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VCE Mathematical Methods ¾
AOS 2 Revision [2.0]
Contour Check (Part 1)

Contour Check

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Section A: [2.1] - Differentiation I (Checkpoints)

Sub-Section [2.1.1]: Find Instantaneous Rate of Change and Average Rate of Change

Question 1

a. Find the average rate of change of $f(x) = x^3 + 3x - 2$ over the interval $[0, 2]$.

b. Let $f(x) = \sqrt{x} - e^x$. Find $f'(x)$.

c. Find the gradient of the graph of $y = \sin(x) + 3\cos(x)$ at the point $\left(\frac{\pi}{3}, \frac{3+\sqrt{3}}{2}\right)$.

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Question 2

a. Let $y = \tan(x)$, use the quotient rule to show that $\frac{dy}{dx} = \frac{1}{\cos^2(x)}$.

b. Find the gradient of $y = \sqrt{4 - x^2}$ at the point $(-1, \sqrt{3})$.

c. Let $f(x) = -x \log_e(x)$. At what point is the gradient of f equal to 2?

d. Let $f(x) = e^{x^2+2}$, find $f'(x)$.

e. Let $f(x) = \cos^2(x)$. Find $f'\left(\frac{\pi}{3}\right)$.

Question 3

a. Let $y = \frac{e^{-x}}{\sin(2x^2)}$. Find and simplify $\frac{dy}{dx}$.

b. Let $f(x) = (x-3)^4(x^3 - 5x^2 + 1)$. Find $f'(2)$.

c. Let $f(x) = \sqrt{\sin(4x) + 2}$. Find all values of $x \in [0, \pi]$ such that $f'(x) = 0$.

d. Evaluate $\frac{d}{dx}(\log_e(x)\log_e(x^2 + 3x + 4))$.

e. Let $f(x) = \frac{(xe^x)^2}{x-1} + 2x$. Solve $f'(x) = 2$ for x.

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Question 4

Let $f(x) = \frac{\cos(e^{-x}\log_e(x))}{\sin(e^{-x}\log_e(x))}$.

Show that $f'(a) = 0$ implies that $\frac{1}{a} = \log_e(a)$.

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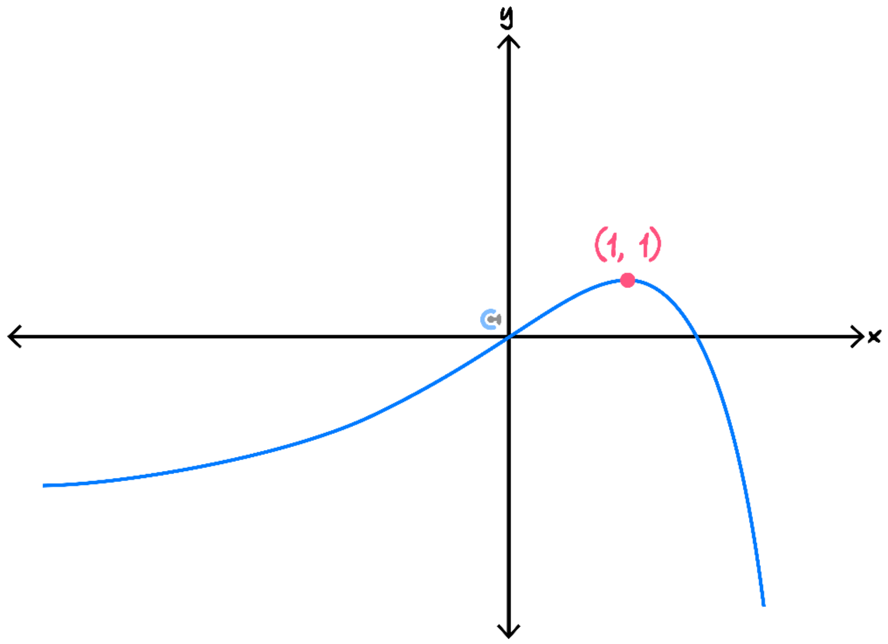
Sub-Section [2.1.2]: Identify the Nature of Stationary Points and Trend

Question 5

The graph of $f(x)$ is drawn below.



a. State the nature of the stationary point when $x = 1$.

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b. State the values of x for which $f(x)$ is strictly increasing.

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Question 6

Let $f(x) = 2x^3 + 3x^2 - 12x + 5$.

a. Find the stationary points of f.

b. State the nature of the stationary points.

c. Hence, state the values of x for which $f(x)$ is strictly decreasing.

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Question 7

Let $f(x) = e^{1+4x-3x^2}$.

a. Find the stationary points of $f'(x)$.

b. State the nature of the stationary points of $f'(x)$.

c. Hence, state the values of x for which $f'(x)$ is strictly increasing.

Question 8

Let $f(x) = x^{\frac{10}{3}}$.

State the values for which $g(x) = f'(x) - f(x)$ is strictly increasing.

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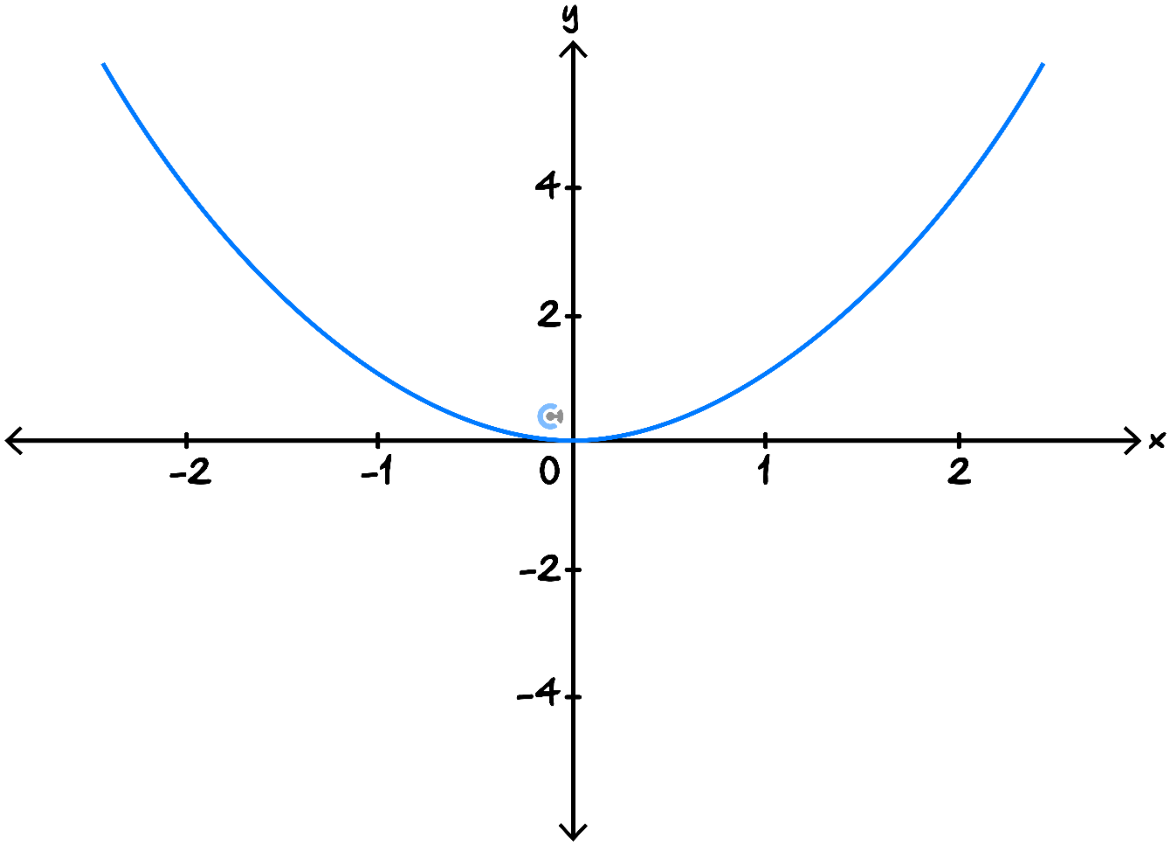
Sub-Section [2.1.3]: Graph Derivative Functions

Question 9

The graph of $f(x)$ is drawn below.

Draw the graph of $f'(x)$ on the same axes.



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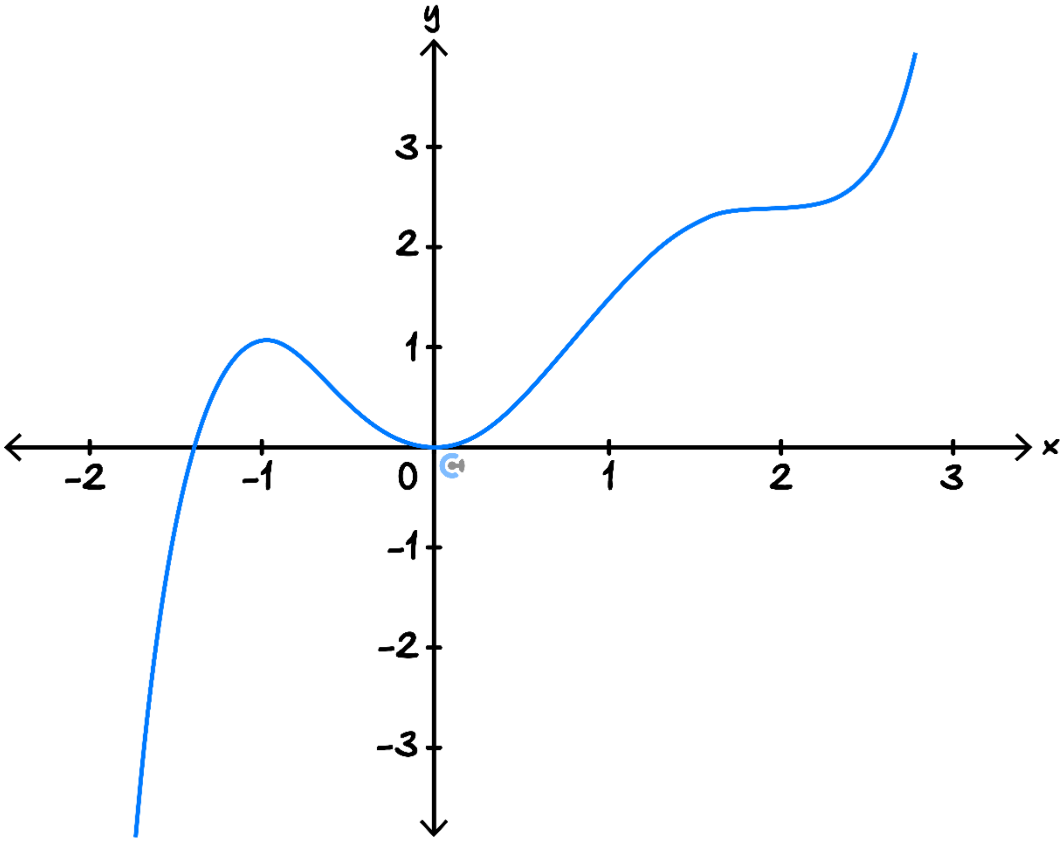
Question 10

The graph of $f(x)$ is drawn below.

Draw the graph of $f'(x)$ on the same axes.



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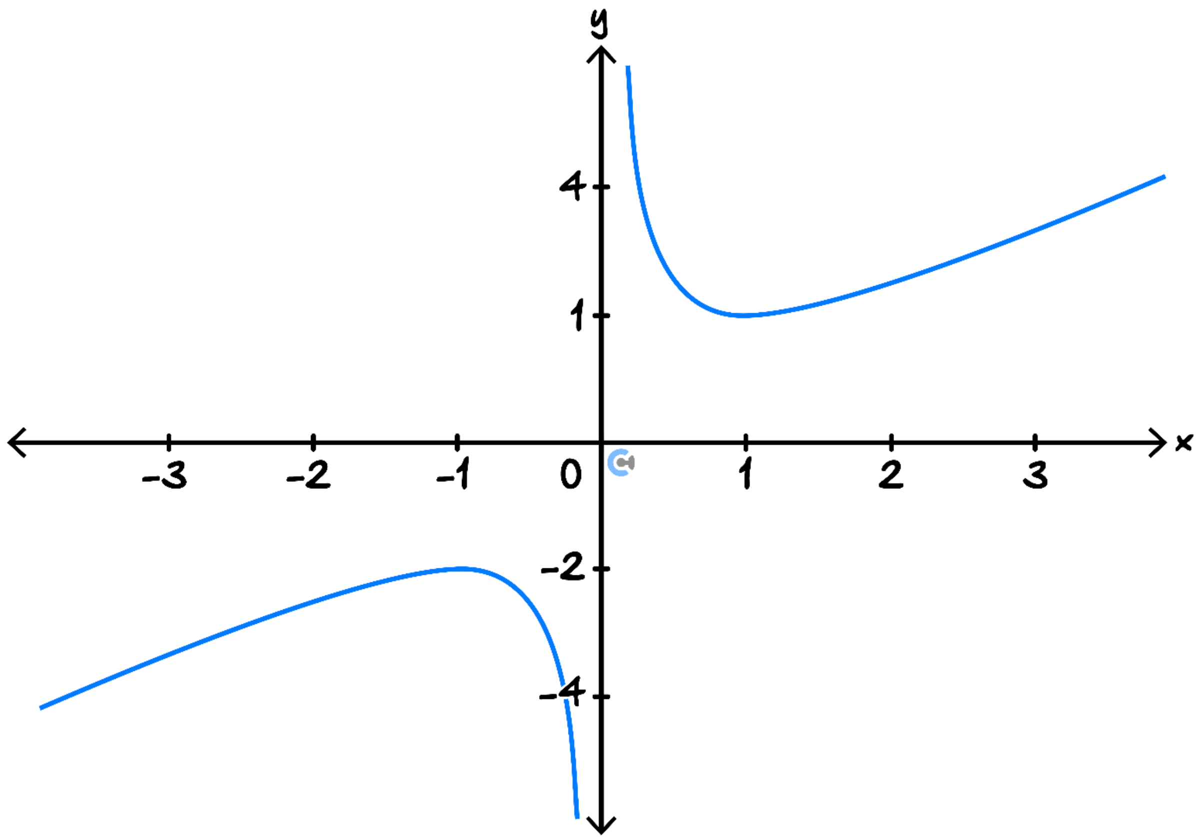
Question 11

The graph of $f(x)$ is drawn below.

Draw the graph of $f'(x)$ on the same axes.



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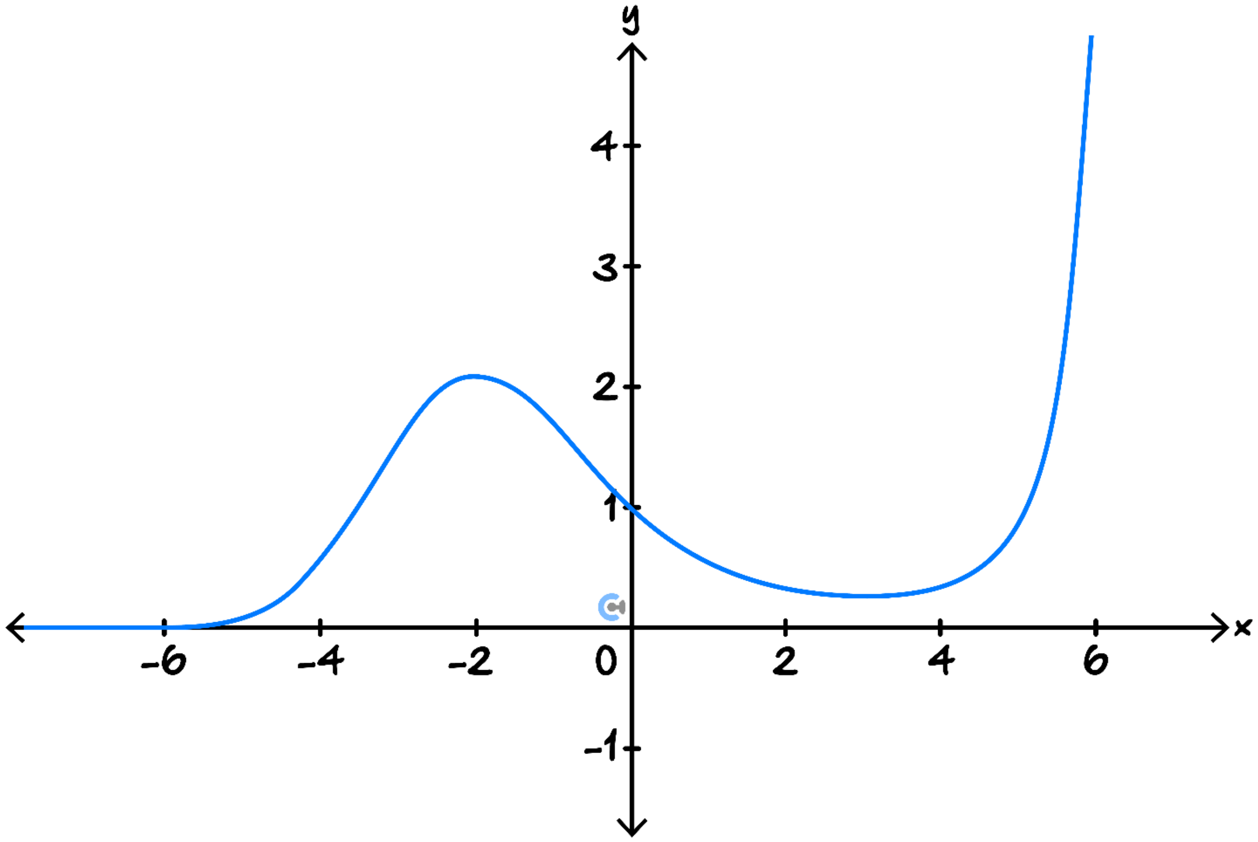
Question 12

The graph of $f(x)$ is drawn below.

Draw the graph of $f'(x)$ on the same axes.



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Section B: [2.2] - Differentiation II (Checkpoints)

Sub-Section [2.2.1]: Evaluate Limits and Find Points Where the Function is not Continuous

Question 13

Evaluate the following limits:

a. $\lim_{x \to 3}(x^3 - 2x^2 + 5)$

b. $\lim_{x \to 4}(2^{\sqrt{x}} + \log_3(x^3 + 2x))$

c. $\lim_{x \to 3}(f(x))$, where:

$$f(x) = \begin{cases} 2x + 1, & x < 3 \\ 3x - 2, & x \geq 3 \end{cases}$$

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Question 14

Find the points x for which the following functions are **discontinuous** and state a reason as to why they are discontinuous.

a. $f(x) = \begin{cases} x, & x < 1 \\ x + 1, & x \geq 1 \end{cases}$

b. $f(x) = \frac{50}{x^2 - 7x + 6}$

c. $f(x) = \frac{x^2 - 4x + 3}{x - 3}$

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Question 15

Consider the following function $f(x)$ with the rule:

$$f(x) = \begin{cases} 3^{x-2} + 5x, & x < 2 \\ ax + 6, & x \geq 2 \end{cases}$$

Find the value of a such that $f(x)$ is continuous for all $x \in \mathbb{R}$.

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Question 16

Consider the following function $f(x)$ with the rule:

$$f(x) = \begin{cases} x^2 - 4x - 12, & x < 7 \\ a^2 - ax + 1, & 7 \le x < 10 \\ -x - 5, & x \ge 10 \end{cases}$$

Find the value of a such that $f(x)$ is continuous for all $x \in \mathbb{R}$.

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Sub-Section [2.2.2]: Apply Differentiability to Find Points Where Functions are not Differentiable, Domain of the Derivative and Unknowns of a Function

Question 17

Find the values of x such that the following functions are not differentiable.

a. $f(x) = \begin{cases} -x+5, & x < 2 \\ x+1, & x \geq 2 \end{cases}$

b. $f(x) = \frac{1}{x^2-4x+3}$

c. $f(x) = \begin{cases} 2x, & x < 0 \\ 2x+1, & x \geq 0 \end{cases}$

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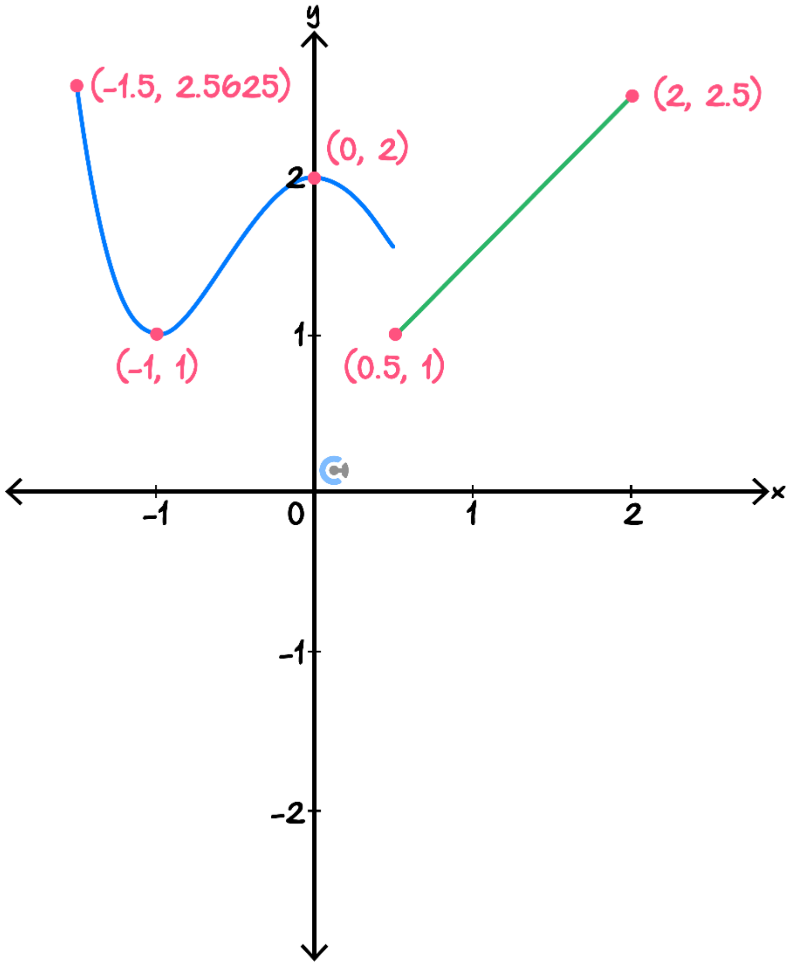
Question 18

Consider the following function.



a. Sketch the corresponding derivative function on the same set of axes above.

b. Furthermore, state the domain of the derivative function.

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Question 19

Consider the following function $f(x)$ with the rule:

$$f(x) = \begin{cases} 2x^2 - 6x + 5, & x < 2 \\ ax + b, & x \geq 2 \end{cases}$$

Find the value of a and b such that $f(x)$ is differentiable at $x = 2$.

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Question 20

Consider the following function $f(x)$ with rule:

$$f(x) = \begin{cases} x^3 - 3x + 5, & x < -1 \\ g(x), & -1 \leq x < 1 \\ x^2 - 5x + 2, & x \geq 1 \end{cases}$$

The goal for this question is to find a suitable rule $g(x)$ making $f(x)$ differentiable for all $x \in \mathbb{R}$.

a. State the four equations that $g(x)$ and $g'(x)$ must satisfy at $x = 1$ and $x = -1$.

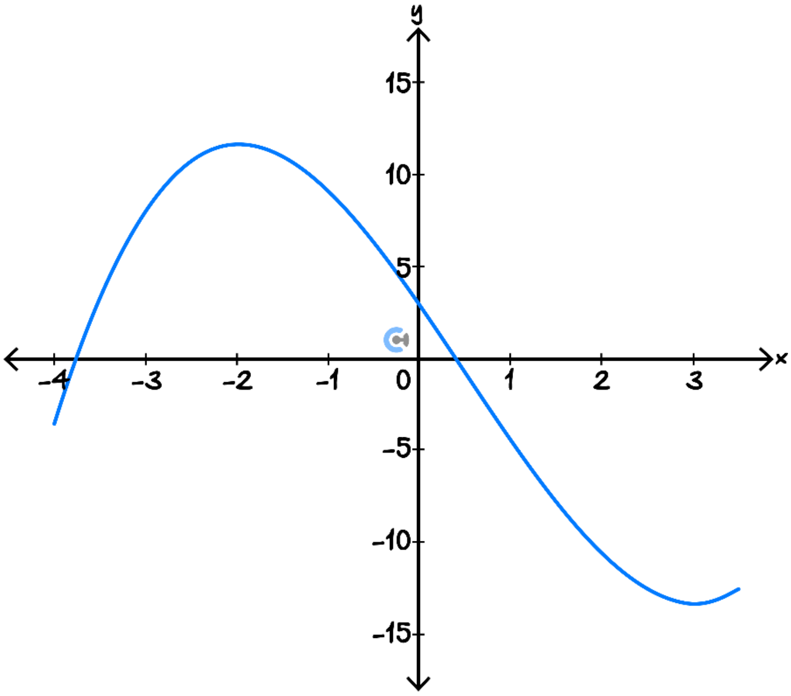
b. A natural choice would be to let $g(x)$ be a polynomial. As there are four equations that need to be satisfied, explain why it is suitable to set $g(x)$ to be a cubic polynomial.

c. Hence, find a suitable rule for $g(x) = ax^3 + bx^2 + cx + d$ assuming $g(x)$ is a polynomial. It may be necessary to use a CAS to solve the system of equations obtained in the working.

Sub-Section [2.2.3]: Identify Concavity and Find Inflection Points

Question 21

Consider the following graph for $f(x)$.



a. Circle the point of inflection on the above graph.

b. State the values of x such that the function is concave up.

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c. State the values of x such that the function is concave down.

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Question 22

Consider a function $f : \mathbb{R} \to \mathbb{R}, f(x) = x^4 - 2x^3 - 36x^2 + 5x + 1$.

a. Calculate the second derivative of the function $f(x)$.

b. Find the points of inflection of the function $f(x)$.

c. Find the values of x where the function is concave up.

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Question 23

Suppose that a function $f(x)$ is double differentiable for all $x \in (0,2)$, and satisfies the following properties:

- $f''(1) = 0$
- $f'(0) = 1$
- $f'(0.5) = 0$
- $f'(0.75) = -0.71$
- $f'(1) = -1$
- $f'(1.25) = -0.71$
- $f'(1.5) = 0$

Find the values of x such that the function is concave up.

Question 24

Find a rule of a polynomial $g(x)$ so that $g(0) = 12,\ g(1) = 9,\ g(2) = 0$, and so that there is a point of inflection when $x = 2$.

Section C: [2.3] - Differentiation Exam Skills (Checkpoints)

Sub-Section [2.3.1]: Find General Derivatives with Functional Notation

Question 25

If f is a differentiable function, find $\frac{dy}{dx}$ for the following:

a. $y = f(x)\tan(x)$

b. $y = \sqrt{f(x)}$

Question 26

If f and g are differentiable functions, find $\frac{dy}{dx}$ for the following:

a. $y = f(e^x) \cdot g(x)$

b. $y = f(g(\cos(3x))$

Question 27

If f and g are differentiable functions, find $\frac{dy}{dx}$ for the following:

a. $y = \sqrt{f(3x^2) + g(2x + f(x))}$

b. $y = \frac{e^{f(x^2)}}{g(f(x^2)) + f(x^2)}$

Question 28

If f and g are differentiable increasing functions, with $g'(x)$ also being one-to-one, what is the maximum amount of stationary points that $y = f(x) + 3x + g(-f(x) - 3x)$ has?

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Sub-Section [2.3.2]: Apply Differentiability to Join Two Functions Smoothly

Question 29

A hybrid function is defined as:

$$f(x) = \begin{cases} e^{2x} - 2, & x < 0 \\ ax + b, & x \geq 0 \end{cases}$$

Find the values of a and b such that $f(x)$ is smooth and continuous at $x = 0$.

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Question 30

A hybrid function is defined as:

$$f(x) = \begin{cases} \log_e(ax), & x < 1 \\ bx^2, & x \geq 1 \end{cases}$$

Where $a > 0$. Find the values of a and b such that $f(x)$ is both continuous and differentiable at $x = 1$.

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Question 31

A hybrid function, $f : \mathbb{R} \to \mathbb{R}$, is defined as:

$$f(x) = \begin{cases} 2x + 4 & x < -2 \\ ax^3 + bx^2 + cx + d & -2 \leq x \leq 2 \\ x^2 - 6x + 10 & x > 2 \end{cases}$$

Find the values of a, b, c and d such that $f(x)$ is both continuous and smooth over its entire domain.

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Question 32 Tech-Active.

a. A hybrid function $f : \mathbb{R} \to \mathbb{R}$, is defined as:

$$f(x) = \begin{cases} \sin(x) + 3 & x < 0 \\ g_1(x) & 0 \le x < 1 \\ g_2(x) & 1 \le x < 2 \\ g_3(x) & 2 \le x < 3 \\ \log_e\left(\frac{e^2x^3}{27}\right) & x \ge 3 \end{cases}$$

Where g_1, g_2 and g_3 are cubic polynomials. Find g_1, g_2, g_3 if both f and f' are smooth on $\mathbb{R}$.

b. A different hybrid function, $h : \mathbb{R} \to \mathbb{R}$, is defined as:

$$h(x) = \begin{cases} \sin(x) + 3 & x < 0 \\ g_4(x) & 0 \le x < 3 \\ \log_e\left(\frac{e^2x^3}{27}\right) & x \ge 3 \end{cases}$$

Where g_4 is a polynomial. If both h and h' are smooth on $\mathbb{R}$, what is the minimum degree of $g_4(x)$?

Section D: [2.4] - Applications of Differentiation (Checkpoints)

Sub-Section [2.4.1]: Find Tangents and Normals

Question 33

Find the equation of the normal to the graph of $f(x) = \cos(5x)$ at the point $x = \frac{\pi}{4}$.

Question 34

Find the equation of the normal to the graph of $f(x) = x^2 - 3x - 1$ which has a gradient of $-\frac{1}{5}$.

Question 35

Find the equation of the normal to the graph of $f: (2, \infty) \rightarrow \mathbb{R},\ f(x) = x^2 - 2x$ at the point $x = a$. Hence by using a CAS, obtain the equation of the normal that passes through the point $(-1,4)$.

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Question 36

Consider the function given by $f(x) = e^{x^2} - \cos(x)$.

a. Find the equation of the tangent to the graph of $f(x)$ at the point $x = 1$.

b. Without needing to do any further differentiation / solving, find the equation of the normal that passes through the point $x = -1$.

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Sub-Section [2.4.2]: Find Minimum and Maximum

Question 37

Find the maximum and minimum values of the function $f(x) = x^3 - 3x^2 - 24x + 15$ with domain $x \in [0,5]$.

Question 38

Find the maximum area of a rectangle with a perimeter equal to 18 m.

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Question 39

Find the maximum rate of change of the function $f(x) = -x^3 + 6x^2 + 10x - 5$.

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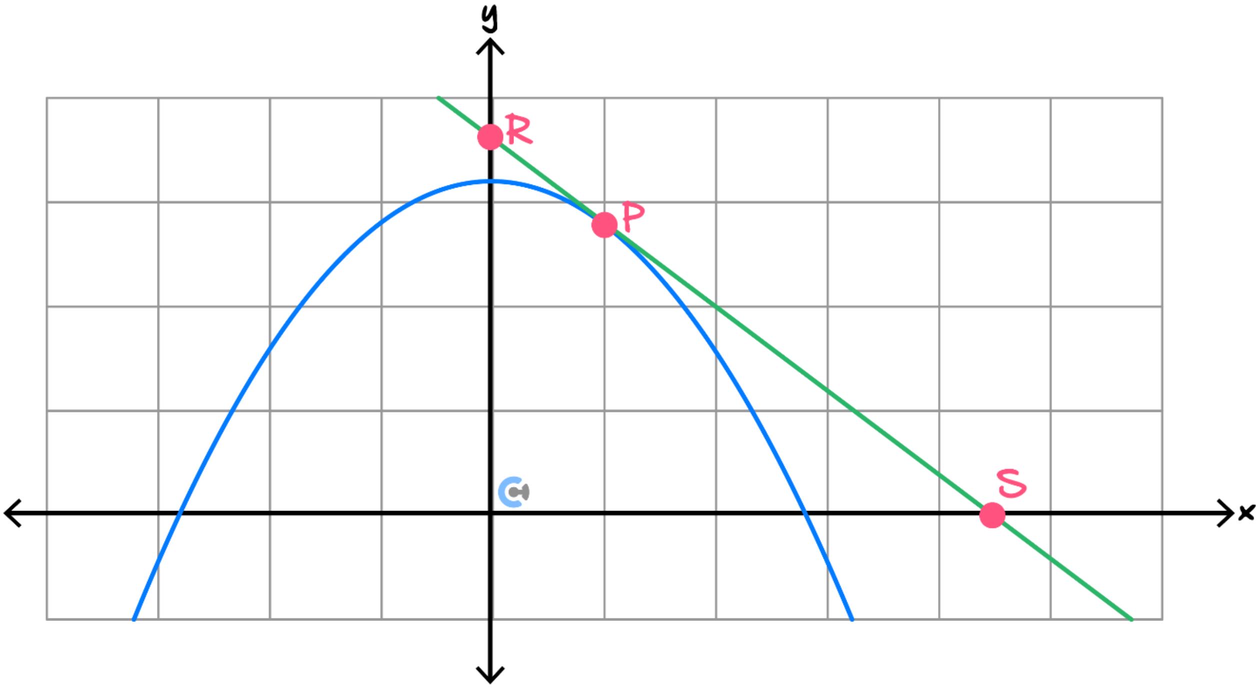
Question 40 Tech-Active.

The diagram below shows the graph of the function $f(x) = 16 - 2x^2$.



The graph of the tangent to the curve at the point $P(p. f(p))$, where $p \in \left[\frac{1}{2}, \frac{5}{2}\right]$ is also shown.

Determine the equation of the tangent line in terms of p.

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Sub-Section [2.4.3]: Apply Newton's Method to Find the Approximation of a Root and its Limitations

Question 41

Approximate the root of the equation $x^3 - 2x^2 + 5x - 6$ using Newton's method with an initial value of $x_0 = 1.2$ and a tolerance level of 0.01. Leave your answer correct to two decimal places.

Question 42

Approximate a solution of the equation $e^x = \cos(2x - 1)$ using Newton's method with an initial value of $x_0 = -2$. Use only one iteration for your approximation.

Question 43

Consider the function $f(x) = \sin(x) - e^{2x}$. Explain why it would be unsuitable to choose an initial value that solves the equation $\cos(x) - 2e^{2x}$.

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Question 44

An issue that can arise when using Newton's method is that the derivative may not be easy to calculate.

a. Explain why Newton's method is impractical for approximating the roots of $f(x) = \sin^{-1}\left(x^2 - \frac{\pi}{2}\right)$ within the context of VCE Mathematical Methods Units 3 and 4.

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b. Nevertheless, we can try to use a similar method known as the secant method with an initial guess of $x_0 = 1.5$. Approximate the tangent to $x_0 = 1.5$ by finding the equation of secant (i.e. the straight line) passing through the points $(1.5, 0.7467)$ and $(1.51, 0.7885)$. Your answer should be given to two decimal places.

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c. Hence, obtain an approximation for a root of $f(x) = \sin^{-1}\left(x^2 - \frac{\pi}{2}\right)$ based on the line obtained above.

Note that this method approximates the root which is $x = \sqrt{\pi/2} \approx 1.2533$.

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Section E: [2.5] - Applications of Differentiation Exam Skills (Checkpoints)

Sub-Section [2.5.1]: Advanced Tangents and Normal Questions

Question 45

a. Find the equation of the tangent to the function $f(x) = 2x^2 + 2x - 3$ that is parallel to the line $y = 3 - 2x$.

b. Find the equation of the tangent to the function $f(x) = 3x^2 - 13x + 8$, that is perpendicular to the line $y + x = 3$.

c. Find the equation of a tangent to the function $f(x) = e^x$, which makes an angle of $45°$ with the positive x-axis.

Question 46

a. Find the equation of the tangent to the function $f(x) = \frac{2}{x}$ that is parallel to the tangent of f when $x = -1$.

b. Find the equation of the tangent to the function $f(x) = \frac{1}{3}x^3 - 3x$, that is parallel to the tangent of f when $x = 3$.

c. Find the equation of the tangents to the curve $y = x^2 + 2x + 2$ that passes through the point $(-1, 0)$.

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Question 47

a. A tangent to $f(x) = \frac{x^3}{3} - x^2 - 2x + 4$ makes an angle of $45°$ with the positive x-axis and passes through the point $(3, b)$, where $b < 0$. Find the value of b.

b. A tangent to the function $f(x) = \cos(2x) + 2$, makes an angle of $120°$, with the positive x-axis and passes through the point $\left(0, \frac{\pi}{6} + \frac{5}{2\sqrt{3}}\right)$.

Determine the point on f that this tangent is drawn to.

c. Find the equation of a tangent to the function $f(x) = x^2 - 3x + 3$ that makes an angle of $15°$ with the line $\sqrt{3}y = x$. The slope of the tangent is greater than the slope of the line.

Question 48 Tech-Active.

a. Find the equation of the tangent to $f(x) = x^2 + 2x - 5$ when $x = 1$.

b. Find the equation of the normal to $y = 2x^3 - 3x + 1$ when $x = -2$.

c. Find the equation of the normal to the function $f(x) = x^2 - 2x - 1$ that passes through the point $(4, -2)$.

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Sub-Section [2.5.2]: Advanced Maximum / Minimum Questions

Question 49

a. The sum of three positive numbers is 26. The second number is 3 times as large as the first. If the sum of the squares of these numbers is minimum, find the numbers.

b. Find the maximum area of a field that can be enclosed by 40 metres of fencing.

c. Find the minimum distance from the origin to a point on the line $y = x - 3$.

Question 50

a. Water is being poured into a container. The volume of the container at time t seconds, in mL, is given by:

$$V(t) = 20t^2 - \frac{1}{3}t^3, 0 \leq t \leq 430$$

At what time is the rate of increase in volume the greatest, and what is this rate of increase?

b. Part of a roller coaster track can be described by the rule $y = 8\sin\left(\frac{\pi x}{20}\right) + 5, x \in [0, 40]$.

State the coordinates of the point on the track for which the magnitude of the gradient is maximum.

c. The population of foxes $P(t)$ on an island at time t in years is given by $P(t) = 20te^{-\frac{t}{2}}$. Find the maximum rate of increase in the population and the time at which this occurs.

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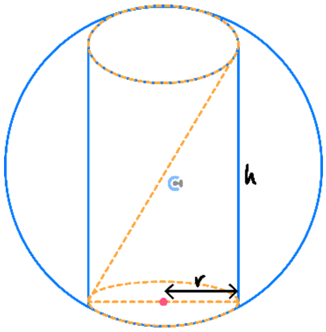
Question 51

A cylinder fits inside a sphere of radius $3\sqrt{3}\ cm$.



a. If the radius of the cylinder is r and the height of the cylinder is h, show that $r = \frac{1}{2}\sqrt{108 - h^2}$.

b. The volume of a cylinder is given by $V = \pi r^2 h$. Find the maximum volume of the cylinder.

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Section F: [2.6] - Families of Functions & Its Exam Skills (Checkpoints)

Sub-Section [2.6.1]: Applying Family of Functions

Question 52

Consider the following family of functions $f(x) = e^{ax} - 1, a > 0$.

a. Identify the "surname" (common aspect(s) of the family) and the "first name" (unique aspect(s) of the family).

b. Hence, state what happens to the graph of f in terms of a transformation when the value of a increases.

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Question 53

Consider the following family of functions $f(x) = (x-2)^2 + k, k \in \mathbb{R}$.

a. Show that the graph of f always has a stationary point at $x = 2$ and find the nature of this stationary point.

b. Hence, identify the "surname(s)" and "first name(s)" of the family.

Question 54

Consider the family of functions $f(x) = \sin\left(kx + \frac{\pi}{2}\right), k \in \mathbb{R}$.

a. Identify the effects of k on the graph.

b. Hence, identify the "surname" and "first name(s)" of the family.

c. Express $f(x)$ without using sin when $k = \pm 1$.

Hint: List out the transformations and sketch the resulting graph if you get stuck!

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Question 55 Tech-Active.

Consider the family of functions $f(x) = e^{ax} - ax + 1, a \in \mathbb{R}^+$.

a. State one transformation that maps the graph of $g(x) = e^x - x + 1$ onto the graph of $f(x)$.

b. Identify a "surname" of the family.

c. Describe what happens to the shape of $f(x)$ as a increases.

d. By plotting $h(x) = f(x) - e^{ax}$ on the same axes as $f(x)$, state the equation of the asymptote of $f(x)$.

Space for Personal Notes

Sub-Section [2.6.2]: Finding Unknowns for a Certain Number of Intersections

Question 56

Find the value of a where $a \in \mathbb{R}$ such that the graph of $f(x) = e^x + a$ intersects the line $y = x$ exactly once.

Question 57

Consider the functions $f: \mathbb{R} \to \mathbb{R}, f(x) = e^x + a$ where $a \in \mathbb{R}$ and $g(x) = x - 3$.

a. Find the inverse function of $f(x)$.

b. Find the value of a such that the graph of $f^{-1}(x)$ intersects with $g(x)$ exactly once.

Question 58

Consider the functions $f: [-4,4) \to \mathbb{R}, f(x) = x^2 + 1$ and $g(x) = mx, m \in \mathbb{R}$. Find the value(s) of m where f and g intersect exactly once.

Space for Personal Notes

Question 59 Tech-Active.

Consider the function $f: [k, \infty) \rightarrow \mathbb{R}, f(x) = (x - k)^2 + 4$. Find the value(s) of k such that $f(x)$ and its inverse intersect exactly once.

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Sub-Section [2.6.3]: Finding Unknowns for Maximums and Minimums

NOTE: This entire section can be done tech-active.

Question 60

For what value of $k \in \mathbb{R}$, will the function, $f(x) = 2 + e^{kx}$ have a minimum on the x-axis?

Question 61

For what value(s) of $k \in \mathbb{R}$ will the function $f(x) = (x - k)^2 \log_e(x)$ have a minimum at $x = 4$?

Space for Personal Notes

Question 62

Find the value(s) of $k \in \mathbb{R}$ such that the minimum of the function $f: [-4,10] \to \mathbb{R}, f(x) = 3\left(\frac{x}{2} - 3k\right)^2 - 6$ occurs at $x = -4$.

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Section G: [2.7] - Pseudocode & Its Exam Skills (Checkpoints)

Question 63

The following pseudocode segments each print a value. Write down the value that is printed.

a. $x \leftarrow 8$
if $x > 5$
  $x \leftarrow x - 3$
end if
print(x)

---

b. $y \leftarrow 4$
if $y < 3$
  $y \leftarrow y + 6$
else
  $y \leftarrow y - 1$
end if
print (y)

---

c. $z \leftarrow 2$
while $z < 6$
  $z \leftarrow z + 2$
end while
print (z)

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Question 64

The following pseudocode segments each print a value. Write down the value that is printed.

a.

```
a ← 2
for i ← 1 to 4
    if a is even
        a ← a + 3
    else
        a ← a + 2
    end if
end for
print(a)
```

---

b.

```
b ← 10
while b > 4
    if b is even
        b ← b − 2
    else
        b ← b − 3
    end if
end while
print(b)
```

---

c. $c \leftarrow 1$
for $j \leftarrow 1$ **to** 5
  if j is even
    $c \leftarrow c + 2$
  else
    $c \leftarrow c + 4$
  end if
end for
print(c)

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Question 65

The following pseudocode segments each print values. Write down the values that are printed.

a.

```
a ← 2
b ← 3
for i from 1 to 2
    for j from 1 to 3
        if a is even
            b ← b + j
            a ← a + 1
        else
            a ← a + 2
        end if
    end for
end for
print(a, b)
```

b.

$x \leftarrow 5$
$y \leftarrow 12$
while $y > 6$
    for j **from** 1 **to** 3
        if x is even
            $y \leftarrow y - 2$
        else
            $x \leftarrow x + 1$
        end if
    end for
end while
print(x, y)

c. $p \leftarrow 1$
$q \leftarrow 10$
for j **from** 1 **to** 4
  for k **from** 1 **to** 2
    if q is even
      $p \leftarrow p + k$
      $q \leftarrow q - 2$
    else
      $q \leftarrow q - 1$
    end if
  end for
end for
print(p, q)

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Sub-Section [2.7.1]: Evaluate and Understand the Pseudocode for Different Implementations of Newton's Method

Question 66

An implementation of Newton's method is shown below.

define newton($f(x), x_0, n$):
$df(x)$ ← the derivative of $f(x)$
for i **from** 1 **to** n **do**
  if $df(x_0) = 0$ **then**
    return "Error: Division by zero"
  else
    $x_0 \leftarrow x_0 - \frac{f(x_0)}{df(x_0)}$
  end if
end while
return x_0

Consider calling the function newton $(x^2 - 5, 2, 5)$.

a. How many iterations are performed?

b. What is the final value of x_0. Give your answer correct to four decimal places.

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Question 67

An implementation of Newton's method is shown below.

define newton($f(x), x_0, n, tol$):
$df(x)$ ← the derivative of $f(x)$
$i \leftarrow 0$
$x_n \leftarrow x_0$
while $i < n$ **do**
  if $df(x_n) = 0$ **then**
    return "Error: Division by zero"
  $x_{n+1} \leftarrow x_n - \frac{f(x_n)}{df(x_n)}$
  if $-tol < x_{n+1} - x_n < tol$ **then**
    return x_{n+1}
  $x_n \leftarrow x_{n+1}$
  $i \leftarrow i + 1$
end while
return x_n

Consider calling the function newton $(x^3 - x^2 + 5, 2, 30, 0.0001)$.

a. Find the final return value. Give your answer correct to four decimal places.

b. State the value of i when the algorithm terminates.

Question 68

An implementation of Newton's method is shown below.

define newton($f(x), x_0, n, tol$):
$df(x)$ ← the derivative of $f(x)$
$i \leftarrow 0$
$x_n \leftarrow x_0$
while $i < n$ **do**
  if $df(x_n) = 0$ **then**
    return "Error: Division by zero"
  $x_{n+1} \leftarrow x_n - \frac{f(x_n)}{df(x_n)}$
  if $-tol < x_{n+1} - x_n < tol$ **then**
    return x_{n+1}
  $x_n \leftarrow x_{n+1}$
  $i \leftarrow i + 1$
end while
return x_n

Consider calling the function.

a. Find the final return value of newton $(x^3 - 2x^2 + 4, 1, 2, 0.001)$.

b. The function newton $\left(x(x+7)(x-7), \frac{7}{\sqrt{5}}, 5001, 0.0001\right)$ is called. Find the final return value.

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