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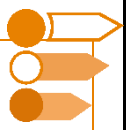
VCE Mathematical Methods ¾
Transformations Exam Skills [1.4]
Homework

Homework Outline:

Compulsory	Pg 2 – Pg 29
Supplementary	Pg 30 – Pg 61

Section A: Compulsory



Sub-Section [1.4.1]: Apply Quick Method to Find Transformations

Question 1

Consider the transformation, $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by the following sequence of transformations:

- A dilation by a factor of 3 from the y-axis, followed by,
- A dilation by a factor of $\frac{1}{2}$ from the x-axis, followed by,
- A translation 3 units upwards and 2 units left.

T maps the graph of $f(x) = \sqrt{x}$ onto the graph of g. Find the rule for g.

Question 2

A transformation, $T(x, y) = (ax + b, cx + d)$ maps the graph of $y = f(x)$ onto the graph of $y = 4 - 2f(3 - x)$. Find the values of a, b, c and d.

Question 3

Describe a sequence of transformations that maps the graph of $y = e^{2x+3} + 2$ onto the graph of $y = 1 - 3e^{x}$.

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Sub-Section [1.4.2]: Apply Transformations of Functions to Find its Domain and range

Question 4

The function $f\ [-1, 3) \to \mathbb{R}$ has a range of $(-3, 5]$.

Find the domain and range of $g(x) = -2f\left(\frac{x}{2} - 1\right)$.

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Question 5

The function $f: [0, 1] \to \mathbb{R}$ has a range of $[0, 5]$.

The following sequence of transformations maps the graph of f onto the graph of g:

- A dilation by a factor of 2 from the x-axis, followed by,
- A reflection in the y-axis, followed by,
- A translation of 3 units left and 1 unit up.

Find the domain and range of g.

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Question 6

Consider the function, $f : [-2, 4) \rightarrow \mathbb{R}, f(x) = 3 - x^2$.

The function $g(x) = af\big(b(x + c)\big) + d$ has a domain of $(-1, 1]$ and a range of $[-1, 3)$.

Find the values of a, b, c and d.

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Sub-Section [1.4.3]: Apply Transformations of Functions to Find Transformed Points and Tangents

Question 7

Find the image of the point $A(2, 3)$ under the transformation:

$$T: \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = \left(2x - 3, -\frac{y}{2} + 1\right)$$

Question 8

The equation of the tangent to the graph of $f(x)$ at the point $(-3, 5)$ is $y = \frac{1}{3}x + 6$.

The transformation, $T(x, y) = (2x + 1, -y)$ maps the graph of f onto the graph of g.

Find the equation of the tangent to the graph of g when x is equal to -5.

Question 9

The equation of the tangent to the graph of $f(x)$ when $x = 2$ is $y = -5x - 2$.

The following sequence of transformations maps the graph of f to the graph of g:

- A dilation by a factor of 2 from the y-axis, followed by,
- A dilation by a factor of 3 from the x-axis, followed by,
- A reflection in the y-axis, followed by,
- A translation of 3 units in the positive direction of the x-axis, followed by,
- A translation of 2 units in the negative direction of the y-axis.

a. Find a point, A on the graph of g.

b. Find the tangent to g at the point A.

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Sub-Section [1.4.4]: Find Transformations with Constraints

Question 10

Consider the transformation, $T: \mathbb{R}^2 \to \mathbb{R}^2$ given by the following sequence of transformations:

- A translation by a factor of a in the positive direction of the x-axis, followed by,
- A dilation by a factor of b from the y-axis.

T maps the graph of $f(x) = x^2 + 1$ onto the graph of $g(x) = 4(x-1)^2 + 1$. Find the values of a and b.

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Question 11

Describe a sequence of two translations, followed by a dilation and a reflection that maps the graph of $y = x^2$ onto the graph of $y = 3(10 - 5x)^2 + 4$.

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Question 12

Let $f(x) = 2(x + 1)^2$ and $g(x) = 2 - 3x$.

A transformation, $T(x, y) = (x + a, by + c)$ maps the graph of $f \circ g$ onto the graph of $g \circ f$.

Find the values of a, b and c.

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Sub-Section [1.4.5]: Find Transformations of the Inverse Functions

Question 13

Let $f: [0, \infty) \rightarrow \mathbb{R}, f(x) = \sqrt{x}$.
Describe a sequence of transformations that maps the graph of f onto the graph of g, where the inverse function of g is defined as such:

$$g^{-1}: [-1, \infty) \rightarrow \mathbb{R}, g(x) = 2(x+1)^2$$

Question 14

Consider the one-to-one functions, $f(x)$ and $g(x)$. The transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2, T(x, y) = (2x + 3, y + 5)$ maps the graph of f onto the graph of g.

Describe a sequence of transformations that maps the graph of f^{-1} onto the graph of g^{-1}.

Question 15

Consider the functions, $f : \mathbb{R} \to \mathbb{R}, f(x) = 2^x + 1$ and $g : \mathbb{R} \to \mathbb{R}, g(x) = 2^{2-6x} - 3$.

The transformation,

$$T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (ax + b, cy)$$

maps the graph of f^{-1} onto the graph of g^{-1}. Find the values of a, b, and c.

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Sub-Section [1.4.6]: Find Opposite Transformations

Question 16

Describe a sequence of transformations that maps the graph of $y = 2(x - 3)^2 + 4$ onto the graph of $y = x^2$.

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Question 17

The following sequence of transformations maps the graph of $y = f(x)$ onto the graph of $y = 2\log_e(3 - x) + 4$:

- A reflection in the y-axis, followed by,
- A dilation by a factor of $\frac{1}{2}$ from the x-axis, followed by,
- A translation of 3 units right and 4 units up.

Find the rule of f.

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Question 18

Let $f: [2, \infty) \to \mathbb{R}, f(x) = 4x^2 - 16x - 1$ and $g: (-\infty, 0] \to \mathbb{R}, g(x) = x^2$.

The transformation,

$$T: \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (ax + b, y + c)$$

maps the graph of f onto the graph of g. Find the values of a, b, and c.

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Sub-Section: Exam 1 Questions

Question 19

a. A translation T maps the graph of $y = x\cos(x)$ onto the graph of $y = (\pi - x)\cos(x)$, where,

$$T: \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (x + a, y)$$

And a is a real constant.

State the value of a.

b. The equation of the tangent to the graph of $y = x\cos(x)$ when $x = 0$ is $y = x$.

Find the equation of the tangent to the graph of $y = (x - \pi)\cos(x)$ when $x = \pi$.

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Question 20

Let $f: (-\infty, -2) \to \mathbb{R}, f(x) = \frac{1}{2x+4}$ and $g: (-\infty, 0) \to \mathbb{R}, g(x) = \frac{1-4x}{2x}$.

a. Show that $f\big(g(x)\big) = x$.

b. Describe a sequence of **translations** that maps the graph of f onto the graph of g.

c. Let $k: (-\infty, -1) \to \mathbb{R}, k(x) = f(2x)$.

Describe a transformation that maps the graph of g onto the graph of k^{-1}, the inverse function of k.

d. Let $h: (-2, \infty) \to \mathbb{R}, f(x) = \frac{1}{2x+4}$ have the same rule as f but with a different domain.

Describe a sequence of transformations that maps the graph of g onto the graph of h.

Question 21

Let $f : \mathbb{R} \to \mathbb{R}, f(x) = x^2 - 9$.

a. Find the co-ordinates of the axis intercepts of f.

b. Let the graph of h be a transformation of the graph of f where the transformations have been applied in the following order:

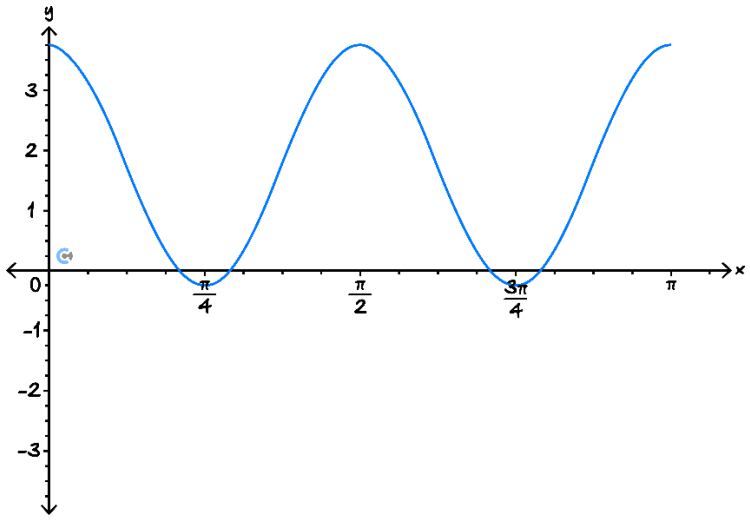
- Dilation by a factor of $\frac{1}{3}$ from the y-axis.
- Dilation by a factor of 2 from the x-axis.
- Translation by two units to the right.

State the co-ordinates of the axis intercepts of h.

Question 22

The graph of $y = f(x)$, where $f : [0, \pi] \to \mathbb{R}, f(x) = 2\cos(4x) + \sqrt{3}$ is shown below.

a. On the axes below, draw the graph of $y = g(x)$, where $g(x)$ is the reflection of f in the horizontal axis.



b. Let $h: D \rightarrow \mathbb{R}, h(x) = 2\cos(4x) + \sqrt{3}$, where $h(x)$ has the same rule as $f(x)$ with a different domain.

The graph of $y = h(x)$ is translated a units in the positive horizontal direction and b units in the negative vertical direction so that it is mapped onto the graph of $y = g(x)$, where $a, b \in (0, \infty)$.

i. Find the value for b.

ii. Find the smallest positive value for a.

iii. Hence, or otherwise, state the domain, D, of $h(x)$.

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Question 23

Describe a sequence of transformations that maps the graph of $y = 3\sin(2x)$ onto the graph of $y = \cos(x)$.

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Sub-Section: Exam 2 Questions

Question 24

The point $A(1, 5)$ lies on the graph of the function f. A transformation maps the graph of f to the graph of g, where $g(x) = 2f(3 - x) + 2$. The same transformation maps the point A to the point B.

The coordinates of the point P are:

A. $f(x) = (2, 12)$

B. $f(x) = (4, 12)$

C. $f(x) = (2, 8)$

D. $f(x) = (4, 8)$

Question 25

The point $A(u, v)$ is transformed by $T(x, y) = \left(3x - 1, -\frac{1}{5}y + 2\right)$.

If the image of A is $(1, 1)$, then A is:

A. $\left(2, \frac{9}{5}\right)$

B. $(2, 5)$

C. $\left(\frac{2}{3}, 5\right)$

D. $\left(\frac{2}{3}, \frac{9}{5}\right)$

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Question 26

The sequence of transformations that maps the graph of $y = e^x$ onto the graph of $y = e^{3x+6}$ is:

A. A translation of 6 units right followed by a dilation by a factor of 3 from the y-axis.

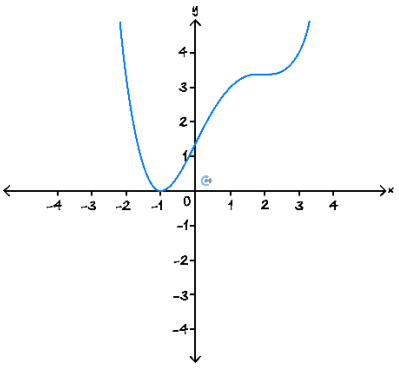
B. A translation of 6 units left followed by a dilation by a factor of $\frac{1}{3}$ from the y-axis.

C. A translation of 2 units left followed by a dilation by a factor of 3 from the y-axis.

D. A dilation by a factor of $\frac{1}{3}$ from the y-axis followed by a translation of 6 units right.

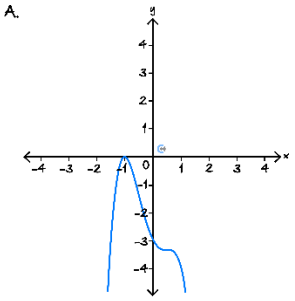
Question 27

The graph of $y = f(x)$ is shown below.

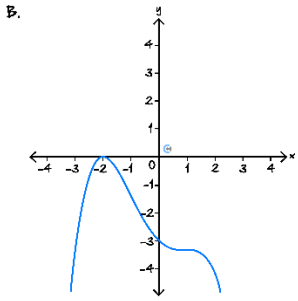


The corresponding part of the graph of the inverse function $f(1 - 2x)$ is best represented by:

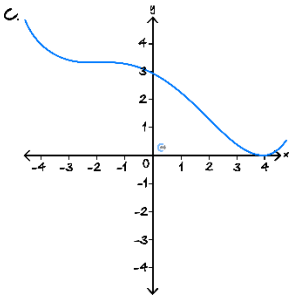
A.



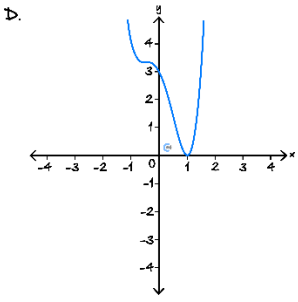
B.



C.



D.



Question 28

The line $y = -\frac{1}{3}x + 5$ is tangent to the graph of f when $x = 3$.

The following sequences of transformations map the graph of f onto the graph of g:

1. A dilation by a factor of 2 from the y-axis, followed by,

2. A translation of 3 units in the negative direction of the y-axis.

Which of the following statements is true?

A. The line $y = -\frac{1}{6}x + 2$ is tangent to g at the point $(3, 4)$.

B. The line $y = -\frac{2}{3}x + 2$ is tangent to g at the point $(6, 1)$.

C. The line $y = -\frac{1}{6}x + 2$ is tangent to g at the point $(6, 1)$.

D. The line $y = -\frac{2}{3}x + 2$ is tangent to g at the point $\left(\frac{3}{2}, 1\right)$.

Question 29

Consider the functions,

$$f : \mathbb{R} \to \mathbb{R}, f(x) = 4x^3 - 3x^2 - 6x + 5$$

$$g : \mathbb{R} \to \mathbb{R}, g(x) = (4x - 5)(x + 1)^2$$

a. Find the co-ordinates of the axial intercepts of f.

_____

_____

_____

b.

i. Hence, or otherwise, describe a sequence of **reflections and dilations**, T that maps the graph of f onto the graph of g.

ii. Describe a sequence of **translations**, S that maps the graph of f onto the graph of g.

c. The image of a point $P(u, v)$ under both S and T is the same.

Find the values of u and v.

d. Show that $h(x) = f(x) + g(x)$ has the property that $h(-x) = -h(x)$.

Question 30

Consider the function, $f : \mathbb{R} \to \mathbb{R}, f(x) = 2^x$.

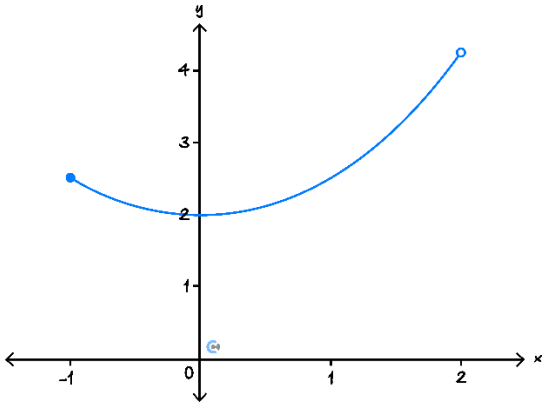
a. The transformation,

$$T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (x + 3, 2y)$$

maps the graph of f onto the graph of $g : \mathbb{R} \to \mathbb{R}, g(x) = af(x)$. Find the value of a.

b. Hence, describe another transformation that maps the graph of f onto the graph of g.

c. Let $h : [-1, 2) \rightarrow \mathbb{R}, h(x) = f(x) + f(-x)$. The graph of h is drawn below.



i. State the range of h.

ii. Hence, or otherwise, state the domain and range of the image of the graph of h under T.

iii. Describe a sequence of possible transformations that maps the graph h onto a graph with a domain of $[-1, 2)$ and a range of $\left(2, \frac{17}{4}\right]$.

d. The equation of the tangent to the graph of f when $x = a$ is $y = 2^a\left(1 + \log_e(2)(x - a)\right)$.

i. Find the equation of the tangent to the graph of h when $x = a$.

ii. Let $k(x) = h(4 - x)$.

Find the equation of the tangent to the graph of k when $x = a$.

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Section B: Supplementary

Sub-Section [1.4.1]: Apply Quick Method to Find Transformations

Question 31

Find the image of the graph of $y = x^2$ under the transformation, $T : \mathbb{R}^2 \to \mathbb{R}^2$, $T(x, y) = (1 - 2x, y + 5)$.

Question 32

Describe a sequence of transformations that maps the graph of $y = x^3$ onto the graph of $y = 2(3x + 2)^3 - 3$.

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Question 33

Find the image of the graph of $y = 2\log_2(x) - 3$ under the following sequence of transformations:

- A dilation by a factor of 3 from the x-axis, followed by,
- A translation of 2 units left and 3 units up, followed by,
- A reflection in the y-axis, followed by,
- A dilation by a factor of 5 from the y-axis.

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Question 34

Consider four linear functions, $p_1(x)$, $p_2(x)$, $q_1(x)$ and $q_2(x)$.

A transformation,

$$T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (x', y')$$

maps the graph of $y = f(x)$ onto the graph of $y = (p_1 \circ p_2 \circ f \circ q_2 \circ q_1)(x)$. Express x' in terms of x and y' in terms of y.

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Sub-Section [1.4.2]: Apply Transformations of Functions to Find its Domain and Range

Question 35

The function $f : \mathbb{R} \rightarrow \mathbb{R}$ has a range of $[2, \infty)$.

The transformation, $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2, T(x, y) = (5 - 2x, 3 + y)$ maps the graph of f onto the graph of g. State the domain and range of g.

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Question 36

The function $f: (-\infty, -1] \rightarrow \mathbb{R}$ has a range of $[-2, \infty)$.

Describe a sequence of transformations that maps the graph of f onto a graph of a function with a domain of $[0, \infty)$ and a range of $(-\infty, 2]$.

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Question 37

Consider the function, $f: \mathbb{R}\backslash\{-2\} \rightarrow \mathbb{R}, f(x) = \frac{3}{(x+2)^2} - 5$.

The following sequence of transformations maps the graph of f onto the graph of g:

- A reflection in the x-axis, followed by,
- A dilation by a factor of 3 from the x-axis, followed by,
- A dilation by a factor of $\frac{1}{2}$ from the y-axis, followed by,
- A translation of 3 units up and 2 units left.

State the domain and range of g.

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Question 38

Let $f: (-2, 1] \to \mathbb{R}, f(x) = 2(x + 1)^2 - 3$.

Consider the transformation, $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (ax + b, cy + d)$ where a and c are both non-zero.

The transformation T maps the graph of f onto the graph of g.

a. Explain why the range of g will always be of the form $[p, q]$ for some real $p < q$.

b. Explain why the domain of g will always be of the form $(p, q]$ or $[p, q)$ for some real $p < q$.

c. For what values of a is the domain of g of the form $(p, q]$.

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Sub-Section [1.4.3]: Apply Transformations of Functions to Find Transformed Points and Tangents

Question 39

The equation of the tangent to the graph of $f(x)$ at the point $(1, 3)$ is $y = 2x + 1$.

The transformation, $T(x, y) = \left(x, \frac{y}{3} + 1\right)$ maps the graph of f onto the graph of g.

Find the equation of the tangent to the graph of g at the point $(1, 2)$.

Question 40

The points $(2, 4)$ and $(4, 7)$ lie on the graph of $f(x)$.

Evaluate $g(2)$, where $g(x) = 3f(6 - x) + 5$.

Question 41

Consider the transformation, $T: \mathbb{R}^2 \to \mathbb{R}^2$ described by the following sequence of transformations:

- A dilation by a factor of 2 from the x-axis, followed by,
- A translation by a factor of 4 in the negative direction of the x-axis, followed by,
- A dilation by a factor of $\frac{1}{3}$ from the y-axis, followed by,
- A translation by a factor of 5 in the positive direction of the y-axis.

The image of $A(u, v)$ under T is $(3, 7)$. Find the values of u and v.

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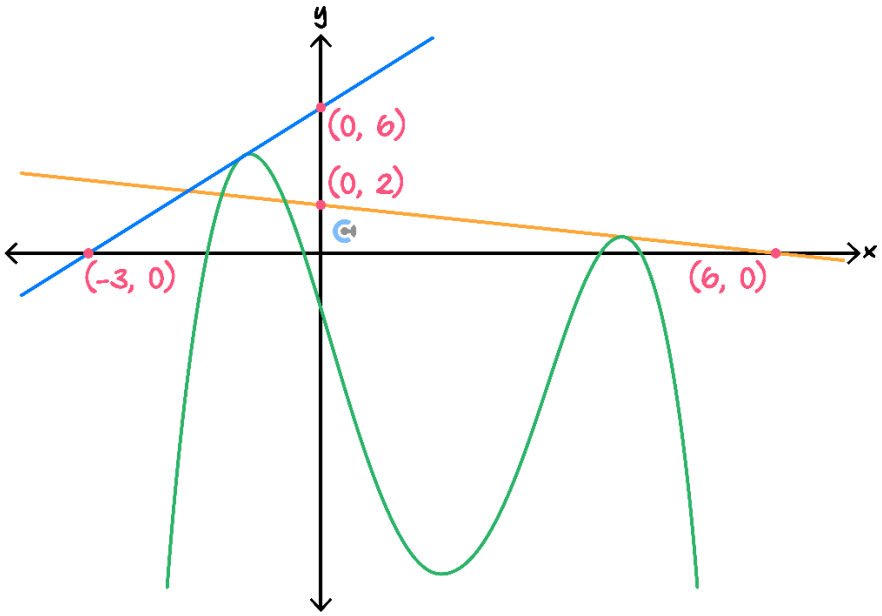
Question 42

The graph of $y = f(x)$ is drawn below along with two tangents at $x = 4$ and at $x = -1$.



Find the equation of the tangent to the graph of $g(x) = 1 - 3f(2 - 2x)$ when $x = -1$.

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Sub-Section [1.4.4]: Find Transformations with Constraints

Question 43

Consider the transformation, $T: \mathbb{R}^2 \to \mathbb{R}^2$ given by the following sequence of transformations:

- A dilation by a factor of a from the x-axis.
- A translation by a factor of b in the positive direction of the y-axis.

T maps the graph of $f(x) = \sqrt{x}$ onto the graph of $g(x) = \sqrt{9x} + 6$.

Find the values of a and b.

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Question 44

The transformation, $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (ax + b, y + c)$ maps the graph of $y = 2^x$ onto the graph of $y = 8 \times 2^{3x-1} - 5$.

Find the values of a, b and c.

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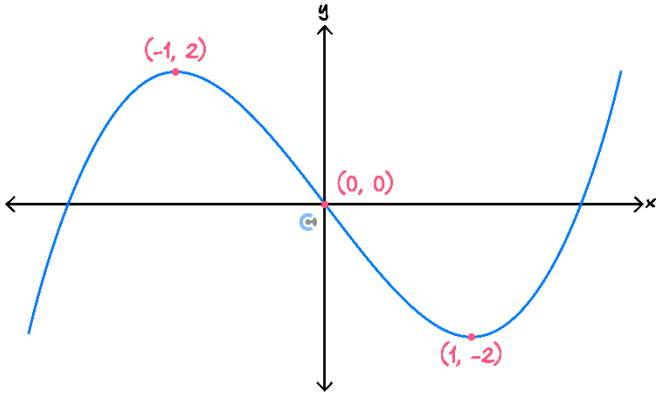
Question 45

The graph of $y = x^3 - 3x$ is drawn below.



The transformation,

$$T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (a - x, b - y)$$

maps the graph of $y = x^3 - 3x$ onto the graph of $y = (x - 1)^3 - 3x + 5$.

Find the values of a and b.

Question 46

Consider the functions,

$$f : [-1, \infty) \to \mathbb{R}, f(x) = x^2 + 2x + 2$$

$$g : (-\infty, 1] \to \mathbb{R}, g(x) = 4(2x - 1)^2 + 3$$

Describe a sequence of a dilation followed by two translations and lastly a reflection that maps the graph of f onto the graph of g.

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Sub-Section [1.4.5]: Find Transformations of the Inverse Functions

Question 47

Consider the function, $f: \mathbb{R}\backslash\{1\} \rightarrow \mathbb{R}, f(x) = \frac{2}{x-1} + 4$.

The transformation, $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2, T(x, y) = (x + a, y + b)$ maps the graph of f onto the graph of its inverse function. Find the values of a and b.

Question 48

Consider the one-to-one functions, $f(x)$ and $g(x)$. The transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2, T(x, y) = (3 - x, 2y + 7)$ maps the graph of f onto the graph of g.

Describe a sequence of transformations that maps the graph of f^{-1} onto the graph of g^{-1}.

Question 49

Let $f: (-\infty, 2] \to \mathbb{R}, f(x) = 3x^2 - 12x + 11$ and $g: [-3, \infty) \to \mathbb{R}, g(x) = 2\sqrt{x+3} + 4$.

a. Describe a sequence of transformations that maps the graph of f onto the graph of g^{-1}.

b. Hence, or otherwise, describe a sequence of transformations that maps the graph of g onto the graph of f^{-1}.

Question 50

Consider the function f which has the property that $f(x-3)-3 = f^{-1}(x)$.

The transformation, $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x,y) = (4x + 1, 2 - y)$ maps the graph of f onto the graph of g.

Describe a sequence of basic transformations (translations, dilations and reflections in the x and y-axis only) that maps the graph of g onto the graph of g^{-1}.

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Sub-Section [1.4.6]: Find Opposite Transformations

Question 51

Describe a sequence of transformations that maps the graph of $y = 3e^{2x+1} - 4$ onto the graph of $y = e^x$.

Question 52

The transformation, $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = \left(2x + 3, \frac{1}{3}y - 4\right)$ maps the graph of $y = f(x)$ onto the graph of $y = x^3$.

Find the rule of f.

Question 53

The following sequence of transformations maps the graph of f onto the graph of $y = \sqrt{x}$, for $x \in (2, \infty)$:

- A dilation by a factor of 3 from the x-axis, followed by,
- A translation of 2 units left and 4 units up, followed by,
- A reflection in both the x-axis and the y-axis.

State the rule and domain of f.

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Question 54

Describe a transformation different from $(x, y) \mapsto (x, y)$, that maps the graph of $y = a(x - k)^5 + b(x - k)^3 + h$ onto itself.

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Sub-Section: Exam 1 Questions

Question 55

The following sequence of transformations maps the graph of $y = f(x)$ onto the graph of $y = \frac{1}{2}\cos\left(\frac{\pi}{3} - 2x\right)$:

- A translation of $\frac{\pi}{6}$ units in the positive direction of the x-axis, followed by,
- A dilation by a factor of $\frac{1}{2}$ in from the y-axis, followed by,
- A dilation by a factor of 2 from the x-axis.

Find the rule of f.

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Question 56

Let $f: \mathbb{R} \to \mathbb{R}, f(x) = 2 - \frac{1}{2}x^3$, and let $g: \mathbb{R} \to \mathbb{R}, g(x) = 6 - 2x$.

a.

i. Find $(g \circ f)(x)$.

ii. Find $(f \circ g)(x)$ and express it in the form $k + m(x - h)^3$, where m, k and h are integers.

b. The transformation, $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (x + b, ay + c)$, where a, b and c are integers, maps the graph of $y = (f \circ g)(x)$ onto the graph of $y = (g \circ f)(x)$.
Find the values of a, b and c.

Question 57

Let $f: [1, \infty) \to \mathbb{R}, f(x) = 4(x - 1)^2 - 3$ and let $g: [2, \infty) \to \mathbb{R}, g(x) = 1 - \sqrt{x - 2}$.

a. Let g^{-1} be the inverse function of g.

i. State the domain and range of g^{-1}.

ii. Find the rule of g^{-1}.

b. The transformation, $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (ax + b, y + c)$ maps the graph of f onto the graph of g^{-1}.

Find the values of a, b and c.

Question 58

Let $f: \mathbb{R}\backslash\{a\} \to \mathbb{R}, f(x) = \frac{1}{x-a} + b$.

a. Find the rule and domain for the graph of f^{-1} in terms of a and b.

b. The following sequence of transformations maps the graph of f to the graph of f^{-1}:

- A translation of 4 units in the positive direction of the x-axis, followed by,
- A translation of 4 units in the negative direction of the y-axis.

Find the value of a in terms of b.

c. Let $g(x) = \frac{1}{x-c} + d$. A transformation,

$$T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (x + h, y + k)$$

maps the graph of g onto the graph of g^{-1}.

What restrictions are there on the values of h and k?

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Sub-Section: Exam 2 Questions

Question 59

The graph of the function f passes through the point $(2, -3)$.

If $h(x) = 3f(x - 2)$, then the graph of the function h must pass through the point:

A. $(0, -1)$

B. $(4, -9)$

C. $(0, -9)$

D. $(4, -1)$

Question 60

The graph of the function $f: \mathbb{R} \to \mathbb{R}, f(x) = 2^x - 1$, is reflected in the y-axis and then translated 2 units to the left and then 3 units up.

Which one of the following is the rule of the transformed graph?

A. $y = 2^{2-x} + 2$

B. $y = 2^{2+x} + 2$

C. $y = \left(\frac{1}{2}\right)^{-2-x} + 2$

D. $y = \frac{1}{4}\left(\frac{1}{2}\right)^{x} + 2$

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Question 61

The transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$, which maps the graph of $y = 4 - \log_e\left(\frac{x-1}{2}\right)$ onto the graph of $y = \log_e(x)$, has the rule:

A. $T(x,y) = \left(\frac{x-1}{2}, 4-y\right)$

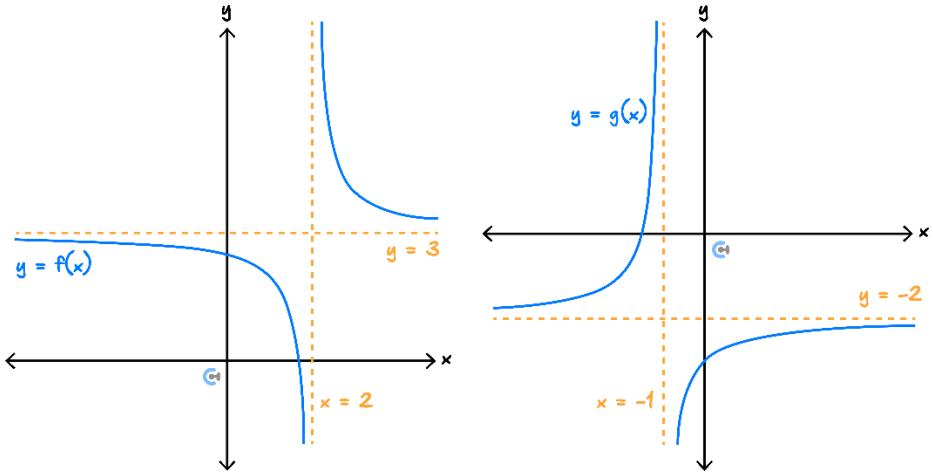
B. $T(x,y) = (2x+1, -y-4)$

C. $T(x,y) = (2x+1, 4-y)$

D. $T(x,y) = \left(\frac{x-1}{2}, -y-4\right)$

Question 62

Consider the graph of f and g below, which have the same scale,



If T transforms the graph of f onto the graph of g, then:

A. $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x,y) = (1-x, y-5)$

B. $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x,y) = (x-3, y-5)$

C. $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x,y) = (x-3, 5-y)$

D. $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x,y) = (1-x, 2-y)$

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Question 63

The graph of the function g is obtained from the graph of the function $f: [-2,3] \to \mathbb{R}, f(x) = 2x^2 - 4x + 5$, by a dilation of factor 2 from the y-axis, followed by a dilation of factor $\frac{1}{3}$, from the x-axis, followed by a reflection in the y-axis, and finally, followed by a translation of 1 unit in the negative direction of the y-axis.

The domain and range of g are respectively:

A. $[-6,4]$ and $\left[\frac{8}{3}, 6\right]$

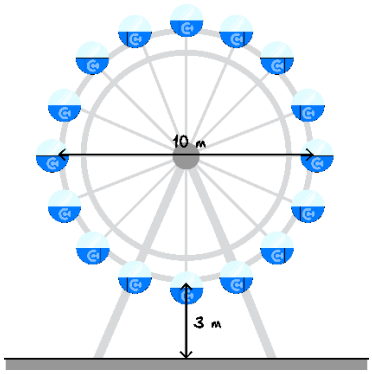
B. $\left[-1, \frac{2}{3}\right]$ and $[21,41]$

C. $[-6,4]$ and $\left[\frac{2}{3}, \frac{17}{3}\right]$

D. $[-6,4]$ and $[0,6]$

Question 64

The Contour Ferris Wheel pictured below takes 30 minutes to complete a trip.



Thus, the height of the bottom of a carriage t minutes after the start of a trip is given by,

$$h(t) = 8 - 5\cos\left(\frac{\pi t}{15}\right)$$

a. Describe a sequence of transformations that maps the graph of $\sin(t)$ onto the graph of h.

b. The horizontal displacement, d from the bottom of the carriage to the centre of the roller coaster t minutes after the start of a trip is,

$$d(t) = 5\sin\left(\frac{\pi t}{15}\right)$$

The transformation, $T(t, y) = (t + a, y + b)$ maps the graph of h onto the graph of d.

i. Find b.

ii. Find a possible value of a.

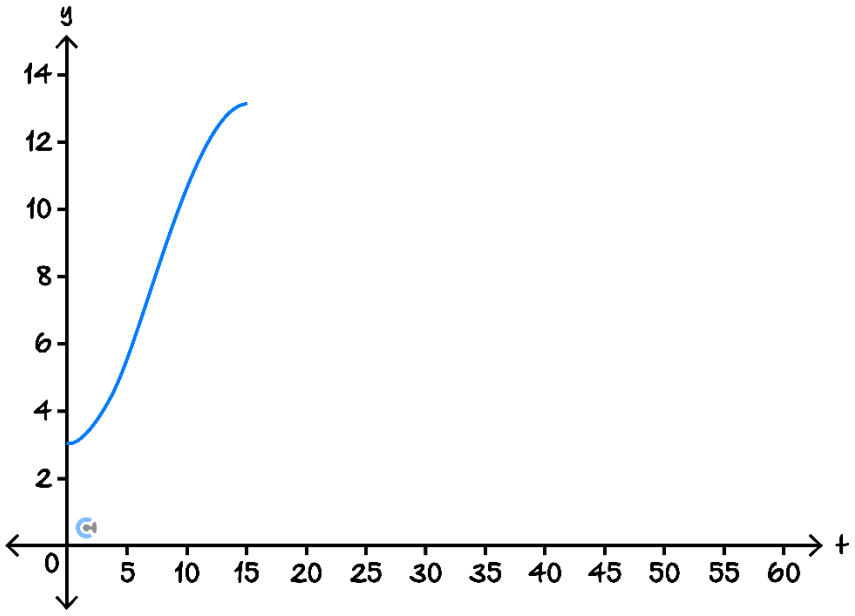
c. 15 minutes into a trip on the Ferris Wheel, Caitlin crashes her car into the Ferris Wheel. This causes the Ferris Wheel to stop for 5 minutes before starting again at half speed.

The height of the Ferris wheel in this trip, $h_1 : [0, r] \rightarrow \mathbb{R}$ is given by the following function:

$$h_1(t) = \begin{cases} h(t) & 0 \leq t < 15 \\ k & 15 \leq t < 20 \\ h(pt + q) & 20 \leq t \leq r \end{cases}$$

Find a set of possible values of p, q, k and r.

d. Part of the graph of h_1 is drawn on the axis below. Draw the rest of the graph of h_1 labelling endpoints with their co-ordinates.



Question 65

Consider the function, $f : (-1, 1) \to \mathbb{R}, f(x) = (2x - 1)^2 (x + 1)$.

a. State the range of f.

b. The following sequence of transformations, T, maps the graph of f onto the graph of g:

- A dilation by a factor of 3 from the x-axis, followed by,
- A translation of 2 units down and 5 units left, followed by,
- A reflection in the y-axis.

i. State the rule of g.

ii. State the domain of g.

iii. State the range of g.

c. The tangent to the graph of f at the point $A\left(-\frac{1}{4}, \frac{27}{16}\right)$ is given by the equation,

$$y = \frac{9}{8} - \frac{9x}{4}.$$

i. Find B, the image of A under T.

ii. Find the equation of the tangent to the graph of g at point B.

d. A transformation, $S : \mathbb{R}^2 \to \mathbb{R}^2, S(x, y) = (-x, a - y)$ maps the graph of f onto itself.

i. State the value of a.

ii. Hence, or otherwise, describe a sequence of transformations in terms of S and T as required, that maps the graph of g to itself, but does not map A to itself.

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