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# VCE Mathematical Methods ¾
## Coordinate Geometry Exam Skills [1.6]
## Homework Solutions

### Homework Outline:

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| Compulsory Questions | Pg 2 – Pg 20 |
| Supplementary Questions | Pg 21 – Pg 41 |

## Section A: Compulsory Questions

### Sub-Section [1.6.1]: Apply Midpoint to Find a Reflected Point

**Question 1**

Find the reflection of the point $(4, 2)$ about the line $x = 6$.

$(8, 2)$

**Question 2**

The point $(2, 3)$ is reflected in the line $y = b$ to become the point $(2, 9)$. Find the value of $b$.

$b = 6$

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**Question 3**

Find the perpendicular bisector between the points $(3, 6)$ and $(-2, -9)$.

Midpoint is $\left(\frac{1}{2}, -\frac{3}{2}\right)$ and the line joining the points is $y = 3x - 3$.

So we want a line with gradient $-\frac{1}{3}$ through the point $\left(\frac{1}{2}, -\frac{3}{2}\right)$. Therefore, the perpendicular bisector is

$$y = -\frac{1}{3}x - \frac{4}{3}$$

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## Sub-Section [1.6.2]: Apply Parallel and Perpendicular Lines to Geometric Problems

**Question 4**

Find the equation of the line that is parallel to $y = 2x + 3$ that passes through the point $(1, 4)$.

$y = 2x + 2$

**Question 5**

Find the area of the triangle formed by the lines $y = x + 2$, $y = 8 - x$ and the $y$-axis.

Lines intersect at $(3, 5)$. Triangle base $= 8 - 2 = 6$ and triangle height $= 3$.
Therefore, triangle area is $\frac{1}{2} \times 6 \times 3 = 9$

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**Question 6**

Find the minimum distance between the line $y = 3 - x$ and the point $(4, 3)$.

Want the equation of line perpendicular to $y = 3 - x$ and through the point $(4, 3)$.
Therefore line with gradient 1 and through $(4, 3)$

$$y = x - 1$$

Now $y = x - 1$ and $y = 3 - x$ intersect at the point $(2, 1)$.
Therefore, the minimum distance is the distance between the points $(4, 3)$ and $(2, 1)$

$$d = \sqrt{2^2 + 2^2} = 2\sqrt{2}$$

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## Sub-Section [1.6.3]: Solve Coordinate Geometry Problems With Transformations

**Question 7**

The area bounded by the lines $y = x$, $y = -x + 10$ and the $x$-axis is 25 square units. Use this to find the area bounded by:

**a.** The lines $y = 2x$, $y = -2x + 20$ and the $x$-axis.

Both lines have been dilated by a factor of 2 from the $x$-axis. Therefore area is 50 square units.

**b.** The lines $y = x - 5$, $y = -x + 15$ and the $x$-axis.

Both lines have been shifted 5 units to the right. The area does not change. Therefore area is 25 square units.

**c.** The lines $y = \frac{2}{3}x$, $y = -\frac{2}{3}x + 20$ and the $x$-axis.

The lines have both undergone a dilation by factor 3 from the $y$-axis followed by a dilation by factor 2 from the $x$-axis. Therefore the bound area has increased by a factor $2 \times 3 = 6$.
The bound area is now $25 \times 6 = 150$ square units.

**Question 8**

Find the equation of the tangent line to the transformed graphs in the following scenarios.

**a.** The original function is $f(x) = x^3$, and the tangent line to the graph of $y = f(x)$ at $x = 1$ is $y = 3x - 2$. The graph of $f$ is dilated by a factor 2 from the $x$-axis, then translated up by 4 units. Find the equation of the tangent to the transformed graph when $x = 1$.

$y = 2(3x - 2) + 4$. Tangent is $y = 6x$

**b.** The original function is $f(x) = \sqrt{x}$, and the tangent line to the graph of $y = f(x)$ at $x = 4$ is $y = \frac{1}{4}x + 1$. The graph of $f$ is reflected about the $y$-axis, then translated 3 units to the left. Find the equation of the tangent line when $x = -7$.

$g(x) = \frac{1}{4}x + 1$. The tangent when $x = -7$ is given by

$$g(-(x+3)) = 1 + \frac{1}{4}(-x-3) = \frac{1}{4} - \frac{1}{4}x$$

**c.** Let $f(x) = (x-2)^2 + 1$, the graph of $y = f(x)$ has a tangent $y = 2x - 4$ when $x = 3$. Find the equation of the tangent to $y = \frac{1}{2}x^2 - 4x + 9$ when $x = 6$.

Let $g(x) = \frac{1}{2}x^2 - 4x + 9 = 2\left(\frac{1}{2}x - 2\right)^2 + 1 = 2\left(\left(\frac{1}{2}x - 2\right)^2 + 1\right) - 1$

Therefore $g(x) = 2f\left(\frac{1}{2}x\right) - 1$

A dilation by factor 2 from the $y$-axis maps $x = 3$ to $x = 6$. Therefore the equation of the tangent is

$$y = 2\left(2x \times \frac{1}{2} - 4\right) - 1 = 2x - 9$$

**Question 9**

**a.** Find the values of $a$ such that the area bounded by the graphs of $y = x$, $y = -x + a$ and the $x$-axis is 9 square units.

The lines intersect at when $x = -x + a \implies x = \frac{a}{2}$. Therefore intersect at $\left(\frac{a}{2}, \frac{a}{2}\right)$.

Take the triangle height as $\frac{a}{2}$, then the base is $a$. Solve

$$\begin{aligned}\frac{1}{2} \times \frac{a}{2} \times a &= 9 \\ a^2 &= 36 \\ a &= \pm 6.\end{aligned}$$

**b.** Find the values of $a$ such that the area bounded by the graphs of $y = 2x$, $y = -\frac{x}{2} + a$ and the $x$-axis is 20 square units.

The lines intersects at when $2x = -\frac{x}{2} + a \Rightarrow x = \frac{2}{5}a$. Therefore, intersect at $\left(\frac{2}{5}a, \frac{4}{5}a\right)$.
Take the triangle height as $\frac{4}{5}a$, then the base is $2a$.
Solve

$$\frac{1}{2} \times \frac{4}{5}a \times 2a = 20$$
$$a = \pm 5$$

**c.** Find the values of $a$ such that the area bounded by the graphs of $y = x + 2$, $y = -x + a$ and the $y$-axis is 9 square units.

The lines intersect at when $x + 2 = -x + a \implies x = \frac{a-2}{2}$. Therefore intersect at $\left(\frac{a-2}{2}, \frac{a+2}{2}\right)$. Take the triangle height as $\frac{a-2}{2}$, then the base is $a - 2$. Solve

$$\frac{1}{2} \times \frac{a-2}{2} \times (a-2) = 9$$
$$(a-2)^2 = 36$$
$$a - 2 = \pm 6$$
$$a = 8, -4$$

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## Sub-Section: Exam 1 Questions

**Question 10**

Consider the simultaneous linear equations:

$$\frac{k}{2}x + 3y = 4$$

$$6x + (2k + 1)y = 12$$

where $k$ is a real constant.

**a.** Find the values of $k$ for which there is a unique solution to the simultaneous equations.

The lines must have different gradients. Solve

$$\frac{k/2}{3} = \frac{6}{2k+1}$$

$$\implies k = -\frac{9}{2}, 4$$

Therefore, unique solution for $k \in \mathbb{R} \setminus \left\{-\frac{9}{2}, 4\right\}$

**b.** Find the value of $k$ for which there are infinitely many solutions.

The $y$-intercepts must be equal. Therefore solve,

$$\frac{4}{3} = \frac{12}{2k+1}$$

$$\implies k = 4$$

infinitely many solutions if $k = 4$.

**c.** Find the value of $k$ for which there are no solutions.

$$k = -\frac{9}{2}$$

**Question 11**

Consider the line segment $AB$ with coordinates $A(1, 0)$ and $B(7, 12)$.

**a.** Find the coordinates of $M$, the midpoint of $AB$.

$M(4, 6)$

**b.** Find the equation of the perpendicular bisector of the line segment $AB$.

$AB$ has gradient 2. So perpendicular bisector has gradient $-\frac{1}{2}$ and goes through $M(4, 6)$. Therefore,

$$y = -\frac{1}{2}x + 8$$

**c.** Let $D$ be the point $(16, 0)$. Find the area of the triangle $AMD$.

$AMD$ is a right angled triangle.
$|AM| = \sqrt{3^2 + 6^2} = \sqrt{45} = 3\sqrt{5}$
$|MD| = \sqrt{12^2 + 6^2} = \sqrt{180} = \sqrt{45 \times 4} = 6\sqrt{5}$
Therefore area $AMD = \frac{1}{2} \times 3\sqrt{5} \times 6\sqrt{5} = 45$

**d.** Let $E(2,0)$, $F(8,12)$ and $G(32,0)$. Find the area of the triangle $EFG$.

The points, $A, M, D$ are dilated by factor 2 from $x$-axis and by factor 2 from $y$-axis. Therefore, area $EFG = 45 \times 2 \times 2 = 180$

**Question 12**

The point $P(2,3)$ is reflected in the line $y = 7 - x$ to become the point $P'$.

**a.** Find the coordinates of $P'$.

Want the line with gradient 1 passing through the point $P(2,3)$. This line is

$$y = x + 1$$

Now we find the intersection of the lines $y = 7 - x$ and $y = x + 1$

$$7 - x = x + 1 \implies x = 3$$
$$\therefore \text{Intersect at } (3,4)$$

Therefore we have $P'(4,5)$

**b.** The point $P$ can also be mapped to $P'$ if it undergoes a reflection in the line $x = a$, followed by a reflection in the line $y = b$. State the values of $a$ and $b$.

$a = 3$ and $y = 4$

**Question 13**

Consider a function $f(x)$, the graph of $y = f(x)$ has a tangent line given by $y = 2x - 5$ and a normal line given by $y = -\frac{1}{2}x + 10$, when $x = 6$.

**a.** Find the area bounded by the tangent line, normal line and the $y$-axis.

When $x = 6$, $y = 7$.
Normal line has $y$-intercept $(0, 10)$ and tangent has $y$-intercept $(0, -5)$.
Therefore the triangle has area $\frac{1}{2} \times 6 \times 15 = 45$.

The graph of $y = f(x)$ is dilated by a factor of 2 from the $y$-axis and by a factor of $\frac{3}{2}$ from the $x$-axis. Let this tranformed graph be given by $y = g(x)$.

**b.** Find the equation of the tangent to the graph of $y = g(x)$ when $x = 12$.

Let $t(x) = 2x - 5$, then the equation of the tangent at $x = 12$ is given by

$$y = \frac{3}{2}t\left(\frac{x}{2}\right) = \frac{3}{2}(x - 5)$$

**c.** Consider the graph of $y = g(x)$, a tangent and normal line are drawn to the graph at the point where $x = 12$. Find the area bounded by the tangent line, normal line, and the $y$-axis.

Equation of normal line passing through $\left(12, \frac{21}{2}\right)$ is $y = -\frac{2}{3}x + \frac{37}{2}$.
Corner points of triangle formed by the tangent line, normal line and $y - axis$ are $\left(0, \frac{37}{2}\right), \left(0, \frac{15}{2}\right)$, and $\left(12, \frac{21}{2}\right)$
Now, area$= \frac{1}{2} \times 12 \times \left(\frac{37}{2} + \frac{15}{2}\right) = 156$

## Sub-Section: Exam 2 Questions

**Question 14**

The perpendicular bisector of the points $(2, 4)$ and $(5, -2)$ is:

**A.** $y = 2x + 3$

**B.** $y = \frac{1}{2}x - \frac{3}{4}$

**C.** $y = -\frac{1}{2}x + 3$

**D.** $y = -2x + \frac{4}{3}$

**Question 15**

It is known that the lines $y = mx + 3$ and $y = 2x - 4$ make an angle of $45°$ when they intersect.

The possible values of $m$ are:

**A.** $m = -\frac{1}{3}$ only

**B.** $m = 3$ only

**C.** $m = -3, \frac{1}{3}$

**D.** $m = -3, -\frac{1}{3}$

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**Question 16**

The tangent to the graph of $y = f(x)$ when $x = 2$ is $y = 3x - 2$. Find the equation of the tangent to the graph of $y = 2f\left(\frac{x}{3}\right)$ when $x = 6$.

**A.** $y = 6x - 10$

**B.** $y = 2x - 4$

**C.** $y = 3x - 2$

**D.** $y = 12x - 4$

**Question 17**

The simultaneous linear equations:

$$2x + (k + 3)y = 4$$

$$(2 - k)x + 2y = 1$$

where $k$ is a real constant has infinitely many solutions for:

**A.** $k = 1$

**B.** $k = -2$

**C.** $k = -2, 1$

**D.** No value of $k$.

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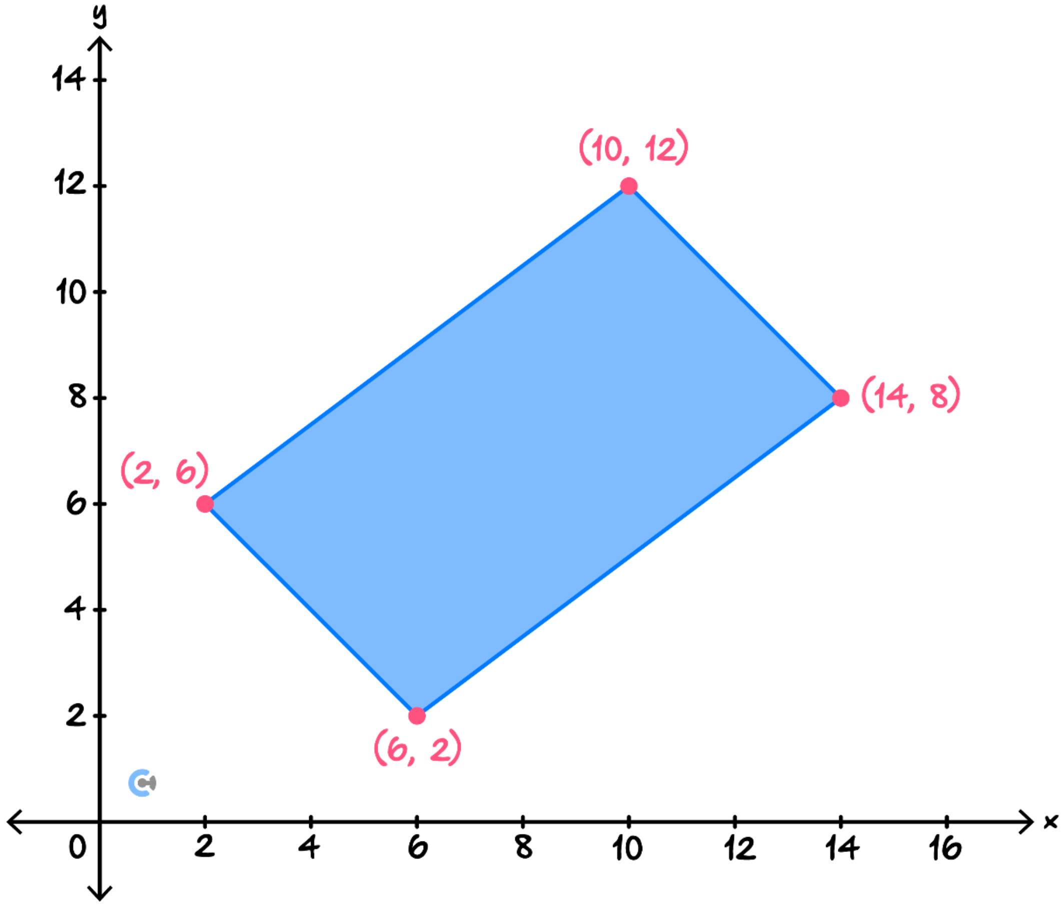
**Question 18**

Find the area, in square units, of the parallelogram shown below:




**A.** 48

**B.** 72

**C.** 56

**D.** 54

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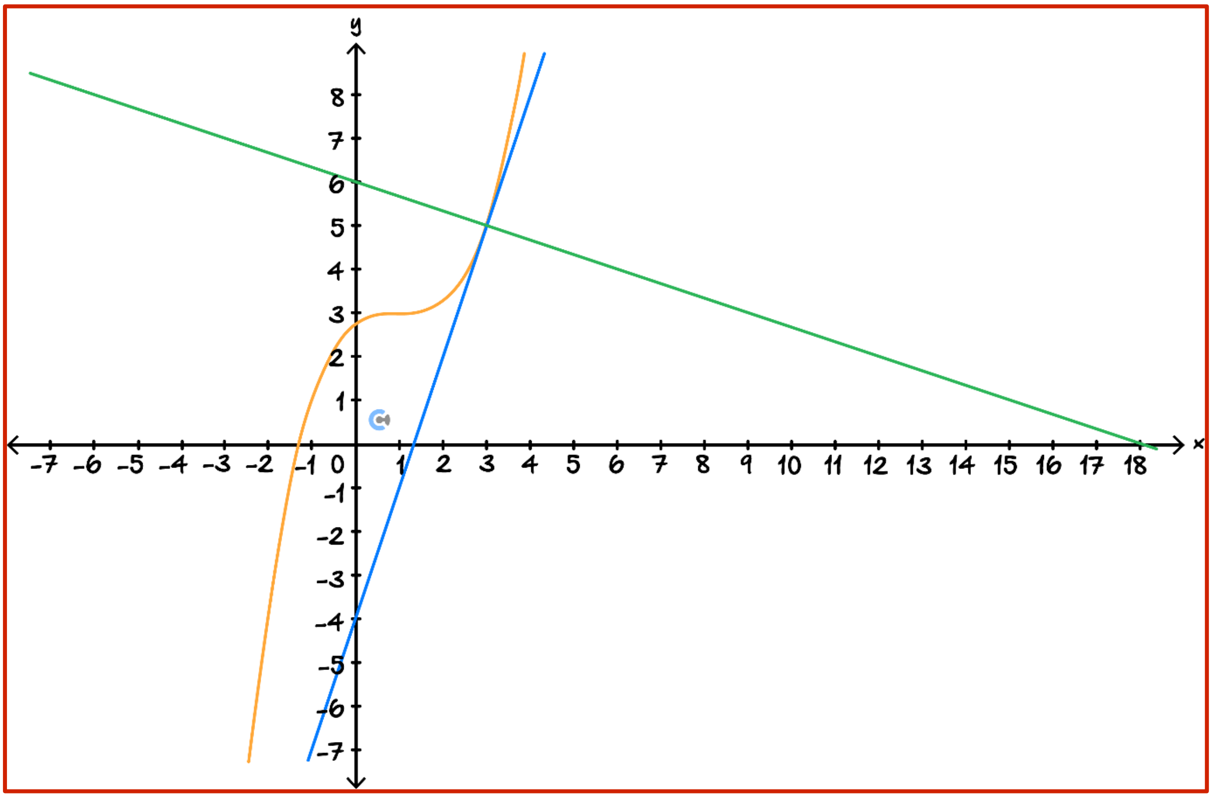
**Question 19**

Consider the function $f(x) = \frac{1}{4}(x-1)^3 + 3$. The graph of $y = f(x)$ and it tangent line at the point where $x = 3$ is sketched on the axes below. The tangent line has a $y$-intercept at $(0, -4)$.




**a.** State the equation of the tangent line to the graph of $y = f(x)$ when $x = 3$.

Line through $(3, 5)$ and $(0, -4)$

$y = 3x - 4$

**b.** State the angle that this tangent line makes with the positive $x$-axis, correct to the nearest degree.

$\arctan(3) = 72°$

**c.** Find the equation of the normal line to the graph of $y = f(x)$ when $x = 3$, and sketch it on the axes at the start of this question.

Line with gradient $-\frac{1}{3}$ and passing through the point $(3, 5)$

$$y = -\frac{x}{3} + 6$$

**d.** Find the area of the triangle bounded by the tangent line, the normal line, and the $y$-axis.

$$\text{Area} = \frac{1}{2} \times 10 \times 3 = 15$$

**e.** The graph of $y = f(x)$ undergoes a dilation by factor 2 from the $y$-axis, a dilation by factor 3 from the $x$-axis, and is translated 2 units to the right. A tangent and normal line are drawn to this new graph at the point where $x = 8$.

Find the area of the triangle bounded by this tangent line, normal line, and the $x$-axis.

New tangent line: $y = \frac{9}{2}x - 21$.

New normal line: $y = \frac{151}{9} - \frac{2x}{9}$

Tangent and normal intersect at $(8, 15)$

$$\text{Area} = \frac{1}{2} \times 15 \times \left(\frac{151}{2} - \frac{14}{3}\right) = \frac{2125}{4}$$

**Question 20**

A soccer field in the shape of a parallelogram is being constructed. As part of the planning phase, the field is modelled on the cartesian plane.

Two adjacent sides of the field are modelled by the equations $y = x + 35$ and $y = \frac{305}{6} - \frac{7x}{12}$.

The corner diagonally opposite to the corner formed by these two lines is the point $C(140, 80)$. All measurements are in metres.

**a.** Show that the field has vertices $A(10, 45)$, $B(80, 115)$ and $D(70, 10)$.

Vertex at the intersection of $y = x + 35$ and $y = \frac{305}{6} - \frac{7x}{12}$. Solve,

$$x + 35 = \frac{305}{6} - \frac{7x}{12}$$

$$\implies x = 10 \implies y = 10 + 35 = 45$$

Therefore vertex at $(10, 45)$

Line through $(140, 80)$ with gradient $-\frac{7}{12}$ is given by

$$y = \frac{485}{3} - \frac{7x}{12}$$

Vertex at the intersection of the line $y = x + 35$ and $y = \frac{485}{3} - \frac{7x}{12}$ is the point $(80, 115)$

Now the line through $(140, 80)$ with gradient 1 is

$$y = x - 60$$

Vertex at the intersection of the line $y = x - 60$ and $y = \frac{305}{6} - \frac{7x}{12}$ is the point $(70, 10)$

**b.** Find the exact dimensions of the field.

Length $= |AB| = 70\sqrt{2}$
Width $= |AD| = 5\sqrt{193}$
The field is $70\sqrt{2} \times 5\sqrt{193}$ metres.

**c.** Find the angle $\angle BAD$ in degrees, correct to two decimal places.

$$\angle BAD = \left|\arctan(1) - \arctan\left(-\frac{7}{12}\right)\right| = 75.26^\circ$$

**d.** Find the area of the soccer field.

The height is perpendicular. Line with gradient $-1$ through the point $(10, 45)$ is

$$y = 55 - x$$

The line $y = 55 - x$ and $y = x - 60$ intersect at $\left(\frac{115}{2}, -\frac{5}{2}\right)$.
Therefore the parallelogram height is given by

$$h = \sqrt{\left(\frac{115}{2} - 10\right)^2 + \left(45 + \frac{5}{2}\right)^2} = \frac{95}{\sqrt{2}}$$

So the area is $A = 70\sqrt{2} \times \frac{95}{\sqrt{2}} = 6650$ square metres.

**e.** The vertices that make up the soccer field are all dilated by a factor of 2 from the $x$-axis and by a factor of 2 from the $y$-axis. What is the area of the field formed from these tranformed vertices?

$2 \times 2 \times 6650 = 26600$ square metres.

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# Section B: Supplementary Questions

## Sub-Section [1.6.1]: Apply Midpoint to Find a Reflected Point

**Question 21**

The point $(-1, 5)$ is reflected in the line $y = 2$. Find the coordinates of the reflected point.

$(-1, -1)$

**Question 22**

The point $(2, -3)$ is reflected about a line to become the point $(-10, -3)$. State the equation of the line.

$x = -4$

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**Question 23**

Find the perpendicular bisector of the line segment joining the points $(4, -2)$ and $(-1, 0)$.

Midpoint is $\left(\frac{3}{2}, -1\right)$ and the line joining the points is $y = -\frac{2}{5}x - \frac{2}{5}$

So we want a line with gradient $\frac{5}{2}$ through the point $\left(\frac{3}{2}, -1\right)$. Therefore, the perpendicular bisector is

$$y = \frac{5}{2}x - \frac{19}{4}$$

**Question 24**

The point $(1, -6)$ is reflected in a line to become the point $(5, -4)$. Find the equation of the line.

The line is the perpendicular bisector between $(1, -6)$ and $(5, -4)$

Midpoint is $(3, -5)$ and the line joining the points is $y = \frac{1}{2}x - \frac{13}{2}$

So we want a line with gradient $-2$ through the point $(3, -5)$. Therefore, the perpendicular bisector is

$$y = -2x + 1$$

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## Sub-Section [1.6.2]: Apply Parallel and Perpendicular Lines to Geometric Problems

**Question 25**

Find the equation of the line that passes through the point $(-2, 3)$ and is perpendicular to $y = x + 7$.

$y = -x + 1$

**Question 26**

Find the area of the triangle formed by the lines $y = 2x - 8$, $y = 6x - 4$, and $y = 2$.

Lines intersect at $(5,2)$, $(1,2)$ and $(-1,-10)$. Triangle base $= 5 - 1 = 4$ and triangle height $= 2 - (-10) = 12$
Therefore, triangle area is $\frac{1}{2} \times 4 \times 12 = 24$

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**Question 27**

Find the distance between the point $(2, 7)$ and the line $y = 3x - 1$.

Want the equation of line perpendicular to $y = 3x - 1$ and through the point $(2,7)$
Therefore the line with gradient $-\frac{1}{3}$ and through $(2,7)$

$$y = -\frac{1}{3}x + \frac{23}{3}$$

Now $y = 3x - 1$ and $y = -\frac{1}{3}x + \frac{23}{3}$ intersect at the point $\left(\frac{13}{5}, \frac{34}{5}\right)$

Therefore, the minimum distance is the distance between the points $(2,7)$ and $\left(\frac{13}{5}, \frac{34}{5}\right)$

$$d = \sqrt{\left(2 - \frac{13}{5}\right)^2 + \left(7 - \frac{34}{5}\right)^2} = \frac{\sqrt{10}}{5}$$

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**Question 28**

Consider the points $A(2,1)$, $B(1,-2)$, $C(5,0)$ and $D(m,n)$, where $m, n \in \mathbb{R}^+$. It is known that $\angle ABC = 45°$. Find the values of $m$ and $n$ such that $\angle BCD = 135°$.

$\angle ABC$ is supplementary to $\angle BCD$. Therefore, $ABCD$ is a parallelogram

Equation of $AB$ is $y = 3x - 5$ and equation of $BC$ is $y = \frac{1}{2}x - \frac{5}{2}$

$AD$ is parallel to $BC$ and goes through $A$. Therefore, $AD$ is the line with gradient $\frac{1}{2}$ through (2,1)

$$y = \frac{1}{2}x$$

$CD$ is parallel to $AB$ and goes through $C$. Therefore, $CD$ is the line with gradient 3 through (5,0)

$$y = 3x - 15$$

Now $y = \frac{1}{2}x$ and $y = 3x - 15$ intersect at (6,3)

$m = 6$ and $n = 3$

**Space for Personal Notes**

## Sub-Section [1.6.3]: Solve Coordinate Geometry Problems With Transformations

**Question 29**

The area bound by the lines $y = 2x - 4$, $y = -1 - x$, and $y = \frac{1}{2}x + 2$ is $\frac{27}{2}$ square units. Hence, find the area bound by:

**a.** The lines $y = 8x - 4$, $y = -1 - 4x$ and $y = 2x + 2$.

All lines have been dilated by a factor of $\frac{1}{4}$ from the $y$ axis. Therefore the area is $\frac{27}{8}$ square units

**b.** The lines $y = -2x + 4$, $y = 1 + x$ and $y = -\frac{1}{2}x - 2$.

All lines have been reflected in the $x$ axis. The area does not change. Therefore the area is $\frac{27}{2}$ square units

**c.** The lines $y = 6x - 4$, $y = 5 - 3x$, $y = \frac{3}{2}x + 14$.

All lines have undergone a dilation by a factor of 3 from the $x$ axis followed by a translation 8 units up. Therefore the area is $\frac{81}{2}$ square units

**Question 30**

**a.** The original function is $f(x) = \frac{2}{(x-5)^2} - 16$, and the tangent line to the graph of $y = f(x)$ at $x = 6$ is $y = -4x + 8$. The graph of $f(x)$ is reflected in the $x$-axis, translated 2 units down, then dilated by a factor of $\frac{1}{2}$ from the $x$ axis. Find the equation of the tangent to the transformed graph when $x = 6$.

$y = \frac{1}{2}(-(-4x + 8) - 2)$. Tangent is $y = 2x - 5$

**b.** The graph of $f(x) = 2x^2 - 3x + 1$ has a tangent line at $x = -1$ with an equation of $y = -7x - 1$. $f(x)$ undergoes a translation 3 units right, followed by a dilation by a factor of 4 from the $x$-axis. Find the equation of the tangent to the transformed graph when $x = 2$.

$y = 4(-7(x - 3) - 1)$. Tangent is $y = -28x + 80$

**c.** Consider the graph $f(x) = x^2 - 6x + 4$. The line $y = 2x - 12$ is a tangent to $f(x)$ at $x = 4$. Find the equation of the tangent to $y = 4x^2 - 28x + 32$ at $x = 4$.

$f(x) = (x-3)^2 - 5$

Let $g(x) = 4x^2 - 28x + 32 = (2x-7)^2 - 17 = \left((2x-4)-3\right)^2 - 5 - 12$

Therefore $g(x) = f(2x-4) - 12$

A dilation by a factor of $\frac{1}{2}$ from the $y$ axis and translation 2 units right maps $x = 4$ to $x = 4$. Therefore the equation of the tangent is

$$y = 2(2x-4) - 12 - 12 = 4x - 32$$

**Question 31**

**a.** Find the value of $a$ such that the area bound by the graphs $y = x - 2$, $y = ax + a$ and the $y$ axis is 2 square units.

The lines intersect when $x - 2 = ax + a \Rightarrow x = \frac{a+2}{1-a}$. Therefore intersect at $\left(\frac{a+2}{1-a}, \frac{3a}{1-a}\right)$

Take the triangle height as $\frac{a+2}{1-a}$, then the base is $a + 2$. Solve

$$\frac{1}{2} \times \frac{a+2}{1-a} \times (a+2) = 2$$

$$a = -8, 0$$

**b.** It is known that the triangle formed by the lines $y = 2x + 6$, $y = -x - a$, and the $x$-axis has an area of 5. Find the values of $a$.

The lines intersect when $2x + 6 = -x - a$

Therefore, intersect at $\left(\frac{-a-6}{3}, \frac{-2a-6}{3}\right)$

Take the triangle height as $\frac{-2a-6}{3}$

Then the base is $-3 - a$.

Solve

$$\frac{1}{2} x \frac{-2a - 6}{3} x (-3 - a) = 5$$

$$a = -2\sqrt{6}$$

**c.** Find the values of $a$ where the area between the lines $y = ax$, $y = x - 4$ and the $y$ axis is 12.

The lines intersect when $ax = x - 4 \Rightarrow x = \frac{4}{1-a}$. Therefore intersect at $\left(\frac{4}{1-a}, \frac{4a}{1-a}\right)$

Take the triangle height as $\frac{4}{1-a}$, then base is 4. Solve

$$\frac{1}{2} \times \frac{4}{1-a} \times 4 = 12$$

$$a = \frac{1}{3}$$

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**Question 32**

**a.** The shape bound by the lines $y = -\frac{1}{2}x - 1$, $y = x + 5$ and $y = ax - 1$ has an area of 8 square units. Find the value of $a$ if $a \in (-\infty, 1)$.

The lines intersect at $(0, -1)$, $(-4, 1)$ and $\left(\frac{6}{a-1}, \frac{5a+1}{a-1}\right)$

Take the base of the triangle as $\sqrt{\left(-4 - \frac{6}{a-1}\right)^2 + \left(1 - \frac{5a+1}{a-1}\right)^2} = \frac{2\sqrt{2}(a+1)}{a-1}$

Height of triangle is perpendicular to base and goes through $(0, -1)$. Therefore, want a line with a

Gradient $-1$ and through $(0, -1)$

$$y = -x - 1$$

Intersection between $y = x + 5$ and $y = -x - 1$ is $(-3, 2)$

Height of triangle is $\sqrt{(0 - (-3))^2 + ((-1) - 2)^2} = 3\sqrt{2}$

Solve $\frac{1}{2} \times \frac{2\sqrt{2}(2a+1)}{a-1} \times 3\sqrt{2} = 8$

$$a = -\frac{7}{2}$$

**b.** Hence or otherwise, find the values of $m$ and $c$ such that the area bound by the graphs $y = -2x + 2$, $y = 4x + 8$, and $y = mx + c$ is 2 square units. Assume $m, c \in (1, \infty)$.

All lines have undergone a dilation by a factor of 1/4 from the y-axis and a translation 3 units up, making the new area $\frac{1}{4} \times 8 = 2$

The equation of the original lines was $y = -\frac{7}{2}x - 1$.

Therefore, the equation of the new line is $y = 7 - \frac{7}{2}(4x) - 1 + 3 = -14x + 2$

$$m = -14 \text{ and } c = 2$$

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## Sub-Section: Exam 1 Questions

**Question 33**

Consider the simultaneous linear equations:

$$2ax - (a+1)y = -1$$

$$\frac{x}{2a+1} + 3y = 4a + 5$$

where $a$ is a real constant.

**a.** Find the values of $a$ for which there is a unique solution to the set of equations.

The lines must have different gradients. Solve

$$\frac{2a}{a+1} \neq -\frac{1}{6a+3}$$

$$a \neq -\frac{1}{3}, -\frac{1}{4}$$

Therefore, unique solution for $a \in \mathbb{R} \backslash \left\{-\frac{1}{3}, -\frac{1}{4}\right\}$

**b.** Find the value of $a$ for which there are no unique solutions.

The $y$ intercepts must not be equal. Solve

$$\frac{1}{a+1} \neq \frac{4a+5}{3}$$

$$a \neq -2, -\frac{1}{4},$$

Therefore, unique solution for $a = -\frac{1}{3}$

**c.** Find the value of $a$ for which there are infinitely many solutions.

$a = -\frac{1}{4}$

**Question 34**

Consider the points $A(8, -2)$ and $B(2, 6)$.

**a.** Find the equation of the line that is parallel to the line segment $AB$, and also contains the point $C(6,9)$.

Gradient of $AB$ is $\frac{-2-6}{8-2} = -\frac{4}{3}$

Line with gradient $-\frac{4}{3}$ and through $(6,9)$

$y = -\frac{4}{3}x + 17$

**b.** Find the equation of the perpendicular bisector of $AB$.

Midpoint of $AB$ is $(5,2)$

Perpendicular bisector has gradient $\frac{3}{4}$ and goes through $(5,2)$

$y = \frac{3}{4}x - \frac{7}{4}$

**c.** Find the coordinates of $D$, the point of intersection between the lines found in **part a.** and **b**.

$(9, 5)$

**d.** Find the area of the quadrilateral $ABCD$.

$ABCD$ is a trapezium

$|AB| = \sqrt{(8-2)^2 + (-2-6)^2} = 10$

$|CD| = \sqrt{(9-6)^2 + (5-9)^2} = 5$

Height $= \sqrt{(9-5)^2 + (5-2)^2} = 5$

Therefore area $ABCD = \frac{1}{2}(10+5) \times 5 = \frac{75}{2}$

**e.** Let $E\left(\frac{8}{3}, -4\right)$, $F\left(\frac{2}{3}, 12\right)$, $G(2, 18)$, and $H(3, 10)$. Find the area of $EFGH$.

The points $A, B, C, D$ are dilated by a factor of 2 from the $x$ axis and by a factor of $\frac{1}{3}$ from the $y$ axis

Therefore, area $EFGH = \frac{75}{2} \times 2 \times \frac{1}{3} = 25$

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**Question 35**

The point $P(4, 1)$ is reflected in the line $y = 2x - 2$ to become the point $P'$.

**a.** Find the coordinates of $P'$.

Want the line with gradient $-\frac{1}{2}$ passing through the point $(4,1)$.

$$y = -\frac{1}{2}x + 3$$

Now we find the intersection of the lines $y = 2x - 2$ and $y = -\frac{1}{2}x + 3$

$\therefore$ Intersection at $(2,2)$

Therefore we have $P'(0, 3)$

**b.** Find the point of intersection between the lines $y = 2x - 2$ and $y = 7x - 27$.

$(5, 8)$

**c.** The line $y = 7x - 27$ is reflected in the line $2x - 2$. Find the equation of the new line.

The line passes through the points $(5,8)$ and $(0,3)$

$$y = x + 3$$

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**Question 36**

At $x = -2$, the graph $y = f(x)$ has a tangent line with the equation $y = 3 - 2x$, and a normal line given by $y = \frac{1}{2}x + 8$.

**a.** Find the area bounded by the tangent line, normal line, and the $x$-axis.

When $x = -2, y = 7$

Normal line has $x$ −intercept $(-16, 0)$ and tangent has $x$ −intercept $\left(\frac{3}{2}, 0\right)$.

Therefore, area of triangle is $\frac{1}{2} \times 7 \times \left(\frac{3}{2} + 16\right) = \frac{245}{4}$

The graph of $f(x)$ is translated down 3 units, dilated by a factor of 2 from the $x$-axis, and dilated by a factor of 5 from the $y$-axis to become the graph $g(x)$.

**b.** Find the equation of the normal line to $y = g(x)$ at $x = -4$.

Let $t(x) = \frac{1}{2}x + 8$, then the equation of the tangent at $x = -4$ is given by

$$y = 2\left(t\left(\frac{1}{5}x\right) - 3\right) = \frac{1}{5}x + 10$$

**c.** Find the area bounded by the $x$-axis, the tangent line and normal line of the graph $y = g(x)$ at $x = -4$.

Area $= 2 \times 5 \times \frac{245}{4} = \frac{1225}{4}$

## Sub-Section: Exam 2 Questions

**Question 37**

The set of simultaneous equations:

$$\frac{5}{3k-4}y - \frac{x}{2} = \frac{3}{8}k + \frac{3}{2}$$

$$(k-6)x + 2ky = \frac{4}{3} - k$$

has no solutions for:

**A.** $k = 3$ or $k = -\frac{10}{3}$

**B.** $k = -\frac{10}{3}$

**C.** $k = 3$

**D.** $k \neq -\frac{2}{3}$ or $k \neq -\frac{10}{3}$

**Question 38**

The area of the triangle formed by the points $(2, 3)$, $(-4, 7)$ and $(4, 6)$ is:

**A.** 13 square units.

**B.** 25 square units.

**C.** 26 square units.

**D.** 19 square units.

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**Question 39**

The graph $f(x) = x^2 - 4x + 3$ has a tangent line and normal line constructed at $x = 1$. The area bound by the tangent line, the normal line, and the $y$-axis is $\frac{5}{4}$ square units. The area bound by the $y$-axis, tangent line, and normal line to the graph $y = -\frac{1}{2}x^2 + 4x - 3$ at $x = -2$ is:

**A.** $\frac{5}{8}$ square units.

**B.** $\frac{5}{4}$ square units.

**C.** 5 square units.

**D.** 8 square units.

**Question 40**

The acute angle formed between the lines $y = 3x - 1$ and $y = mx + 5$ is at least $45°$ when:

**A.** $m \in \left[\frac{1}{2}, \infty\right)$

**B.** $m \in \left[-2, \frac{1}{2}\right]$

**C.** $m \in (-\infty, -2] \cup \left[\frac{1}{2}, \infty\right)$

**D.** $m \in \left[-2, 0\right) \cup \left(0, \frac{1}{2}\right]$

**Question 41**

The equation of the tangent line to $f(x)$ at $x = 2$ is $y = 1 - 4x$. The equation of the normal line to $f(x)$ at $x = 2$ is:

**A.** $y = \frac{1}{4}x - \frac{15}{2}$

**B.** $y = -\frac{1}{4}x + 1$

**C.** $y = 4x - 2$

**D.** Cannot be determined

**Question 42**

Consider the points $A(6, -2)$ and $B(3, 4)$.

**a.** Find the perpendicular bisector of $AB$.

Midpoint is $\left(\frac{9}{2}, 1\right)$ and the line joining the points is $y = -2x + 10$.

So we want a line with gradient $\frac{1}{2}$ through the point $\left(\frac{9}{2}, 1\right)$. Therefore, the perpendicular bisector is

$$y = \frac{1}{2}x - \frac{5}{4}$$

**b.** Find the values of $m$ such that the line $y = mx$ forms a $45°$ angle with the line segment $AB$.

$|\arctan(-2) - \arctan(m)| = 45$

$m = -\frac{1}{3}, 3$

**c.** Point $C(m, n)$ and point $D(p, q)$ are different points that lie on the perpendicular bisector of $AB$, where $m$, $n \in \mathbb{R}^+$. Find the coordinates of $C$ and $D$ such that the triangles $ABC$ and $ABD$ are both right angle triangles.

We want a line with gradient 3 and goes through $(6, -2)$. Therefore, the line is $y = 3x - 20$

Intersection between $y = \frac{1}{2}x - \frac{5}{4}$ and $y = 3x - 20$ is $\left(\frac{15}{2}, \frac{5}{2}\right)$

We want a line with gradient $-\frac{1}{3}$ and goes through $(6,2)$. Therefore, the line is $y = -\frac{1}{3}x$

Intersection between $y = \frac{1}{2}x - \frac{5}{4}$ and $y = -\frac{1}{3}x$ is $\left(\frac{3}{2}, -\frac{1}{2}\right)$

Therefore, $C\left(\frac{15}{2}, \frac{5}{2}\right)$ and $D\left(\frac{3}{2}, -\frac{1}{2}\right)$

**d.** The point $C$ can be mapped onto point $D$ by a reflection in the line $y = a$ followed by a reflection in the line $x = b$. State the values of $a$ and $b$.

$a = 1$ and $b = \frac{9}{2}$

**e.** Find the area of $ACBD$.

$|AC| = \sqrt{\left(6 - \frac{15}{2}\right)^2 + \left(-2 - \frac{9}{2}\right)^2}$

$|AC| = \frac{\sqrt{178}}{2}$

$\text{Area} = \left(\frac{\sqrt{178}}{2}\right)^2 = \frac{89}{2}$

**f.** Find the area of the square that has opposite corners at $(7, -4)$ and $(1, 8)$.

$A, B$ have been dilated by a factor of 2 from the $x$ axis and dilated by a factor of 2 from the $y$ axis and translated 5 units left

$\text{Area} = \frac{89}{2} \times 2 \times 2 = 178$

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**Question 43**

The function $f(x) = 2(x+3)^2 - 5$ has a tangent line with the equation $y = 4x + 5$.

**a.** Show that $y = 4x + 5$ is a tangent to $f(x)$ at the point $(-2, -3)$.

$f(x)$ and the tangent line will intersect at the point of tangency
$2(x+3)^2 - 5 = 4x + 5$
$2x^2 + 8x + 8 = 0$
$2(x+2)^2 = 0$
$x = -2$
$f(-2) = -3$
Therefore, the point of tangency is $(-2, -3)$

**b.** Find the equation of the normal line to $f(x)$ at $x = -2$.

Normal line has a gradient $-\frac{1}{4}$ and passes through $(-2, -3)$

$$y = -\frac{1}{4}x - \frac{7}{2}$$

**c.** State the obtuse angle formed between the line $y = 4x + 5$ and the $x$-axis, correct to 2 decimal places.

$\theta = 180 - \arctan(4) = 104.04^o$

**d.** Find the area enclosed by the tangent line, the normal line, and the $x$-axis.

Normal line has $x$-intercept $(-14, 0)$ and tangent has $x$-intercept $\left(-\frac{5}{4}, 0\right)$
Therefore, the triangle has area $\frac{1}{2} \times 3 \times \frac{51}{4} = \frac{153}{8}$

The graph of $y = f(x)$ is translated 4 units right, dilated by a factor of 4 from the $x$-axis, and dilated by a factor of $\frac{2}{3}$ from the $y$-axis to become the graph $y = g(x)$.

**e.** Find the equation of the tangent line to $y = g(x)$ at $x = \frac{4}{3}$.

$g(x) = 4f\left(\frac{3}{2}x - 4\right)$. The tangent when $x = \frac{4}{3}$ is given by

$$y = 4\left(4\left(\frac{3}{2}x - 4\right) + 5\right) = 24x - 44$$

**f.** State the obtuse angle formed between the new tangent of $y = g(x)$ at $x = \frac{4}{3}$, correct to 2 decimal places.

$\theta = 180 - \arctan(24) = 92.39^o$

**g.** Find the area of the triangle formed between the $x$-axis, the tangent, and normal line to $y = g(x)$ at $x = \frac{4}{3}$.

$\text{Area} = 4 \times \frac{2}{3} \times \frac{153}{8} = 51$

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