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VCE Mathematical Methods $\frac{3}{4}$ Transformations [1.3] Workbook

Outline:



Introduction To Transformations

Pg 2-8

- Image And Pre-Image
- Dilation
- Reflection
- Translation

Transformation Of Point

Pg 9-19

- Basic Transformation Of Points
- The Order Of Transformations
- Interpreting The Transformation of Points

Transformation Of Functions

Pg 20-26

- Applying Transformations To Functions
- Finding The Applied Transformations

Learning Objectives:

- MM34 [1.3.1] - Applying x' and y' notation to find transformed points, find the interpretation of transformations and altered order of transformations.
- MM34 [1.3.2] - Find transformed functions.
- MM34 [1.3.3] - Find transformations from transformed function (Reverse Engineering).



Section A: Introduction To Transformations

Sub-Section: Image And Pre-Image

Context: Transformations

➤ Transformation is a super important topic in MM34.

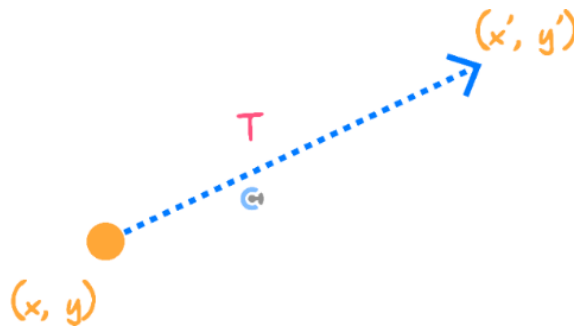
➤ It sets the foundation for the hardest topic:

Topic: Family of Functions

➤ It is important to learn the foundations well for future questions!

What do we call an Original Coordinate and a Transformed Coordinate?

Image and Pre-Image



➤ The original coordinate is called the _____.

➤ The transformed coordinate is called the _____.

Pre-Image: (x, y)

Image: (x', y')

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Question 1

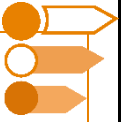
It is known that $(2,3)$ transformed into $(4,5)$. State the value of x' and y' .

NOTE: The x' and y' notation will be used quite heavily!



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Sub-Section: Dilation

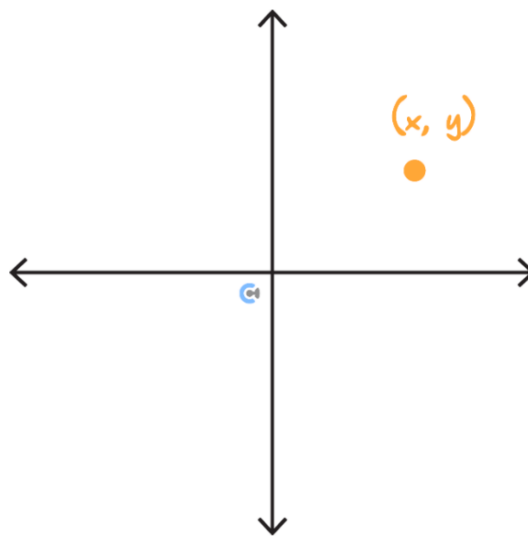


Let's do a quick revision of Dilation!




Exploration: Dilation

➤ Consider the point below:




➤ Let's plot the coordinates:

 P1: Dilation by a factor 2 from the x -axis.

 P2: Dilation by a factor $\frac{1}{2}$ from the x -axis.

 P3: Dilation by a factor 2 from the y -axis.

 P4: Dilation by a factor $\frac{1}{2}$ from the y -axis.

Dilation



Dilation by a factor a from the x -axis: $y' = ay$

Dilation by a factor b from the y -axis: $x' = bx$

Question 2

Find the image (x', y') after applying the following transformations to the point (x, y) .

Dilation by a factor 3 from the x -axis.

Dilation by a factor $\frac{1}{5}$ from the y -axis.

NOTE: We are applying the transformations on (x, y) not (x', y') .



Misconception

"Shouldn't we do the opposite to x ?"

TRUTH: Transformation applies the same for x and y .

How it is represented on the function is different, however. (More on this later!)



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Sub-Section: Reflection

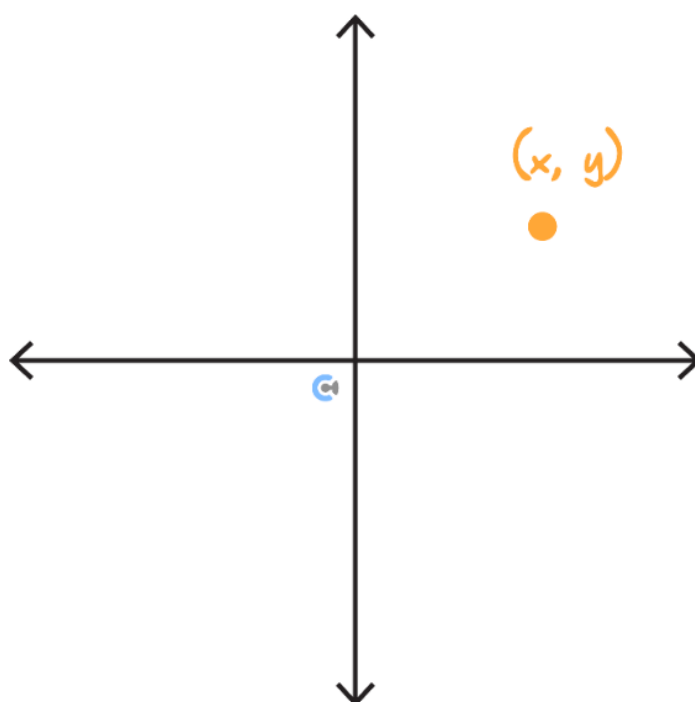


Let's do a quick revision of Reflection!



Exploration: Reflection

➤ Consider the point below:



➤ Let's plot the coordinates:

 P1: Reflection in the x -axis.

 P2: Reflection in the y -axis.

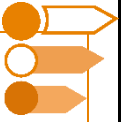
Reflection



Reflection in the x -axis: $y' = -y$

Reflection in the y -axis: $x' = -x$

Sub-Section: Translation

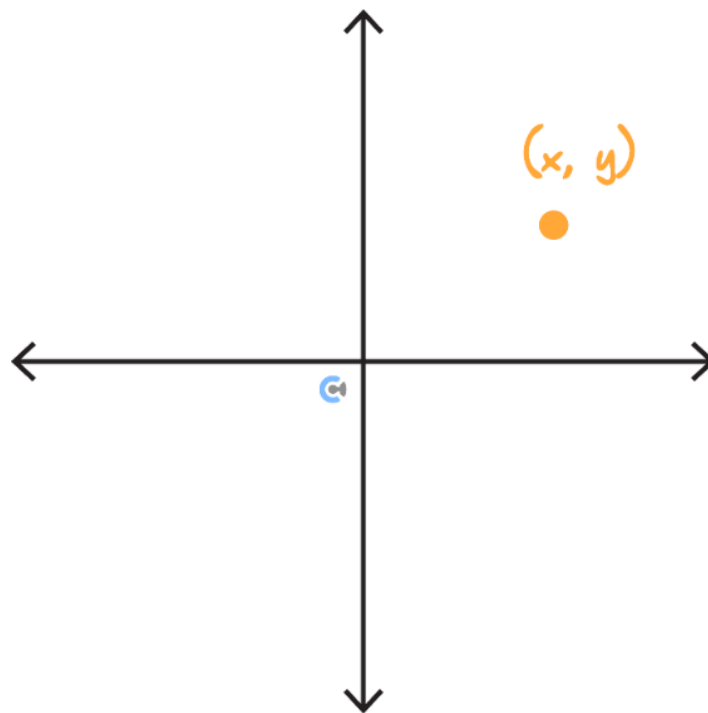


Let's do a quick revision of Translation!





Exploration: Translation

➤ Consider the point below:



➤ Let's plot the coordinates (ignore the scale):

 P1: Translation by 2 units in the negative direction of the x -axis.

 P2: Translation by 3 units in the negative direction of the y -axis.

Translation



Translation by c units in the positive direction of the x -axis: $x' = x + c$

Translation by d units in the positive direction of the y -axis: $y' = y + d$

Question 3

Find the image (x', y') after applying the following transformations to (x, y) .

Translation by 2 units in the positive direction of the x -axis.

Translation by 5 units in the negative direction of the y -axis.

Key Takeaways



- ✓ The transformed point is called the image and is denoted by (x', y') .
- ✓ The dilation factor is multiplied by the original coordinates.
- ✓ Reflection makes the original coordinates the negative of their original values.
- ✓ Translation adds a unit to the original coordinates.

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Section B: Transformation Of Points**Sub-Section: Basic Transformation Of Points**

Let's try to apply all types of transformations to a point!

**Question 4 Walkthrough.**

Find the image (x', y') after applying the following transformations to (x, y) .

Dilation by a factor 2 from the x -axis.

Dilation by a factor 3 from the y -axis.

Reflection in the x -axis.

Translation by 3 units in the negative direction of the x -axis.

Translation by 1 unit in the positive direction of the y -axis.

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Question 5

Find the image (x', y') after applying the following transformations to (x, y) .

Translation by 2 units in the positive direction of the x -axis.

Translation by 1 unit in the negative direction of the y -axis.

Dilation by a factor 4 from the x -axis.

Dilation by a factor $\frac{1}{3}$ from the y -axis.

Reflection in the x -axis.

Question 6 Extension.

Find the image (x', y') after applying the following transformations to (x, y) .

Translation by a units in the negative direction of the x -axis.

Translation by b units in the positive direction of the y -axis.

Dilation by a factor c from the x -axis.

Dilation by a factor $\frac{2}{d}$ from the y -axis.

Reflection in the x -axis.

NOTE: Order is important!

➤ Apply the next transformation on top of everything that has already been done!



Sub-Section: The Order Of Transformations



Discussion: From the previous question, what happens when the translation is applied first?



What is the Order of Transformations the same as?



The Order of Transformation



Order = BODMAS Order

Question 7 Walkthrough.

Consider the point (x, y) was transformed into a point $(2x + 8, y)$ by the transformation T .

Jennifer thinks the transformation was:

“A translation by 8 units in the positive direction of the x -axis, followed by a dilation by a factor 2 from the y -axis.”

Meanwhile, David thinks the transformation was:

“A dilation by a factor 2 from the y -axis, followed by a translation by 8 units in the positive direction of the x -axis.”

Who is correct? And why?

Question 8

Consider the point (x, y) was transformed into a point $(3(x - 1), y)$ by the transformation T .

Mary thinks the transformation was:

“A translation by 1 unit in the negative direction of the x -axis, followed by a dilation by a factor 3 from the y -axis.”

Meanwhile, Sam thinks the transformation was:

“A dilation by a factor 3 from the y -axis, followed by a translation of 1 unit in the negative direction of the x -axis.”

Who is correct? And why?

Question 9 Extension.

Consider the point (x, y) was transformed into a point $(2ax + 4a, y)$ by the transformation T .

Jennifer thinks the transformation was:

“A translation by 2 units in the positive direction of the x -axis, followed by a dilation by a factor $2a$ from the y -axis.”

Meanwhile, David thinks the transformation was:

“A dilation by a factor $2a$ from the y -axis, followed by a translation by $2a$ units in the positive direction of the x -axis.”

Who is correct? And why?



Discussion: If the order is the same as the BODMAS order, how do we change the order of transformations?

Question 10 Walkthrough.

The series of transformations, “a dilation by a factor $\frac{1}{3}$ from the x -axis and a translation by 2 units up” yields the same result as the series of transformations, “a translation by a units up and a dilation by a factor b from the x -axis.” Find the values of a and b .

Question 11

The series of transformations, “a dilation by a factor 3 from the y -axis, a reflection in the y -axis and a translation by 6 units left” yields the same result as the series of transformations, “a translation by c units right, a reflection in the y -axis and a dilation by a factor d from the y -axis.” Find the values of c and d .

Question 12 Extension.

The series of transformations, “a dilation by a factor 2 from the y -axis, a reflection in the y -axis, a dilation by a factor 3 from the x -axis, a translation by 6 units left and a translation by 3 units down”, yields the same result as the series of transformations, “a translation by c units right, a reflection in the y -axis, a dilation by a factor d from the y -axis, a translation k units down, and a dilation by a factor m from the x -axis.” Find the values of c , d , k and m .

NOTE: Dilation factors don't change!



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Sub-Section: Interpreting The Transformation Of Points



Active Recall: Order of Transformation



Order = BODMAS Order

Question 13

Consider the transformation which maps:

$$x' = 3x - 6$$

$$y' = -2(y - 2)$$

a. State the transformations in DRT (Dilation, Reflection, Translation) order.

b. State the transformations in the translation in first order.

NOTE: Expanding or factorising changes the order of transformation.



Discussion: Could the order of x and y transformations change?





Analogy: James' Weight

- James says,

"I was 2 kg lighter last year!"

- Did James gain or lose weight over the year?

- Similarly consider:

$$x = x' - 2$$

- Did x increase or decrease by 2?

- It is more evident if we make x' the subject.

$$x' = x + 2$$



Interpretation of Transformations

- When the _____ x' and y' are the subject, we can read the transformation _____.

$$x' = x + 5 \rightarrow 5 \text{ right}$$

- When the _____ x and y are the subjects instead, we must read the transformation in the _____ way.

- This includes the order of transformation!

$$x = x' - 5 \rightarrow 5 \text{ right}$$

NOTE: This includes the order of transformation!



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Discussion: Can you all see where this is going?

Question 14 Walkthrough.

Consider the transformation which maps:

$$x = \frac{1}{3}x' + 2$$

$$y = 2y' + 4$$

a. State the transformations in DRT (Dilation, Reflection, Translation) order.

b. State the transformations in the translation in first order.

TIP: It is best to make x' and y' the subject before you interpret the transformations.



Question 15

Consider the transformation which maps:

$$x = -2x' - 5$$

$$y = \frac{3}{2}y' + 1$$

a. State the transformations in DRT (Dilation, Reflection, Translation) order.

b. State the transformations in the translation in first order.

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Question 16 Extension.

Consider the transformation which maps:

$$x = ax' + b$$

$$y = -c(y' - d)$$

a. State the transformations in DRT (Dilation, Reflection, Translation) order.

b. State the transformations in the translation in first order.

Key Takeaways


- ✓ Transformations should be interpreted when x' and y' are isolated.
- ✓ The order of transformation follows the BODMAS order.
- ✓ To change the order of transformations, we either factorise or expand.

Section C: Transformation Of Functions

Sub-Section: Applying Transformations To Functions

Let's now work with Functions!

Transformation of Functions

- The aim is to get rid of the old variables, x and y , and have the new variables, x' and y' , instead.

$$y = f(x) \rightarrow y' = f(x')$$

- Steps:

1. Transform the points.
2. Make x and y the subjects.
3. Substitute them into the function.

Question 17 Walkthrough.

Apply the following transformations to $y = x^3$.

Reflection in the y -axis.

Translation by 3 units to the right.

Dilation by a factor 3 from the y -axis.



Your turn!

Question 18

Apply the following transformations to the functions below:

a. $f(x) = x^3$.

Dilation by a factor 2 from the x -axis.

Reflection in the y -axis.

Translation by 3 units to the right.

Dilation by a factor 3 from the y -axis.

b. $f(x) = \log_e(x)$.

Dilation by a factor 2 from the y -axis.

Dilation by a factor $\frac{1}{3}$ from the x -axis.

Translation by 5 units to the left.

Translation by 3 units up.

Reflection in the y -axis.

Question 19 Extension.

Apply the following transformations to $y = 2^x$.

Translation by a units to the right.

Reflection in the y -axis.

Dilation by a factor 3 from the y -axis.

Translation by d units up.

A dilation by a factor 2 from the x -axis.

A reflection in the x -axis.

Active Recall: Interpretation of Transformations


- When the new variables x' and y' are the subject, we can read the transformation directly.

$$x' = x + 5 \rightarrow 5 \text{ right}$$

- When the original variables x and y are the subject instead, we must read the transformation in the opposite way.
- This includes the order of transformation!

$$x = x' - 5 \rightarrow 5 \text{ right}$$

Discussion: Which form is the transformation of x stuck in? $x = \dots$ or $x' = \dots$. Hence, would the transformation of x be represented as it is, or in the opposite way?



Calculator Tip: Finding Transformed Functions

- Save the function as $f(x)$.
- Substitute the x and y in terms of x' and y' .
- Solve for y' !

Question 20 Tech-Active.

Apply the following transformations to $y = 2 \sin(3x) + 10$.

Dilation by a factor 2 from the x -axis.

Dilation by a factor $\frac{1}{4}$ from the y -axis.

Reflection in the y -axis.

Translation of 2 units right.

Translation of 5 units down.

Sub-Section: Finding The Applied Transformations



Now let's go backwards!



Reverse Engineering



➤ Steps:

1. Add the dashes (') back to the transformed function.
2. Make $f()$ the subject.
3. Equate the LHS of the original and transformed functions to the RHS of the original and transformed functions.
4. Make x' and y' the subjects and interpret the transformations.

Question 21 Walkthrough.

Find the transformations required for $y = x^2$ to be transformed to $y = 2\left(\frac{x-3}{2}\right)^2 - 1$.

Your turn!



Question 22

State a series of transformations (in order) that allow $f(x)$ to be transformed into $g(x)$.

a. $f(x) = 3e^{2x-1} + 2$ and $g(x) = e^{\frac{1}{3}x+1} + 1$.

b. $f(x) = (x - 2)^3 + 1$ and $g(x) = 2(2x + 4)^3 - 5$.

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Question 23 Extension.

Find a sequence of transformations required for $y = 3 - 4\sqrt{4(x + 1)^2 + 3}$ to be transformed to $y = 2\sqrt{x^2 - 2x + 4}$.

Key Takeaways


- ✓ We transform the coordinates first, then transform the function.
- ✓ To transform the function, replace its old variables with the new ones.
- ✓ To find the transformations, simply equate LHS with RHS after separating the transformations of x and y .



Contour Check

Learning Objective: [1.3.1] - Applying x' and y' notation to find transformed points, find the interpretation of transformations and altered order of transformations.

Key Takeaways

- ☐ The transformed point is called the _____ and is denoted by _____.
- ☐ The dilation factor is _____ to the original coordinate.
- ☐ Reflection makes the original coordinates the _____ of their original values.
- ☐ Translation _____ a unit to the original coordinate.
- ☐ Transformations should be interpreted when _____ are isolated.
- ☐ The order of transformation follows the _____ order.
- ☐ To change the order of transformations, we either _____.

Learning Objective: [1.3.2] - Find transformed functions.

Key Takeaways

- ☐ To transform the function, replace its _____ with the new one.

Learning Objective: [1.3.3] - Find transformations from transformed function (Reverse Engineering).

Key Takeaways

- ☐ To find the transformations, simply equate the _____ after separating the transformations of x and y .



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