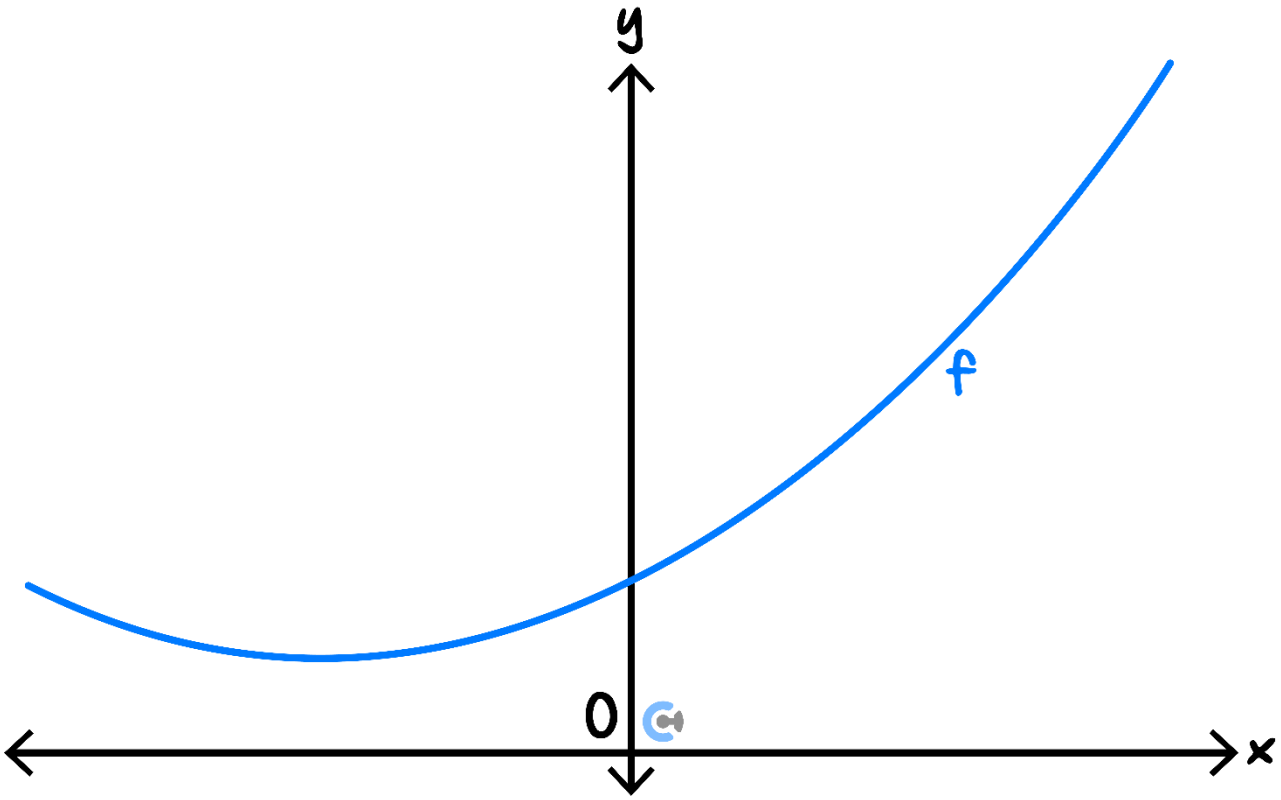
Question 78 (8 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2020
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2020/2020MM1-w.pdf#page=10

Consider the function $f(x) = x^2 + 3x + 5$ and the point $P(1, 0)$. Part of the graph of $y = f(x)$ is shown below.



a. Show that point P is not on the graph of $y = f(x)$. (1 mark)

b. Consider a point $Q(a, f(a))$ to be a point on the graph of f.

i. Find the slope of the line connecting points P and Q in terms of a. (1 mark)

ii. Find the slope of the tangent to the graph of f at point Q in terms of a. (1 mark)

iii. Let the tangent to the graph of f at $x = a$ pass through point P.

Find the values of a. (2 marks)

iv. Give the equation of one of the lines passing through point P that is tangent to the graph of f. (1 mark)

c. Find the value, k, that gives the shortest possible distance between the graph of the function of $y = f(x - k)$ and point P. (2 marks)

Question 79 (3 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2021
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2021/2021MM1-w.pdf#page=2

a. Differentiate $y = 2e^{-3x}$ with respect to x. (1 mark)

b. Evaluate $f'(4)$, where $f(x) = x\sqrt{2x+1}$. (2 marks)

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Question 80 (3 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2022
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2022/2022MM1-w.pdf#page=3

a. Let $y = 3xe^{2x}$.

Find $\frac{dy}{dx}$. (1 mark)

b. Find and simplify the rule of $f'(x)$, where $f: R \to R, f(x) = \frac{\cos(x)}{e^x}$. (2 marks)

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Question 81 (4 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2023
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2023/2023MM1-w.pdf#page=3

a. Let $y = \frac{x^2 - x}{e^x}$.

Find and simplify $\frac{dy}{dx}$. (2 marks)

b. Let $f(x) = \sin(x)\, e^{2x}$.

Find $f'\left(\frac{\pi}{4}\right)$. (2 marks)

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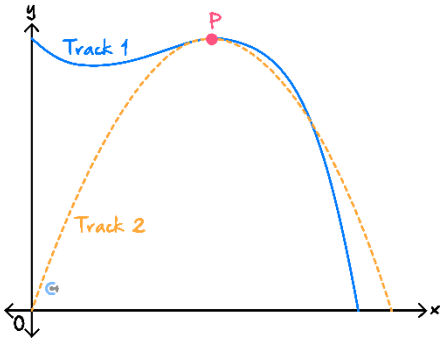
Question 82 (6 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2023
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2023/2023MM1-w.pdf#page=12

The shapes of two walking tracks are shown below.



Track 1 is described by the function $f(x) = a - x(x - 2)^2$.

Track 2 is defined by the function $g(x) = 12x + bx^2$.

The unit of length is kilometres.

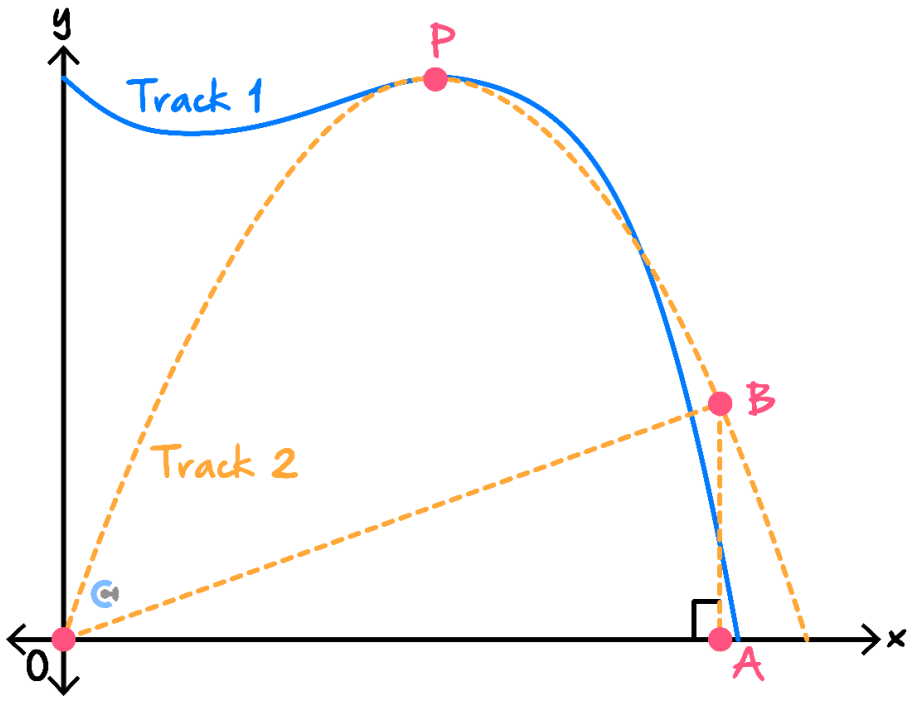
a. Given that $f(0) = 12$ and $g(1) = 9$, verify that $a = 12$ and $b = -3$. (1 mark)

b. Verify that $f(x)$ and $g(x)$ both have a turning point at P.

Give the co-ordinates of P. (2 marks)

c. A theme park is planned whose boundaries will form the triangle ΔOAB where O is the origin, A is at $(k, 0)$ and B is at $\big(k, g(k)\big)$, as shown below, where $k \in (0, 4)$.

Find the maximum possible area of the theme park, in km^2. (3 marks)



Question 83 (3 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/2024MM1-w.pdf#page=2

a. Let $y = e^x \cos(3x)$.

Find $\frac{dy}{dx}$. (1 mark)

b. Let $f(x) = \log_e(x^3 - 3x + 2)$.

Find $f'(3)$. (2 marks)

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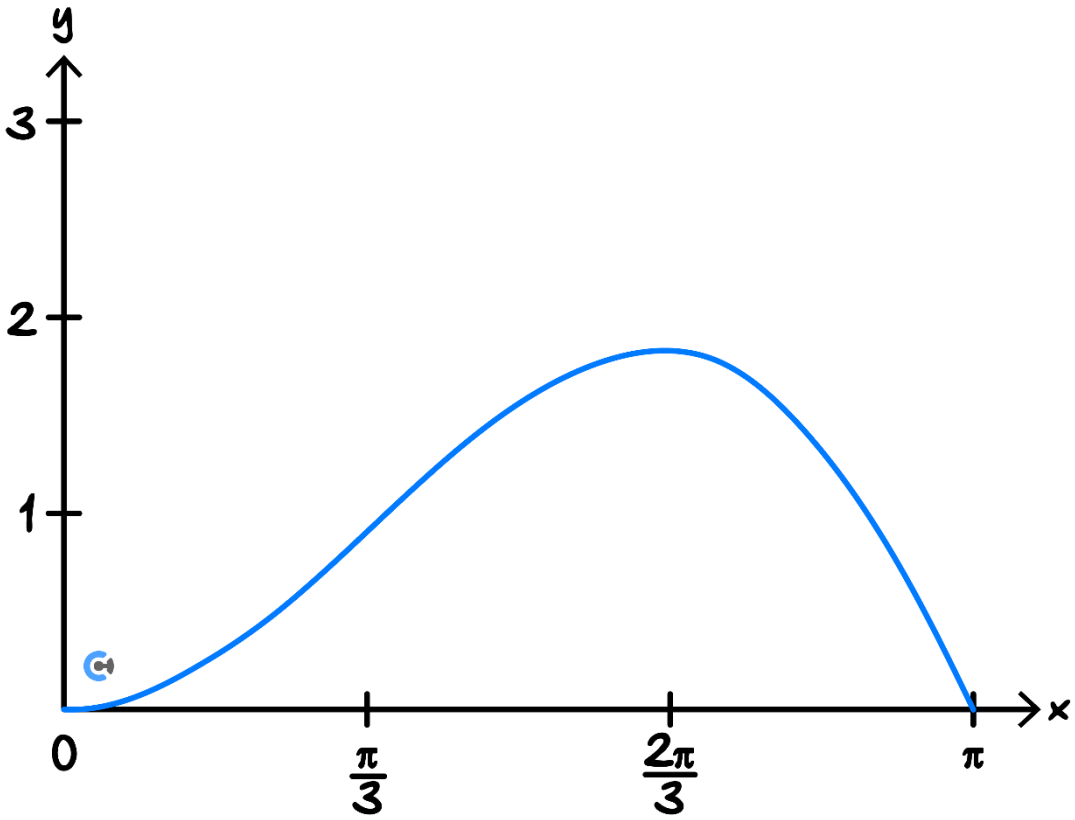
Question 84 (9 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/2024MM1-w.pdf#page=8

Part of the graph of $f: [-\pi, \pi] \to R, f(x) = \sin(x)$ is shown below.



a. Use the trapezium rule with a step size of $\frac{\pi}{3}$ to determine an approximation of the total area between the graph of $y = f(x)$ and the x-axis over the interval $x \in [0, \pi]$. (3 marks)

b.

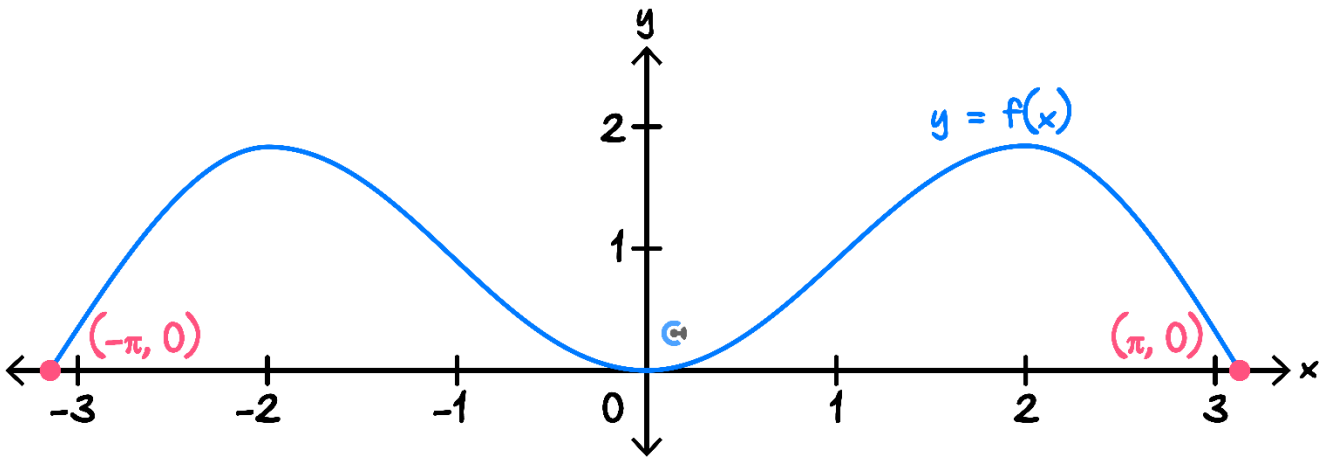
i. Find $f'(x)$. (1 mark)

ii. Determine the range of $f'(x)$ over the interval $\left[\frac{\pi}{2}, \frac{2\pi}{3}\right]$. (1 mark)

iii. Hence, verify that $f(x)$ has a stationary point for $x \in \left[\frac{\pi}{2}, \frac{2\pi}{3}\right]$. (1 mark)

c. On the set of axes below, sketch the graph of $y = f'(x)$ on the domain $[-\pi, \pi]$, labelling the endpoints with their coordinates.

You may use the fact that the graph of $y = f'(x)$ has a local minimum of approximately $(-1.1, -1.4)$ and a local maximum of approximately $(1.1, 1.4)$. (3 marks)



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Question 85 (7 marks)

Inspired from VCAA Mathematical Methods ¾ Exam 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/2024MM1-w.pdf#page=10

Let $g: R \to R, g(x) = \sqrt[3]{x-k} + m$, where $k \in R\backslash\{0\}$ and $m \in R$.

Let the point P be the y-intercept of the graph of $y = g(x)$.

a. Find the coordinates of P, in terms of k and m. (1 mark)

b. Find the gradient of g at P, in terms of k. (2 marks)

c. Given that the graph of $y = g(x)$ passes through the origin, express k in terms of m. (1 mark)

d. Let the point Q be a point different from the point P, such that the gradient of g at points P and Q are equal.

Given that the graph of $y = g(x)$ passes through the origin, find the coordinates of Q in terms of m. (3 marks)

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Question 86 (4 marks)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2017
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2017/nht/2017MM1-nht-w.pdf#page=3

a. Let $y = e^{2x}\cos\left(\frac{x}{2}\right)$.

Find $\frac{dy}{dx}$. (2 marks)

b. Let $f: (0, \pi) \to R$, where $f(x) = \log_e(\sin(x))$.

Evaluate $f'\left(\frac{\pi}{3}\right)$. (2 marks)

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Question 87 (4 marks)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2018
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2018/nht/2018MM1-nht-w.pdf#page=3

a. Let $f(x) = \frac{e^x}{(x^2-3)}$.

Find $f'(x)$. (2 marks)

b. Let $y = (x+5)\log_e(x)$.

Find $\frac{dy}{dx}$, when $x = 5$. (2 marks)

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Question 88 (4 marks)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2019
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2019/NHT/2019MM1-nht-w.pdf#page=3

a. Let $y = \frac{2e^{2x}-1}{e^x}$.

Find $\frac{dy}{dx}$. (2 marks)

b. Let $f(x) = x^2\cos(3x)$.

Find $f'\left(\frac{\pi}{3}\right)$. (2 marks)

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Question 89 (8 marks)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2019
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2019/NHT/2019MM1-nht-w.pdf#page=6

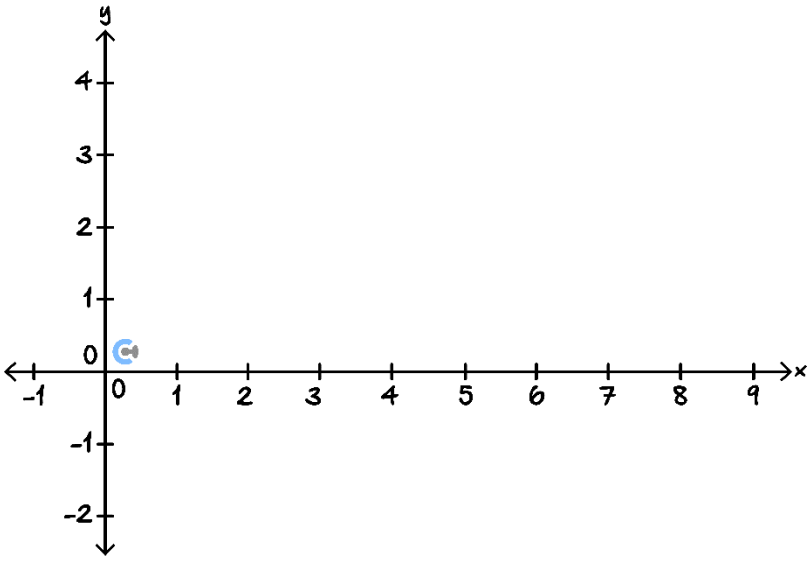
A function g has rule $g(x) = \log_e(x - 3) + 2$.

a. State the maximal domain of g and the range of g over its maximal domain. (2 marks)

b.

i. Find the equation of the tangent to the graph of g at $(4, 2)$. (2 marks)

ii. On the axes on page 7, sketch the graph of the function g, labelling any asymptote with its equation. Also draw the tangent to the graph of g at $(4, 2)$. (4 marks)



Question 90 (4 marks)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2021
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2021/NHT/2021MM1-nht-w.pdf#page=4

a. Find the derivative of $\frac{e^{2x}}{2x+1}$. (2 marks)

b. Let $f: R \to R, f(x) = \sin^4(2x)$.

Evaluate $f'\left(\frac{\pi}{4}\right)$. (2 marks)

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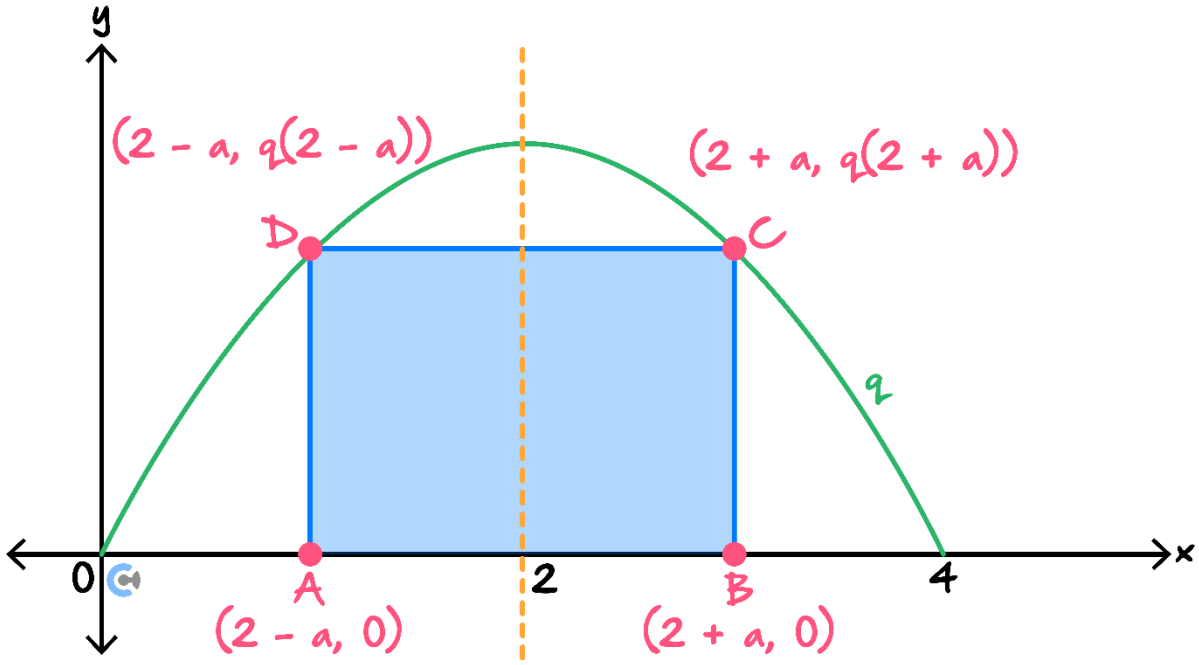
Question 91 (5 marks)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2021
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2021/NHT/2021MM1-nht-w.pdf#page=10

Let $q: [0,4] \to R, q(x) = x(4-x)$.

A rectangle $ABCD$ is inscribed between the graph of the function q and the x-axis. Its vertices are a units, where $a > 0$, from the axis of symmetry, $x = 2$, as shown below.



a. Find the value of a when the rectangle is a square. Give your answer in the form $b + \sqrt{c}$, where b is an integer and c is a positive integer. (2 marks)

b. Find the maximum area of the rectangle $ABCD$. Give your answer in the form $\frac{m\sqrt{n}}{p}$, where m, n and p are positive integers. (3 marks)

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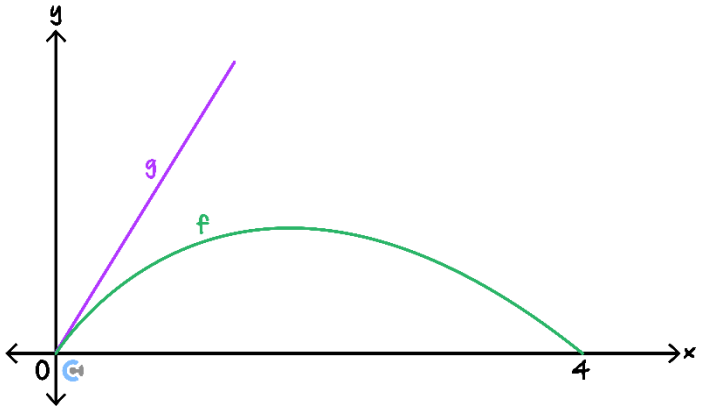
Question 92 (7 marks)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2021
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2021/NHT/2021MM1-nht-w.pdf#page=12

The graph of $f: [0,4] \to R, f(x) = x\left(2 - \sqrt{x}\right)$ and part of the graph of $g: [0, \infty) \to R, g(x) = 2x$ are shown below.



a. Find $f'(x)$. (1 mark)

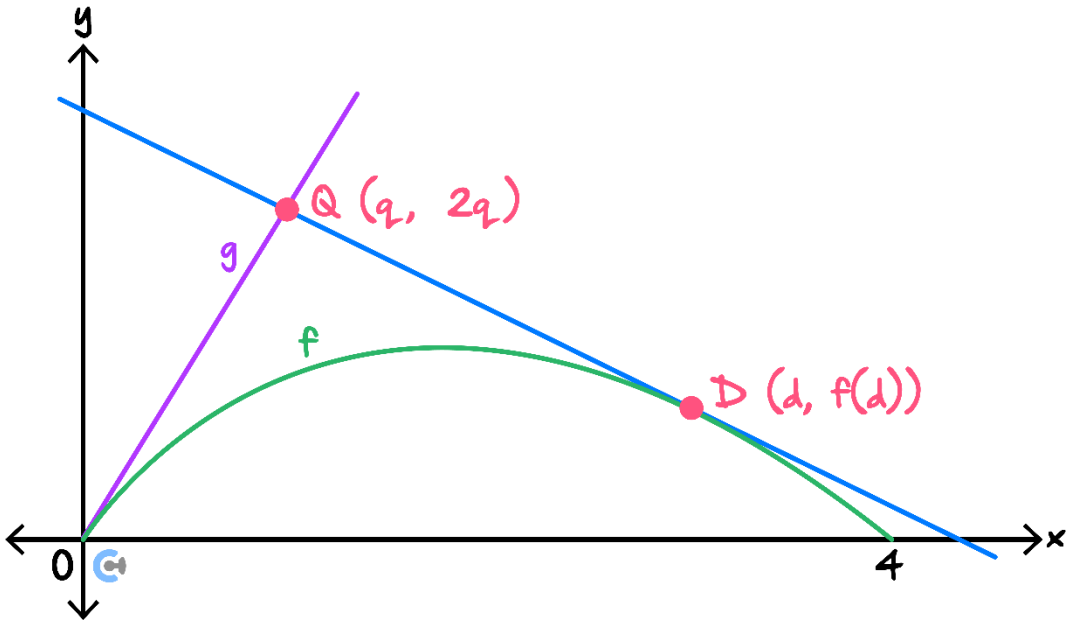
b. The tangent to the graph of f at the point $B\left(b, f(b)\right)$ is perpendicular to the graph of g.

Find the coordinates of B. (3 marks)

c. Show that the graphs of f and g intersect only at the origin. (1 mark)

d. Let $Q\ (q, 2q)$, where $q > 0$, be a point on the graph of g.

The tangent to the graph of f at the point $D\big(d, f(d)\big)$ passes through Q, as shown below.



It can be shown that $d = 3q$.

Determine the values of q for which the tangent to the graph of f passes through Q and has an x-axis intercept greater than 4. (2 marks)

Question 93 (3 marks)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2021
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2021/NHT/2021MM1-nht-w.pdf#page=14

A differentiable function $f : R \to R$ has the following two properties:

- $f'(x) = f(x)\big(4 - f(x)\big)$.
- The range of f is $(0,4)$.

a. Find $f'(0)$ if $f(0) = 1$. (1 mark)

b. Determine, with appropriate justification, the number of stationary points of the graph of f. (1 mark)

c. State the range of f'. (1 mark)

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Question 94 (3 marks)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2022
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2022/NHT/2022mm1-nht-w.pdf#page=3

a. If $y = \sin(x^2 + 1)$, find $\frac{dy}{dx}$. (1 mark)

b. If $f(x) = x^2 \log_e(x)$, find $f'(e)$. (2 marks)

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Question 95 (4 marks)

Inspired from VCAA Mathematical Methods ¾ NHT Exam 2024
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2024/NHT/2024MM1-nht-w.pdf#page=2

a. Let $y = xe^{x^2+1}$.

Find and factorise $\frac{dy}{dx}$. (2 marks)

b. Let $f(x) = \frac{x^3}{\log_e(x)}$.

Evaluate $f'(x)$ at $x = e$. (2 marks)

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