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VCE Mathematical Methods ¾
Transformations Exam Skills [1.4]
Workbook

Outline:

Learning Objectives:

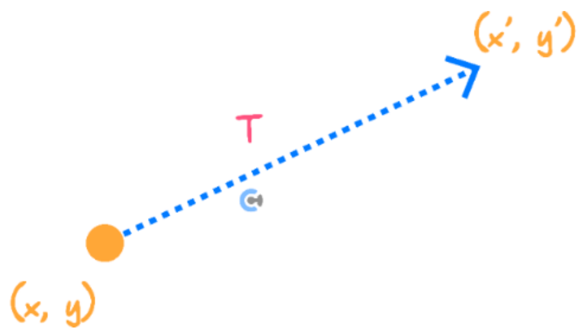
- MM34 [1.3.1] - Applying x' and y' notation to find transformed points, find the interpretation of transformations and altered order of transformations.
- MM34 [1.3.2] - Find transformed functions.
- MM34 [1.3.3] - Find transformations from transformed function (Reverse Engineering).

Section A: Recap of Transformations

Sub-Section: Image and Pre-Image

What do we call an original coordinate and a transformed coordinate?

Image and Pre-Image



- The original coordinate is called the ________________.
- The transformed coordinate is called the ______________.

Pre-Image: (x, y)

Image: (x', y')

NOTE: The x' and y' notation will be used quite heavily!

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Sub-Section: Dilation

Dilation

Dilation by a factor a from the x-axis: $y' = ay$

Dilation by a factor b from the y-axis: $x' = bx$

NOTE: We are applying the transformations on (x, y) not (x', y').

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Sub-Section: Reflection

Reflection

$$\text{Reflection in the } x\text{-axis: } y' = -y$$

$$\text{Reflection in the } y\text{-axis: } x' = -x$$

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Sub-Section: Translation

Translation

Translation by c units in the positive direction of the x-axis: $x' = x + c$

Translation by d units in the positive direction of the y-axis: $y' = y + d$

Question 1

Find the image (x', y') after applying the following transformations to (x, y).

Dilation by a factor 4 from the x-axis.

Dilation by a factor 2 from the y-axis.

Reflection in the x-axis.

Translation by 3 units in the negative direction of the x-axis.

Translation by 5 unit in the positive direction of the y-axis.

$x' = 2x - 3.$ $\qquad \frac{(x'+3)}{2} = x$

$y' = -4y + 5$ $\qquad$ opposite. inverse order

$\frac{(y'-5)}{-4} = y$

Key Takeaways

- The transformed point is called the image and is denoted by (x', y').
- The dilation factor is multiplied by the original coordinates.
- Reflection makes the original coordinates the negative of their original values.
- Translation adds a unit to the original coordinates.

Sub-Section: Basic Transformation of Points

Let's try to apply all types of transformations to a point!

Question 2

Find the image (x', y') after applying the following transformations to (x, y).

Translation by 3 units in the positive direction of the x-axis.

Translation by 2 unit in the negative direction of the y-axis.

Dilation by a factor 4 from the x-axis.

Dilation by a factor $\frac{1}{2}$ from the y-axis.

Reflection in the x-axis.

NOTE: Order is important!

- Apply the next transformation on top of everything that has already been done!

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Sub-Section: The Order Of Transformations

What is the Order of Transformations the same as?

The Order of Transformation

$$\text{Order} = \text{BODMAS Order}$$

Question 3

The series of transformations, "a dilation by a factor 2 from the y-axis, a reflection in the y-axis and a translation by 8 units left" yields the same result as the series of transformations, "a translation by c units right, a reflection in the y-axis and a dilation by a factor d from the y-axis." Find the values of c and d.

NOTE: Dilation factors don't change!

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Sub-Section: Interpreting the Transformation of Points

Interpretation of Transformations

- When the ________________ x' and y' are the subject, we can read the transformation __________.

$$x' = x + 5 \rightarrow 5 \text{ right}$$

- When the ____________________ x and y are the subjects instead, we must read the transformation in the ______________ way.
- This includes the order of transformation!

$$x = x' - 5 \rightarrow 5 \text{ right}$$

NOTE: This includes the order of transformation!

TIP: It is best to make x' and y' the subject before you interpret the transformations.

Question 4

Consider the transformation which maps:

$$x = -3x' - 4$$

$$y = 2y' + 2$$

a. State the transformations in DRT (Dilation, Reflection, Translation) order.

b. State the transformations in the translation in first order.

Key Takeaways

- Transformations should be interpreted when x' and y' are isolated.
- The order of transformation follows the BODMAS order.
- To change the order of transformations, we either factorise or expand.

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Sub-Section: Applying Transformations to Functions

Let's now work with functions!

Transformation of Functions

- The aim is to get rid of the old variables, x and y, and have the new variables, x' and y', instead.

$$y = f(x) \rightarrow y' = f(x')$$

- Steps:

1. Transform the points.
2. Make x and y the subjects.
3. Substitute them into the function.

Question 5

Apply the following transformations to the functions below:

a. $f(x) = (x+1)^3$.

Dilation by a factor 3 from the x-axis.

Reflection in the y-axis.

Translation by 4 units to the right.

Dilation by a factor 2 from the y-axis.

1) $x' = 2(-x+4)$

$y' = 3y$

2) $\frac{x'}{2} = -x+4$

$x = -\frac{x'}{2} + 4$ $\qquad$ $\frac{y'}{3} = y$

3) $y = (x+1)^3$ ← old var

$\frac{y'}{3} = \left(-\frac{x'}{2} + 4 + 1\right)^3$ ← New

$\frac{y}{3} = \left(-\frac{x}{2} + 5\right)^3$

$y = 3\left(-\frac{x}{2} + 5\right)^3$

b. $f(x) = \cos(x)$.

Dilation by a factor 3 from the y-axis.

Dilation by a factor $\frac{1}{2}$ from the x-axis.

Translation by 4 units to the left.

Translation by 2 units up.

Reflection in the y-axis.

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Sub-Section: Finding the Applied Transformations

Now let's go backwards!

Reverse Engineering

- Steps:

1. Add the dashes (') back to the transformed function.
2. Make $f(\ )$ the subject.
3. Equate the LHS of the original and transformed functions to the RHS of the original and transformed functions.
4. Make x' and y' the subjects and interpret the transformations.

Your turn!

Question 6

State a series of transformations (in order) that allow $f(x)$ to be transformed into $g(x)$.

a. $f(x) = 2e^{3x-4} + 1$ and $g(x) = e^{\frac{1}{3}x+2} + 2$.

1) $y = 2e^{3x-4} + 1$

$y' = e^{\frac{1}{3}x'+2} + 2$

2) "where x?"

$\frac{y-1}{2} = e^{3x-4}$

$y' - 2 = e^{\frac{1}{3}x'+2}$

3) $y' - 2 = \frac{y-1}{2}$.

$3x - 4 = \frac{1}{3}x' + 2$.

4) $y' = \frac{y-1}{2} + 2$

$y' = \frac{1}{2}y + \frac{3}{2}$.

$3x - 6 = \frac{1}{3}x'$

$x' = 9x - 18$

Dil $\frac{1}{2}$ from x
Dil 9 from y
Translate $\frac{3}{2}$ up
Translate 18 left.

b. $f(x) = (x-3)^3 + 2$ and $g(x) = 3(2x+5)^3 - 6$.

1) $y = (x-3)^3 + 2$

$y' = 3(2x'+5)^3 - 6$

2) $y - 2 = (x-3)^3$

$\frac{y'+6}{3} = (2x'+5)^3$

3) $\frac{y'+6}{3} = y-2.$ $\quad 2x'+5 = x-3.$

$y'+6 = 3y-6$ $\quad 2x' = x-8$

$y' = 3y-12,$ $\quad x' = \frac{1}{2}x - 4$

$= 3(y-4)$ $\quad = \frac{1}{2}(x-8)$

Key Takeaways

- ☑ We transform the coordinates first, then transform the function.
- ☑ To transform the function, replace its old variables with the new ones.
- ☑ To find the transformations, simply equate LHS with RHS after separating the transformations of x and y.

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Section B: Transformations Exam Skills

Sub-Section: Quick Method

Let's try to do it more quickly!

Active Recall: Interpretation of Transformations

- When the new variables x' and y' are the subject, we can read the transformation directly.

$$x' = x + 5 \rightarrow 5\ right$$

- When the original variables x and y are the subject instead, we must read the transformation in the opposite way.
- This includes the order of transformation!

$$x = x' - 5 \rightarrow 5\ right$$

Active Recall: In the transformed function, was the transformation of x stuck in $x = t(x')$ or $x' = t(x)$ form?

$y = a f\left(\frac{(x'-c)}{d} = x\right) + b$

Dil d

c right

$\frac{x'-c}{d} = x$

$x' = dx$

$x' = dx + c$

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Quick Method

- The transformation of x in the function is represented in the opposite way in the final function.
- For applying transformation in a quick method,

Apply everything for x in the opposite direction.
Including the order!

- For interpreting transformation in a quick method,

Read everything for x in the opposite direction.
Including the order!

Question 7 Walkthrough.

sin(x)

Apply the following transformations to $y = \sin(x)$ using the quick method.

Dilation by a factor 3 from the x-axis

Dilation by a factor 2 from the y-axis

Reflection in the x-axis

Reflection in the y-axis

Translation of 2 units right

Translation of 3 units down

$$y = -3\sin\left(-\frac{1}{2}(x-2)\right) - 3$$

NOTE: For x, simply apply everything in the opposite way and order!

Your turn!

Question 8

Apply the following transformations to $y = \log_e(x)$ using the quick method.

Dilation by a factor $\frac{1}{5}$ from the x-axis

Dilation by a factor 3 from the y-axis

Reflection in the x-axis

Reflection in the y-axis

Translation of 5 units left

Translation of 2 units up

$$y = -\frac{1}{5}\log_e\left(-\frac{1}{3}(x+5)\right) + 2$$

NOTE: For x, simply apply everything in the opposite way and order!

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Now, interpreting transformations!

Question 9 Walkthrough

State the transformations required for $y = \sin(x)$ to transform into $y = 2\sin(3x + \pi) - 1$.

$y = \sin(x)$

$y = 2\sin(3x + \pi) - 1$

x +

Dil 2 fr x

1 down

π left

Dil $\frac{1}{3}$ fr y

NOTE: The order is opposite to BODMAS for x.

Your turn!

Question 10

Trala.

State the transformation required for $y = e^x$ to transform into $y = \frac{1}{3}e^{3(x+1)} + 1$.

x

$3(x'+1)$

$3(x'+1) = x$

$x' + 1 = \frac{1}{3}x$

$x' = \frac{1}{3}x - 1$

Dil $\frac{1}{3}$ fr x

Dil $\frac{1}{3}$ fr y

1 up

1 left

Sub-Section: Finding Opposite Transformations

How can we undo transformations?

Analogy: Untying a shoelace

- Sam is being silly and ties his shoelace when he was meant to take off his shoes at a chocolate restaurant that he's booked 3 years in advance.
- Which knot should he start untying first? **[First Knot, Last Knot]**
- Similarly, which transformations should we undo first? **[First transformation, Last transformation]**

Finding Opposite transformations

- Order is opposite.
- All transformations are opposed.

Question 11

a. Find the transformation from $f(x) = 3(x+1)^2 - 1$ to $g(x) = -2x^2 + 3$.

$y = 3(x+1)^2 - 1$

$y' = -2(x')^2 + 3.$

$\frac{y+1}{3} = (x+1)^2$

$\frac{y'-3}{-2} = (x')^2$

$x' = x + 1$

$\frac{y'-3}{-2} = \frac{y+1}{3}$

$y' - 3 = -\frac{2}{3}y - \frac{2}{3}$

$y' = -\frac{2}{3}y + \frac{7}{3}$

Dil $\frac{2}{3}$ fm x

Reflect in x

Translt $\frac{7}{3}$ unit up

Translate 1 unit right

b. Hence, state the transformation from $g(x)$ to $f(x)$.

$f \downarrow g$

$\boxed{\begin{array}{l} \text{Dil } \frac{2}{3} \text{ fm } x \\ \text{Reflect in } x \\ \text{Translt } \frac{7}{3} \text{ units up} \\ \text{Translate 1 unit right} \end{array}}$

$g \downarrow f$

Translate 1 unit left

Translate $\frac{7}{3}$ units down

Reflection in x

Dil $\frac{3}{2}$ from x axis

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Sub-Section: Finding Domain, Range, Points, and Tangents of Transformed Functions

Analogy: Function, points, and tangents

- Let's say your entire family decides to move 2 units right.

$$\textit{Family}: \textit{Let's go 2 units right}.$$

- What does that mean for you?

$$\textit{You}:$$ 2 right.

- Similarly, if a function moves in a certain way, how should its points, tangents, domain, and range move? [Same way, Different way]

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Finding domain, range, points, and tangents of transformed functions.

- Everything moves together as a function.
- **Steps**

  1. Find the transformations between two functions.
  2. Apply the same transformations to domain, range, points, and tangents.

Question 12 Walkthrough.

It is known that $f(x)$ has a domain of $[2,4]$ and a range of $(0,20]$.

The function has been transformed to $g(x) = -2f(x+5) + 2$.

a. State the transformation from $f(x)$ to $g(x)$.

f(x)

$-2f(x+5)+2.$

Dil 2 from x
Reflect in x
2 up
5 left

b. State the domain of $g(x)$.

Dom f: $[2, 4]$

Dom g: $[-3, -1]$

c. State the range of $g(x)$.

Range f: $(0, 20]$

$(0, 40]$

$[-40, 0)$

Range g: $[-38, 2)$

Question 12 Walkthrough.

It is known that $f(x)$ has a domain of $[2,4]$ and a range of $(0,20]$.

The function has been transformed to $g(x) = -2f(x+5) + 2$.

a. State the transformation from $f(x)$ to $g(x)$.

$f(x)$

$-2f(x+5)+2.$

Dil 2 from x

Reflect in x

2 up

5 left

Question 13

It is known that $f(x)$ has an x intercept at $(3,0)$ and a tangent of $y = 2x - 1$ at $x = 3$.

The function has been transformed to $g(x) = 3f(2x-1)$.

a. State the transformation from $f(x)$ to $g(x)$.

Dil 3 from x

1 right

Dil $\frac{1}{2}$ fm y

Dil $\frac{1}{2}$

$\frac{1}{2}$ right

b. State the x intercept of $g(x)$.

x int f: $(3,0)$

$(4,0)$

x int g: $(2,0)$

c. State the tangent of $g(x)$ at $x = 2$.

tangent of $f(x)$ at $x=3$: $y=2x-1$

transformed

$y = 6x-3$.

$y = 6(2x'-1)-3$

tangent of $g(x)$ at $x=2$: $y=12x-9$

$x' = \frac{1}{2}(x+1)$

$2x'-1 = x$

Question 13

It is known that $f(x)$ has an x intercept at $(3,0)$ and a tangent of $y = 2x - 1$ at $x = 3$.

The function has been transformed to $g(x) = 3f(2x - 1)$.

a. State the transformation from $f(x)$ to $g(x)$.

Dil 3 from x

1 right

Dil $\frac{1}{2}$ fm y

Dil $\frac{1}{2}$

$\frac{1}{2}$ right

NOTE: Everything changes with respect to the transformations.

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Sub-Section: Finding Transformations of Inverse Functions

REMINDER: Don't forget Inverse Relations,

Inverse functions swap x and y.

Discussion: If $f(x)$ moves 2 units right, where would $f^{-1}(x)$ go to?

Increase x by 2

Increase y by 2.

Finding transformation of inverse functions

$$f(x) \to f(x-2): 2\ Right$$

$$f^{-1}(x) \to f^{-1}(x) + 2: 2\ Up$$

- Steps:

1. Find the transformation between two original functions.
2. Inverse the transformations found in **1**.

Question 14 Walkthrough.

It is known that $f(x)$ has been transformed to $g(x) = 2f(x-3) + 1$.

State the transformations required for $f^{-1}(x)$ to transform to $g^{-1}(x)$.

$f \to g$

Dil 2 from x

Translate 3 units right

Translate 1 unit up.

$f^{-1} \to g^{-1}$

Dil 2 from y

Translate 3 units up

Translate 1 unit right

Active Recall: Steps on finding transformations of inverse functions

1. Find the transformation between two original functions.
2. Inverse the transformations found in **1.**

Question 15

$x' + 3 = x - 1$

It is known that $f(x) = 2(x-1)^2 + 3$ has been transformed to $g(x) = 4(x+3)^2 + 1$.

a) State the transformations required for $f^{-1}(x)$ to transform to $g^{-1}(x)$.

$f \to g$

Dil 2 from x axis
Translate 5 units down.
Translate 4 units left

$f^{-1} \to g^{-1}$

Dil 2 from y axis
Translate 5 units left
Translate 4 units down.

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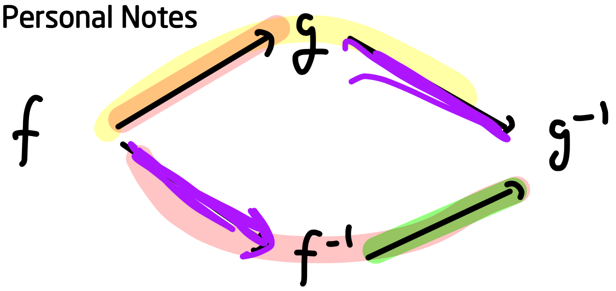
b)



$f \to f^{-1} \to g^{-1}$

Reflection in $y = x$

Dil 2 from y axis
Translate 5 units left
Translate 4 units down.

$f \to g \to g^{-1}$

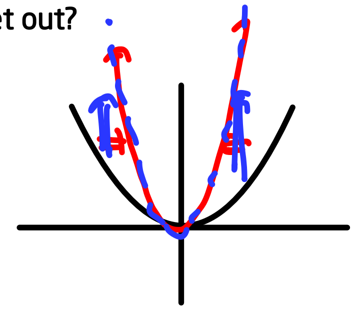
Dil 2 from x axis
Translate 5 units down.
Translate 4 units left
Reflect in $y = x$

Sub-Section: Multiple Pathways for the Same Transformation

Discussion: Consider the transformations required for $f(x) = x^2$ to $g(x) = (2x)^2$. What happens if we take the factor of 2 inside the square bracket out?

$y = x^2$
$\downarrow$
$y = (2x)^2$
$= 4x^2$

Dil $\frac{1}{2}$ fm y
=
Dil 4 from x



Multiple Pathways.

- Same transformations can be done differently by either putting it in or out of the $f()$.
- Commonly, look for basic algebra, index and log laws.

Question 16 Walkthrough.

⇒ MCQ.

Find the transformation for $y = x^3$ to transform into $y = 8x^3$ by using a dilation from the y-axis.

↳ what variable. = x

$y = x^3$

$y = 8x^3$
$= (2x)^3$

Dil $\frac{1}{2}$ fm y axis.

REMINDER: Don't forget Log Law,

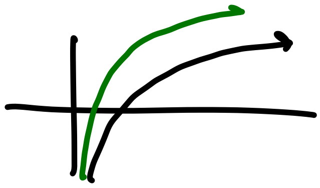
$$\log_a(xy) = \log_a(x) + \log_a(y)$$

Question 17

Dil ≠ fn y

Find the transformation for $y = \log_2(x)$ to transform into $y = \log_2(4x)$ by using translations only.

Translate 2 up.



$\log_2(x) + \log_2(4)$

$\log_2(x) + 2$

NOTE: This skill is important for MCQ questions.

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Sub-Section: Manipulating the Function to Find Appropriate Transformations

Discussion: How can we find transformations between $\sqrt{x^2+1}$ to $\sqrt{(x+1)^2+4}$?

Manipulating the function to find appropriate transformations

- **Steps**

1. Identify the region of x.
2. Identify the region of y.
3. Manipulate the function so that all the changes are within the region of x or y.

Highlighter section

TIP: To find the region of x and y, ask yourself "Where is x inside?" "where is y outside of?"

Question 18 Walkthrough.

Find the appropriate transformations for $\sqrt{x^2+1}$ to transform to $\sqrt{(x+1)^2+4}$.

$y = \sqrt{x^2+1}$

$y = \sqrt{(x+1)^2+4}$

$= \sqrt{4}\sqrt{\frac{(x+1)^2}{4}+1}$

$= 2\sqrt{\left(\frac{x+1}{2}\right)^2+1}$

Dil 2 from x axis

Dil 2 from y

Translate 1 left

NOTE: This was in JMSS SAC 1 of 2024.

<u>Active Recall:</u> Manipulating functions to find appropriate transformations

- **Steps**

1. Identify the ____________________.
2. Identify the____________________.
3. Manipulate the function so that ____________________ are within the region of x or y.

Your turn!

<u>REMINDER</u>: Don't forget Log Law,

$$\log_a(x^y) = y\log_a(x)$$

Question 20

Find the appropriate transformations from $2\log_2((x+1)^3) + 4$ to $\underbrace{\log_2(x^2 - 4x + 4)}_{\log_2((x-2)^2)}$.

$6\log_2(x+1) \cancel{+4}$

$2\log_2(x'-2)$

$x'-2 = x+1$

4 dow.

Dil $\frac{1}{3}$ fm x.

3 rygh

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Section C: Exam 1 (20 Marks)

Question 21 (2 marks)

The series of transformations given by "a dilation by a factor of 4 from the x-axis, reflection in the x-axis, and a translation of 2 units up" yields the same result as the series of transformations given by "a translation by a units down, a reflection in the x-axis, and a dilation by a factor of b from the x-axis." Find the values of a and b.

$-4y+2$ $a = \frac{1}{2}$

$-4(y-\frac{1}{2})$. $b=4$

Question 22 (4 marks)

The following sequence of transformations,

- A translation 2 units up
- A translation 3 units left
- A dilation by factor 2 from the x-axis
- A dilation by factor $\frac{1}{3}$ from the y-axis
- A reflection in the x-axis

$x' = \frac{1}{3}(x-3)$

$x = \frac{1}{3}(x'-3)$

is applied to the function $f(x)$ so that $f(x)$ is mapped to $g(x) = \sqrt{x}$.

a. Find a sequence of transformations that map $g(x)$ to $f(x)$. (2 marks)

Reflection in the x

Dil 3 from the y ax

Dil $\frac{1}{2}$ from x ax

Translate 3 right

Trans 2 down

b. Find the rule for $f(x)$. (2 marks)

$f(x) = -3\sqrt{2(x-3)} - 2$

Question 23 (4 marks)

Consider the functions $f(x) = x^2 - 6x + 11$ and $g(x) = 8(x+1)^2 - 10$.

a. Find a sequence of three transformations in the order DTT that maps $f(x)$ to $g(x)$. (2 marks)

$f: \quad y = (x-3)^2 + 2$

$g: \quad y = 8(x+1)^2 - 10$

$y - 2 = \frac{y' + 10}{8}$

$8y - 16 = y' + 10$

$8y - 20 = y'$

Dil 8 from x axis

Translat 26 down

Translat 4 left

b. Find a different sequence of transfromations in the order DDTT, where one of the dilations is from the y-axis, that also maps $f(x)$ to $g(x)$. (2 marks)

$f: \quad y = (x-3)^2 + 2$

$g: \quad y = 8(x+1)^2 - 10$

$y = 2 \times 4(x+1)^2 - 10$

$= 2(2x+2)^2 - 10$

$2x'+2 = x-3$

$2x' = x-5$

$x' = \frac{1}{2}x - \frac{5}{2}$

4

Dil 2 fm x

Dil ½ fm y

Translat 14 down.

Translate $\frac{5}{2}$ left

Question 24 (5 marks)

Consider the function $f(x) = 3\sqrt{(x-2)^2+3} - 2$ defined on the domain $[0, 6]$.

a. The function g is obtained by applying the following sequence of transformations to f.

- A dilation by factor $\frac{1}{2}$ from the y-axis. ← x
- A dilation by factor 3 from the x-axis.
- A translation 2 units right. ← x
- A reflection in the x-axis.

i. State the domain of g. (1 mark)

$[0, 3]$

Dom g = $[2, 5]$

ii. Find the rule for $g(x)$. (2 marks)

$x' = \frac{1}{2}x + 2$

$y = 3\sqrt{(x-2)^2+3} - 2$

$\downarrow \times 3$

$x = 2(x'-2)$

$y = 9\sqrt{(x-2)^2+3} - 6$

$y = -9\sqrt{(\ \ -2)^2+3} + 6 \rightarrow y = -9\sqrt{(2x-6)^2+3} + 6$

b. Let $h(x) = \sqrt{(x-1)^2+3} + 1$. Write down a sequence of three transformations that map $f(x)$ to $h(x)$. (2 marks)

$f: 3\sqrt{(x-2)^2+3} - 2$ $\quad -2/3$

$h: \sqrt{(x'-1)^2+3} + 1$

$x' - 1 = x - 2$

$x' = x - 1$

Dil $\frac{1}{3}$ from x

Translate $\frac{5}{3}$ up.

Translate 1 left

Question 25 (2 marks)

Consider the function f with inverse function f^{-1}. The function f is transformed to the function g by the following sequence of transformations: A dilation by factor 2 from the x-axis and a translation 1 unit down. Write down the transformations that take f^{-1} to g^{-1}.

Dil 2 from y axis

Translate 1 unit left.

Question 26 (3 marks)

It is known that $f(x)$ has a tangent $y = 2x + 1$ at $x = 2$. $f(x)$ is transformed into $g(x)$ be the following sequence of transformations: A dilation by factor 3 from the x-axis, followed by a dilation by factor $\frac{1}{2}$ from the y-axis followed by a translation 4 units left and a translation 2 units up.

Find the equation of the tangent of $g(x)$ at $x = -3$.

TANGENTS

$y=2x+1$

$x=2$

$\downarrow$

$x=1$

$\downarrow$

$x=-3$

$y=$

$y=2x+1$

$\downarrow$

$y=6x+3$

$\downarrow$

$y=12x+3$

$\downarrow$

$y=12(x+4)+3$

$=12x+51$

$\downarrow$

$y=12x+53$

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Section D: Tech Active Exam Skills

Calculator Tip: Finding Transformed Functions

1.
- Save the function as $f(x)$.

2.
- Substitute the x and y in terms of x' and y'.
- Solve for y!
- Can also apply the transformations directly to $f(x)$. Must make sure you interpret the transformations correctly or you can easily make a mistake doing this.

Question 27 Tech-Active.

Apply the following transformations to $y = 2\sin(2x) + 3$.

1) Save

Dilation by a factor 3 from the x-axis.

$2\sin(2x)+3 \to f(x)$

Dilation by a factor $\frac{1}{2}$ from the y-axis.

Reflection in the y-axis.

2) Transform $f(x)$ using quick method

Translation of 3 units right.

Translation of 4 units down.

$3f(-2(x-3))-4$

Mathematica UDF:

- ApplyTransformList[]

ApplyTransformList[*f[x]* , *{x, y}*, *list of transforms*]
Applies the list of transforms to f[x] in the chronological order.

```
ApplyTransformList[x^2, {x, y}, {x - 1, 2 x, y + 3}]
```

$$4 + x + \frac{x^2}{4}$$

```
ApplyTransformInvList[f[x], {x, y}, {x - 1, 2 x, y + 3}]
```

```
-3 + f[2 (-1 + x)]
```

```
ApplyTransformInvList[Sin[x], {x, y}, {x - π / 2, 2 y, y - 1 }]
```

$$\text{Sin}\left[\frac{x}{2}\right]^2$$

- ApplyTransformInvList[]

ApplyTransformInvList[*f[x]* , *{x, y}*, *list of transforms*]
Applies the list of transforms to f[x] in reverse order and as the inverse to the transforms of *ApplyTransformList.*

```
In[∘]:= ApplyTransformInvList[x^2, {x, y}, {x - 1, 2 * x, y + 3}]
Out[∘]= 1 - 8 x + 4 x^2

In[∘]:= ApplyTransformInvList[f[x], {x, y}, {x - 1, 2 * x, y + 3}]
Out[∘]= -3 + f[2 (-1 + x)]

In[∘]:= ApplyTransformInvList[2 * Cos[x] - 1, {x, y}, {x - Pi / 2, 2 * y, y - 1 }]
Out[∘]= Sin[x]
```

TI UDF:

- transform()

Transform a Function

$$transform\left(\sin(x),x,\left\{x-\frac{\pi}{2},2\cdot y,y-1\right\}\right)$$

- Translation $\frac{\pi}{2}$ units along the neg. x−dir.

$\cos(x)$

- Dilation by factor of 2 from the x−axis

$2\cdot\cos(x)$

- Translation −1 unit along the neg. y−dir.

$2\cdot\cos(x)-1$

Overview:

Apply any sequence of transformations to a function. The program will display the transformed function after each step.

Input:

```
transform(<function>, <variable>,
      <list of transformations>)
```

Other notes:

- The list of transformations can either be presented in a (horizontal or vertical) matrix of expressions or a list of expressions

- transform_inv()

Invert a Transformation

$$transform\_inv\left(x^2,x,\{x-1,2\cdot x,y+3\}\right)$$

- Inverted Transformations:

$\left\{y-3,\frac{x}{2},x+1\right\}$

- Translation −3 units along the neg. y−dir.

x^2-3

- Dilation by factor of $\frac{1}{2}$ from the y−axis

$4\cdot x^2-3$

- Translation 1 unit along the pos. x−dir.

$4\cdot x^2-8\cdot x+1$

Overview:

Find the preimage of a function under a list of transformations. The program will display the list of inverted transformations and the transformed function after each step.

Input:

```
transform_inv(<function>, <variable>,
      <list of transformations>)
```

Other notes:

- The list of transformations can either be presented in a row or column matrix, or a list of expressions

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Section E: Exam 2 (17 Marks)

Question 28 (1 mark)

Let $f: [0,4\pi] \to \mathbb{R}, f(x) = 2\sin\left(\frac{x}{2}\right) + 4$. The graph of f is transformed by a reflection in the x-axis, followed by a dilation of factor 2 from the y-axis, then a dilation by a factor of 2 from the x-axis. The resulting graph is defined by:

A. $g: [0,8\pi] \to \mathbb{R}, g(x) = -4\sin\left(\frac{x}{4}\right) - 8$

B. $g: [0,8\pi] \to \mathbb{R}, g(x) = -8\sin\left(\frac{x}{4}\right) + 4$

C. $g: [0,8\pi] \to \mathbb{R}, g(x) = -8\sin\left(\frac{x}{4}\right) + 8$

D. $g: [0,4\pi] \to \mathbb{R}, g(x) = -4\sin\left(\frac{x}{2}\right) + 8$

1) $f(x)$

2) $2f(\frac{1}{2}x)$

Question 29 (1 mark)

The point $P(2,4)$ lies on the graph of f. The point $Q(6,12)$ lies on the graph of h. A transformation that maps the graph of f to the graph of h also maps the point P to the point Q. The relationship between f and h could be given by:

A. $h(x) = \frac{1}{2}f(x+4)$

B. $h(x) = 2f(x-2)$

C. $h(x) = 3f(x-4)$

D. $h(x) = 3f(x+4)$

P: $f(2) = 4$

$h(6) = 3 \cdot f(6-4)$

$= 3f(2)$

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A, B, C, D.

Question 30 (1 mark)

The transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$, which maps the curve with equation $y = 3\sin(x)$ onto the curve with equation $y = \cos(2x)$, has the rule:

A. $T(x, y) = \left(\frac{x}{2} + \frac{\pi}{2}, -\frac{y}{3}\right)$

B. $T(x, y) = \left(\frac{x}{2} - \frac{\pi}{4}, \frac{y}{3}\right)$

C. $T(x, y) = \left(\frac{x}{2} + \frac{\pi}{2}, \frac{y}{3}\right)$

D. $T(x, y) = \left(-2x + \frac{\pi}{2}, -3y\right)$

1) Save $f(x) = 3\sin(x)$

2) A: $-\frac{1}{3} f\left(2\left(x - \frac{\pi}{2}\right)\right)$ enter.

B: $\frac{1}{3} f\left(2\left(x + \frac{\pi}{4}\right)\right)$ enter

$\Rightarrow \cos(2x)$

Question 31 (1 mark)

A sequence of transformations is applied to create the image rule $y = -2\sqrt{x-3} + \frac{1}{2}$ from the original function $y = \sqrt{x}$, in an appropriate order, could be:

A. A reflection in the x-axis, then a dilation by a factor of 4 from the y-axis, followed by a translation 3 units to the right and finally a translation of $\frac{1}{2}$ unit up.

B. A dilation by a factor of 2 from the y-axis, followed by a reflection in the x-axis, a translation 3 units to the left, and finally a translation of $\frac{1}{2}$ unit up.

C. A reflection in the x-axis, a dilation by a factor of $\frac{1}{4}$ from the y-axis, a translation 3 units to the right, and finally a translation of $\frac{1}{2}$ unit up.

D. A dilation by a factor of 2 from the x-axis, followed by a reflection in the y-axis, a translation 2 units right, and finally a translation of $\frac{1}{2}$ unit up.

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Question 32 (1 mark)

If the graphs of $y = h(x)$ and $y = k(x)$ intersect at (p, q), then the graphs of $y = 2h\left(\frac{x}{3}\right)$ and $y = 2k\left(\frac{x}{3}\right)$ intersect at:

A. $\left(3p, \frac{q}{2}\right)$

B. $\left(\frac{p}{3}, 2q\right)$

C. $(3p, 2q)$

D. $(3p, \frac{q}{3})$

Question 33 (12 marks)

Consider the functions,

$$f: \mathbb{R} \to \mathbb{R}, f(x) = 2x^3 - 3x^2 + 1$$

$$g: \mathbb{R} \to \mathbb{R}, g(x) = (x+1)^2(2x-1)$$ $= 0.$

a.

g: $(-1, 0)$. $(\frac{1}{2}, 0)$ $(0, -1)$

i. Find the coordinates of the axial intercepts of f. (1 mark)

$\boxed{(-\frac{1}{2}, 0). \quad (1, 0).}$ $(0, 1)$

ii. Hence or otherwise, describe a sequence of reflections and dilations that map the graph of f onto the graph of g. (2 marks)

Reflect in y axis.

Reflect in x axis

iii. Describe a sequence of translations, that map the graph of f onto the graph of g. (2 marks)

1 down

1 left.

$g(x) = \underbrace{f(x-a) + b}$.

$= (x+1)^2(2x-1)$

b. The equation to the tangent of g at $x = -2$ is $y = 12x + 19$. Use this to find the equation of the tangent to f when $x = -1$. (2 marks)

$g \rightarrow f$.

1 up

1 right

$y = 12(x-1) + 19 + 1$

$= \boxed{12x + 8}$

Consider the following transformations:

$$T: \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (2x - 1, 3y + 2)$$

$$S: \mathbb{R}^2 \to \mathbb{R}^2, S(x, y) = (-x + 2, 2y - 2)$$

c. Find the rule for the image of g after it has undergone the transformation T followed by the transformation S. (3 marks)

$x' = -(2x-1) + 2. = -2x + 3$

$y' = 2(3y+2) - 2 = 6y + 2$

$x = \frac{x'-3}{-2}$

$y = 6g\left(\frac{x'-3}{-2}\right) + 2$

$= 2 - \frac{3}{2}(x-5)^2(x-2$

d. Find the coordinates of the point $P(u, v)$, if the image of the point P under T and S is the same. (2 marks)

$P: (u, v) \xrightarrow{T} (2u-1, 3v+2)$

$\xrightarrow{S} (-u+2, 2v-2)$

$2u - 1 = -u + 2$

$u = 1.$ $v = -4$

$(1, -4)$

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Contour Check

Learning Objective: [1.3.1] – Applying x' and y' notation to find transformed points, find the interpretation of transformations and altered order of transformations.

Key Takeaways

- The transformed point is called the ______________ and is denoted by ________________.
- The dilation factor is __________________ to the original coordinate.
- Reflection makes the original coordinates the _________________ of their original values.
- Translation _____________ a unit to the original coordinate.
- Transformations should be interpreted when _________________ are isolated.
- The order of transformation follows the ________________ order.
- To change the order of transformations, we either _______________________________.

Learning Objective: [1.3.2] – Find transformed functions.

Key Takeaways

- To transform the function, replace its _______________________ with the new one.

Learning Objective: [1.3.3] – Find transformations from transformed function (Reverse Engineering).

Key Takeaways

- [ ] To find the transformations, simply equate the ____________________ after separating the transformations of x and y.

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