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VCE Mathematical Methods ¾

Polynomials Exam Skills [1.8]

Homework

Homework Outline:

Compulsory Questions	Pg 2 – Pg 28
Supplementary Questions	Pg 29 – Pg 50

Section A: Compulsory Questions

Sub-Section [1.8.1]: Apply Transformations to Restrict the Number of Positive/Negative x-Intercept(s)

Question 1

Consider the following polynomials:

a. Given $f(x) = (x - 4)(x + 3)(x - 6)$, determine the values of k such that $f(x + k)$ has no positive x-intercepts.

b. Given $f(x) = (x - 1)(x + 2)(x - 5)$, determine the values of k such that $f(x - k)$ has exactly one positive x-intercept.

c. Given $f(x) = (x - 2)(x - 7)(x + 1)$, determine the values of k such that $f(x - k)$ has exactly two positive x-intercepts.

Question 2

Consider the following quadratic polynomials:

a. Given $f(x) = x^2 - 4x + 3$, factorise $f(x)$ and determine the values of k such that $f(x - k)$ has exactly one positive x-intercept.

b. Given $f(x) = x^2 + 2x - 3$, factorise $f(x)$ and determine the values of k such that $f(x + k)$ has no positive x-intercepts.

c. Given $f(x) = x^2 - 5x + 6$, factorise $f(x)$ and determine the values of k such that $f(x - k)$ has exactly one negative x-intercept.

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Question 3

Consider the following cubic polynomials:

a. Given $f(x) = x^3 - 4x^2 + x - 4$, factorise $f(x)$ and determine the values of k such that $f(x+k)$ has exactly one positive x-intercept.

b. Given $f(x) = x^3 - 3x^2 - 4x + 12$, factorise $f(x)$ and determine the values of k such that $f(x - k)$ has one negative x-intercept.

c. Given $f(x) = x^3 - 6x^2 + 9x$, factorise $f(x)$, and determine the values of k such that $f(x-k)$ has two positive x-intercepts.

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Sub-Section [1.8.2]: Apply Discriminant to Solve Number of Solutions Questions

Question 4

For each of the following quadratic equations, determine the conditions on k for the equation to have the specified number of solutions.

a. $x^2 + x + 5k = 0$ has exactly two distinct real solutions.

b. $x^2 - 4x + 4(k+1) = 0$ has no real solutions.

c. $kx^2 - 3x + 2k = 0$ has exactly one real solution.

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Question 5

For each of the following quadratic equations, determine the conditions on k for the equation to have the specified number of solutions.

a. $2x^2 + 4x + 2\log_3(k) = 0$ has exactly two distinct real solutions.

b. $\log_2(5)\,x^2 + 3x + \log_2(k) = 0$ has exactly one real solution.

c. $4k^2x^2 - 2kx + 1 = 0$ has no real solutions.

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Question 6

For each of the following equations, determine the conditions on k for the equation to have the specified number of solutions.

a. $x^2 + kx + 3 = 0$ has two real solutions.

b. $2x^2 - 4kx + k^2 + 3 = 0$ has no real solutions.

c. $kx^3 + 4x^2 + 2kx = 0$ has three real solutions.

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Sub-Section [1.8.3]: Apply Shape/Graph to Solve Number of Solutions Questions

Question 7

The cubic function $f(x) = x^3 - 6x^2 + 9x + 2$ has turning points at $(1,6)$ and $(3,2)$. Determine the values of k for which the equation $f(x) = k$ has exactly two solutions.

Question 8

Consider the quadratic function $g(x) = \frac{1}{2}x^2 - kx + 3$. Determine the values of k for which $g(x) = 2$ has exactly two solutions.

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Question 9

The quartic function $f(x) = x^4 - 4x^3 - 2x^2 + 12x + 2$ has turning points at $(-1, -7)$ and $(1,9)$ and $(3, -7)$.

Find the values of k for which the equation $f(x) = k$ has exactly two solutions.

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Sub-Section [1.8.4]: Apply Odd and Even Functions

Question 10

For an odd function $f(x)$, it is known that $f(1) = 2$ and $f'(1) = 3$.

Find the values of $f(-1)$ and $f'(-1)$.

Question 11

An odd function $f(x) = \frac{1}{2}x^3$, has a tangent line of $y = 6x - 8$ at the point $(2,4)$. Find the equation of the line tangent to $f(x)$ when $x = -2$.

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Question 12

Let $f(x) = (x-3)(x-5)(x+1)(x+3)$. Find the value of k such that $f(x+k)$ is an even function.

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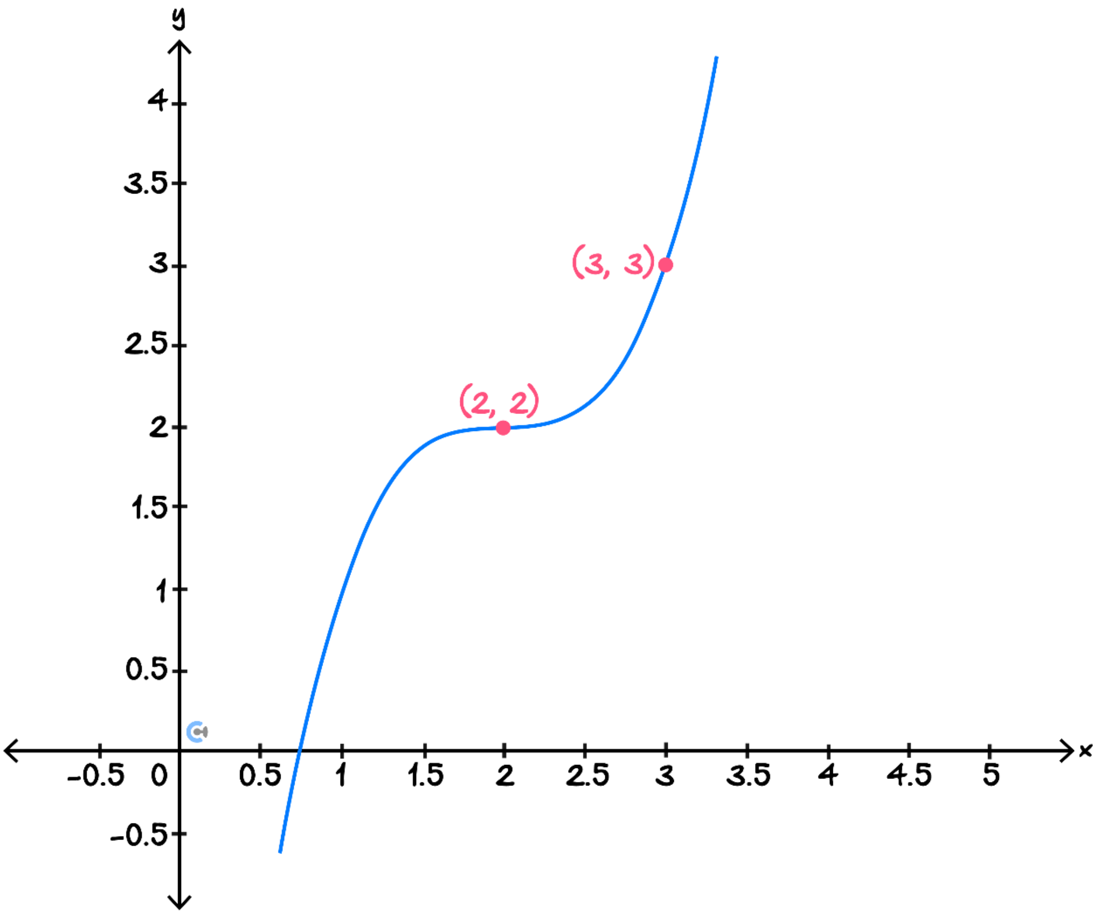
Sub-Section [1.8.5]: Identify Possible Rule(s) From a Graph

Question 13

Part of the graph of $y = f(x)$ is sketched below. The point $(2,2)$ is a stationary point of inflection. Determine the rule for $f(x)$.



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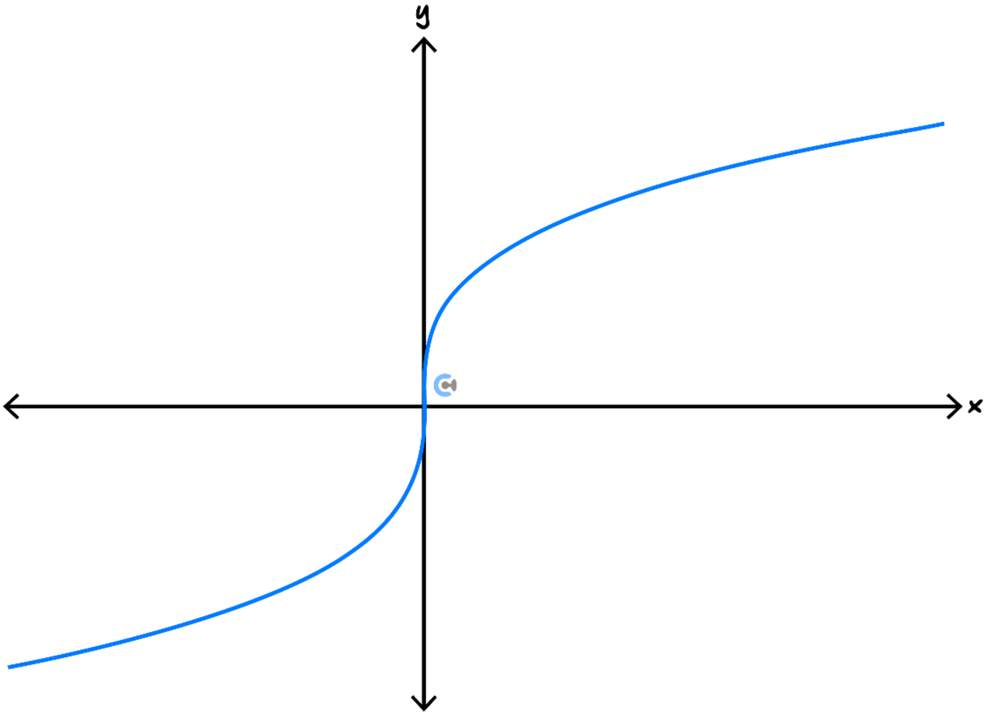
Question 14

Part of the graph of $y = x^{\frac{m}{n}}$, where m and n are positive integers, is shown below.



a. Is it true that $m > n$?

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b. Determine whether m and n are odd or even.

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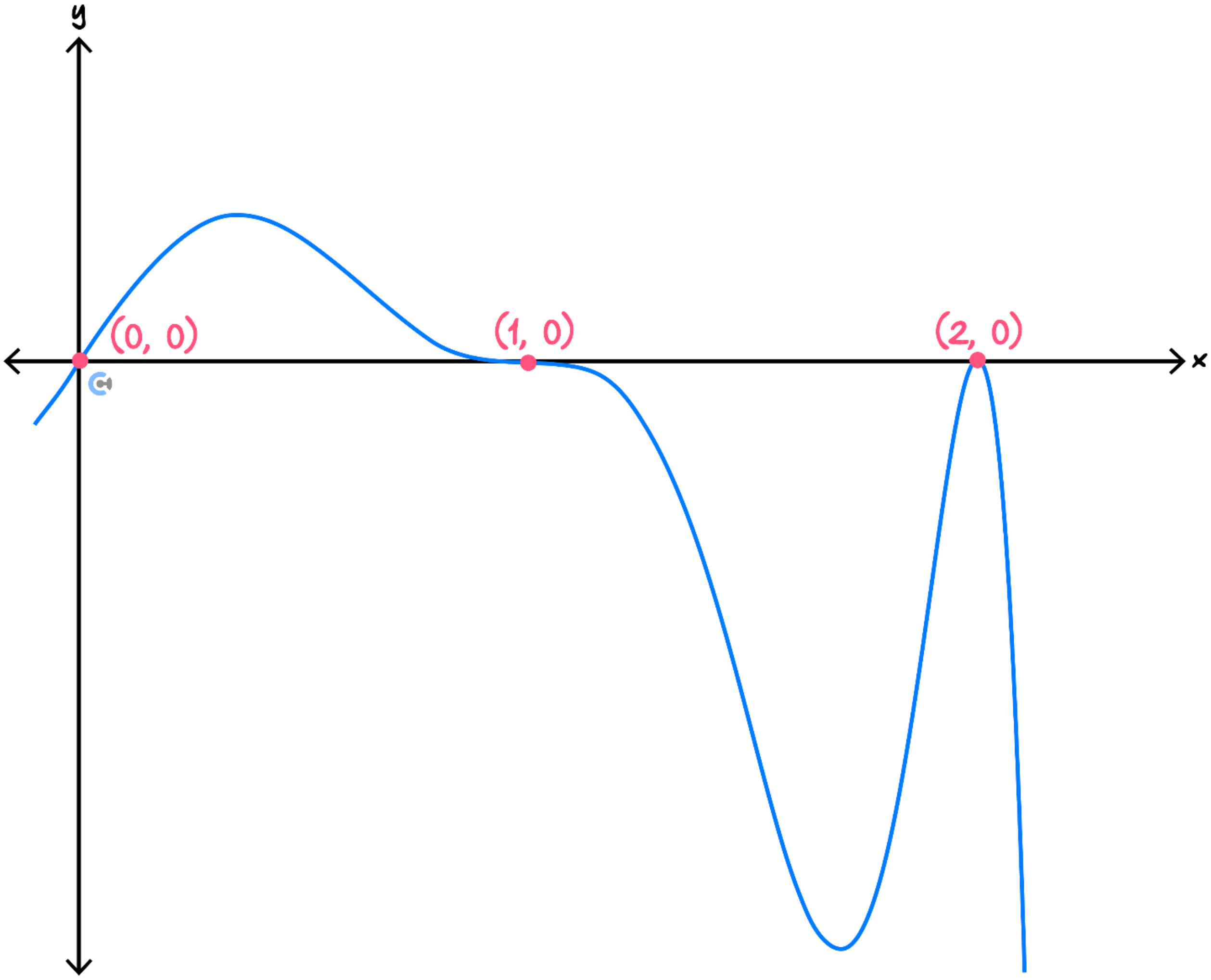
Question 15

Let $f(x)$ be an odd function. Part of the graph of $y = f(x)$ is shown below.

Determine a possible rule for $f(x)$.



Sub-Section: Exam 1 Questions

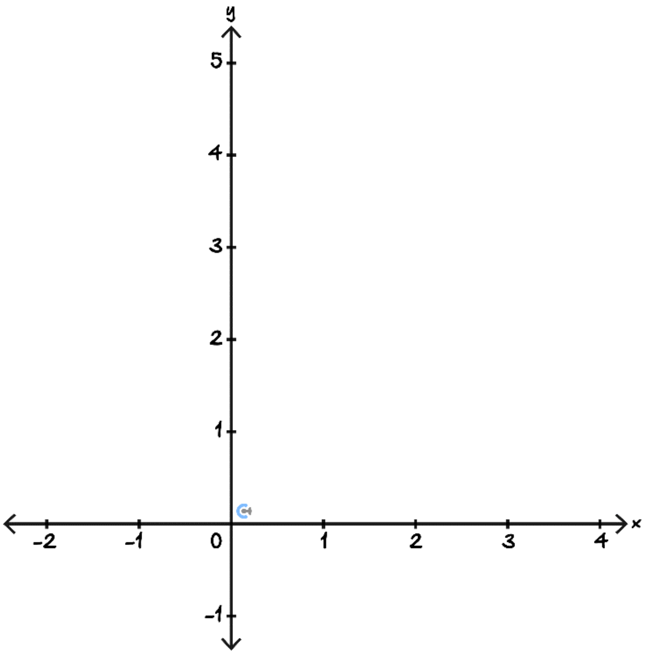
Question 16

Let $f: [-1,3] \to \mathbb{R}, f(x) = x^3 - 3x^2 + 4$.

a. Show that $x - 2$ is a factor of $f(x)$.

b. Fully factorise $f(x)$.

c. It is known that the graph of $y = f(x)$ has a turning point on its y-intercept. Sketch the graph of $y = f(x)$, labelling all axes intercepts, turning points and end points.



d. Let $g: \mathbb{R} \to \mathbb{R}, g(x) = x^3 - 3x^2 + 4$.

Find the values of k such that $g(x - k) = 0$ has two positive solutions.

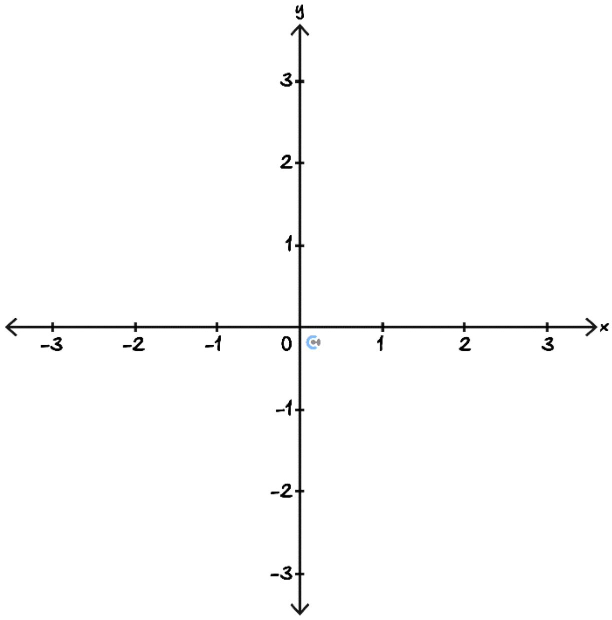
Question 17

Let $f: \mathbb{R} \to \mathbb{R}, f(x) = 3x - x^3$.

It is known that the graph of $y = f(x)$ has a turning point when $x = 1$.

a. Show that f is an odd function.

b. Sketch the graph of $y = f(x)$. Label all axes intercepts and turning points with coordinates.



c. Consider the function $g: \mathbb{R} \to \mathbb{R}, g(x) = 3x - x^3 + k$, where k is a real constant.

i. Find the values of k for which $g(x)$ has exactly two x-axis intercepts.

ii. Find the values of k for which $g(x) = 1$ has exactly one solution.

Question 18

Consider the function $f(x) = x^3 - ax^2 + bx + 8$, where a and b are integers.

It is known that $x - 2$ is a factor of $f(x)$ and that $f(x)$ has a remainder of -24 when divided by $x + 2$.

Find the values of a and b.

Question 19

Consider $f: \mathbb{R} \to \mathbb{R}, f(x) = -x^3 + ax^2$ and $g : \mathbb{R} \to \mathbb{R}, g(x) = ax$ where a is a positive real constant.

a. Find the coordinates of the x-intercepts of the graph of f in terms of a, where appropriate. (1 mark)

b. Find the values of a for which the graphs of f and g have only one point of intersection.

The graphs of f and g have three points of intersection when $a > 4$. Let the x-coordinates of these three points of intersection be r, s and t where $r < s < t$.

c. Find the values of r, s and t, in terms of a, where appropriate.

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Sub-Section: Exam 2 Questions

Question 20

Let $p(x) = x^3 - 3ax^2 + 2x - 2$, where $a \in \mathbb{R}$. When p is divided by $x + 2$ the remainder is 10.

The value of a is:

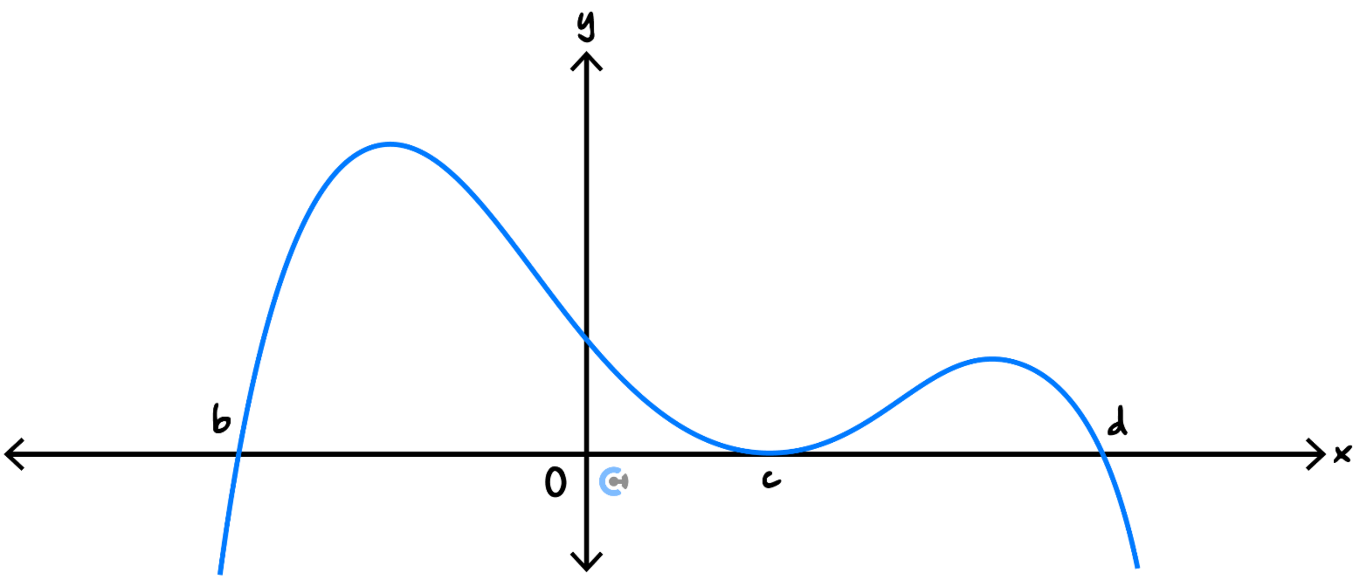
A. -2

B. -1

C. 1

D. 2

Question 21



The rule for a function with the graph above could be:

A. $y = -2(x + b)(x - c)^2(x - d)$

B. $y = 2(x + b)(x - c)^2(x - d)$

C. $y = -2(x - b)(x - c)^2(x - d)$

D. $y = 2(x - b)(x - c)\ (x - d)$

Question 22

A graph with rule $f(x) = x^3 - 3x^2 + c$, where c is a real number, has three distinct x-intercepts.

The set of all possible values of c is:

A. $[0,4]$

B. $\{0,4\}$

C. $(0,4)$

D. $(-\infty, 4)$

Question 23

The equation $x^3 - 3x^2 - 9x + c = 0$ has only one solution for x when:

A. $-5 < c < 27$

B. $c \leq -5$

C. $c < -5$ or $c > 27$.

D. $c \leq -5$ or $c \geq 27$.

Question 24

A set of three numbers that could be the solutions of $x^3 + bx^2 - 22x + 40 = 0$, where $b \in \mathbb{R}$, is:

A. $\{-1,4,5\}$

B. $\{-2,2,4\}$

C. $\{-5,-4,2\}$

D. $\{-5,2,4\}$

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Question 25

Consider the quartic $f: \mathbb{R} \to \mathbb{R}, f(x) = 3x^4 - 4x^3 - 12x^2$.

a. Find the coordinates of the point M at which the minimum value of the function f occurs.

b. State the values of $b \in \mathbb{R}$ for which the graph of $y = f(x) + b$ has no x-intercepts.

A tangent line l is drawn to the graph of f when $x = \frac{1}{2}$ and has the equation $l(x) = -\frac{27}{2}x + \frac{55}{16}$.

c. Find the coordinates of all points where the line l intersects the graph of f.

Let $p: \mathbb{R} \rightarrow \mathbb{R}, p(x) = 3x^4 - 4x^3 - 12x^2 + 2a, a \in \mathbb{R}$.

d. Find the values of a for which:

i. $p(x) = 0$ has three solutions.

ii. $p(x) = 0$ has two solutions.

e. Find the value of k for which the function $g(x) = 3x^4 - (4 - k^2)x^3 - (12 + k)x^2 + (24 - 12k)x + 3k$ is an even function.

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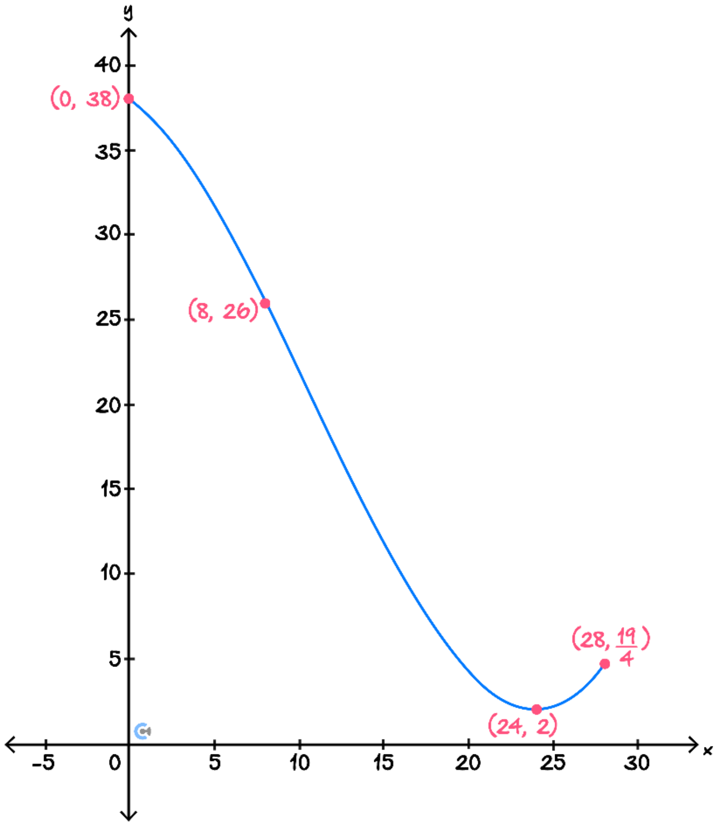
Question 26

James is designing a waterslide that launches you into the water. The waterslide's cross-section is modelled by a function:

$$f: [0,28] \to \mathbb{R}, f(x) = ax^3 + bx^2 + cx + d.$$

The graph of f is shown below.



a. Show that $a = \frac{1}{256}, b = -\frac{1}{8}, c = -\frac{3}{4}, d = 38$.

b. $f(x)$ can be written as $f(x) = g(x)(x-8) + r$ where r is an integer.

Find $g(x)$ and r.

c. The slide is supported by a support beam with equation $s(x) = 38 - ax$ where $a > 0$.

Find the values of a for which:

i. $f(x) = s(x)$ has three solutions.

ii. $f(x) = s(x)$ has one solution.

Let $h: \mathbb{R} \rightarrow \mathbb{R}, h(x) = f(x)$.

d. Describe a sequence of translations that map the graph of $h(x)$ onto the graph of an odd function.

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Section B: Supplementary Questions

Sub-Section [1.8.1]: Apply Transformations to Restrict the Number of Positive/Negative x-Intercept(s)

Question 27

Let $f(x) = (x-1)(x+4)(x-2)^2$. Find the values of k such that $f(x+k)$ has no positive x-intercepts.

Question 28

Let $f(x) = x^3 - 2x^2 - 5x + 6$. Find the values of k such that $f(x+k)$ has exactly one negative x-intercept.

Question 29

Let $f(x) = 2x^2 - 15x + 14$ and $g(x) = x^2 - 10x + 8$. Find the values of k such that $f(x+k)$ and $g(x+k)$ have exactly two intersections with negative x-coordinates.

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Question 30

Let $f(x) = \frac{1}{2}x + 3$ and $g(x) = 2x^2 - 4x - 22$. Find the values of k such that $f\left(g(x + k)\right)$ has exactly one negative x-intercept.

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Sub-Section [1.8.2]: Apply Discriminant to Solve Number of Solutions Questions

Question 31

Find the values of k such that the equation $x^2 - 2^k x + 4$ has no solutions.

Question 32

Find the values of k such that the equation $x^2 - 2kx + 5k$ has exactly two solutions.

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Question 33

Find the values of k such that the equation $(x^2 - kx + 4)(x^2 - 2\sqrt{3}x + k) = 0$ has exactly three solutions.

Question 34

Let $f(x) = x^2 - 4x + 3$ and $g(x) = x^2 - 6x + k$. Find the values of k such that $f(g(x))$ has exactly four solutions.

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Sub-Section [1.8.3]: Apply Shape/Graph to Solve Number of Solutions Questions

Question 35

Suppose $f(x) = x^2 - kx + 3$. Find the value of $k > 0$ so that $f(x) = k$ has exactly one solution.

Question 36

It is known that the quartic $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 8.5$ has turning points at $(1, -0.5)$, $(2, 0.5)$ and $(3, -0.5)$. Find the values of k such that $f(x) = k$ has exactly two solutions.

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Question 37

It is known that the quartic $f(x) = x^4 - 4x^3 - 8x^2 + 48x + 3$ has turning points at $(-2, -77)$, $(2,51)$ and $(3,48)$. Find the values of k such that $f(x) = k$ has exactly two solutions.

Question 38

Let $f(x) = x^4 - 16x^3 + 46x^2 - 48x + 20$ and $g(x) = -x^4 + 2x^2 + 3$. It is known that the quartic $h(x) = 2x^4 - 16x^3 + 44x^2 - 48x + 17$ has turning points at $(1, -1)$, $(2,1)$ and $(3, -1)$. Hence or otherwise, find the value of k such that $f(x) = g(x) + k$ has exactly three solutions.

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Sub-Section [1.8.4]: Apply Odd And Even Functions

Question 39

Show that the function given by $f(x) = x^5 - 2x^2 + 1$ is neither even nor odd.

Question 40

Let $f(x) = x^4 - (k^2 - 5k + 6)x^3 + k^3x^2 + 10$. Find the value(s) of k so that $f(x)$ is an even function.

Question 41

The tangent to the graph of $f(x) = x^2 \ - 4$ at the point $x = 2$ is given by $h(x) = 4x - 8$. Denote the tangent to $f(x)$ at $x = -2$ by $k(x)$. Find the rule for $k(x)$ by applying a reflection to $h(x)$.

Question 42

The tangent to the graph of $f(x) = x^3 - 3x$ at the point $x = 2$ is given by $h(x) = 9x - 16$. Denote the tangent to $f(x)$ at $x = -2$ by $k(x)$. The rule for $k(x)$ can be obtained from the rule of $h(x)$ via the following sequence of transformations:

- A translation of a units in the positive direction of the x-axis.
- A translation of b units in the positive direction of the y-axis.

State the values of a and b and hence or otherwise, find the rule of $k(x)$.

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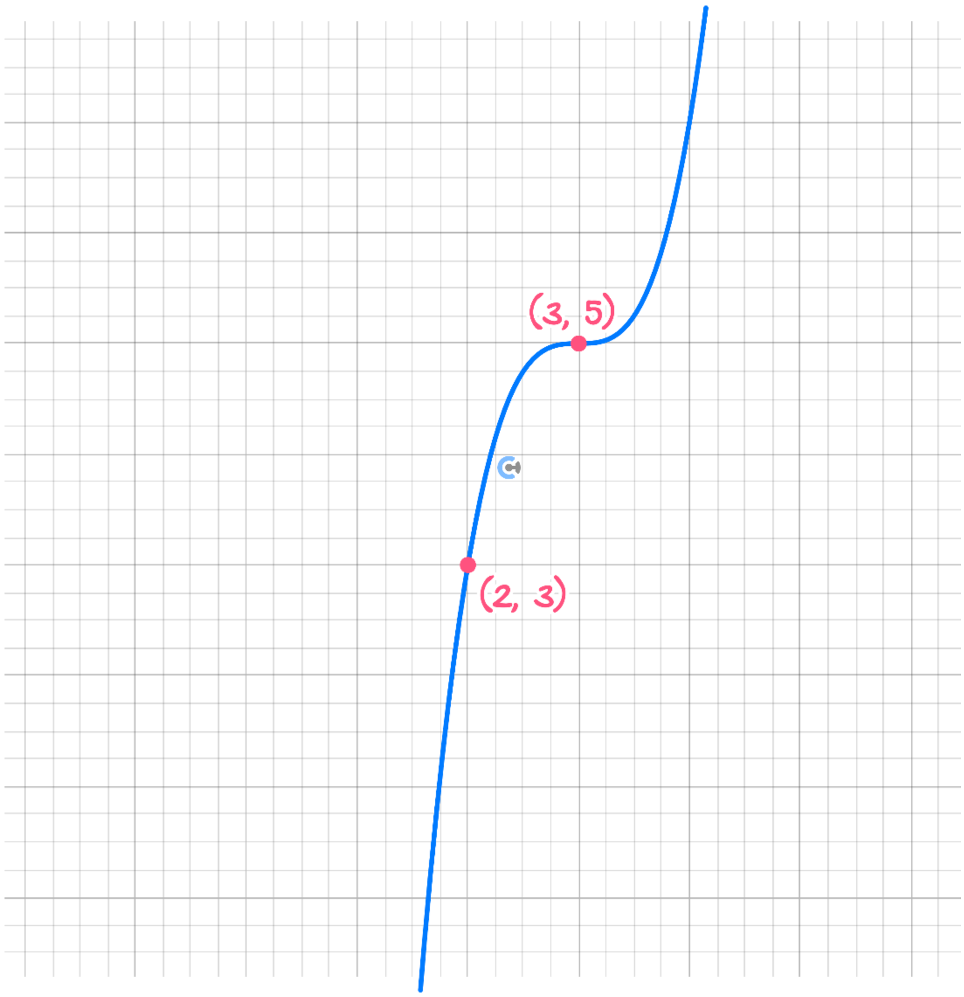
Sub-Section [1.8.5]: Identify Possible Rule(s) From a Graph

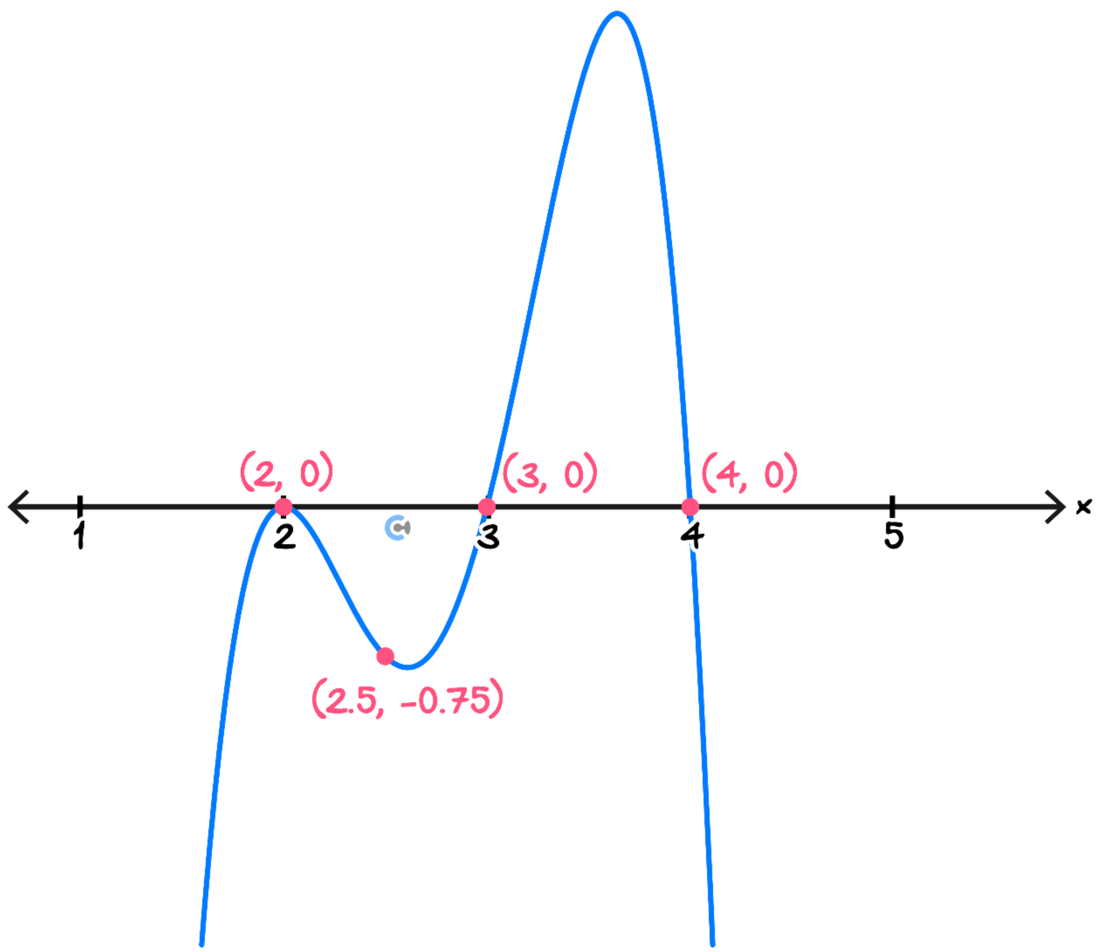
Question 43

Part of the graph of $f(x)$ is plotted below. The point $(3,5)$ is a stationary point of inflection. Find a possible rule for the function.



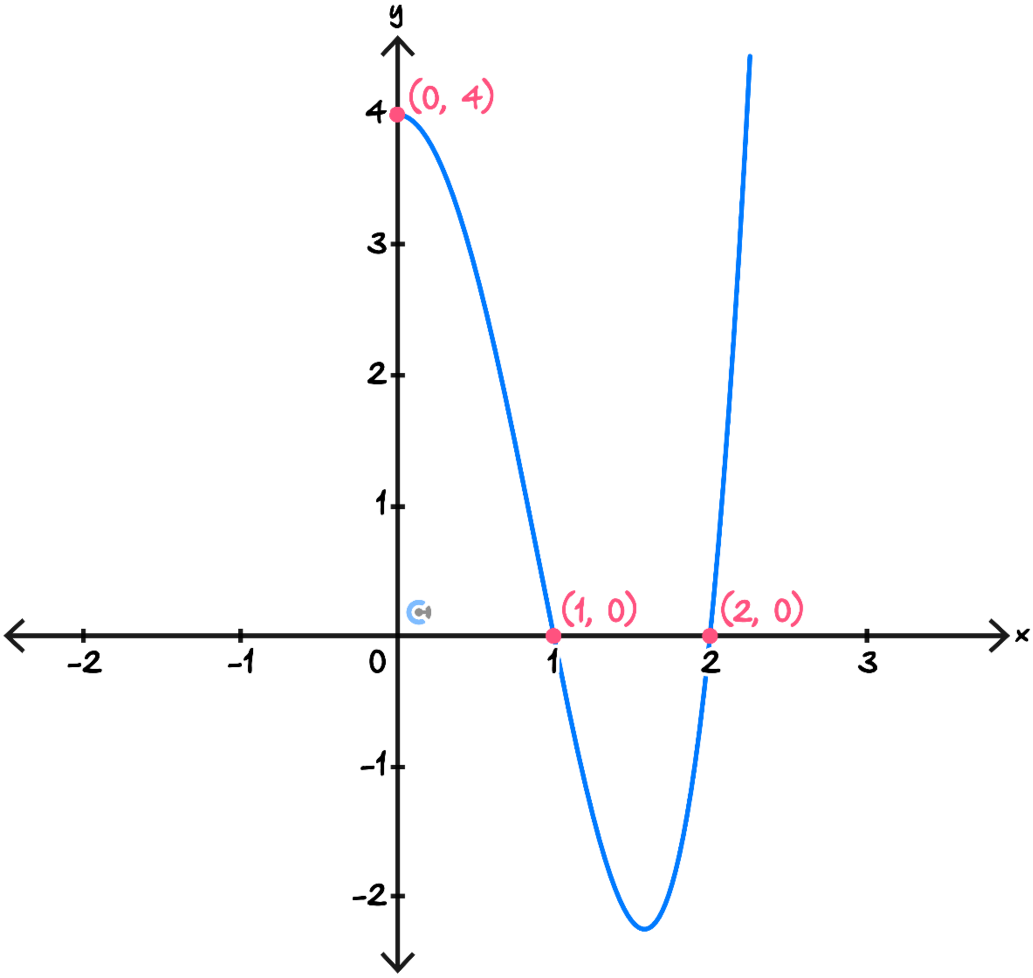
Question 44

Part of the graph of $f(x)$ is plotted below. Find a possible rule for the function.



Question 45

Part of the graph $f(x)$ is plotted below. Find a possible rule for the function if the function is known to be even.



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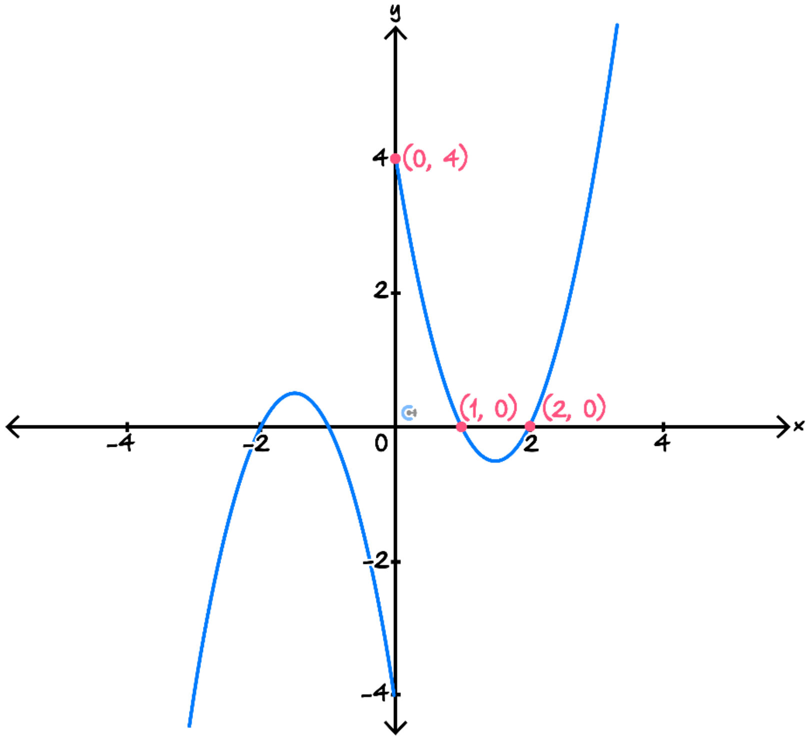
Question 46

Part of the graph $f(x)$ is plotted below.



Find a possible rule for the function if the function is known to be odd. Write your answer in the form.

$$f(x) = \begin{cases} f_1(x), & x < 0 \\ f_2(x), & x > 0 \end{cases}$$

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Sub-Section: Exam 1 Questions

Question 47

Find the value(s) of k so that the equation $(x^2 - kx + 16)(x^2 - 2\sqrt{7}x + k) = 0$ has:

a. Exactly one solution.

b. Exactly four solutions.

Question 48

Suppose that $f(x) = x^2 - 7x + 6$ and $g(x) = x^2 - kx + 1$. Find the values of k so that the equation $f\big(g(x)\big)$ has:

a. Exactly two solutions.

b. Exactly four solutions.

Question 49

Suppose that $f(x)$ is an odd function such that $f(x) = (x-2)^2$ for $x > 0$.

a. Write down a possible rule for $f(x)$ in the form:

$$f(x) = \begin{cases} f_1(x), & x < 0 \\ f_2(x), & x > 0 \end{cases}$$

b. It is known that the tangent to $f(x)$ at the point $x = 3$ is given by the rule $h(x) = 2x - 5$. By applying an appropriate sequence of transformations to $h(x)$, find the rule for the tangent at the point $x = -3$.

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Question 50

Consider a quartic of the form $f(x) = ax^4 + bx^3 + cx^2 + dx + e$. It is known that the quartic satisfies the following conditions:

- $f(1) = 0$.
- $f(2) = 0$.
- $f(0) = 4$.
- Also, $f(x)$ is even.

a. Find the values of a, b, c, d and e.

b. Verify that $f(x)$ can be factorised to $(x - 1)(x + 1)(x - 2)(x + 2)$.

c. Find the values of k so that $f(x + k)$ has exactly two positive x-intercepts.

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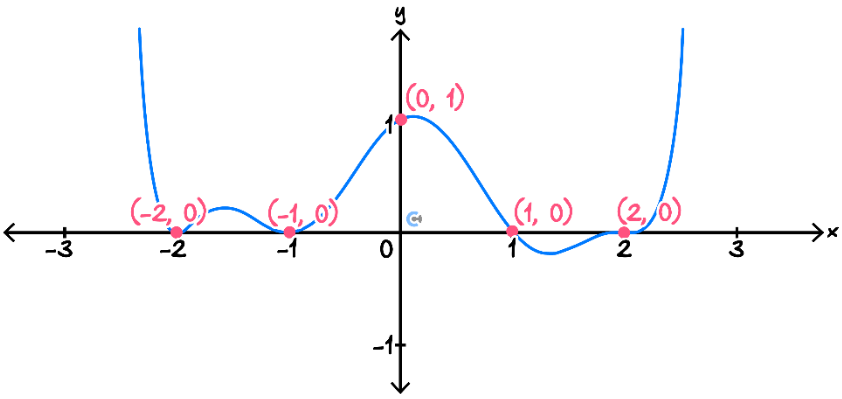
Sub-Section: Exam 2 Questions

Question 51

The minimum degree of the following polynomial is:



A. 2

B. 4

C. 6

D. 8

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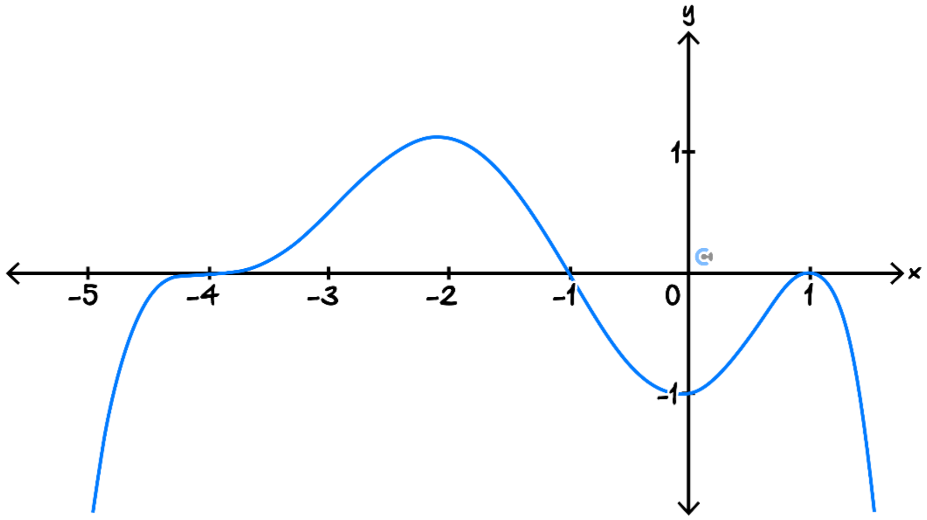
Question 52

A possible rule for the following function given below is:



A. $a(x-1)^3(x+4)^2(x+1)$ where $a<0$.

B. $a(x-1)^3(x+4)^2(x+1)^3$ where $a>0$.

C. $a(x-1)^2(x+4)^3(x+1)$ where $a<0$.

D. $a(x-1)\ (x+4)^3(x+1)$ where $a>0$.

Question 53

Let $f(x)=x^3-(k^2-5k+6)x^2-(k^3+5k)x$. If $f(x)$ is odd, then k must equal:

A. 1 or 3.

B. 1 or 2.

C. 2 or 3.

D. 2 or 6.

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Question 54

Let $g(x) = (x - 1)^2(x - 5)^2 - 4$. There will be exactly four solutions to the equation given by $g(x) = k$ whenever:

A. $-16 < k < 8$

B. $-4 < k < 12$

C. $-4 < k < 0$

D. $-4 < k < 16$

Question 55

Let $h(x) = x^4 - 10x^2 + 9$. The function $h(x + k)$ will have exactly three negative x-intercepts whenever:

A. $1 < k \leq 3$

B. $1 \leq k \leq 3$

C. $-3 < k \leq 1$

D. $-3 \leq k \leq 1$

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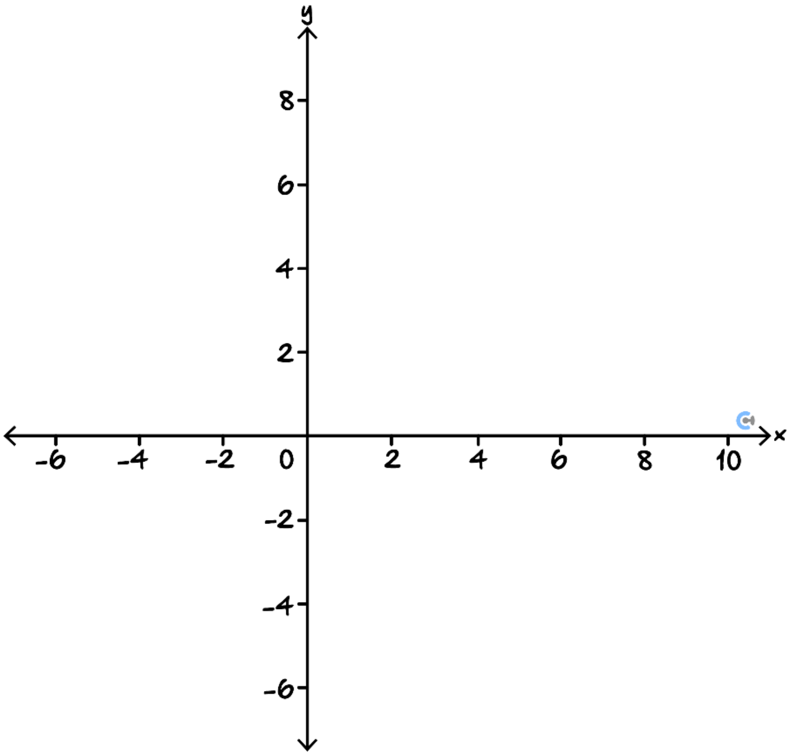
Question 56

Consider a cubic of the form $f(x) = ax^3 + bx^2 + cx + d$. Suppose that $f(x)$ satisfies the following conditions:

- $f(0) = 4$.
- $f(1) = 0$.
- $f(-2) = 0$.
- $f(4) = 0$.

a. Calculate the values of a, b, c and d.

b. Sketch the graph of the function $y = f(x)$, labelling all turning points and intercepts.



c. Find the value(s) of k such that $f(x) - k = 0$ has exactly:

i. 2 solutions.

ii. 3 solutions.

d. Let $g(x) = x^2 - kx + 5$. State the values of k such that $f(g(x)) = 0$ gives the maximum number of solutions possible.

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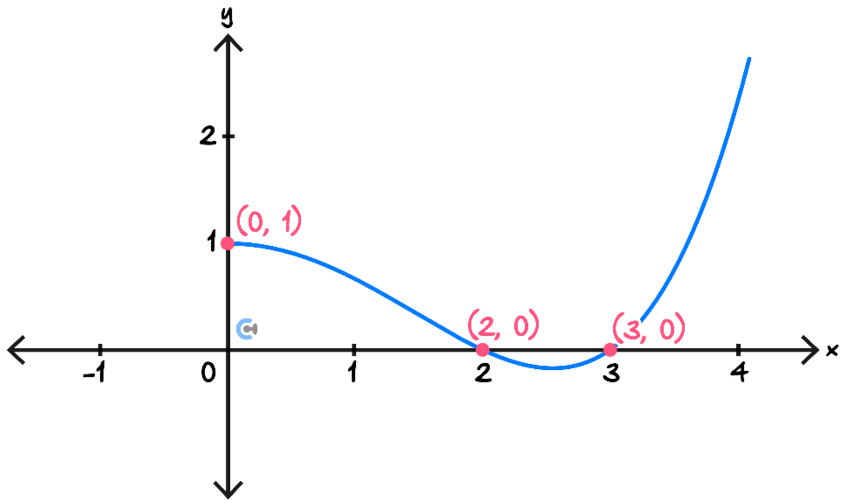
Question 57

The part of the graph of $f(x)$ is shown below. Furthermore, it is known that the function $f(x)$ is a quartic and also even.



a. State the rule for $f(x)$.

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b. The tangent to the graph of $f(x)$ at $x = 3$ is given by $y = \frac{5}{6}x - \frac{5}{2}$.

i. Describe a sequence of transformation(s) that can be applied to $h(x)$ to obtain the tangent to the graph of $f(x)$ at $x = -3$.

---

ii. Hence, write down the rule for the tangent to the graph of $f(x)$ at $x = -3$.

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c. State the values of k so that $f(x-k)$ has exactly:

i. 3 positive x-intercepts.

ii. 3 negative x-intercepts.

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