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VCE Mathematical Methods ¾
Polynomials [1.7]
Homework

Homework Outline:

Compulsory Questions	Pg 2 – Pg 20
Supplementary Questions	Pg 21 – Pg 37

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Section A: Compulsory Questions

Sub-Section [1.7.1]: Applying Factor and Remainder Theorems

Question 1

a. State the remainder when $x^3 - 3x + 2$ is divided by $x - 3$.

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b. State the remainder when $x^4 + 2x + 1$ is divided by $2x + 2$.

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c. Is $x - 3$ a factor of $f(x) = x^3 - 8x + 3$?

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Question 2

Let $f(x) = ax^3 + 4x + 1$. Find the value of a such that $f(x)$ has a factor of $2x + 1$.

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Question 3

a. Let $f(x) = ax^2 + 3x + c$. Find the values of a and b such that f has a factor of $2x + 5$, and when f is divided by $2x + 1$, it has a remainder of -6.

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b. Tech-Active.

Let $g(x) = ax^3 + bx^2 + cx + d$, have the following properties,

1. $g(x)$ has a factor of $x^2 - 1$.

2. $g(x)$ divided by $x - 2$ leaves a remainder of 7.

3. $g(x)$ divided by $2x + 3$ leaves a remainder of -4.

Find the values of a, b, c and d.

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Sub-Section [1.7.2]: Finding Factored Forms of Polynomials

Question 4

Factorise the following polynomials:

a. $8x^3 + 27$.

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b. $x^3 - 4x^2 - x + 4$.

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c. $x^3 + 2x^2 + x$.

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Evaluate the following expression without a calculator:

d. $7^3 - 5^3$.

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Question 5

a. Let $f(x) = x^3 - 2x^2 - 5x + 6$.

i. Show that $f(1) = 0$.

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ii. Hence, or otherwise, write $f(x)$ in the form $f(x) = (x - a)(x - b)(x - c)$ for integers a, b, c.

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b. Factorise $g(x) = x^3 - 2x^2 - 9x + 18$.

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c. Find all of the real roots of $h(x) = x^3 + 2x^2 - 29x - 30$.

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d. Tech-Active.

Factorise $P(x) = 6x^5 + 11x^4 - 49x^3 - 41x^2 + 115x - 42$.

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Question 6

a. Let $f(x) = 9x^3 - 54x^2 - x + 6$.

    i. According to the rational root theorem, what are the possible rational roots of f?

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    ii. Hence, or otherwise, find all of the roots of f.

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b. Show that the polynomial $P(x) = x^3 - 5$ has no rational roots.

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c. Consider $f(x) = x^3 + \frac{7x^2}{4} + \frac{7x}{2} - 1$. It is known that f has only positive roots. Factorise $f(x)$.
Hint: To apply the rational root theorem all of your polynomial coefficients must be integers.

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Sub-Section [1.7.3]: Graphing Factored and Unfactored Polynomials

Question 7

a. Sketch the graph of $y = (x - 2)^4 - 1$ on the axis below.
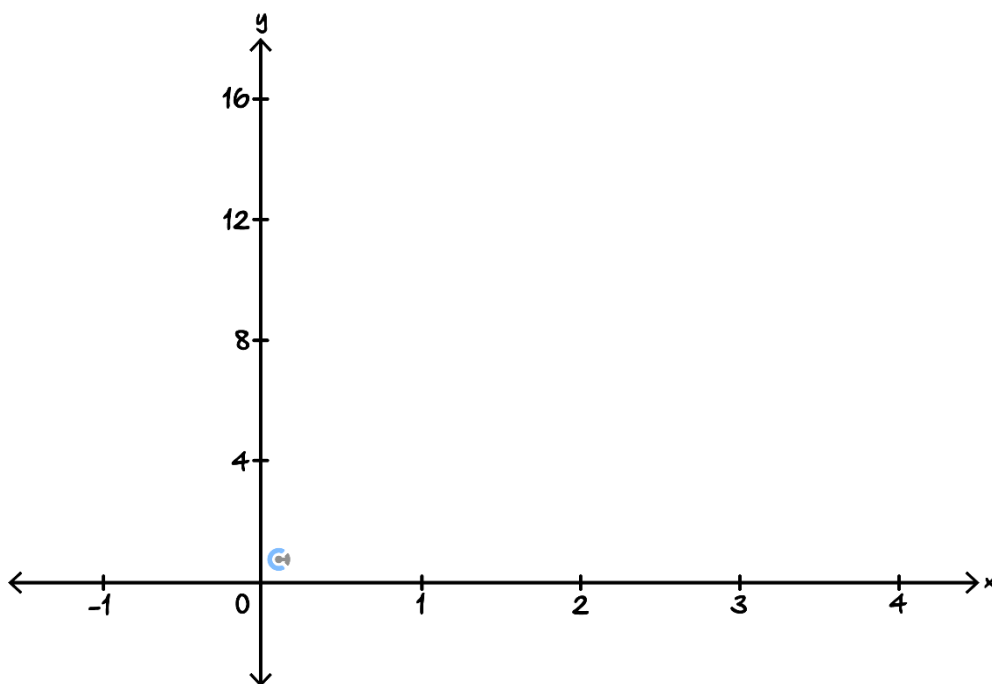
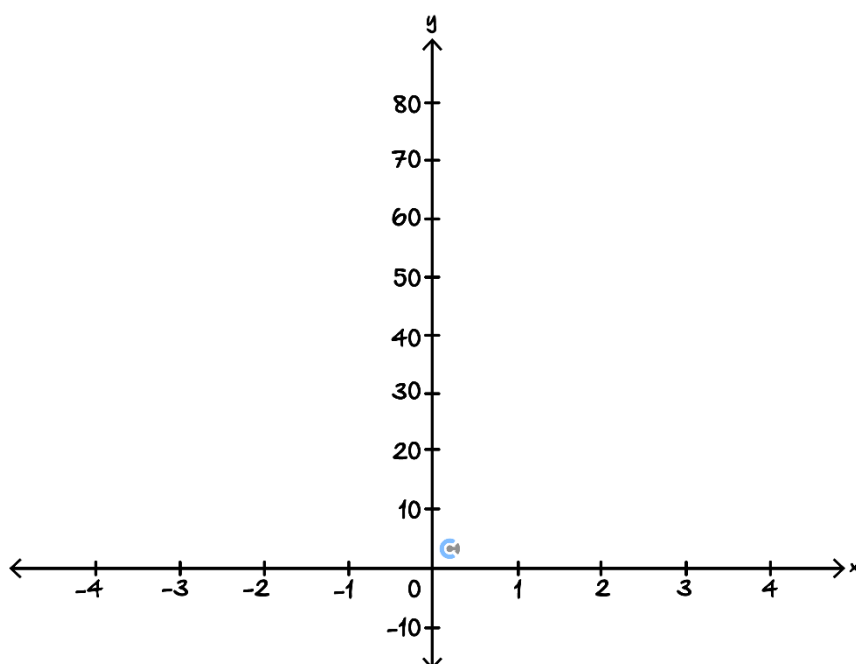**b.** Sketch the graph of $y = (x - 3)^3 - 8$ on the axis below.
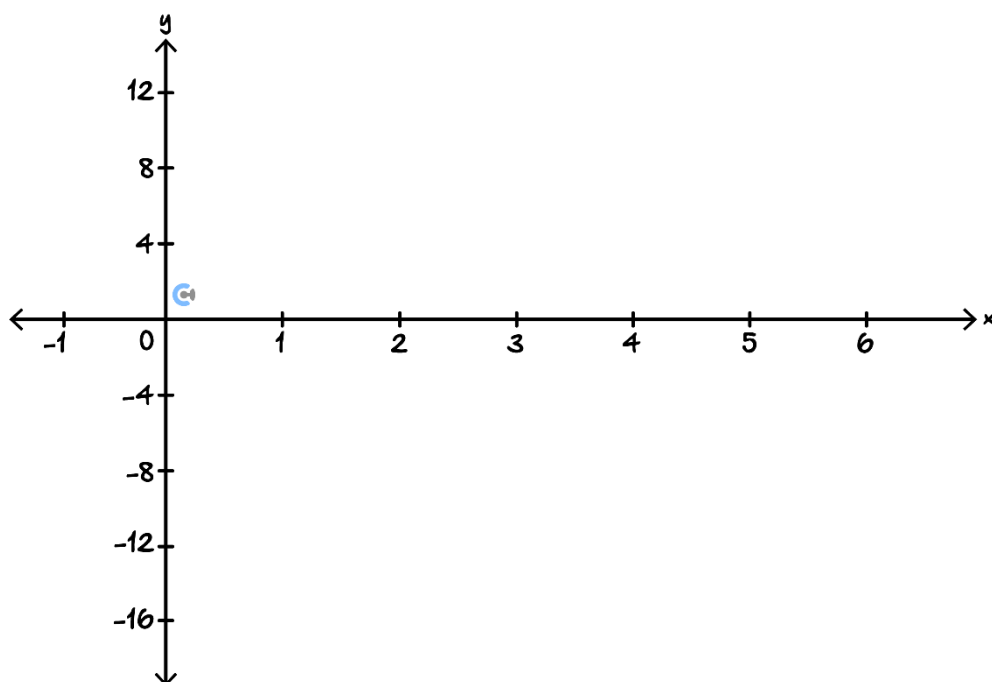
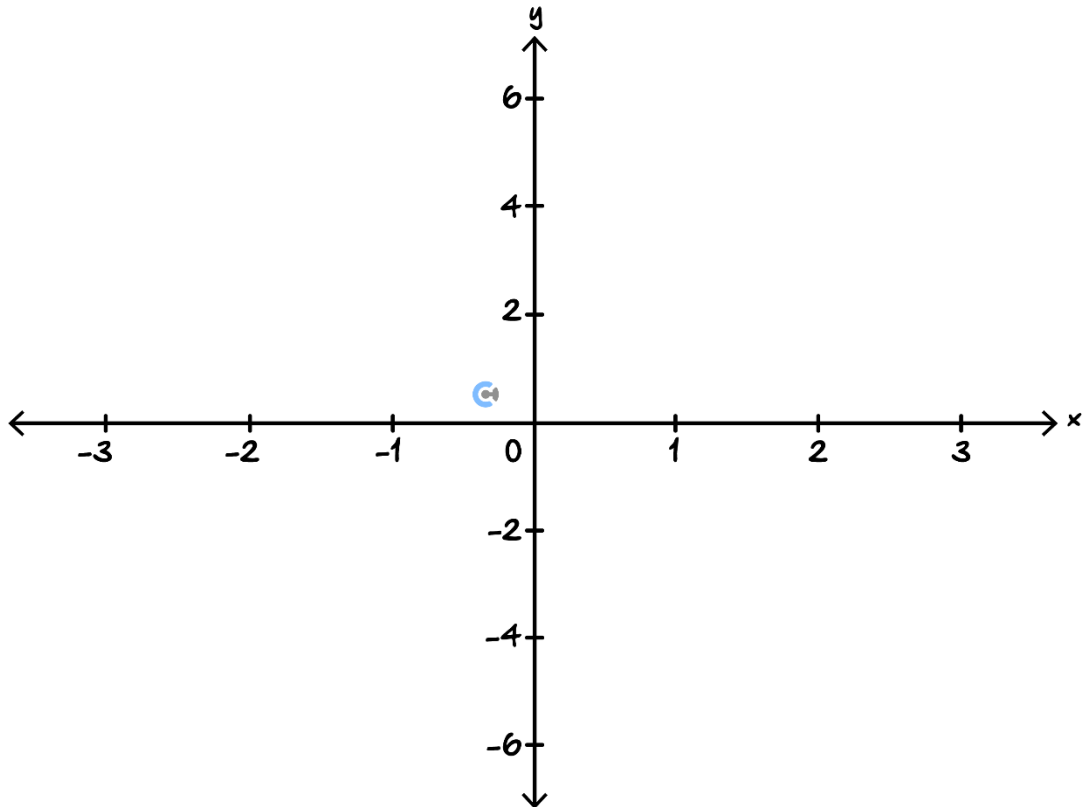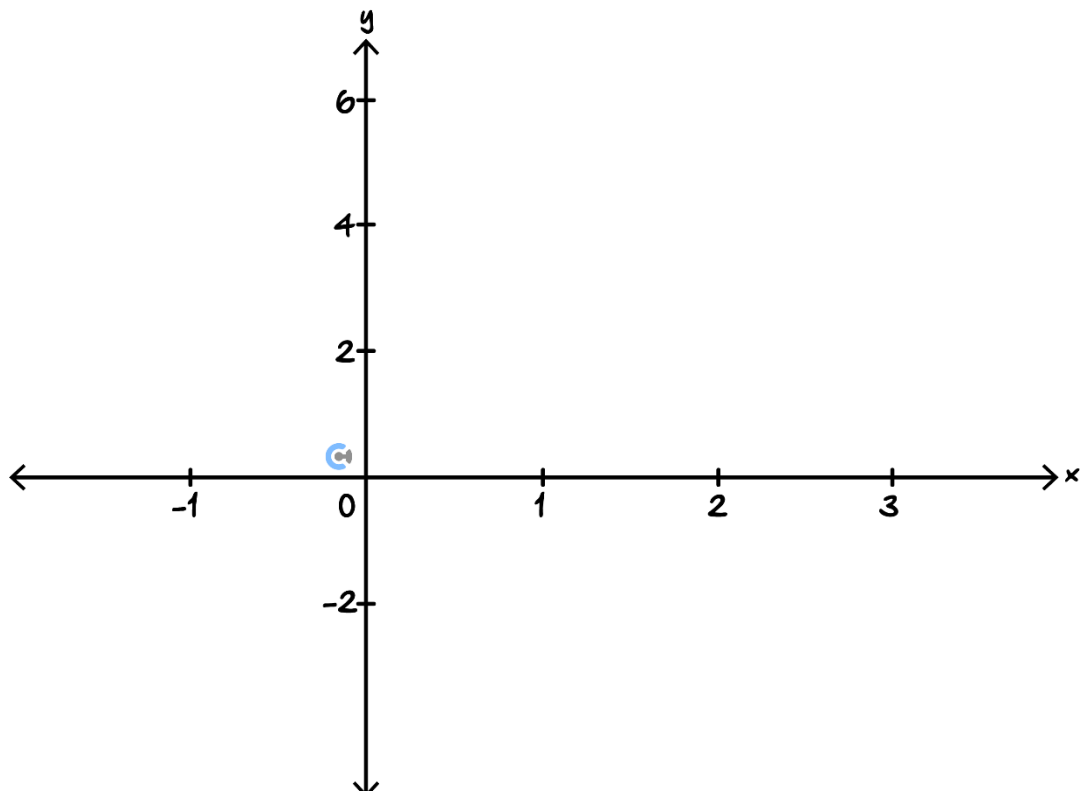**c.** Sketch the graph of $y = 3(x - 2)(x - 1)(x + 5)$ on the axis below.
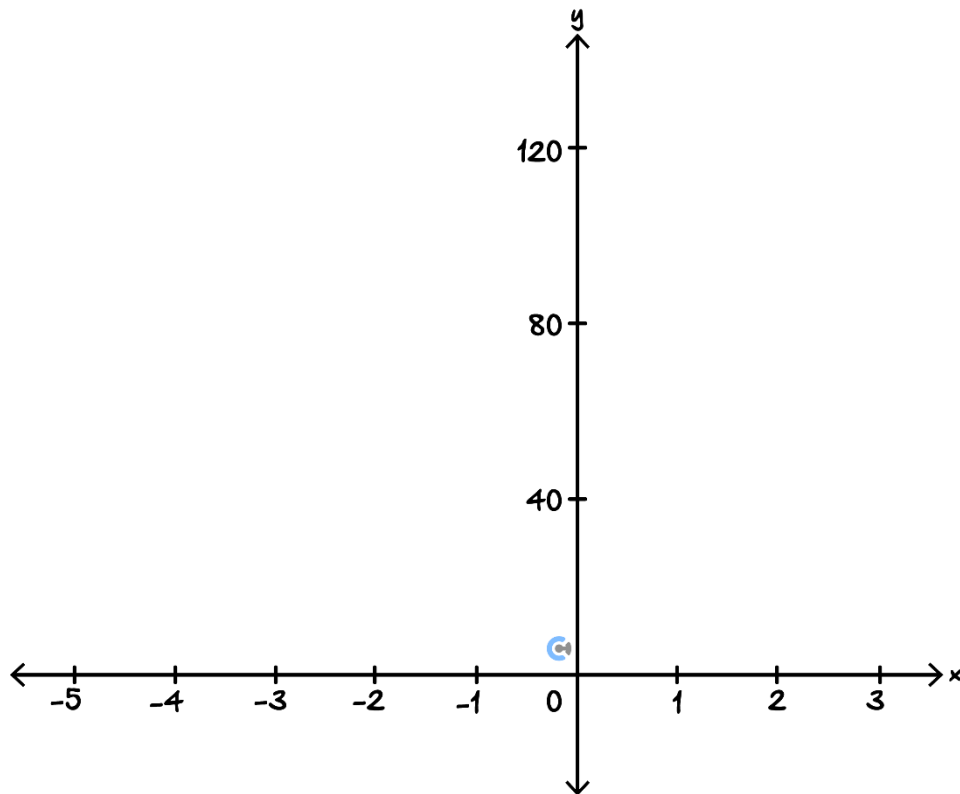
Question 8

a. Sketch the graph of $y = (x^2 - 9)^2 - 4$ on the axis below, labelling axis intercepts with their coordinates.

b. Sketch the graph of $y = x(x^2 - 5)$ on the axis below, labeling axis intercepts with their coordinates.
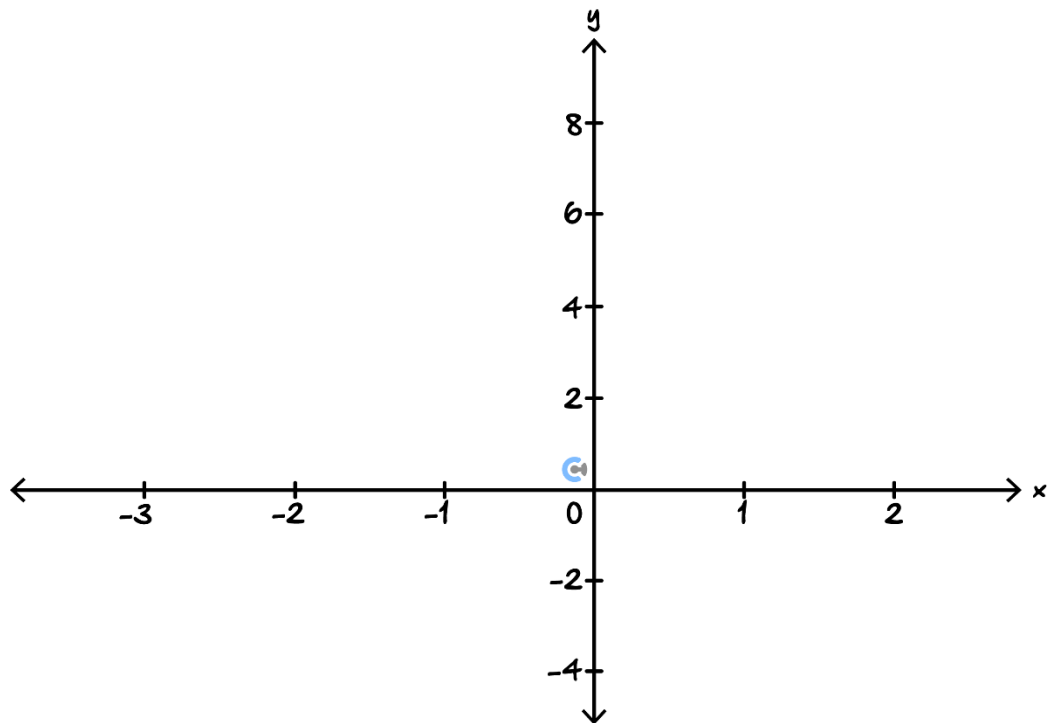

c. Sketch the graph of $y = x^3 - 4x^2 + x + 6$ on the axis below, labeling axis intercepts with their coordinates.
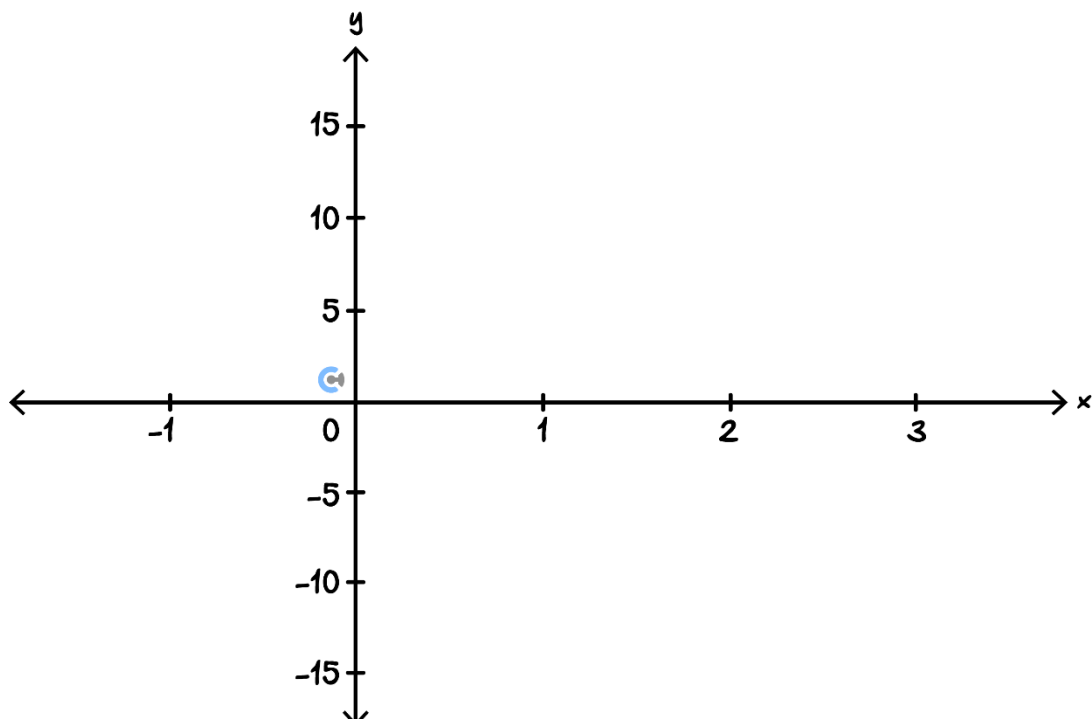
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Question 9

a. Sketch the graph of $y = x^3 + x^2 - 5x + 3$ on the axis below, labeling axis intercepts with their coordinates.
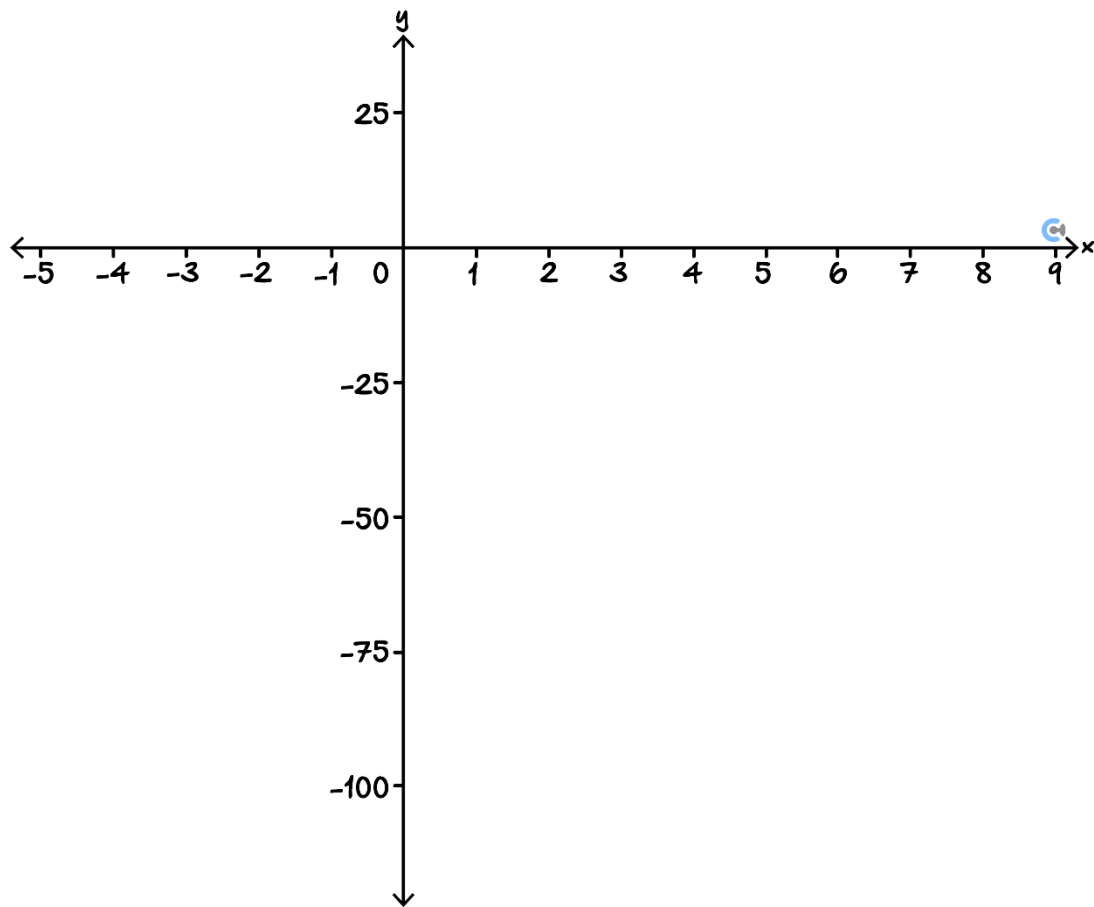

b. Tech-Active.

Sketch the graph of $y = x(x - 2)^3(x + 1)^3(x - 3)$ on the axis below, labeling axis intercepts with their coordinates.

c. Sketch the graph of $y = 5 + 4x - 31x^2 + 6x^3$ on the axis below, labeling axis intercepts with their coordinates.

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Sub-Section [1.7.4]: Identify Odd and Even Functions

Question 10

a. Let $f(x)$ be an even function and $g(x)$ be an odd function.

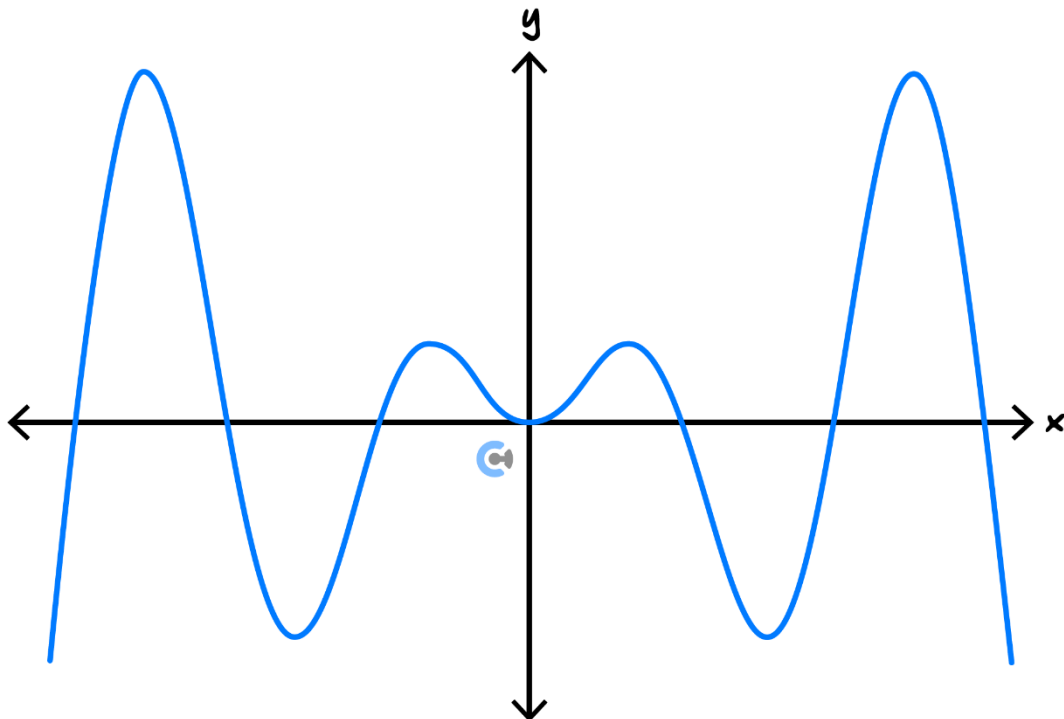   **i.** State whether $f(g(x))$ is an even or an odd function.

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    ii. State whether $f(x) \times g(x)$ is an even or an odd function.

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b. Part of the graph of $f(x)$ is drawn below. State whether f is an odd or an even function.

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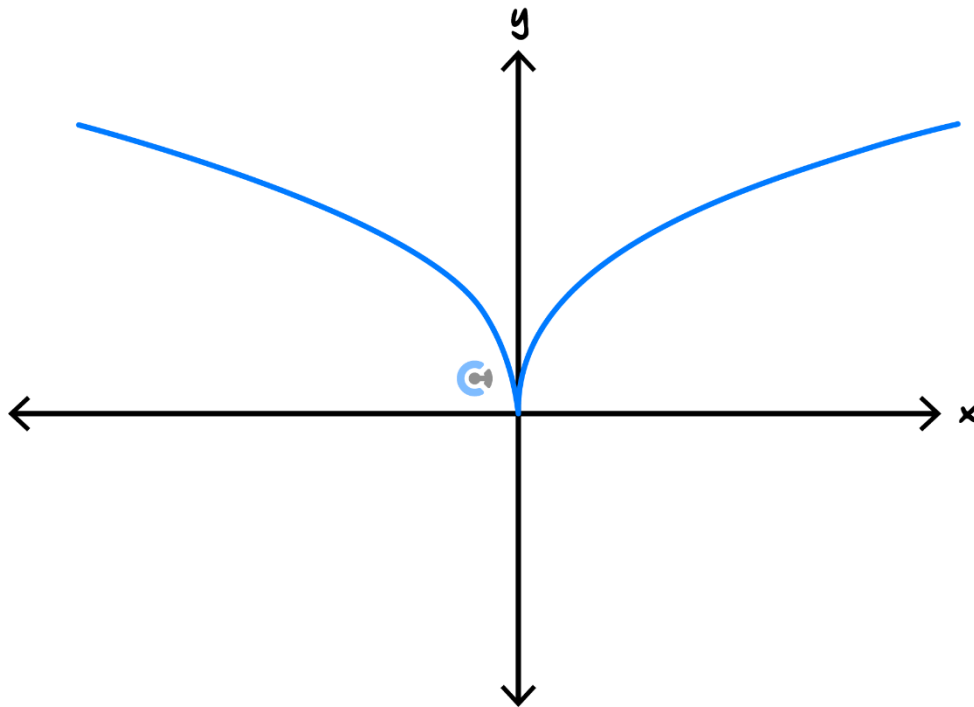CONTOUREDUCATION

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c. Part of the graph of $y = x^{\frac{m}{n}}$ is drawn below where m and n are co-prime.

State whether m and n are even or odd.

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Question 11

a. Let $f(x) = (x-3)^3 + 5$.

Describe a sequence of transformations that map the graph of f onto the graph of an odd function.

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b. Show that $P(x) = 2(x^4 + 3x^2 - 1)^3 - 5$ is an even function.

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c. Consider the function $f(x)$. It is known that $f(x+2)$ is an even function.

If $f(-1) = 3, f(7) = 5$, and $f(3) = 7$, find the value of $2f(-3)$.

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Question 12

a. Let $f(x)$ be an even function and $g(x)$ be an odd one-to-one function.

If $f(3) = 5$, $g(1) = 3$, and $g(3) = 4$. Find $f(-3) + g^{-1}(-3)$.

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b. Tech-Active.

Let $f(x) = x^3 - 9x^2 + 7x$.

A transformation $T(x, y) = (x + a, y + b)$ maps the graph of $f(x)$ onto the graph of an odd function g. Find the values of a and b.

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c. James says that he's found a function, $f(x)$ that is both odd and even.

Show that $f(x) = 0$ for all real x.

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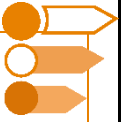
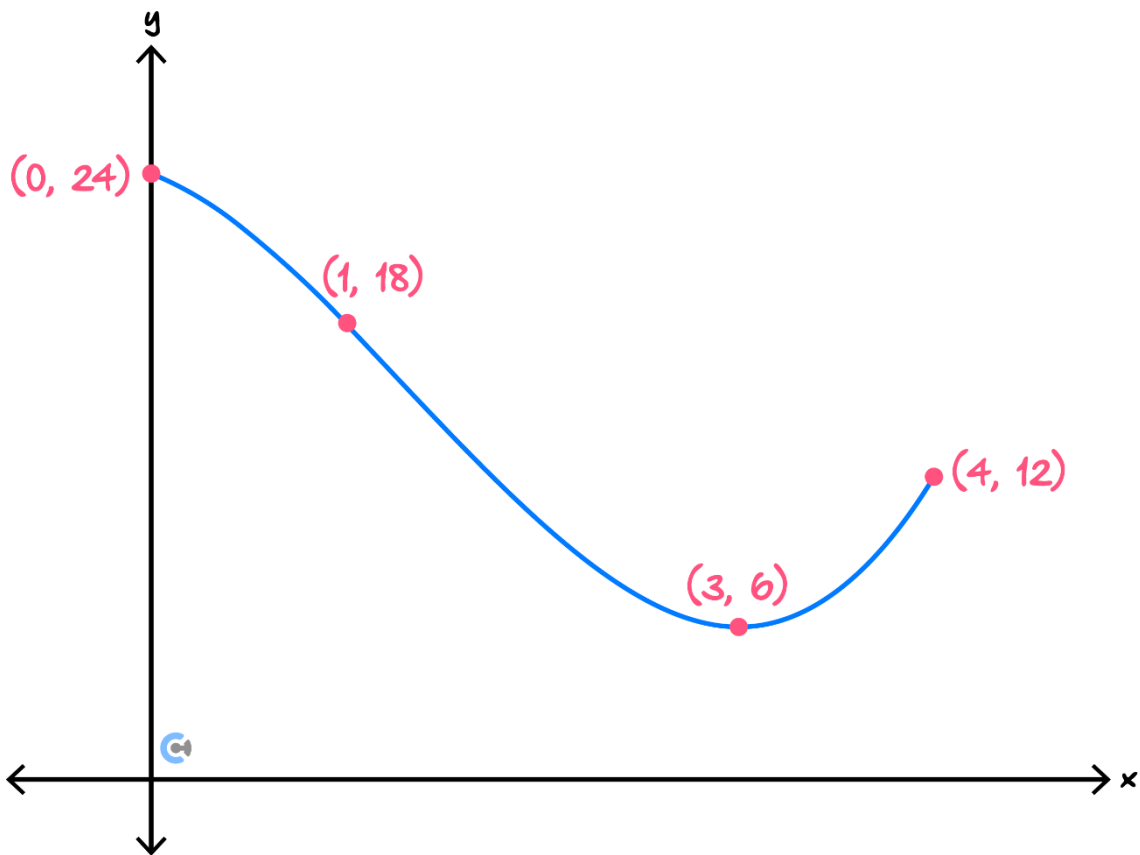VCE Methods ¾ Questions? Message +61 440 138 726

Sub-Section: Boss Question

Question 13

Samuel is building a ramp to throw students off who do not complete their homework.

The cross-section of the ramp is modelled by a function $f: [0, 4] \rightarrow \mathbb{R}, f(x) = ax^3 + bx^2 + cx + d$. The graph of f is shown below.

a. Find the values of a, b, c and d.

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b. $f(x)$ can be written as $f(x) = g(x)(x - 3) + r$ where r is an integer.

    i. State the degree of g.

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    ii. State the value of r.

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Samuel installs a ladder for the students to climb up to the top of the ramp. The cross-section of the ladder is given by the function $L: [0, 4] \to \mathbb{R}, L(x) = 24 - 6x$.

c. Solve $f(x) = L(x)$ for x.

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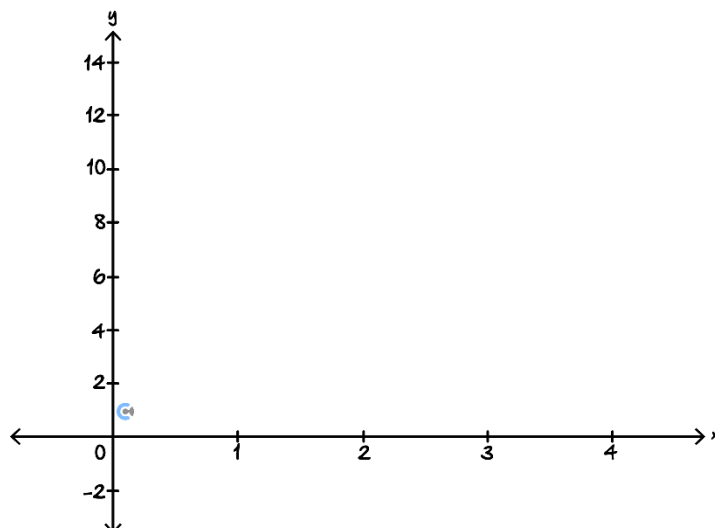**d.** Sketch the graph of $f(x) - L(x)$ on the axis below, labeling axis intercepts and end-points with their co-ordinates.

Let $h(x)$ have the same rule as $f(x)$ but have a domain of all real numbers.

e. How many solutions does the equation $h(x) = 1$ have?

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f. Find a value of a such that $h(x) = a$ has exactly two solutions.

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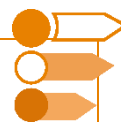**g.** Describe a sequence of translations that map the graph of h onto the graph of an odd function.

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Section B: Supplementary Questions

Sub-Section [1.7.1]: Applying Factor and Remainder Theorems

Question 14

a. State the remainder when $x^2 + 5x - 3$ is divided by $x + 2$.

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b. Is $x - 2$ a factor of $f(x) = x^4 - 16$?

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c. Is $x + 4$ a factor of $g(x) = x^3 + 4x^2 + 2$?

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Question 15

Let $f(x) = 2x^3 + ax^2 + ax + 3$. Find the value of a such that $f(x)$ has a factor of $2x + 3$.

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Question 16

Let $f(x) = x^2 + ax + b$. Find the values of a and b such that f has a factor of -1, and when f is divided by $2x - 3$, it has a remainder of -5.

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Question 17

A cubic polynomial, $g(x)$ has the following properties.

1. $g(x) - 3$ has a factor of $(x - 2)^2$.

2. $g(x)$ divided by $x^2 - 1$ leaves a remainder of 2.

Find the rule for $g(x)$.

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Sub-Section [1.7.2]: Finding Factored Forms of Polynomials

Question 18

Factorise the following polynomials:

a. $x^3 - 8$.

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b. $x^3 - 7x^2 + 10x$.

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c. $x^3 + 3x^2 - 4x - 12$.

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Question 19

a. Factorise $f(x) = x^3 + x^2 - 17x + 15$.

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b. Factorise $g(x) = x^3 - 4x^2 + x + 6$.

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c. Find all of the real roots of $h(x) = x^3 - 3x^2 + 4$.

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Question 20

a. Factorise $f(x) = x^3 - 5x^2 - 29x + 105$.

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b. Factorise $g(x) = 18x^3 - 3x^2 - 28x - 12$.

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c. Factorise $h(x) = 2x^3 + 14x^2 - 10x - 150$.

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Question 21

Let $f(x) = ax^2 + bx + c$ with a, b, c being co-prime non-zero integers, and assume that $\frac{p}{q}$ is a root of f with p and q co-prime and both non-zero.

a. Show that p divides c.

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b. Show that q divides a.

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c. If a, b, c are not co-prime integers, where would your arguments for parts a and b breakdown?

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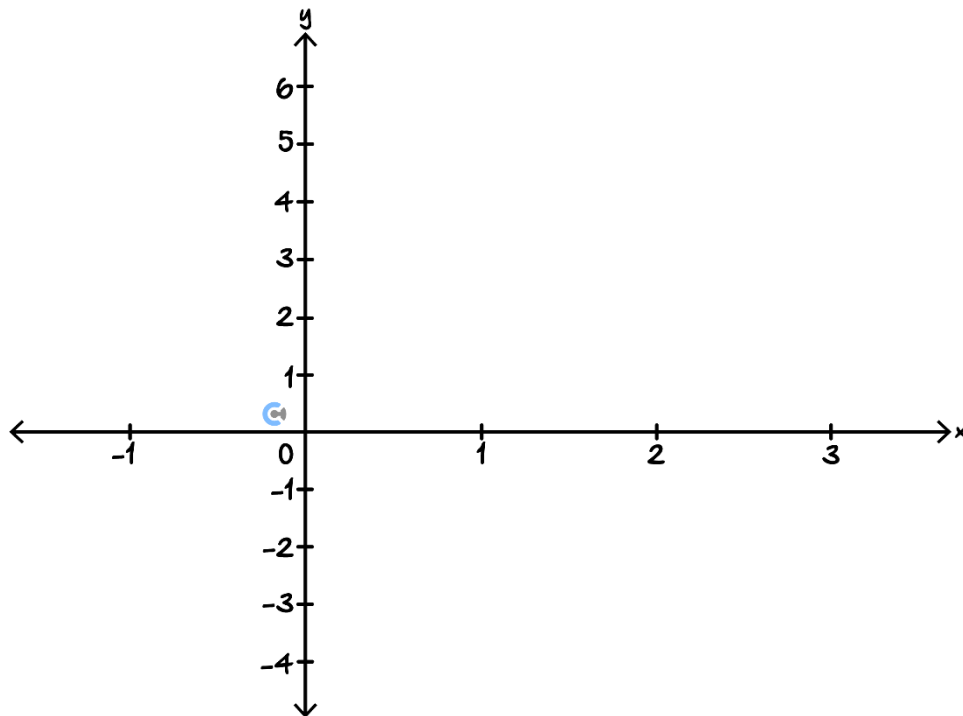## Sub-Section [1.7.3]: Graphing Factored and Unfactored Polynomials

Question 22

a. Sketch the graph of $y = (x + 2)^3 - 1$ on the axis below.
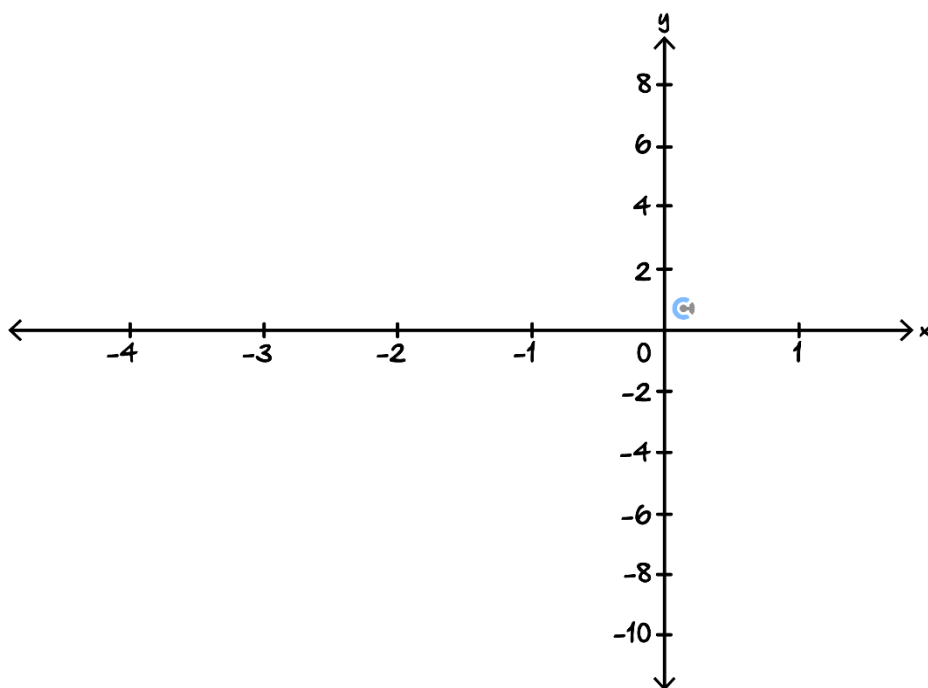
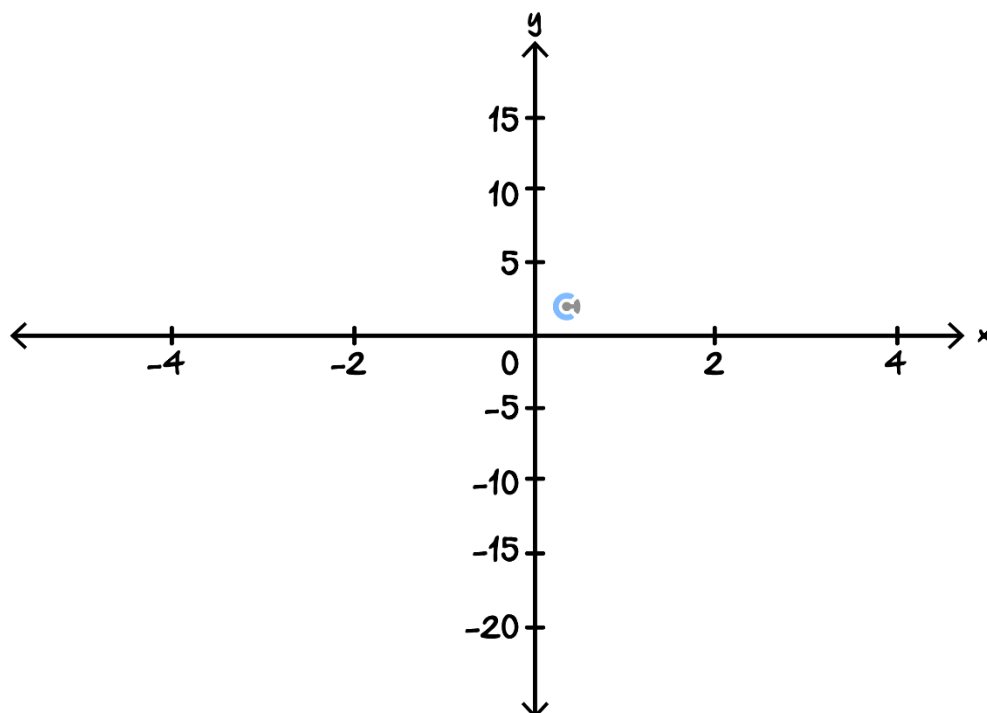**b.** Sketch the graph of $y = x(x - 1)(x + 2)(x - 3)$ on the axis below.

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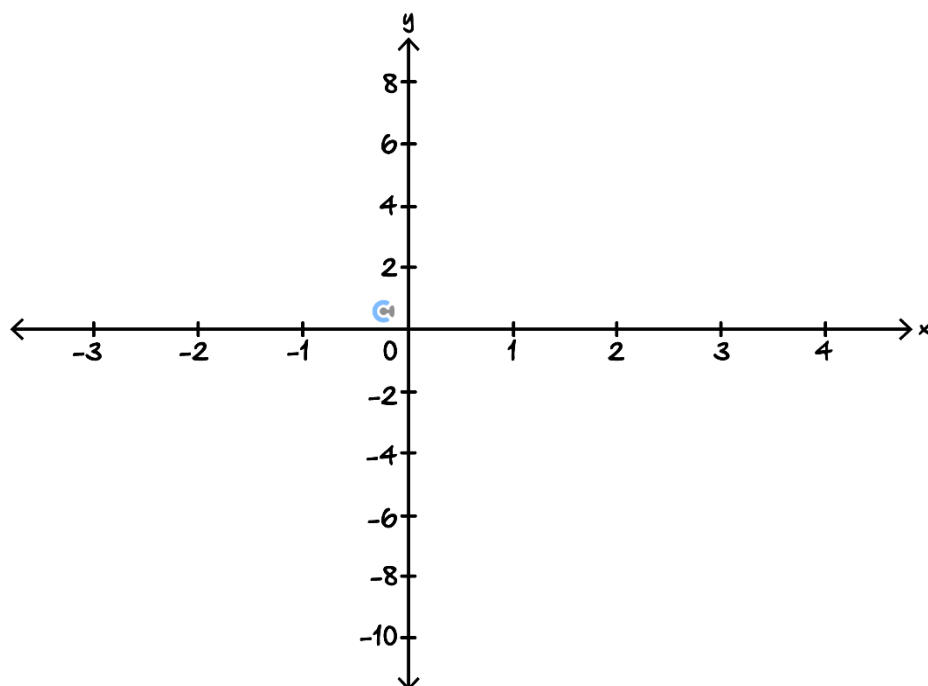
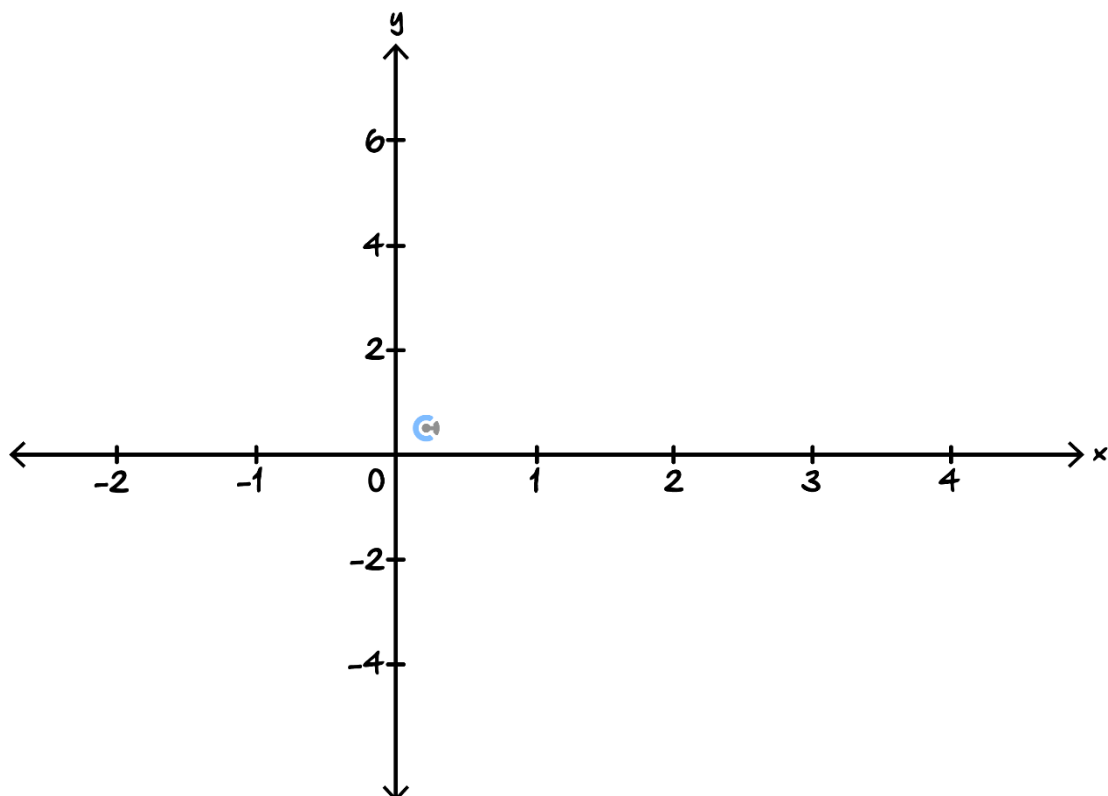**c.** Sketch the graph of $y = 2(x-1)^3 + 2$ on the axis below.

Question 23

a. Sketch the graph of $y = x^3 + 2x^2 - 11x - 12$ on the axis below, labeling axis intercepts with their coordinates.

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b. Sketch the graph of $y = x^4 - 8x^2 + 16$ on the axis below, labeling axis intercepts with their coordinates.
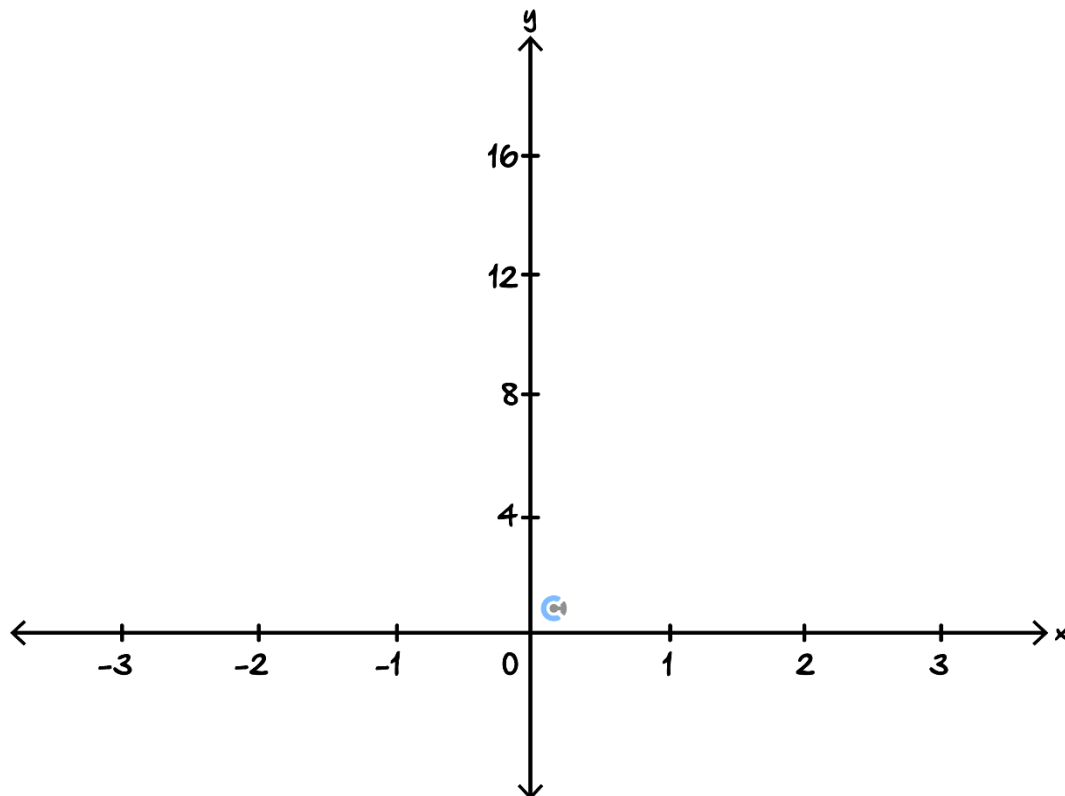
c. Sketch the graph of $y = x^3 - 4x^2 + x + 6$ on the axis below, labeling axis intercepts with their coordinates.
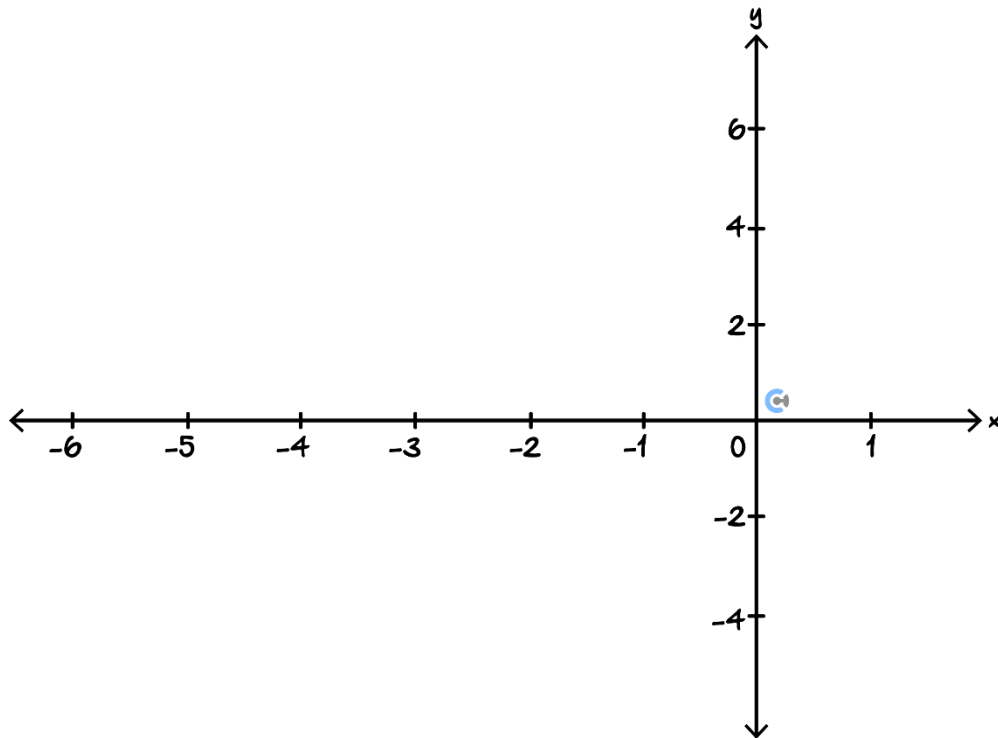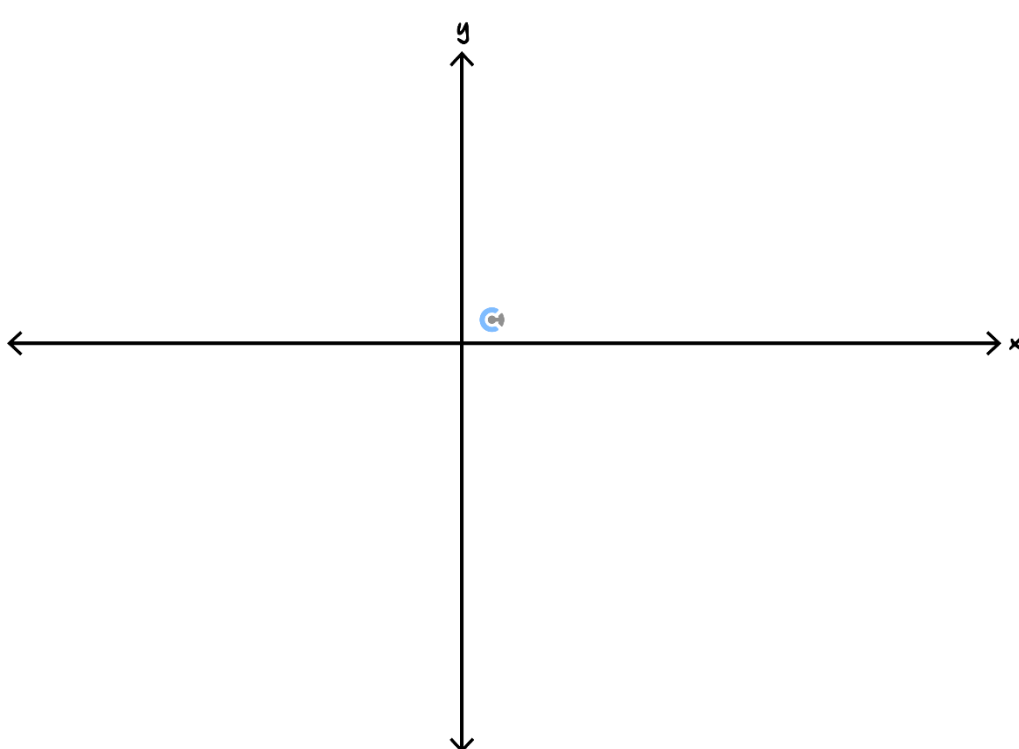
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Question 24

a. Sketch the graph of $y = x^3 + 8x^2 + 16x + 5$ on the axis below, labeling axis intercepts with their coordinates.

b. Sketch the graph of $y = x^2(2x - 3)^3(x + 1)^2$ on the axis below, labeling axis intercepts with their coordinates.
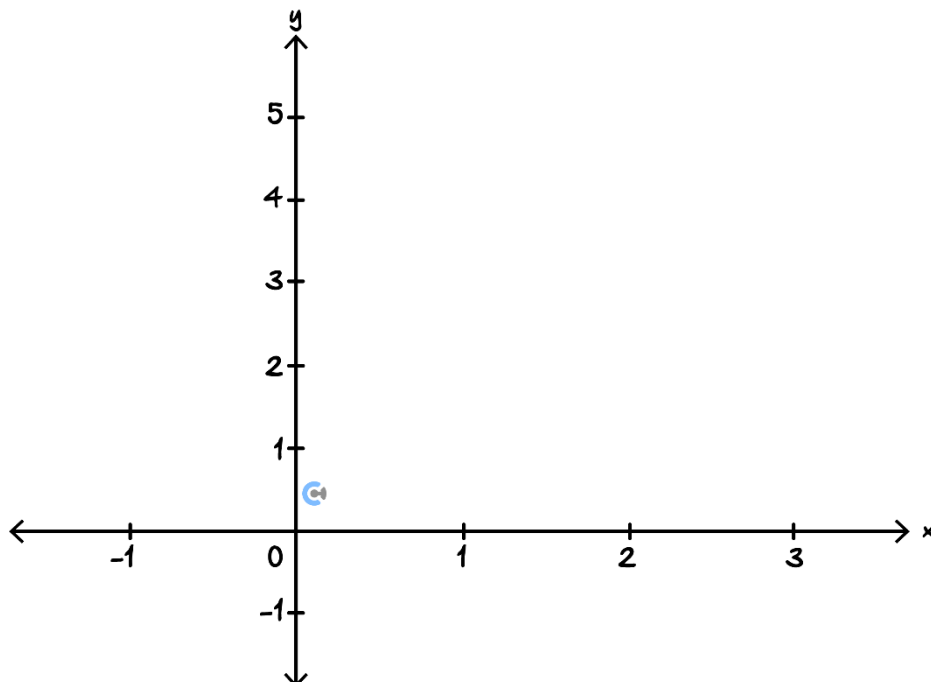
c. Sketch the graph of $y = x^4 + 5x^3 + 3x^2 - 7x - 2$ on the axis below, labeling axis intercepts with their coordinates.
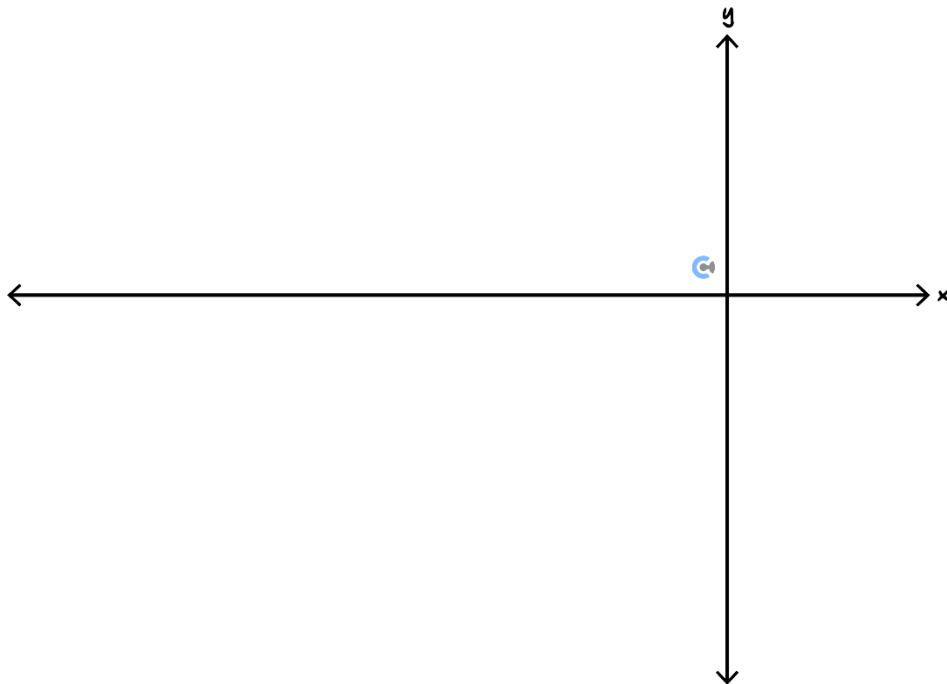
Question 25

Let $f_k(x) = x^4 - 4x^3 + 4x^2 + k$. By considering f_0 and f_{-1}, sketch the graph of f_2 on the axis below, labeling axis intercepts and turning points with their coordinates.

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Sub-Section [1.7.4]: Identify Odd and Even Functions

Question 26

a. Let $f(x)$ and $g(x)$ both be an odd functions.

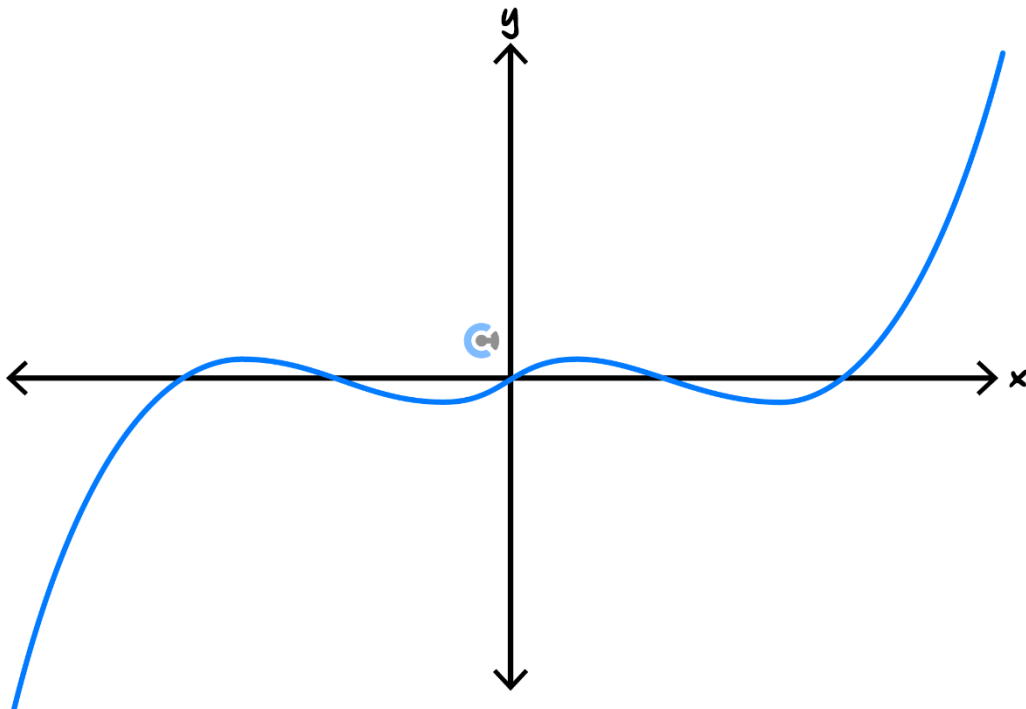   **i.** State whether $f(x) + g(x)$ is an even or an odd function.

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    ii. State whether $(f(x))^2 + 2f(x)g(x) + (g(x))^2$ is an even or an odd function.

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b. Part of the graph of $f(x)$ is drawn below. State whether f is an odd or an even function.

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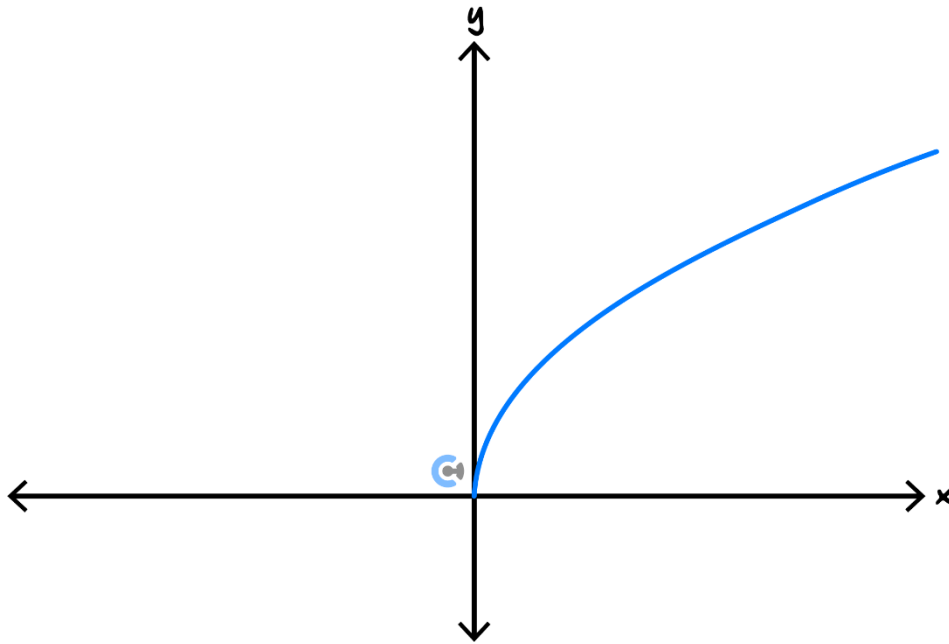CONTOUREDUCATION

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c. Part of the graph of $y = x^{\frac{m}{n}}$ is drawn below where m and n are co-prime.

State whether m and n are even or odd.

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Question 27

a. Show that $f(x) = x^4 - 2x^3$ is neither an even nor an odd function.

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b. Describe a translation that maps the graph of $y = x^2 + 6x + 7$ onto the graph of an even function.

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c. Consider the function $f(x)$. It is known that $f(2x + 3)$ is an odd function.

If $f(5) = 4$ and $f(-1) = -3$, find the value of $f(1)$.

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Question 28

a. Let $f(x)$ be a strictly increasing function with $f(0) = 0$.

If $\left(f(x)\right)^2$ is an even function, show that $f(x)$ is an odd function.

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b. Let $f(x) = x^4 + 2x^3 + x^2$.

Describe a transformation that maps the graph of f onto the graph of an even function.

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c. Let $f(x)$ be an even function.
The function,

$$g(x) = \begin{cases} f(x) + c & x \geq 0 \\ -f(x) + d & x < 0 \end{cases}$$

is an odd function.

Find the values of c and d.

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Question 29

Let $f(x) = x^4 - 4x^3 + x^2 + 6x + k$, where k is a real number.
The function $g(x) = f(x - h)$ is an even function.
Find the value of h.

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