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VCE Mathematical Methods ¾
Coordinate Geometry [1.5]
Workbook

Outline:

Learning Objectives:

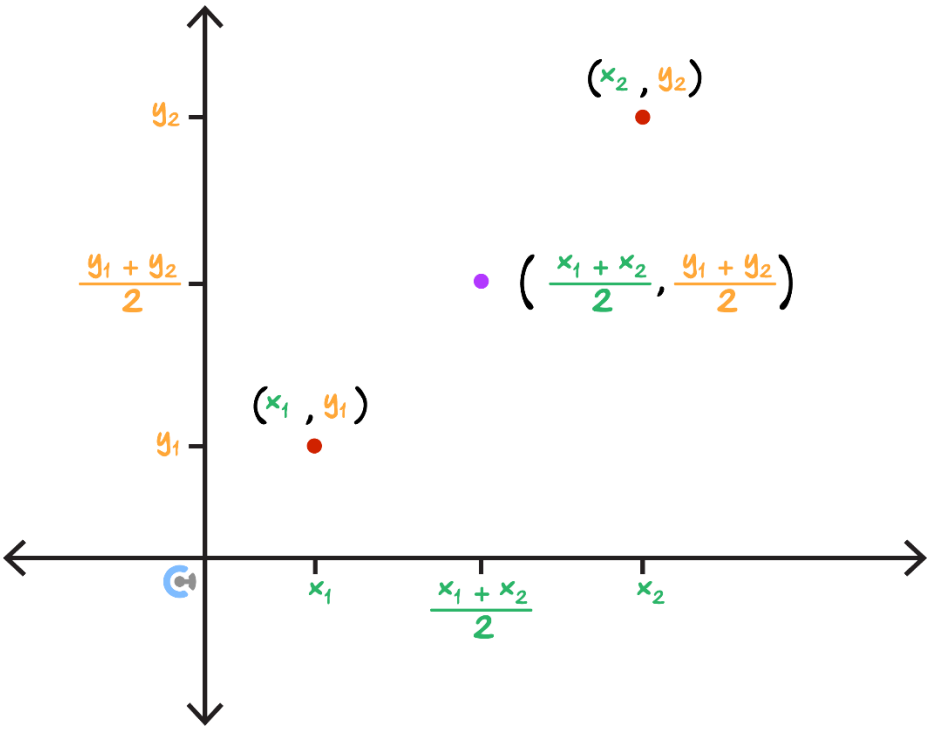
- ☐ **MM34 [1.5.1]** - Find Midpoint, Distance (Horizontal & Vertical) Between Two Points Or Functions
- ☐ **MM34 [1.5.2]** - Find Parallel and Perpendicular Lines
- ☐ **MM34 [1.5.3]** - Find the Angle Between a Line and x-axis or Two Lines
- ☐ **MM34 [1.5.4]** - Find The Unknown Value for Systems of Linear Equations
- ☐ **MM34 [1.5.5]** - Sketching the sum of two function's graph by using the addition of ordinates

Section A: Simple Geometry

Sub-Section: Midpoint

Discussion: How might we find a midpoint between two points?

Midpoint



- The midpoint, M, of two points A and B is simply the point halfway between A and B.

$$M(x_m, y_m) = \left(\qquad \right)$$

- The midpoint can be found by taking the _____ of the x-coordinate and y-coordinate of the two points.

Sub-Section: Distance Between Two Points

Distance Between Two Points

- The distance between two points (x_1, y_1) and (x_2, y_2) can be found using Pythagoras' theorem:

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Question 1 Walkthrough.

Find the points on the line $y = x - 2$ which have a distance of 4 from the point $(2,4)$.

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Question 2

a. Find the points on the line $y = 2x - 6$ which has a distance of $\sqrt{5}$ from the point (2,1).

b. Give a reason as to why there are more than 1 points found in **part a.**

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Question 3 Extension.

Find the points on the line $y = 2x - 3$ which have a distance of $2\sqrt{2}$ from the point $(3,5)$.

TIP: Don't hesitate to define a point by letting its y value be the function (linear in the above question!)

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Sub-Section: Vertical Distance vs Horizontal Distance

Discussion: How can we find a horizontal distance between two points?

Horizontal Distance

$$\text{Horizontal Distance} = x_2 - x_1 \text{ where } x_2 > x_1$$

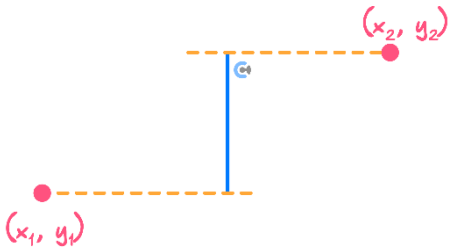
- Find the difference between their x-values.

Question 4

Find the horizontal distance between the two points (7,3) and (11,4).

What about vertical distance then?

Vertical Distance



Vertical Distance$= y_2 - y_1$ where $y_2 > y_1$

- Find the difference between their y values.

Question 5

Find the vertical distance between the two points $(7,4)$ and $(1,9)$.

Key Takeaways

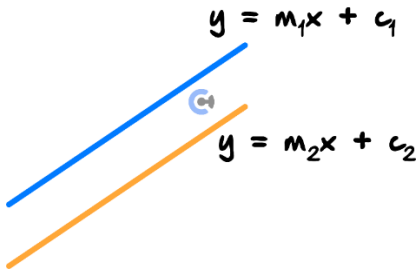
- ☑ Midpoint is simply an average point.
- ☑ Midpoint can be used to find the point reflected around any axis.
- ☑ Distance between two points is derived from Pythagoras theorem.
- ☑ Horizontal distance is simply a difference in their x values.
- ☑ Vertical distance is simply a difference in their y values.

Section B: Line Geometry

Sub-Section: Parallel and Perpendicular Lines

Discussion: What do we need for the two lines to be parallel?

Parallel Lines

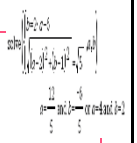


- Parallel lines have the _____ gradient.

$$m_1 = m_2$$

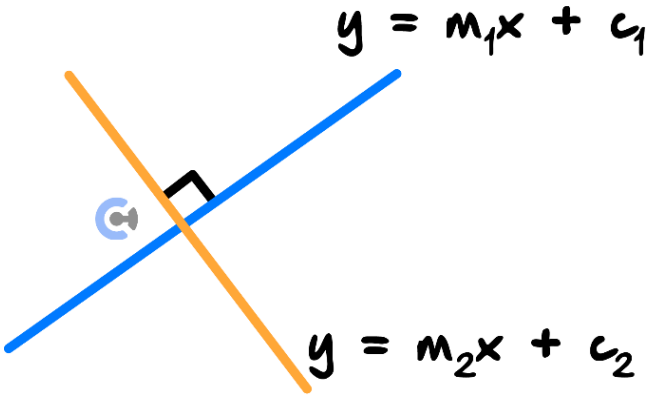
Question 6

Find a line that is parallel to $y = 5x - 1$ passing through the point $(-2, -6)$.



Discussion: What about perpendicular lines?

Perpendicular Lines



$$m_1 = -\frac{1}{m_2}$$

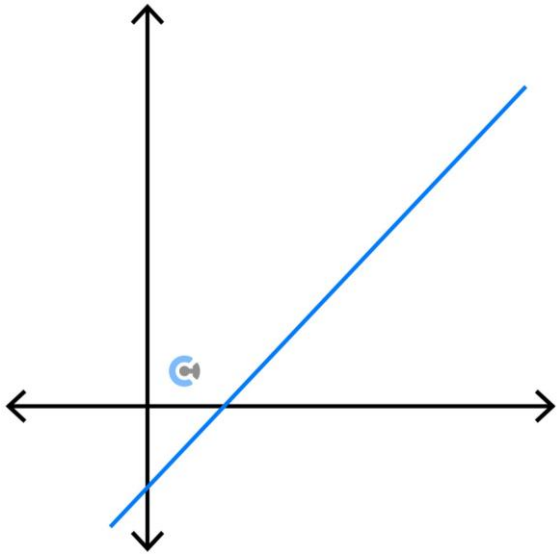
Question 7

Find a line that is perpendicular to $y = 5x - 1$ passing through the point $(1,0)$.

Sub-Section: Angle Between a Line and the x-axis

How do we find the angle between a line and the x-axis?

Angle between a Line and the x-axis



- The angle between a line and the _____ **direction of the x-axis** (anticlockwise) is given by

$$\tan(\theta) = m$$

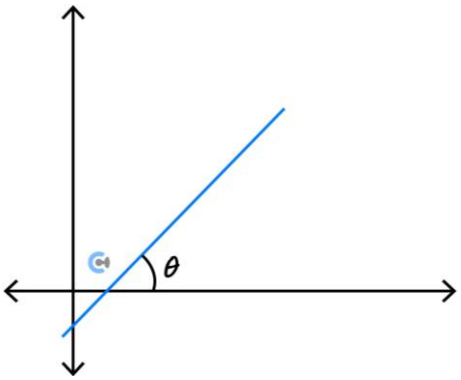
Question 8

Find the angle made between the line $y = -x + 2$ and the x-axis measured in the anticlockwise direction.

How does this formula work?

Exploration: Angle between a line and x-axis.

- **Consider a line in the visual below.**



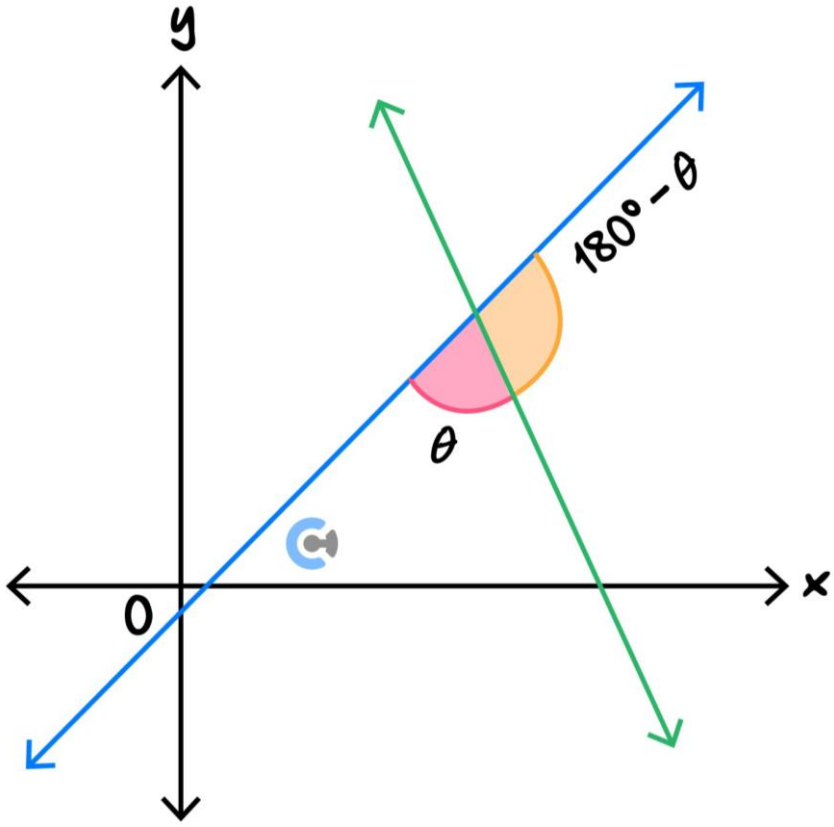
- Construct a right-angle triangle with the angle θ.

- Consider the opposite and adjacent sides of the right-angle triangle. What can we call them?

- Hence, what does $\tan(\theta)$ equal to given that $\tan = opposite/adjacent$?

Sub-Section: Angle Between Two Lines

Acute Angle Between Two Lines



$$\theta = |\tan^{-1}(m_1) - \tan^{-1}(m_2)|$$

- **Alternatively:**

$$\tan(\theta) = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

- For your understanding, note that this formula is derived from the $\tan$ compound angle formula covered in SM34.

NOTE: $|x|$ just takes the positive value of x.

Question 9 Tech-Active.

Find the acute angle between the lines $x - 3y = 2$ and $y = \frac{4}{5}x - 2$. Give your answer in degrees correct to two decimal places.

TIP: Make sure your CAS is in degrees.

Let's see if the formula is consistent with parallel lines!

Exploration: Understanding parallel lines using the angle between two lines formula

- When two lines are parallel, what must be the angle θ between them?

$$\tan(\theta) = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

- Let's substitute the value of θ and see what we get!

- This looks rather familiar!

And now perpendicular lines!

Exploration: Understanding perpendicular lines using the angle between two lines formula

- When two lines are perpendicular, what must be the angle θ between them?

$$\tan(\theta) = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

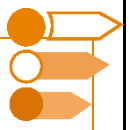
- Let's substitute the value of θ and see what we get! (Note: $\tan(90) = Undef$)

- This looks rather familiar, doesn't it?

Key Takeaways

- ☑ Parallel lines have the same gradient.
- ☑ Perpendicular lines have negative reciprocal gradients.
- ☑ Angle between a line and x-axis is given by $\tan^{-1}(m)$.
- ☑ Angle between two lines is given by $\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$.
- ☑ The parallel lines and perpendicular lines formula is consistent with the angle between the two lines formula.

Section C: Simultaneous Equations

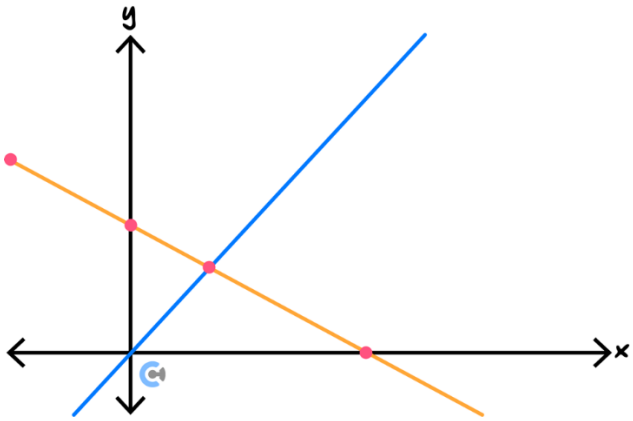


Sub-Section: Systems of Linear Equations

Exploration: Geometry of the number of solutions between linear graphs

- **Unique Solution**

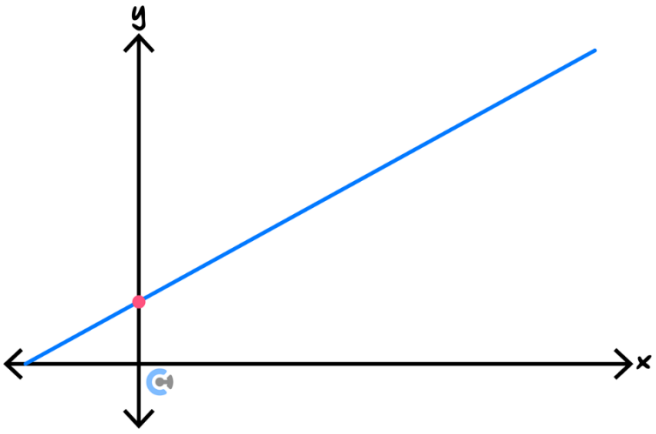
$$m_1 \neq m_2$$



- They just need to have _____.

- **Infinite Solutions**

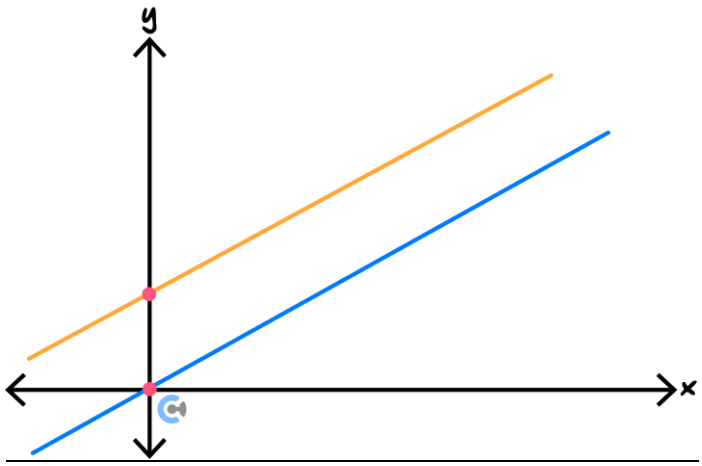
$$m_1 = m_2 \; AND \; c_1 = c_2$$



- They just need to have the same _____ and the same _____.
- In other words, they have to be the _____.

- **No Solutions**

$$m_1 = m_2 \; AND \; c_1 \neq c_2$$



- They need to have the _____ but _____ $+c$.
- They have to be two different _____ lines.

<u>General Solutions of Simultaneous Linear Equations</u>

- Two linear equations are either:
  - The same line is expressed in a different form. In this case, they have infinitely many solutions.
  - Unique lines which are parallel. In this case, they have no solutions.
  - Unique lines which are not parallel. In this case, they have exactly one solution.

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Question 10 Walkthrough.

Consider the following pair of simultaneous equations in terms of $a \in R\backslash\{0\}$:

$$(a+1)x + 2y = 1$$

$$6x + (a-3)y = 1$$

a. Find the value(s) of a for which there are no solutions to the simultaneous equations.

b. Find the value(s) of a for which there is a unique solution to the simultaneous equations.

c. Find the value(s) of a for which there are infinite solutions to the simultaneous equations.

TIP: Substitute your answer back into the equations to see if the criteria are met for each part.

Question 11

Consider the following pair of simultaneous equations in terms of $a \in R\backslash\{0\}$:

$$ax + 3y = 1$$

$$2x + (a + 1)y = 1$$

a. Find the value(s) of a for which there are no solutions to the simultaneous equations.

b. Find the value(s) of a for which there is a unique solution to the simultaneous equations.

c. Find the value(s) of a for which there are infinite solutions to the simultaneous equations.

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Question 12 Extension.

Consider the following pair of simultaneous equations in terms of $a \in R\backslash\{0\}$:

$$(1-2a)x-3y=a+1$$

$$-2x+(a+3)y=2a+2$$

a. Find the value(s) of a for which there are no solutions to the simultaneous equations.

b. Find the value(s) of a for which there is a unique solution to the simultaneous equations.

c. Find the value(s) of a for which there are infinite solutions to the simultaneous equations.

Sub-Section: Finding Simultaneous Equation for Three Variables

Solving Systems of Linear Equations with Parameters

- Occurs when solving for three variables with two equations. We simply,

$$\text{Let } x = k, \text{ or}$$

$$\text{Let } y = k, \text{ or}$$

$$\text{Let } z = k$$

- And solve simultaneously.

Question 13 Walkthrough.

Solve the following system of linear equations with the parameter of k.

$$2x - y = 4$$

$$x + z = 3$$

NOTE: We can let any variable equal to k.

Discussion: How many solutions did we find in the question above?

Question 14

Solve the following system of linear equations with the parameter of k.

$$x + 3z = 1$$

$$x + y = 2$$

Discussion: Why did we get infinite solutions?

Key Takeaways

- ☑ Simultaneous equations can be solved using elimination or substitution methods.
- ☑ Two lines can have either unique, no or infinite solutions.
- ☑ For unique solutions, we just need different gradients.
- ☑ For no solutions, we need the same gradient but different c value.
- ☑ For infinite solutions, we need the same gradient and same c value.
- ☑ When solving two simultaneous equations for three variables, we can let any variable equal to the parameter of k.
- ☑ When we have infinite solutions, it is also called general solutions.
- ☑ We always get infinite solutions when there are fewer equations than a number of variables.

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Section D: Addition of Ordinates

Sub-Section: Addition of Ordinates

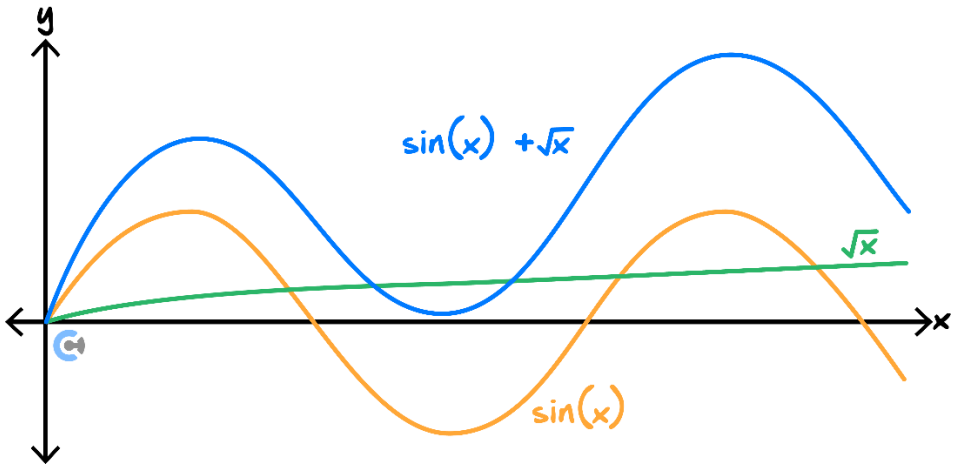
REMINDER: Don't forget Function is always equal to its _____ value.

Discussion: How can we sketch $\sin(x) + \sqrt{x}$?

Addition of Ordinates

- **Definition:**
  - Technique used to graph the sum/difference of two functions.

$$\text{e.g. } y = \sin(x) + \sqrt{x}$$



- Addition of ordinates involves adding the _____ of two functions.

Add two y values

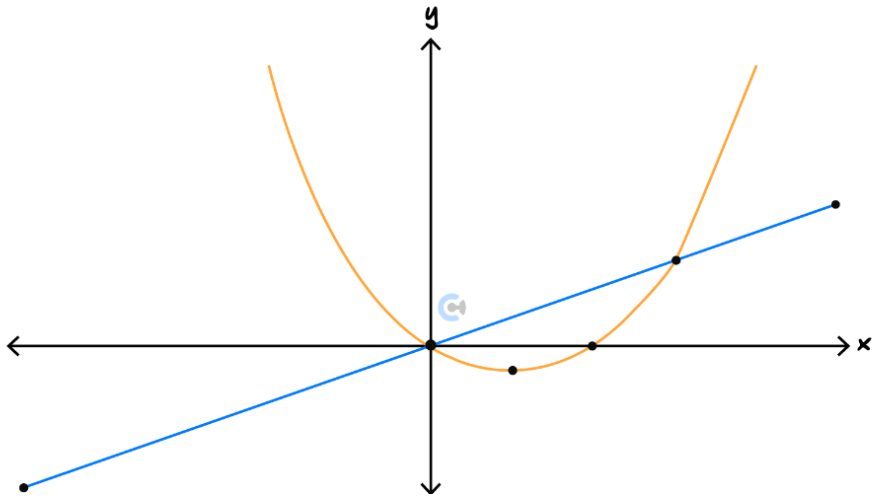
- **Steps to sketching $f(x) + g(x)$**

1. Sketch $f(x)$ and $g(x)$ on the same axes.
2. Plot points for $f(x) + g(x)$ by adding the **y values** of $f(x)$ and $g(x)$.
   - At x-intercepts, the sum equals to the _____. **Why?** _____
   - At intersections, the sum equals to _____ the y value. **Why?** _____
   - When functions are equidistance from x-axis, sum equals to _____. **Why?** _____
3. Join the plotted points.

Question 15 Walkthrough.

Two functions, f and g, are shown below.

Sketch the function $f(x) + g(x)$ on the same axes, without finding or using the rule for either function.



NOTE: We always add their y values.

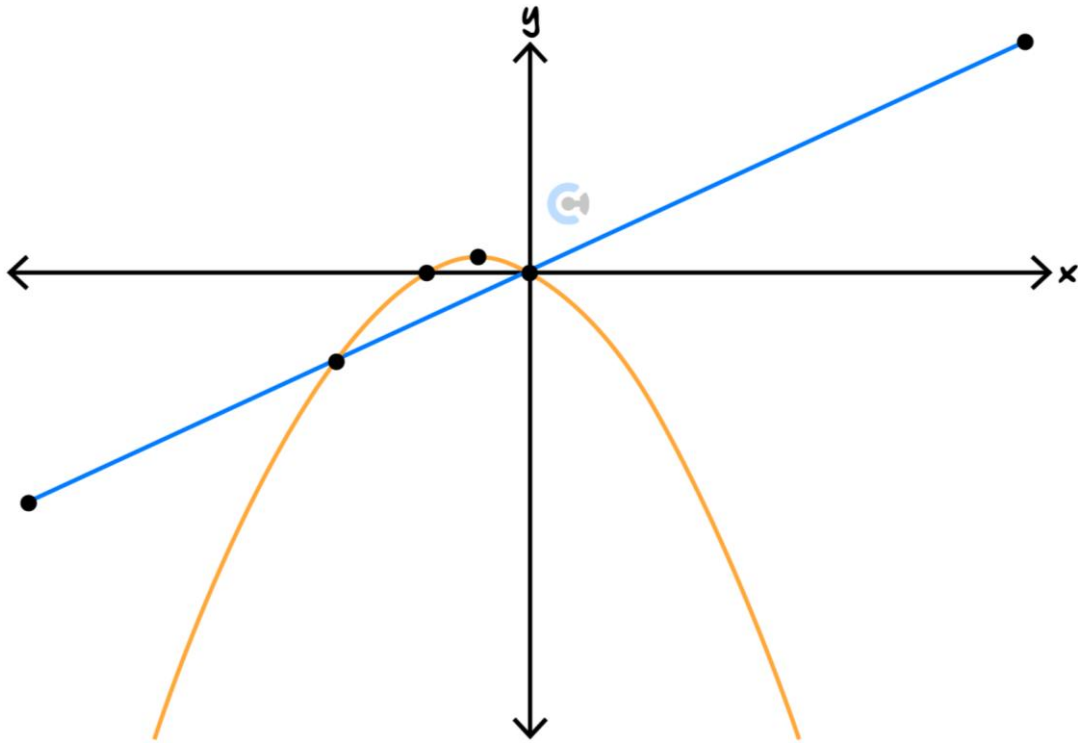
Your turn!

Active Recall: Steps to sketching $f(x) + g(x)$

1. Sketch $f(x)$ and $g(x)$ on the same axes.
2. Plot points for $f(x) + g(x)$ by adding the _____ of $f(x)$ and $g(x)$.
   - At x intercepts, the sum equals to the _____.
   - At intersections, the sum equals to _____ the y value.
   - When functions are equidistance from x-axis, sum equals to ____.
3. Join the plotted points.

Question 16

Plot the sum of the two functions given below, using the addition of ordinates.



Key Takeaways

- ☑ Addition of Ordinates is used to sketch the sum of two functions.
- ☑ We always add their y values.
- ☑ When we have an x-intercept for one graph, the sum graph intersects the other graph.
- ☑ When we have an intersection between two graphs, the sum graph equals to double their y value.
- ☑ When we have an equidistance from the x-axis, the sum graph has an x-intercept.

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Contour Check

Learning Objective: [1.5.1] – Find Midpoint, Distance (Horizontal & Vertical) Between Two Points Or Functions

Key Takeaways

- Midpoint is simply the _____ of 2 points.
- Distance formula is derived from _____.
- Horizontal distance is the distance between ____ values.
- Vertical distance is the distance between _____ values.

Learning Objective: [1.5.2] – Find Parallel and Perpendicular Lines

Key Takeaways

- Parallel lines have the _____ gradient.
- Perpendicular lines have _____ gradient.

Learning Objective: [1.5.3] – Find the Angle Between a Line and x-axis or Two Lines

Key Takeaways

- To find the angle between a line and the x-axis we can use equation $m =$ _____.
- To find the angle between two lines we can use $\theta =$ _____ or $\tan(\theta) =$ _____.

Learning Objective: [1.5.4] – Find The Unknown Value for Systems of Linear Equations

Key Takeaways

- Two linear equations have unique solution if they have _____ gradients.
- Two linear equations have infinitely many solutions when they have _____ gradient and _____ constant.
- Two linear equations have no solution when they have _____ gradient and _____ constant.

Learning Objective: [1.5.5] – Sketching the sum of two function's graph by using the addition of ordinates

Key Takeaways

- Addition of Ordinates is used to sketch the _____ of two functions.
- We always add their _____ values.
- When we have an x intercept for one graph, sum graph _____ the other graph.
- When we have an intersection between two graphs, the sum graph equals to _____ their _____ value.
- When we have an equidistance from the x-axis, sum graph has an _____ intercept.

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