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# VCE Mathematical Methods ¾
## Coordinate Geometry [1.5]
## Homework Solutions

### Homework Outline:

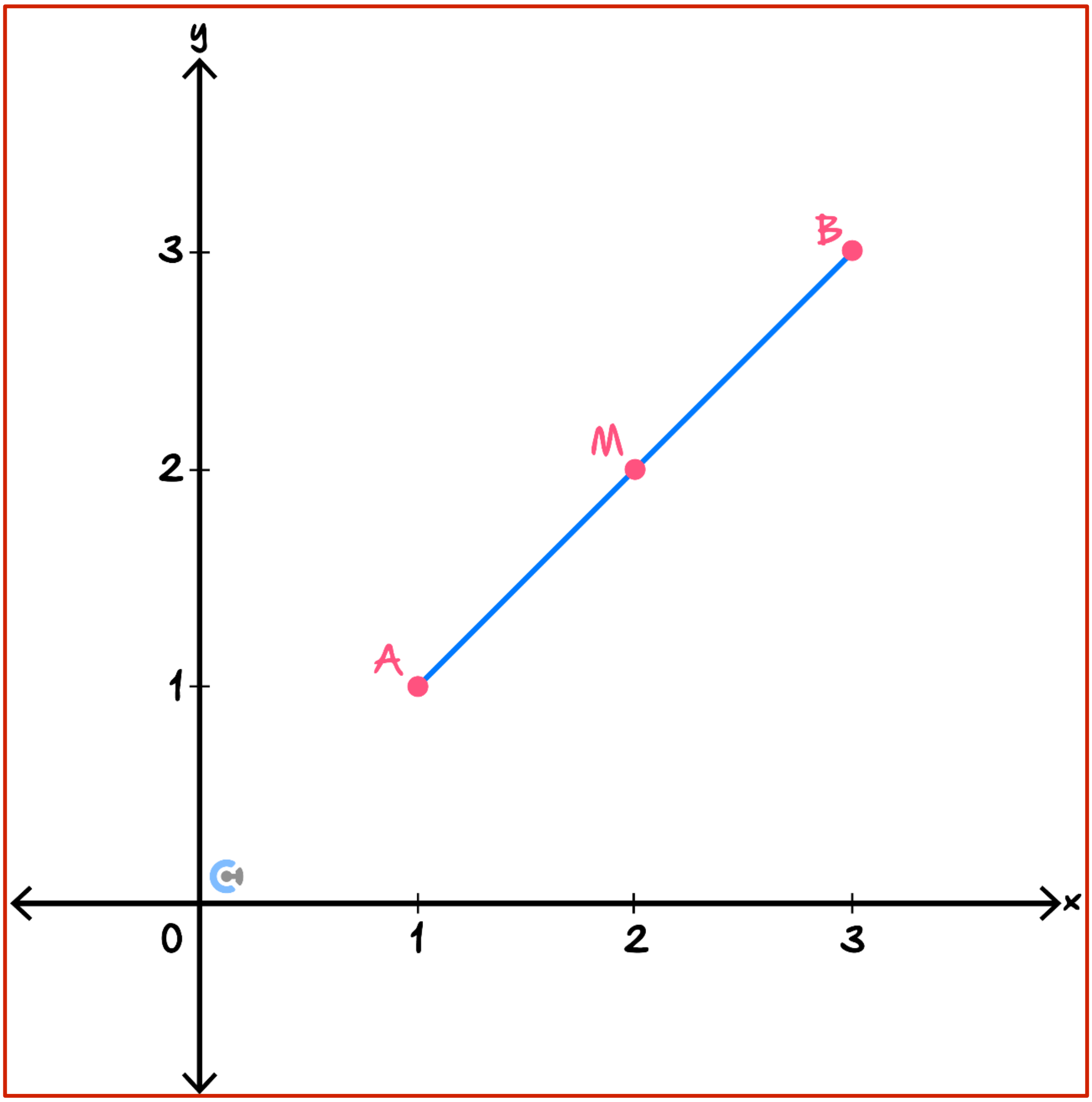
| | |
|---|---|
| Homework Questions | Pg 2 – Pg 25 |

## Section A: Homework Questions

### Sub-Section [1.5.1]: Finding the Midpoint and Distance Between Points and Functions

**Question 1**

The line segment $AB$ is shown on the axis below. Draw the midpoint, $M$ of $AB$.

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**Question 2**

Find the midpoints of the following points.

**a.** $A(3, 7)$ and $B(5, 9)$.

$$\left(\frac{3+5}{2}, \frac{7+9}{2}\right) = (4, 8)$$

**b.** $C(-2, -3)$ and $D(6, 4)$.

$$\left(\frac{-2+6}{2}, \frac{-3+4}{2}\right) = \left(2, \frac{1}{2}\right)$$

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**Question 3**

The midpoint of points $A$ and $B$ is $M(2,2)$.

**a.** If the coordinates of $A$ are $(6,-4)$, find the coordinates of $B$.

Let $B$ have the co-ordinates $(x,y)$.
Then,
$\frac{6+x}{2} = 2 \implies 6 + x = 4 \implies x = -2$ and $\frac{-4+y}{2} = 2 \implies -4 + y = 4 \implies y = 8$
Thus, the coordinates of $B$ are $(-2,8)$.

Consider the points $C(c,5)$ and $D(-3,d)$. The midpoint of the line $CD$ is the origin.

**b.** Find the values of $c$ and $d$.

We know that $\frac{c-3}{2} = 0$, thus $c = 3$.
Similarly, $\frac{5+d}{2} = 0$, thus $d = -5$.

**c.** Find the midpoint of $E(x_1,y_1)$ and $F(x_2,y_2)$ in terms of $x_1$, $x_2$, $y_1$, and $y_2$.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

**d.** The graph of $y = x^2 + k$ and the line $y = 1$ has a minimum vertical distance of 4. Find the value of $k$.

The parabola $y = x^2 + k$ has lowest point at $(0,k)$. Therefore $k - 1 = 4 \implies k = 5$.

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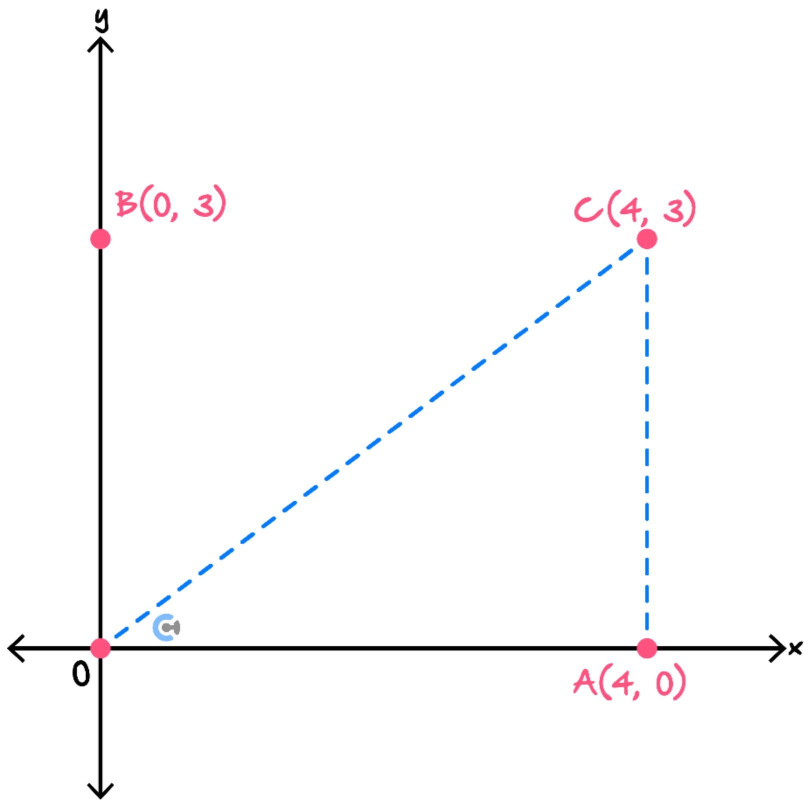
## Sub-Section [1.5.2]: Finding Distances Between Points

**Question 4**

Consider the points, $A, B, C$ as well as the origin drawn below.




**a.** Find the distance between the origin and point $A$.

4 units.

**b.** Find the distance between the origin and point $B$.

3 units.

**c.** Use Pythagoras' theorem to find the distance between the origin and point $C$.

$\sqrt{3^2 + 4^2} = 5$ units.

### Question 5

Find the distance between the following pairs of points.

**a.** $A(2,5)$ and $B(-2,2)$.

$$\sqrt{(2-(-2))^2+(5-2)^2}=\sqrt{4^2+3^2}=\sqrt{25}=5 \text{ units.}$$

**b.** $C(-1,-7)$ and $D(4,5)$.

$$\sqrt{(-1-4)^2+(-7-5)^2}=\sqrt{(-5)^2+(-12)^2}=\sqrt{25+144}=\sqrt{169}=13 \text{ units.}$$

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**Question 6**

A point $P(u, v)$ lies on the line $y = 3 - x$.

**a.** Express the distance between $P$ and the origin in terms of $u$ only.

We know that $v = 3 - u$, thus the distance of $OP$ is,

$$\sqrt{u^2 + (3-u)^2} = \sqrt{2u^2 - 6u + 9}$$

Consider the points $A(-1, -1)$, $B(5, 7)$ and $C(x, y)$.

The length of $AC$ is equal to the length of $BC$ which is equal to halve the length of $AB$.

**b.** Find the coordinates of $C$.

The two conditions provided in the question ensure that $C$ is the midpoint of $A$ and $B$.

Thus the coordinates of $C$ are $(2, 3)$.

**c.** **Tech-Active.** The distance between the point $P(u, v)$ is 3 units away from the origin and 4 units away from the point $Q(1, 4)$. Find the coordinates of $P$.

The distance between $P$ and the origin is, $\sqrt{u^2 + v^2} = 3$.

The distance between $P$ and $Q$ is, $\sqrt{(u-1)^2 + (v-4)^2} = 4$.

We solve these two equations simultaneously to get the value(s) of $u$ and $v$. Thus,

$$P = \left(\frac{5 - 32\sqrt{2}}{17}, \frac{20 + 8\sqrt{2}}{17}\right) \quad \text{or} \quad P = \left(\frac{5 + 32\sqrt{2}}{17}, \frac{20 - 8\sqrt{2}}{17}\right)$$

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## Sub-Section [1.5.3]: Finding Parallel and Perpendicular Lines

**Question 7**

State whether the following lines are parallel or perpendicular to each other.

**a.** $y = 2x + 1$ and $y = 2x + 5$.

$$m_1 = m_2 \Longrightarrow \text{parallel}$$

**b.** $y = 3x + 2$ and $y = -\frac{1}{3}x - 2$.

$$m_1 \times m_2 = -1 \Longrightarrow \text{perpendicular}$$

**c.** $2x + 3y = 5$ and $4x + 6y = 12$.

$$2x + 3y = 5 \Longrightarrow y = -\frac{2}{3}x + \frac{5}{3}$$
$$4x + 6y = 12 \Longrightarrow y = -\frac{4}{6}x + \frac{12}{6} \Longrightarrow y = -\frac{2}{3}x + 2$$
$$m_1 = m_2 \Longrightarrow \text{parallel}$$

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**Question 8**

A line $l_1$ goes through the points $(2, 3)$ and $(3, 5)$.

**a.** Find the gradient of $l_1$.

Let $m_1$ be the gradient of $l_1$.
Then, $m_1 = \frac{5-3}{3-2} = 2$.

**b.** Find the equation of $l_1$.

$$y = 2(x-2) + 3$$
$$y = 2x - 1$$

The line $l_2$ is perpendicular to $l_1$ and goes through the point $(2, 3)$.

**c.** Find the gradient of $l_2$.

$$y - 3 = -\frac{1}{2}(x-2)$$
$$y = \frac{-1}{2}(x-2) + 3$$
$$y = \frac{-x}{2} + 4$$

**d.** Find the equation of $l_2$.

$$y = \frac{-1}{2}(x-2) + 3 = \frac{-x}{2} + 4$$

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**Question 9**

The line $l_1$ is parallel to the line $l_2 = \{(x, y) \in \mathbb{R}^2 : 2y + 3x = 5\}$ and goes through the origin.

**a.** Find the equation of $l_1$.

The equation for $l_2$ is $y = \frac{-3}{2}x + \frac{5}{2}$. Hence the gradient for $l_2$ is $\frac{-3}{2}$.

Hence the equation for $l_1$ is $y = \frac{-3}{2}x$

**b.** Find the equation of the line that is perpendicular to the line with the equation $y = -5x + 7$ and passes through the point $(2, -5)$.

$$y = -5x + 7$$
$$Slope = -5$$
$$Slope\ of\ the\ perpendicular\ line = \frac{1}{5}$$
$$Required\ line\ y + 5 = \frac{1}{5}(x - 2)$$
$$y = \frac{x}{5} - \frac{27}{5}$$

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**Question 10**

**a.** Find the perpendicular bisector of the points $A(2,3)$ and $B(4,9)$.

The line $AB$ has gradient $\frac{9-3}{4-2} = 3$ and midpoint, $\left(\frac{4+2}{2}, \frac{9+3}{2}\right) = (3,6)$.

Hence the perpendicular bisector goes through the point $(3,6)$ and has a gradient of $\frac{-1}{3}$.

Hence the equation of the perpendicular bisector is,

$$y = \frac{-1}{3}(x-3) + 6 = \frac{-x}{3} + 7$$

**b.** A point $P(u,v)$ lies on the line $y = 2x$.

Find the value of $u$ and $v$ for which the distance between $P$ and the point $Q(0,1)$ is minimum.

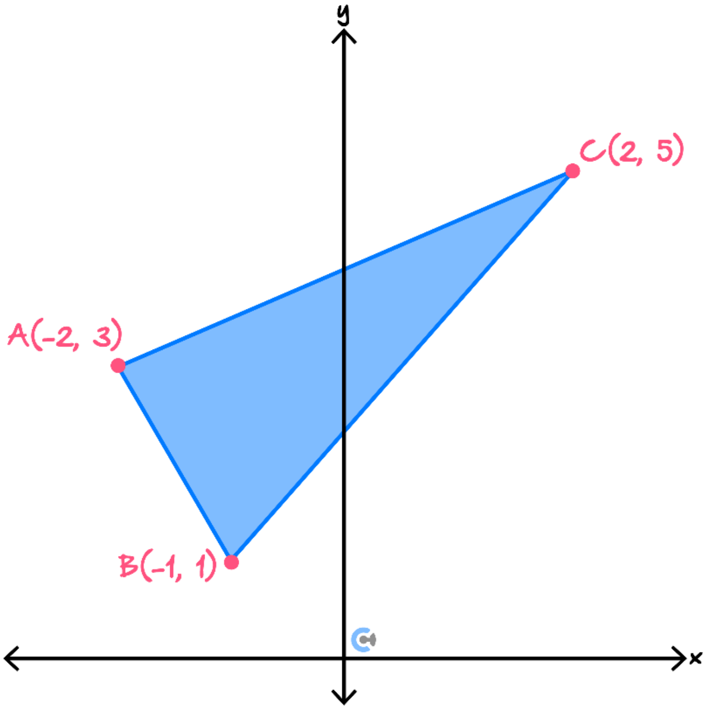
**Hint:** The line $PQ$ is perpendicular to the line $y = 2x$.

The gradient of $PQ$ must be $\frac{-1}{2}$. As $PQ$ goes through the point $Q$, it's equation is,

$$y = \frac{-x}{2} + 1.$$

Thus, $v = \frac{-u}{2} + 1 = 2u \implies \frac{5u}{2} = 1 \implies u = \frac{2}{5} \implies v = \frac{4}{5}$.

**c.** Consider the triangle $ABC$ drawn below.




**i.** Show that the line $AB$ is perpendicular to the line $AC$.

The line $AC$ has gradient $\frac{5-3}{2+2} = \frac{1}{2}$.
The line $AB$ has gradient $\frac{3-1}{-2+1} = -2$ As $\frac{1}{2}$ is the negative reciprocal of $-2$, the lines $AB$ and $AC$ are perpendicular.

**ii.** Hence, find the area of the triangle $ABC$.

The length of $AB$ is $\sqrt{2^2+1^2} = \sqrt{5}$.
The length of $AC$ is $\sqrt{2^2+4^2} = \sqrt{20}$
Thus the area of the triangle is $\frac{\sqrt{5}\sqrt{20}}{2} = 5$

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## Sub-Section [1.5.4]: Angles Between Lines

**Question 11**

**a.** Find the angle of the line $y = x + 1$ makes with the positive direction of the $x$-axis.

$\tan^{-1}(1) = 45°$

**b.** Find the equation of the line that passes through the origin and makes an angle of 30 degrees with the positive direction of the $x$-axis.

The line will have a gradient of $\tan(30°) = \frac{1}{\sqrt{3}}$.

Thus the equation of the line is $y = \frac{x}{\sqrt{3}}$

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**Question 12**

**a.** Find the acute angle between the lines $y = \frac{1}{\sqrt{3}}x + 2$ and $y = \frac{-1}{\sqrt{3}}x$.

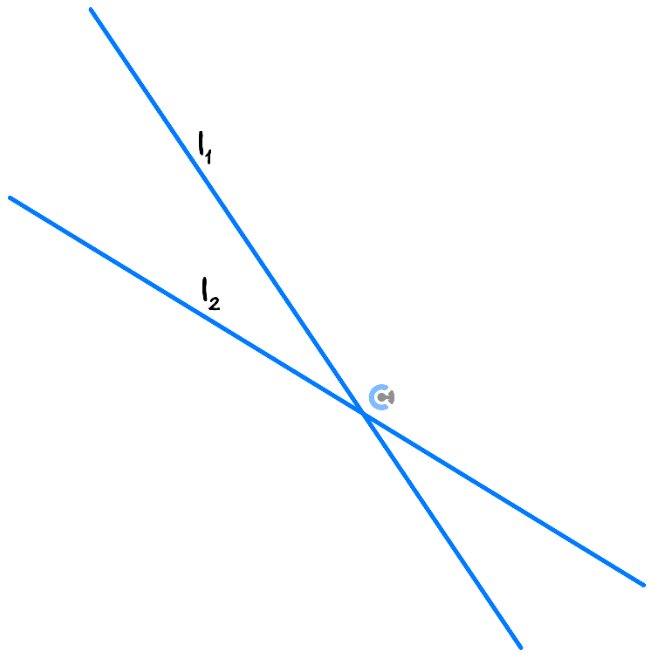
The angle $y = \frac{1}{\sqrt{3}}x + 2$ makes with the $x$-axis is $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30°$.

The angle $y = \frac{-x}{\sqrt{3}}$ makes with the $x$-axis is $\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = -30°$.

Thus the acute angle between our lines is 60°

**b.** **Tech-Active.** Consider the line $l_1$, with the equation $2y + 3x = 5$.

The line $l_2$ intersects $l_1$ at an acute angle 25°. Both $l_1$ and $l_2$ are drawn below.




Find the slope of $l_2$ correct to 2 decimal places.

The angle $l_1$ makes with the positive direction of the $x$-axis is $180° + \tan^{-1}\left(\frac{-3}{2}\right) = 180° - 56.31° = 123.69°$.

Thus $l_2$ makes an angle of 148.69° with the positive direction of the $x$-axis.

Hence the slope of $l_2$ is $\tan(148.69°) = -0.608.. \approx -0.61$

**c.** **Tech-Active.** Find the acute angle of intersection between the lines $y = 3x + 5$ and $-2x + 3y = 7$.

Give your answer in degrees correct to the nearest degree.

$$m_1 = 3, m_2 = \frac{2}{3}$$

Angle between lines :

$$\tan(\theta) = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\tan\theta = \left|\frac{3 - \frac{2}{3}}{1 + 1}\right|$$

$$\tan\theta = \frac{7}{6}$$

$$\theta = 49.398.. \approx 49°$$

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**Question 13**

The line $l$ intersects the positive y-axis at 30°

**a.** Find the gradient, $m$ of $l$ if $m < 0$.

The acute angle between $l$ and the $x$-axis is 60°.

If $m$ is the gradient of $l$, then $m = \tan(\pm 60°) = \pm\sqrt{3}$.

As $m$ is negative then $m = -\sqrt{3}$.

**b.** **Tech-Active.** Find the acute angle of intersection between the lines $y = 2x + 3$ and $3x + 5y = -4$.

Give your answer in degrees correct to the nearest degree.

The angle $y = 2x + 3$ makes with the positive direction of the $x$-axis is, $\tan^{-1}(2) = 63.43°$.

The angle $3x + 5y = -4$ makes with the positive direction of the $x$-axis is, $\tan^{-1}\left(\frac{-3}{5}\right) = -30.96°$.

Hence an angle between our two lines is 94°.

As this angle is greater than 90, it's supplementary angle of 86° is the acute angle of intersection between our two lines.

**c.** Find the equation of all lines that intersect the line $y = x + 3$ at the point $(1, 4)$ at an acute angle of 15°.

The angle $y = x + 3$ makes with the positive direction of the $x$-axis is 45°.

For line to intersect $y = x + 3$ at 15° it needs to,

a. Make an angle of 60 degrees with the positive direction of the $x$-axis, thus have a gradient of $\sqrt{3}$.

b. Make an angle of 30 degrees with the positive direction of the $x$-axis, thus have a gradient of $\frac{1}{\sqrt{3}}$.

Hence our lines are,

$$y = \sqrt{3}(x-1) + 4 = \sqrt{3}x + 4 - \sqrt{3} \quad \text{and} \quad y = \frac{1}{\sqrt{3}}(x-1) + 4 = \frac{x}{\sqrt{3}} + 4 - \frac{1}{\sqrt{3}}$$

## Sub-Section [1.5.5]: Simultaneous Equations

**Question 14**

Solve the following equations simultaneously.

**a.** $3x + 4y = 7$ and $5x - 2y = 3$.

We add $2 \times$ the right equation to the left equation to get,

$$3x + 10x = 7 + 6 \implies 13x = 13 \implies x = 1$$

We substitute it into the left equation to get,

$$3 + 4y = 7 \implies y = 1$$

Hence $x = y = 1$.

**b.** $y = 5x + 3$ and $3y + 4x = 8$.

We substitute the left equation into the right equation, yielding,

$$15x + 9 + 4x = 8 \implies 19x = -1 \implies x = \frac{-1}{19}.$$

Substituting this into the left equation yields,

$$y = 5\left(\frac{-1}{19}\right) + 3 = \frac{52}{19}$$

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**Question 15**

**a.** Find the point of intersection between the lines $y = 3x + 7$ and $2x + 5y = 1$.

We substitute the first line into the second, yielding,

$$15x + 35 + 2x = 1 \implies 17x = -34 \implies x = -2.$$

We substitute this into the left equation to get $y = 1$.

Hence the point of intersection is $(-2, 1)$

**b.** Explain why the equations $2x + 4y = 6$ and $3x + 6y = 5$ have no solutions.

Both lines have slope $-\frac{1}{2}$ but the $y$-intercepts are different. Hence they are parallel lines. No solution.

**c.** **Tech-Active.** For each pair of simultaneous equations, state whether they have, no solution, a unique solution or infinitely many solutions.

**i.** $2x + 5y = 7$ and $3x + 2y = 8$.

Solve on calc, you get 1 solution.
Hence, a unique solution.

**ii.** $y = -3x + 6$ and $2y + 6x = 6$.

Solve on calc, you get false.
Hence, no solution.

**iii.** $6x + y = 2$ and $y = -6x + 2$.

Solve on calc, you get $y = 2 - 6x$.
Hence, you have infinitely many solutions.

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**Question 16**

**a.** Consider the following pair of simultaneous equations,

$$kx - y = 6$$
$$7x + (k - 8)y = 4$$

For what value(s) of $k$ do they have:

**i.** A unique solution.

**ii.** No solution.

For the equations to have a unique solution, their gradients must be different.

The gradient of the first equation is $k$, whilst the gradient for the second equation is $\frac{7}{8-k}$.

We solve,

$$k = \frac{7}{8-k} \implies k^2 - 8k + 7 = (k-7)(k-1) = 0 \implies k = 1, 7$$

Hence our equations have a unique solution if $k \neq 1, 7$.
If $k = 1$ the lines are $x - y = 6$ and $7x - 7y = 4$ and hence there is no solution.
If $k = 7$ the lines are $7x - y = 6$ and $7x - y = 4$ and hence there is no solution.

**b.** Consider the following pair of simultaneous equations,

$$ax + 3y = 6$$
$$x + (4 - a)y = 2$$

For what value(s) of $a$ do they have:

**i.** No solution.

**ii.** Infinitely many solutions.

**iii.** A unique solution.

The gradient of our first line is $\frac{-a}{3}$, whilst the gradient of our second line is $\frac{1}{a-4}$. Hence we solve,

$$\frac{-a}{3} = \frac{1}{a-4} \implies a^2 - 4a + 3 = (a-3)(a-1) = 0 \implies a = 1, 3$$

If $a = 1$ then our equations are $x + 3y = 6$ and $x + 3y = 2$, which have no solutions.

If $a = 3$, then our equations are $3x + 3y = 6$ and $x + y = 2$, which have infinitely many solutions.

Finally, there is a unique solution for $a \in \mathbb{R} \setminus \{1, 3\}$.

**c.** **Tech-Active.** Consider the following pair of simultaneous equations,

$$3x + (1 - a)y = 2$$
$$ax - 2y = b$$

Find all pairs $(a, b)$ such that the equations have infinitely many solutions.

The gradient of our first line is $\frac{3}{a-1}$, whilst the gradient of our second line is $\frac{a}{2}$.

We equate these two gradients yielding, $a = -2, 3$.

Since the gradients of our two lines will be the same for $a = -2, 3$ if the two lines also share a point they will have infinitely many solutions.

If $a = -2$, the $y$-axis intercept of the first line is $\left(0, \frac{2}{3}\right)$, thus $b = -2(0) - 2\frac{2}{3} = \frac{-4}{3}$.

If $a = 3$, the $y$-axis intercept of the first equation is $(0, -1)$, thus $b = 3(0) - 2(-1) = 2$.

Hence our pairs of values are $(a, b) = \left(-2, \frac{-4}{3}\right)$ and $(3, 2)$.

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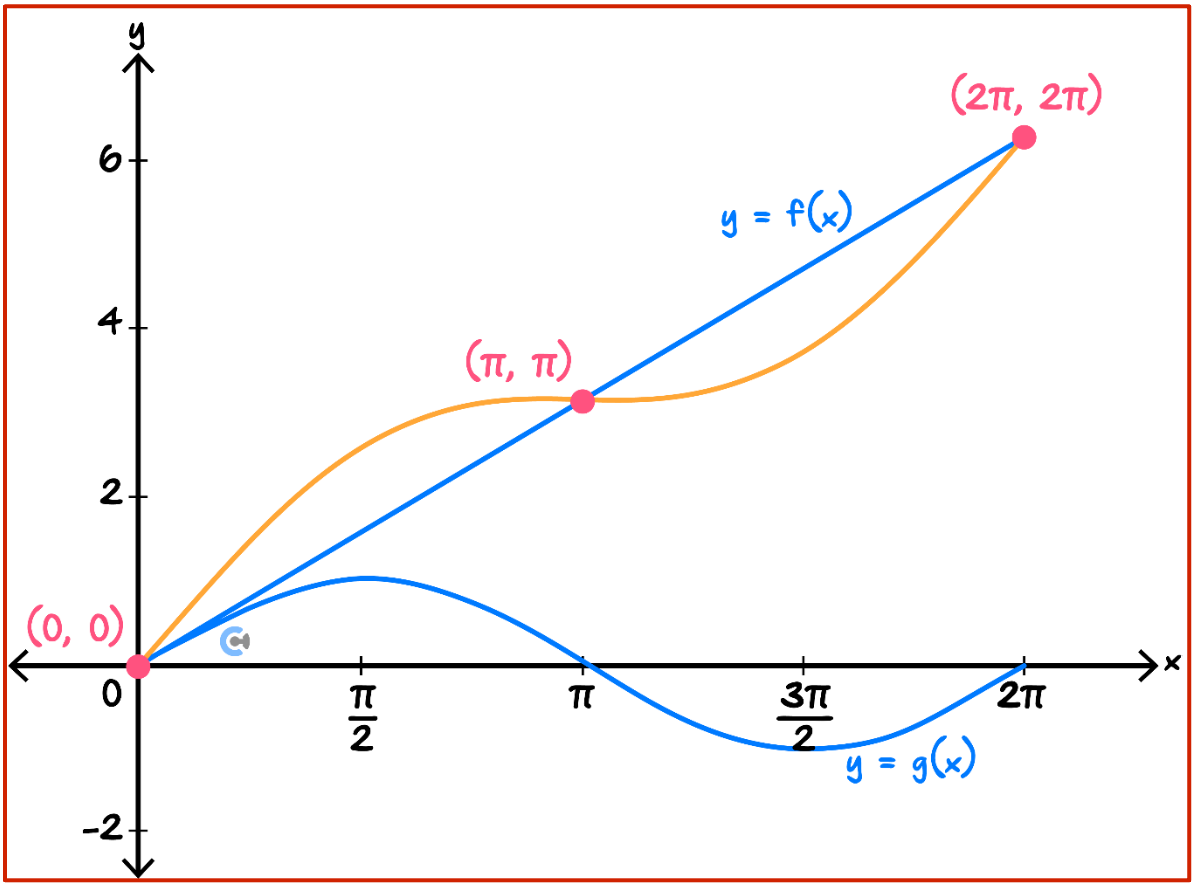
## Sub-Section [1.5.6]: Addition of Ordinates

**Question 17**

The graphs of $f : [0, 2\pi] \longrightarrow \mathbb{R}, f(x) = x$, and $g : [0, 2\pi] \longrightarrow \mathbb{R}, g(x) = \sin(x)$ are drawn below.

Sketch the graph of $h(x) = f(x) + g(x)$ on the axis below, labelling all points of intersection between $f$ and $h$ with their co-ordinates.

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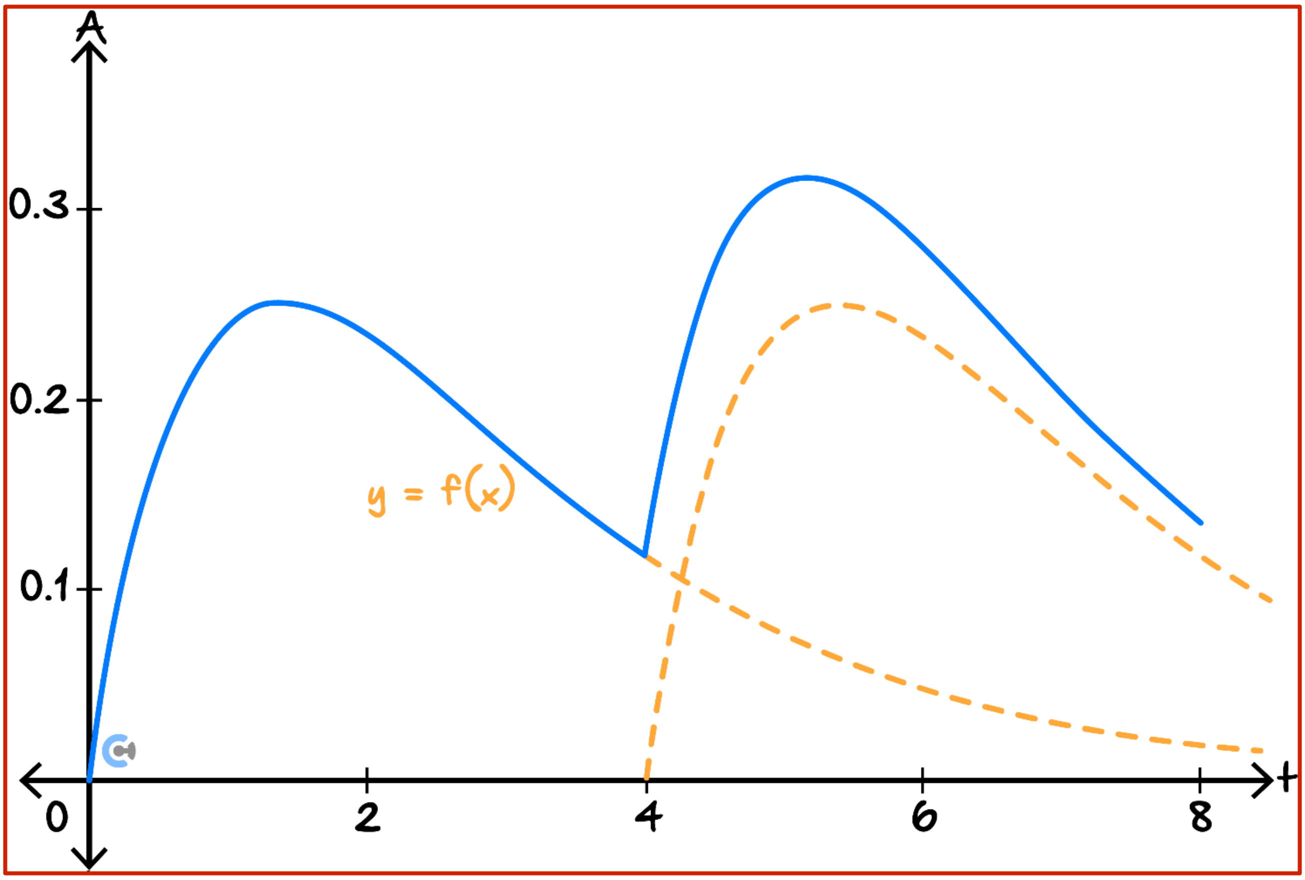
**Question 18**

$t$ hours after taking a mystery pill, the concentration of dopamine in a patient's bloodstream is $A = f(t)$ milligrams per litre. The graph of $f$ is shown below.

4 hours after taking one mystery pill, the patient takes another mystery pill.

On the axis below, sketch the concentration of dopamine in the patient's bloodstream during the first 8 hours after they take the first mystery pill.




**Question 19 Tech-Active.**

Let $f(x) = e^x - e^{-2x}$ and $g(x) = e^{x-x^2}$.

How many solutions does the equation $f(x) + g(x) = 0$ have?

Sketch both graphs. For negative $x$, both graphs are strictly positive. For positive $x$, $g(x)$ is positive however asymptotes towards 0, whilst $f(x)$ tends towards negative infinity.

By addition of ordinates we see that $f(x) + g(x)$ will have one solution.

## Sub-Section [1.5.7]: Boss Question

**Question 20**

Consider the points $A(1, 0)$ and $B(4, 3)$.

**a.** Find the equation of the line segment $AB$.

The gradient of $AB$ is $\frac{3-0}{4-1} = 1$. Hence the equation of $AB$ is,

$$y = 1(x-1) + 0 = x - 1$$

There is another point $C$, such that $A$ is the midpoint of the line segment $CB$.

**b.** Find the coordinates of $C$.

Let $(x, y)$ be the coordinates of $C$. Then,

$$\frac{x+4}{2} = 1 \implies x = -2 \quad \text{and} \quad \frac{y+3}{2} = 0 \implies y = -3,$$

thus the co-ordinates of $C$ are $(-2, -3)$

**c.** Hence or otherwise, find the length of $BC$.

The length of $BC$ is $\sqrt{(4-(-2))^2 + (3-(-3))^2} = \sqrt{6^2 + 6^2} = 6\sqrt{2}$.

**d.** Another point $D(u, v)$ has the following properties,

- The length of $AD$ is equal to twice the length of $AB$.
- The angle between $AD$ and $AB$ is 30°.
- The gradient of $AB$ is larger than the gradient of $AD$.
- Both $u$ and $v$ are positive.

Find the values of $u$ and $v$ correct to 3 decimal places.

The line segment $AB$ makes an angle of 45° with the $x$-axis, as the gradient of $AB$ is larger than the gradient of $AD$, $AD$ must make an angle of 15° with the $x$-axis. Thus a point $(x, y)$ on the line segment $AD$ satisfies,

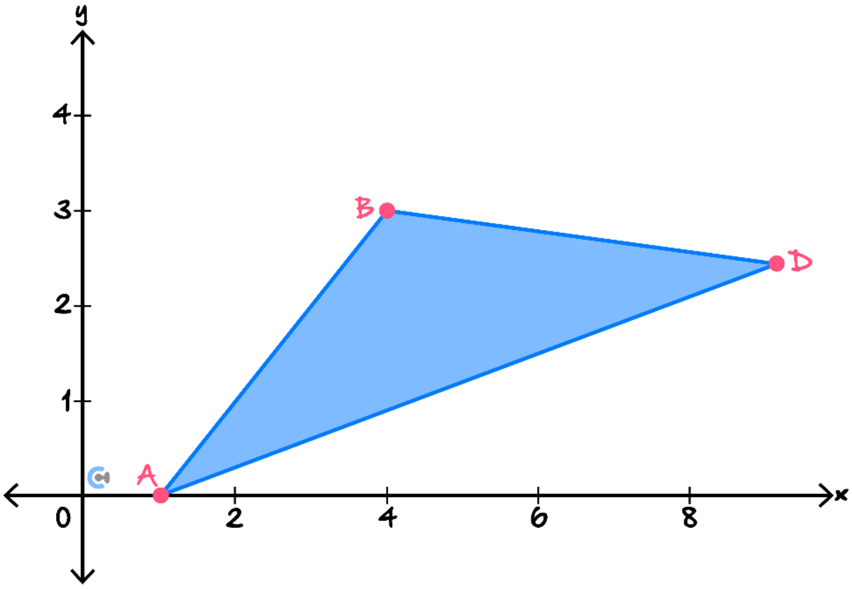
$$y = \tan(15°)(x-1) = (2-\sqrt{3})(x-1)$$

As $(u, v)$ is such a point, along with the fact that the length of $AD$ is $6\sqrt{2}$ we can solve the following two equations simultaneously for $u$ and $v$,

$$v = (2-\sqrt{3})(x-1) \qquad \text{and} \qquad (u-1)^2 + v^2 = 72$$

Thus, $u = 9.196$ and $v = 2.196$.

**e.** The triangle $ABD$ is drawn below.




**i.** Find the equation of the line, $l$ perpendicular to $AD$ that goes through $B$.

The gradient of $AD$ is $2-\sqrt{3}$, thus the gradient of $l$ is $\frac{1}{\sqrt{3}-2}$. Thus the equation of $l$ is,

$$y=\frac{1}{\sqrt{3}-2}(x-4)+3$$

**ii.** Hence or otherwise, find the area of $ABD$ correct to the nearest integer.

We find the point of intersection of $l$ and the line $AD$ by solving,

$$y=\frac{1}{\sqrt{3}-2}(x-4)+3 \quad \text{and} \quad y=(2-\sqrt{3})(x-1)$$

simultaneously. This yields $(x,y)=(4.549, 0.951)$.

Thus the "height" of the triangle, the distance between $B$ and this point is 2.12132 units, whilst the base of the triangle is $6\sqrt{2}$ units.

Hence the area of the triangle is 9 units

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