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VCE Mathematical Methods ¾
Transformations [1.3]
Homework Solutions

Homework Outline:

Compulsory Questions	Pg 2 – Pg 19
Supplementary Questions	Pg 20 – Pg 34

Section A: Compulsory Questions

Sub-Section [1.3.1]: Applying Transformations to Points

Question 1

Consider the following transformations of the plane.

- S, a dilation by a factor of 2 from the x-axis.
- T, a translation of 2 units in the positive direction of the x-axis, and 3 units in the negative direction of the y-axis.
- W, a reflection in the y-axis, followed by a dilation by a factor of 2 from the y-axis.

a. Find $S(x, y) = (x', y')$.

$S(x, y) = (x, 2y)$

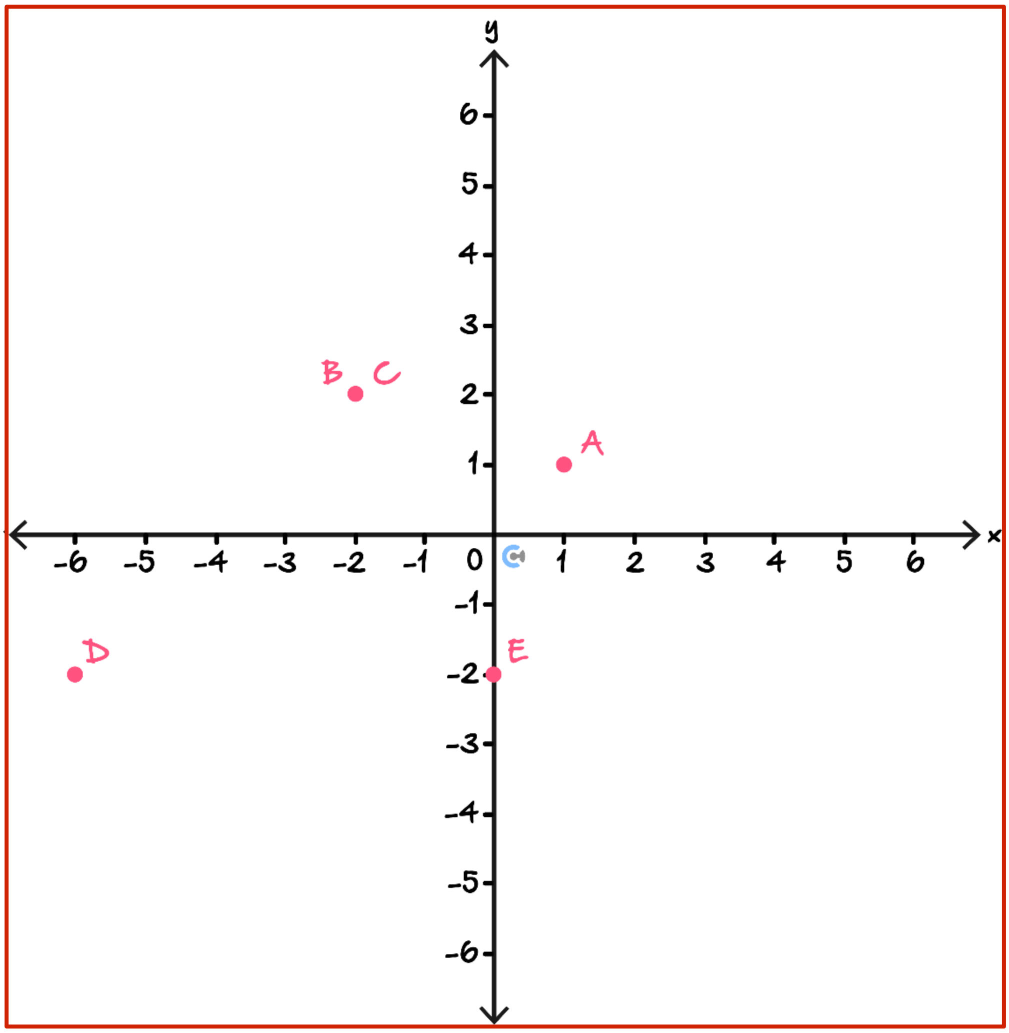
b. Find $T(x, y)$.

$T(x, y) = (x + 2, y - 3)$

c. Find $W(x, y)$.

$W(x, y) = (-2x, y)$

d. The point $A(1,1)$ is drawn on the axis below.



Label the following points on the axis above.

i. B which is the image of A after having S and then W applied to it.

ii. C which is the image of A after having W and then S applied to it.

iii. D which is the image of A after having T and then W applied to it.

iv. E which is the image of A after having W and then T applied to it.

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Question 2

Consider the following transformation, $T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (3x + 6, -4y + 4)$.

T can be described using the following sequence of transformations,

- A dilation by a factor of a from the x-axis, followed by,
- A dilation by a factor of b from the y-axis, followed by,
- A reflection in the x -axis, followed by,
- A translation of c unit in the positive direction of the x-axis, followed by,
- A translation of d unit in the positive direction of the y-axis.

a. Find a, b, c and d.

We need to turn x into $3x$ and y into $4y$ using our two dilations, since the reflection will take $4y$ to $-4y$ and then we can worry about the translations.

Hence $a = 4$ and $b = 3$.

Now we simply translate our sequence to the right point, meaning $c = 6$ and $d = 4$.

b. Describe T as a sequence of two translations, followed by two dilations, and a reflection.

We can rewrite our transformation as follows, $T(x, y) = (3(x + 2), -4(y - 1))$. From here we see that we must get our translations to map (x, y) to $(x + 2, y - 1)$ before applying our dilations / reflections. Hence our sequence of transformations is as follows,

- A translation of 2 units in the positive direction of the x-axis, followed by,
- A translation of 1 unit in the negative direction of the y-axis, followed by,
- A dilation by a factor of 3 from the y-axis, followed by,
- A dilation by a factor of 4 from the x-axis, followed by,
- A reflection in the x-axis.

c. Find the pre-image of $(3, -8)$ under T.

We solve $(3x + 6, -4y + 4) = (3, -8)$ for x and y.

Thus, $3x + 6 = 3 \implies x = -1$ and $-4y + 4 = -8 \implies y = 3$.

Hence the pre-image of $(3, -8)$ under T is $(-1, 3)$.

Question 3

Consider the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ described by the following sequence of transformations.

- A reflection in the line $y = x$, followed by,
- A translation of 6 units in the negative direction of the x-axis, followed by,
- A dilation by a factor of $\frac{1}{3}$ from the y-axis, followed by,
- A dilation by a factor of 5 from the x-axis, followed by,
- A translation of 7 units in the positive direction of the y-axis, followed by,
- A reflection in the x-axis.

a. Let (x', y') be the image of (x, y) under T.

Express x and y in terms of x' and y'.

In order, the transformations take the point (x, y) to,

$$(x, y) \mapsto (y, x) \mapsto (y - 6, x) \mapsto \left(\frac{y-6}{3}, x\right)$$

$$\mapsto \left(\frac{y-6}{3}, 5x\right) \mapsto \left(\frac{y-6}{3}, 5x + 7\right)$$

$$\mapsto \left(\frac{y-6}{3}, -(5x + 7)\right) = \left(\frac{y}{3} - 2, -5x - 7\right) = (x', y')$$

Hence $x' = \frac{y}{3} - 2$ and $y' = -5x - 7$. We re-arrange for x and y to get,

$$x = -\frac{y' + 7}{5} \qquad \text{and} \qquad y = 3x' + 6$$

b. The transformation T can also be described using the following sequence of transformations.

- A dilation by a factor of $\frac{1}{3}$ from the x-axis, followed by,
- A dilation by a factor of 5 from the y-axis, followed by,
- A reflection in the y axis, followed by,
- A reflection in the line $y = x$, followed by,
- A translation of -2 units in the positive direction of the x-axis, followed by,
- A translation of -7 units in the positive direction of the y-axis.

Fill in the blanks.

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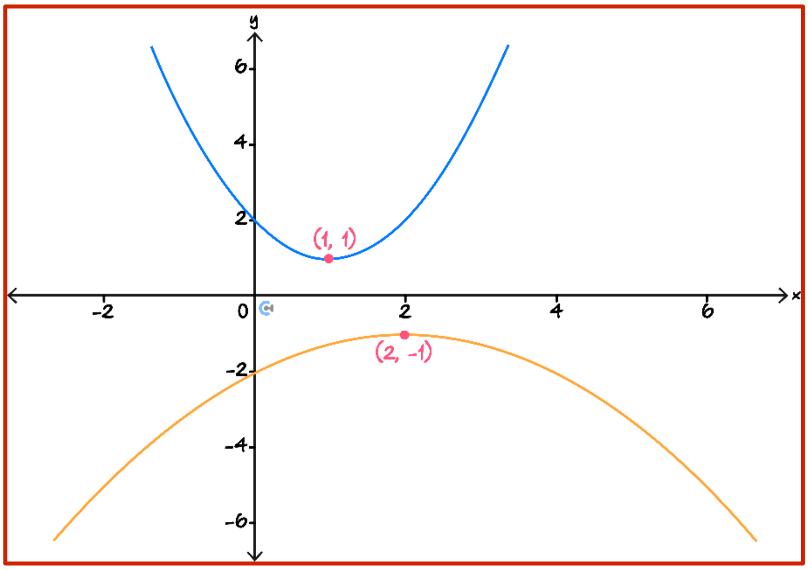
Sub-Section [1.3.2]: Transforming Graphs of Functions.

Question 4

a. The graph of $f(x)$ is shown below.



On the same axes, sketch the graph of $g(x) = -f\left(\frac{x}{2}\right)$.

b. Let $f(x) = \log_e(x)$. The transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, $T(x, y) = (x + 1, y + 3)$ maps the graph of $f(x)$ onto the graph of $g(x)$. Find the rule for $g(x)$.

Consider some points (x', y') on the graph of $g(x)$.

We observe that $(x', y') = T(x, y) = (x + 1, y + 3)$ for some point (x, y) on the graph of $f(x)$. To relate x' with y' we express x in terms of x' and y in terms of y', specifically,

$$x' = x + 1 \implies x = x' - 1 \quad \text{and} \quad y' = y + 3 \implies y = y' - 3$$

We substitute the above two into $y = \log_e(x)$ to relate x' with y'. Hence

$$y' - 3 = \log_e(x' - 1) \implies y' = \log_e(x' - 1) + 3$$

Thus the rule for $g(x)$ is, $g(x) = \log_e(x - 1) + 3$

c. Find the rule for the image of the graph of $y = \sin(x)$ under the transformation,

$$S : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = \left(\frac{x}{2}, -y\right).$$

We apply the same logic as in part b.

Observe that $x' = \frac{x}{2} \implies x = 2x'$, and $y' = -y \implies y = -y'$. Substituting these into $y = \sin(x)$ yields,

$$-y' = \sin(2x') \implies y' = -\sin(2x')$$

Thus the rule for the image of the graph of $y = \sin(x)$ under S is, $y = -\sin(2x)$

Question 5

a. Let $f(x) = 2x^2 + 4$. The transformation $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x,y) = (4x + 2, -y)$ maps the graph of $f(x)$ onto the graph of $g(x)$. Find the rule for $g(x)$.

Consider some points (x', y') on the graph of $g(x)$.

We observe that $(x', y') = T(x, y) = (4x + 2, -y)$ for some point (x, y) on the graph of $f(x)$. To relate x' with y' we express x in terms of x' and y in terms of y', specifically,

$$x' = 4x + 2 \implies x = \frac{x' - 2}{4} \quad \text{and} \quad y' = -y \implies y = -y'$$

We substitute the above two into $y = 2x^2 + f$ to relate x' with y'. Hence

$$-y' = 2\left(\frac{x' - 2}{4}\right)^2 + 4 \implies y' = -\frac{(x' - 2)^2}{8} - 4$$

Thus the rule for $g(x)$ is, $g(x) = -\frac{(x - 2)^2}{8} - 4$

b. Find the rule for the image of the graph of $y = -\sqrt{x+1} + 3x$ under the transformation,

$$S: \mathbb{R}^2 \to \mathbb{R}^2, T(x,y) = (-x + 5, y + 1).$$

We apply the same logic as in part a.

Observe that $x' = -x + 5 \implies x = 5 - x'$ and $y' = y + 1 \implies y = y' - 1$. We substitute the following two values into $y = -\sqrt{x+1} + 3x$ to get,

$$y' - 1 = -\sqrt{6 - x'} + 3(5 - x') \implies y' = -\sqrt{6 - x'} + 16 - 3x'$$

Thus the rule for the image of the graph of $y = -\sqrt{x+1} + 3x$ under S is, $y = -\sqrt{6 - x} + 16 - 3x$

c. Let $f(x) = x^2 + 5$, and let $g(x) = 3(f(x+2) - 6)$.

i. Find and simplify $g(x)$.

$$\begin{aligned} g(x) &= 3f(x+2) - 18 \\ &= 3\left((x+2)^2 + 5\right) - 18 \\ &= 3(x+2)^2 - 3 \end{aligned}$$

ii. Solve $g(x) = 0$.

$$\begin{aligned} g(x) = 0 &\implies 3(x+2)^2 = 3 \\ &\implies (x+2) = \pm 1 \\ &\implies x = -2 \pm 1 = -3, -1 \end{aligned}$$

Question 6

a. Consider the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that the following sequence of transformations can describe.

- A dilation by a factor of 3 from the x-axis, followed by,
- A reflection in the y-axis, followed by,
- A translation of 2 units up and 4 units left.

Find the rule for the image of the graph of $y = e^{2x+3}$ under T.

We see that under T,

$$(x, y) \mapsto (x, 3y) \mapsto (-x, 3y) \mapsto (-x-4, 3y+2) = (x', y')$$

We use algebra to express x in terms of x' and y in terms of y', specifically,

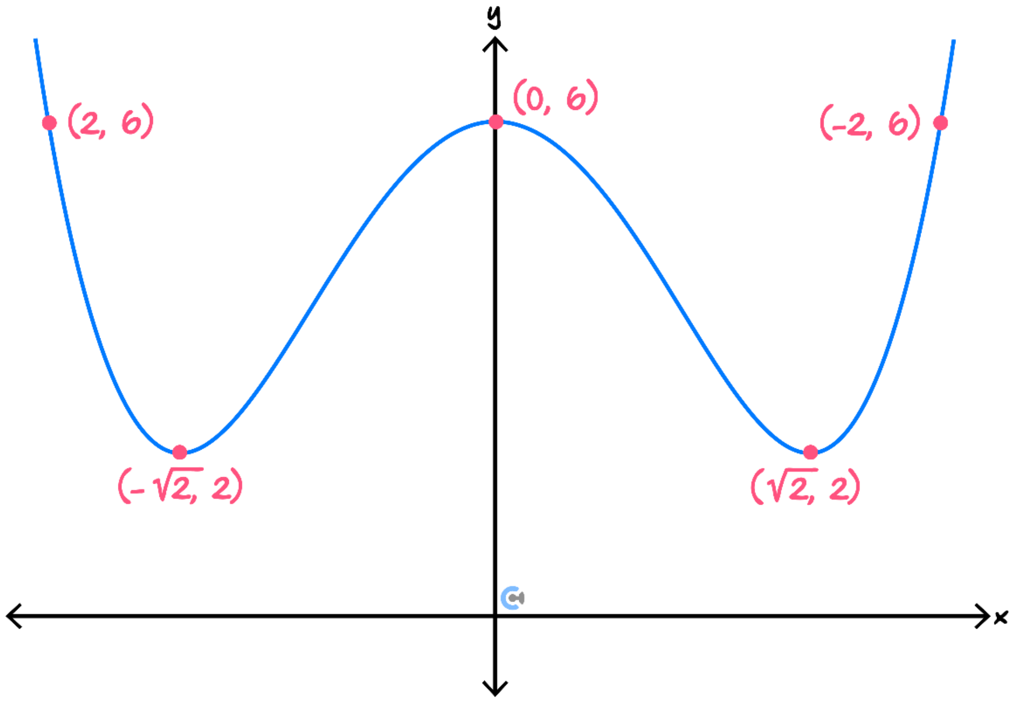
$$x' = -x - 4 \implies x = -x' - 4 \qquad \text{and} \qquad y' = 3y + 2 \implies y = \frac{y'-2}{3}.$$

We substitute the following values into the equation $y = e^x$ to get,

$$\frac{y'-2}{3} = e^{2(-x'-4)+3} \implies y' = 3e^{-2x'-5} + 2.$$

Thus the rule for the image of the graph of $y = e^{2x+3}$ under T is, $3e^{-2x-5} + 2$.

b. The graph of $f(x)$ is shown below.



The function $g(x)$ has a rule, $g(x) = -f(x) + a$.

For what values of a does the equation $g(x) = f(x)$ have 4 solutions?

We rewrite the equation $g(x) = f(x)$ in terms of f. This yields,

$$f(x) = -f(x) + a \implies f(x) = \frac{a}{2}$$

From the graph below, we see that $f(x) = b$ has four solutions for $b \in (2, 6)$, hence $f(x) = g(x)$ will have four solutions for $a \in (4, 12)$.

c. The transformation $S(x, y) = (-5x + 3, 3y - 2)$ maps the graph of $f(x)$ onto the graph of $g(x)$.

If the rule for $g(x) = \sqrt{x}$, find the rule for $f(x)$.

Consider some point (x', y') on the graph of g, then $(x', y') = (-5x + 3, 3y - 2)$ for some (x, y) on the graph of f.

We substitute $x' = -5x + 3$ and $y' = 3y - 2$ into $y' = \sqrt{x'}$ to relate x with y. Thus,

$$3y - 2 = \sqrt{-5x + 3} \implies y = \frac{\sqrt{-5x + 3} + 2}{3}$$

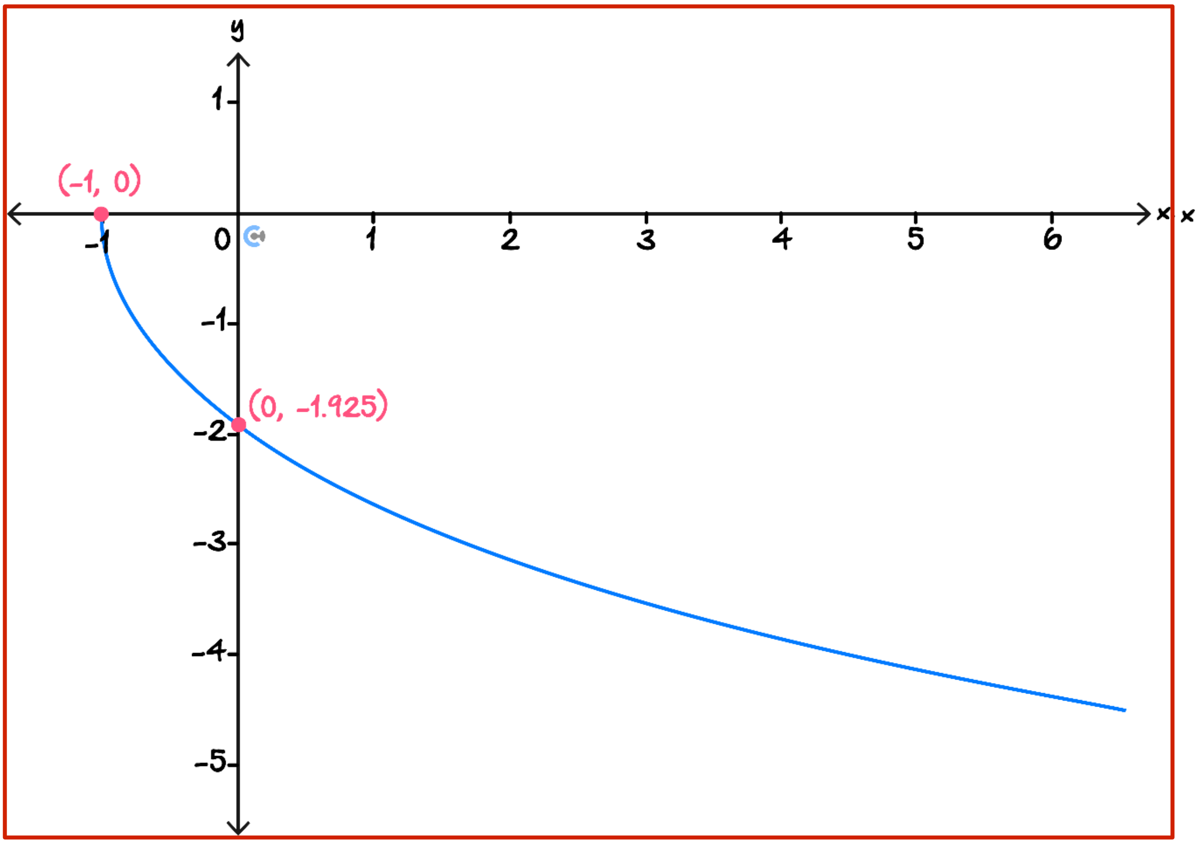
Hence the rule for f is, $f(x) = \dfrac{\sqrt{-5x + 3} + 2}{3}$.

d. (Tech-Active.)

Let $f: [0, \infty) \to \mathbb{R}, f(x) = e^x + e^{-x}$.

The transformation $T: \mathbb{R}^2 \to \mathbb{R}^2, (x, y) \mapsto (y - 3, -2x)$ maps the graph of $f(x)$ onto the graph of $g(x)$.

Sketch the graph of $g(x)$ on the axis below, labelling endpoints and axis intercepts with their coordinates, correct to 3 decimal places.



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Sub-Section [1.3.3]: Find Transformations From Transformed Function

Question 7

a. Let $f(x) = x^2$ and $g(x) = 2x^2 + 1$.

Describe a transformation that maps the graph of f onto the graph of g.

Choose some point (x', y') on the graph of $g(x)$. Then $\frac{y'-1}{2} = f(x')$, hence there is some point (x, y) on the graph of $f(x)$ such that,

$$\left(x', \frac{y'-1}{2}\right) = (x, y) \implies (x', y') = (x, 2y+1)$$

This gives us our transformation, $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (x, 2y+1)$

b. The transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$, $(x, y) \mapsto (ax + b, cy + d)$ maps the graph of $y = e^x$ to the graph of $y = 2e^{x-4} + 3$.

Find the values of a, b, c and d.

We first apply T to the graph of $y = e^x$.

Let (x', y') be a point on the image of $y = e^x$ under T. Then there is some pair (x, y) on the graph of $y = e^x$ such that,

$$(x', y') = (ax + b, cx + d) \implies (x, y) = \left(\frac{x'-b}{a}, \frac{y'-d}{c}\right).$$

We substitute this into the equation $y = e^x$ to get,

$$y' = ce^{\frac{x'-b}{a}} + d = 2e^{x'-4} + 3$$

By comparing coefficients, we see that, $a = 1, b = 4, c = 2$ and $d = 3$.

c. A transf[...]raph of $g(x) = 2(x+1)^2 + 3$.

- A dilation by a factor of 2 from the x-axis, followed by
- A translation of -1 unit(s) in the positive direction of the x-axis, followed by
- A translation of 3 units in the positive direction of the y-axis.

T can be[...]

- A dilation by a factor of _________ from the x-axis, followed by,
- A translation of _________ unit(s) in the positive direction of the x-axis, followed by,
- A translation of _________ units in the positive direction of the y-axis.

Fill in the blanks.

We observe that $g(x) = 2f(x+1) + 3$. Thus any pair (x', y') on the graph of $g(x)$ satisfies,

$$\frac{y'-3}{2} = f(x'+1)$$

Hence we can relate some pair (x, y) on the graph of $f(x)$, to (x', y') by,

$$(x, y) = \left(x'+1, \frac{y'-3}{2}\right) \implies (x', y') = (x-1, 2y+3)$$

We can then describe our transformation as above.

Question 8

a. Let $f(x) = \frac{1}{2x+2}$.

The transformations:

$$S : \mathbb{R}^2 \to \mathbb{R}^2, (x, y) \mapsto (x + a, by),$$
and
$$T : \mathbb{R}^2 \to \mathbb{R}^2, (x, y) \mapsto (c(x + d), y).$$

Both map the graph of $y = \frac{1}{x}$ onto the graph of f.

Find the values of a, b, c and d.

We first apply S onto the graph of $y = \frac{1}{x}$, this yields the graph of,

$$y = \frac{b}{x - a}$$

Comparing coefficients to $f(x) = \frac{1}{2x+2}$ we see that $b = \frac{1}{2}$ and $a = -1$.

Now we apply T onto the graph of $y = \frac{1}{x}$, this yields the graph of,

$$y = \frac{1}{\frac{x}{c} - d}.$$

Comparing coefficients to $f(x) = \frac{1}{2x+2}$ we see that $c = \frac{1}{2}$ and $d = -2$.

b. The function $s : [0, 365] \to \mathbb{R}$, $s(t) = \frac{200}{t+1}$ models the number of minutes per day James smiles t days after the start of the school year.

A new function $s_1(t)$ models the number of minutes Sam smiles. It is known that $s_1(0) = s(0)$, but s_1 decreases at half the rate of s at any point in time.

State a sequence of two transformations that maps s to this new model s_1.

We first deal with the statement, "s_1 decreases at half the rate of s at any point in time". This can be achieved by a dilation by a factor of $\frac{1}{2}$ from the t-axis.

However now we have the rule $\frac{100}{t+1}$, which evaluated at $t = 0$ is 100. Since we require that $s_1(0) = s(0)$ we translate our model 100 minutes upwards. Hence the sequence of transformations is,

- A dilation by a factor of $\frac{1}{2}$ from the t-axis, followed by,
- A translation 100 minutes upwards.

c. Let $f(x) = \tan(x)$ and $g(x) = -2\tan(3x + 6) + 8$.

Fill in ... e graph of $f(x)$ onto the graph ...

- A ...
- A translation of ________units in the positive direction of the x-axis, followed by,
- A translation of ______ ...
- A dilation by a factor o ...
- A reflection in the x -a ...

- A dilation by a factor of $\frac{1}{3}$ from the y-axis, followed by,
- A translation of -2 units in the positive direction of the x-axis, followed by,
- A translation of -4 units in the positive direction of the y-axis, followed by,
- A dilation by a factor of 2 from the x-axis, followed by,

Choose a point (x', y') on the graph of g. We observe that,

$$\frac{y' - 8}{-2} = f(3x' + 6)$$

Hence there is some point (x, y) on the graph of f such that,

$$(x, y) = \left(3x' + 6, \frac{y' - 8}{-2}\right) \implies (x', y') = \left(\frac{x - 6}{3}, -2y + 8\right) = \left(\frac{x}{3} - 2, -2(y - 4)\right)$$

The last equation is useful for us since we are first dilating then translating x, but we are first translating then dilating y. Hence we can read off the required transformations from the last equation.

Question 9

a. Describe a sequence of three transformations that map the graph of $f(x) = \sqrt{4x - x^2}$ onto the graph of $g(x) = \sqrt{1 - x^2}$.

We first complete the square in $f(x)$ to get, $f(x) = \sqrt{-(4 - 4x + x^2) + 4} = \sqrt{4 - (x - 2)^2}$.

Geometrically, this is the top half of a circle centered at $(2, 0)$ with radius 2, whilst the graph of g is the top half of a circle centered at the origin with radius 1. So we should first translate f to be centered at the origin, and then dilate f from both the x and y-axis by a factor of $\frac{1}{2}$ to shrink it's radius to 1. Hence our transformation is,

- A translation of -2 units in the positive direction of the x-axis, followed by,
- A dilation by a factor of $\frac{1}{2}$ from the x-axis, followed by,
- A dilation by a factor of $\frac{1}{2}$ from the y-axis.

We can check our intuition, since our transformation takes (x, y) to $\left(\frac{x-2}{2}, \frac{y}{2}\right) = (x', y')$. Hence,

$$(x, y) = (2x' + 2, 2y')$$

Plugging those numbers into the graph of f yields,

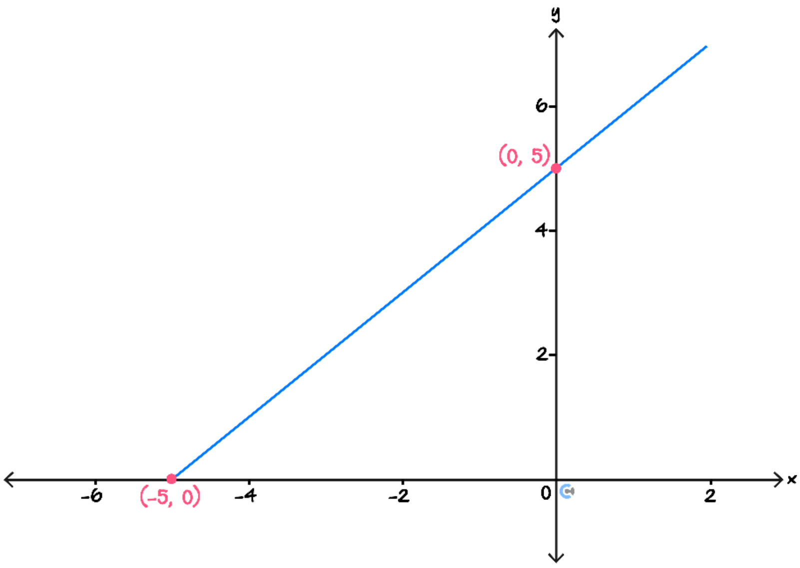
$$2y' = \sqrt{8x' + 8 - 4(x' + 1)^2} = \sqrt{8x' + 8 - 4(x')^2 - 8x' - 4} = \sqrt{4 - 4(x')^2} = 2\sqrt{1 - (x')^2}.$$

From here it is obvious that the point (x', y') sit on the graph of $y = g(x)$, hence our answer is correct.

b. Let $f : (-\infty, -1] \to \mathbb{R}, f(x) = x^2 + 2x$.

A transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, $(x, y) \mapsto (ax + b, y + c)$ maps the graph of $f(x)$ onto the graph of $g(x)$.

The graph of $y = \sqrt{g(x)}$ is shown below.



Find the values of a, b and c.

Let $h(x) = \sqrt{g(x)}$. From the graph above we see that the function h is $h : [-5, \infty) \to \mathbb{R}, h(x) = x + 5$.

Hence the function g is, $g : [-5, \infty) \to \mathbb{R}, g(x) = (x + 5)^2$.

We complete the square for f to get, $f(x) = (x + 1)^2 - 1$.

Let (x', y') be a point on the graph of g. Hence $(x', y') = (ax + b, y + c)$ for some point on the graph of f. Hence,

$$(x, y) = \left(\frac{x' - b}{a}, y' - c\right) \implies y' = f\left(\frac{x' - b}{a}\right) + c = \left(\left(\frac{x' - b}{a} + 1\right)^2 - 1\right) + c = (x' + 5)^2$$

Now, since the domain of g is on the positive side of the line $y = -5$, but the domain of f is on the negative side of the line $y = -1$ we will require a reflection in the y-axis. Hence $a < 0$, and by comparing magnitudes we see that $a = -1$, thus,

$$(b - x' + 1)^2 = (x' - b - 1)^2 = (x' + 5)^2 \implies -b - 1 = 5 \implies b = -6$$

Lastly we see that $c = 1$.

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c. Describe 3 different transformations of the plane that map the graph of $y = x^3$ onto the graph of $y = 3(x - 1)^3 + 5$.

Consider a transformation $T(x, y) = (ax + b, cy + d)$. We apply it to the graph of $y = x^3$ to get,

$$y = c\left(\frac{x-b}{a}\right)^3 + d = \frac{c}{a^3}(x-b)^3 + d$$

From here it is obvious that $b = 1, d = 5$ and $\frac{c}{a^3} = 3$. If we choose $a = 1$ then $c = 3$. If we choose $a = -1$, then $c = -3$ and if we choose $a = \frac{1}{\sqrt[3]{3}}$, then $c = 1$. Hence 3 possible transformations are,

$$T_1 : \mathbb{R}^2 \to \mathbb{R}^2, (x, y) \mapsto (x + 1, 3y + 5)$$

or $$T_2 : \mathbb{R}^2 \to \mathbb{R}^2, (x, y) \mapsto (-x + 1, -3y + 5)$$

or $$T_2 : \mathbb{R}^2 \to \mathbb{R}^2, (x, y) \mapsto \left(\frac{x}{\sqrt[3]{3}} + 1, y + 5\right).$$

d. **(Tech-Active)**

Let $f(x) = \cos(\pi(x^2 + 16x))$.

State a transformation that maps the graph of $y = f(x)$ onto the graph of $y = 2\cos(\pi x^2)$.

We observe that $\cos(2k\pi + x) = \cos(x)$ for all x and integer k. We observe that $x^2 + 16x = (x + 8)^2 - 64$, thus,

$$\cos(\pi(x^2 + 16)) = \cos(\pi(x + 8)^2)$$

From here it is clear that our transformation is given by, $(x, y) \mapsto (x - 8, 2y)$

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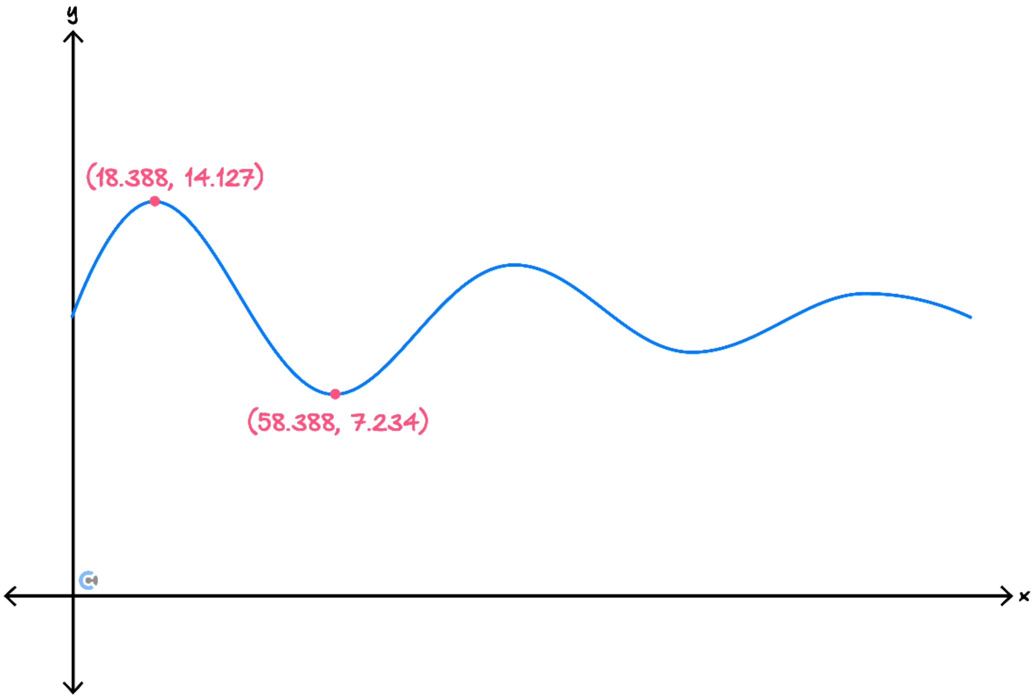
Question 10 Tech-Active.

Part of the graph of $f : [0, \infty) \to \mathbb{R}, f(x) = 4e^{-0.01x} \sin\left(\frac{\pi x}{40}\right) + 10$ is shown below.



Let $g(x) = 2f(5 - x) - 4$.

a. Complete a possible sequence of transformations to map f to g.

1. A dilation by a factor of 2 from the x-axis.

2. 2. A translation of 4 units in the negative direction of the y-axis.

3. 3. A translation of 5 units in the negative direction of the x-axis.

4. A reflection in the y-axis.

b. Find the value of x which,

i. Minimises g correct to 3 decimal places.

We observe that g is minimised when $f(5-x)$ is minimised.

Hence $5 - x = 58.388 \implies x = -53.388$

ii. Maximises g correct to 3 decimal places.

We observe that g is maximised when $f(5-x)$ is maximised.

Hence $5 - x = 18.388 \implies x = -13.388$

c. State the range of g correct to 2 decimal places.

$\text{Ran}(g) = [2 \times 7.234 - 4, 2 \times 14.127 - 4] = [10.47, 24.25]$

A transformation $T(x, y) = (x, cy + d)$ maps the graph of $f(x)$ onto the graph of $h(x)$. The graph of h has the following properties:

- The global minimum of h is at $(18.388, 7.937)$.
- The global maximum of h is at $(58.388, 11.383)$.

d. Find the values of a and b correct to 1 decimal place.

As $x' = x$, we know that

$$T((18.388, 14.127)) = (18.388, c14.127 + d) = (18.388, 7.937)$$

and

$$T(58.388, 7.234) = (58.388, c7.234 + d) = (58.388, 11.383)$$

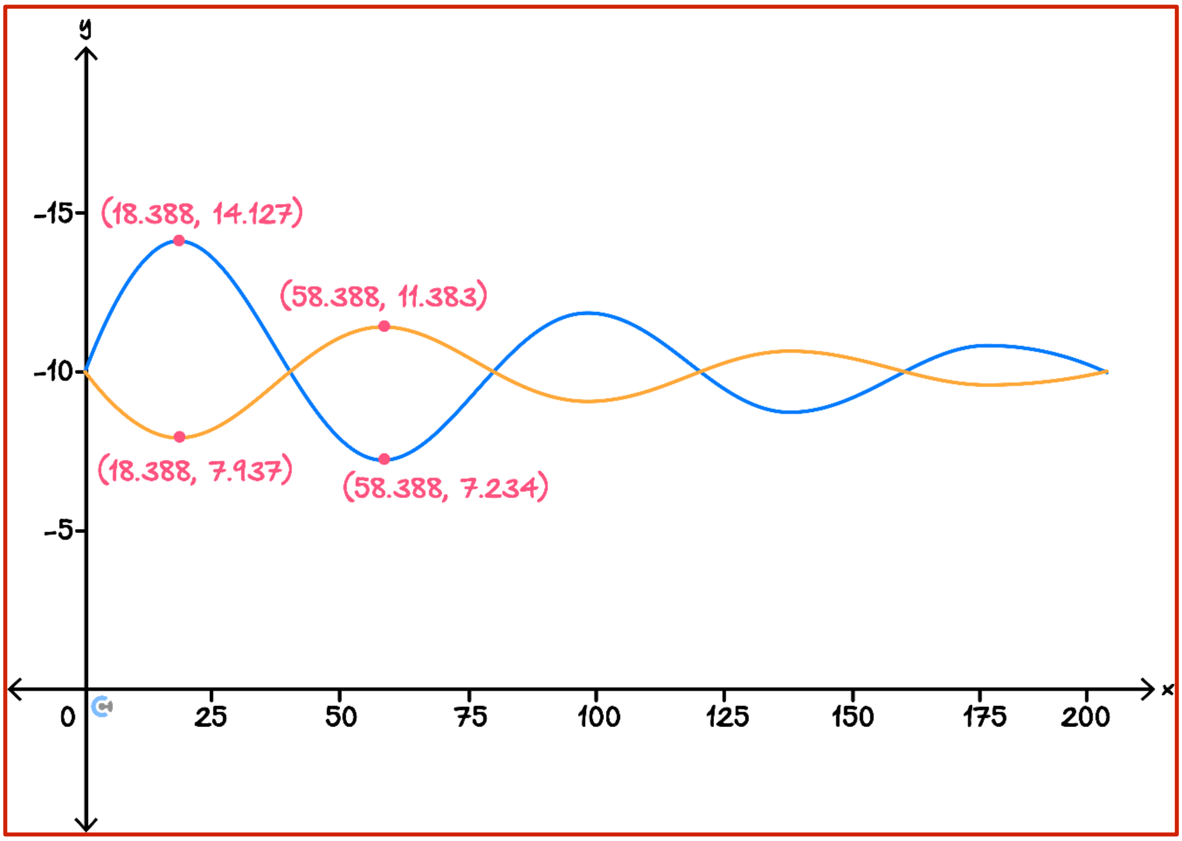
Hence we solve,

$$7.234c + d = 11.383 \quad \text{and} \quad 14.127c + d = 7.937$$

simultaneously for c and d.

Hence $c = -0.5$ and $d = 15.0$

e. Sketch the graph of h on the axis below, labelling its global minimum and maximum.



Section B: Supplementary Questions

Sub-Section [1.3.1]: Applying Transformations to Points

Question 11

Consider the following transformations of the plane.

- S, a dilation by a factor of 2 from the y-axis, followed by a translation of 3 units up.
- T, a translation of 2 units left and 1 unit up.
- W, a reflection in the line $y = x$.

a. Find $S(x, y)$.

$$S(x, y) = (2x, y + 3)$$

b. Find $T(x, y) = (x', y')$. Express x and y in terms of x' and y'.

$T(x, y) = (x - 2, y + 1) = (x', y')$.

Hence $x' = x - 2 \implies x = x' + 2$, and $y' = y + 1 \implies y = y' - 1$.

c. Find $W(x, y)$.

$$W(x, y) = (y, x).$$

Space for Personal Notes

Question 12

Consider the following transformation, $T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (-2x + 4, 5(y + 3))$.

T can be described using the following sequence of transformations:

- A dilation by a factor of a from the x-axis, followed by,
- A dilation by a factor of b from the y-axis, followed by,
- A reflection in the y-axis, followed by,
- A translation c units in the positive direction of the x-axis, followed by,
- A translation of d units in the positive direction of the y-axis.

a. Find a, b, c and d.

We need to turn x into $2x$ and y into $5y$ using our two dilations, since the reflection will take $2x$ to $-2x$ and then we can worry about the translations.

Hence $a = 5$ and $b = 2$.

Now we simply translate our sequence to the right point, meaning $c = 4$ and $d = 15$.

b. Describe T as a sequence of two translations, followed by two dilations, and a reflection.

We can rewrite our transformation as follows, $T(x, y) = (-2(x - 2), 5(y + 3))$. From here we see that we must get our translations to map (x, y) to $(x + 2, y - 1)$ before applying our dilations / reflections. Hence our sequence of transformations is as follows,

- A translation of 2 units in the negative direction of the x-axis, followed by,
- A translation of 3 units in the positive direction of the y-axis, followed by,
- A dilation by a factor of 2 from the y-axis, followed by,
- A dilation by a factor of 5 from the x-axis, followed by,
- A reflection in the y-axis.

c. The image of $(p, -5)$ under T is $(2, q)$. Find p and q.

We apply T to $(p, -5)$ getting, $(-2p+4, -10) = (2, q)$.
Hence $q = -10$ and $-2p + 4 = 2 \implies p = 1$.

Question 13

Consider the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ described by the following sequence of transformations:

- A dilation by a factor of $\frac{1}{5}$ from the x-axis, followed by,
- A translation of 2 units in the positive direction of the x-axis, followed by,
- A reflection in the y-axis, followed by,
- A translation of 3 units in the positive direction of the x-axis, followed by,
- A translation of 5 units in the negative direction of the y-axis, followed by,
- A dilation by a factor of 5 from the x-axis, followed by,
- A reflection in the x-axis, followed by,
- A dilation by a factor of 3 from the y-axis.

a. Find (x', y'), the image of (x, y) under T.

In order, the transformations take the point (x, y) to,

$$(x,y) \mapsto \left(x, \frac{y}{5}\right) \mapsto \left(x+2, \frac{y}{5}\right) \mapsto \left(-x-2, \frac{y}{5}\right) \mapsto \left(-x+1, \frac{y}{5}\right)$$
$$\mapsto \left(-x+1, \frac{y}{5} - 5\right) \mapsto (-x+1, y-25) \mapsto (-x+1, 25-y) \mapsto (-3x+3, 25-y)$$

Thus $(x', y') = (-3x+3, 25-y)$

b. Express x in terms of x' and y in terms of y'.

As $x' = -3x + 3$ we get $x = \frac{x' - 3}{-3} = \frac{3 - x'}{3}$.
As $y' = 25 - y$ we get $y = 25 - y'$.

c. A transformation $S : \mathbb{R}^2 \to \mathbb{R}^2$ maps $T(x, y) = (x', y')$ to (x, y).

Describe S as a sequence of 2 translations followed by 2 reflections followed by a dilation.

- A translation of 3 units in the negative direction of the x-axis, followed by,
- A translation of 25 units in the negative direction of the y-axis, followed by,
- A reflection in the x-axis, followed by,
- A reflection in the y-axis, followed by,
- A dilation by a factor of $\frac{1}{3}$ from the y-axis.

Question 14

a. Describe a reflection in the line $y = x + b$ using elementary transformations.

- A translation of b - units in the negative direction of the y -axis, followed by,
- A reflection in the line $y = x$, followed by,
- A translation of b - units in the positive direciton of the y-axis.

A reflection in the line $y = ax$ can be described via the following transformation,

$$T(x, y) = \left(\frac{x(1-a^2)+2ay}{1+a^2}, \frac{y(a^2-1)+2ax}{1+a^2}\right).$$

b. Describe a reflection in the line $y = ax + b$ using elementary transformations and T.

- A translation of b - units in the negative direction of the y -axis, followed by,
- T, followed by,
- A translation of b - units in the positive direciton of the y-axis.

c. Find the image of the point $(2, 4)$ when it is reflected in the line $y = 3x + 5$.

We apply the transformations in b to our point, noting that $T(x, y) = \left(\frac{-8x + 6y}{10}, \frac{8y + 6x}{10}\right)$.

Hence in order, our transformations map $(2, 4)$ onto,

$$(2, -5) \mapsto (2, -1) \mapsto \left(\frac{-16 - 6}{10}, \frac{-8 + 12}{10}\right) = (-2.2, 0.4) \mapsto (-2.2, 5.4)$$

d. Show using coordinate geometry that T describes a reflection in the line $y = ax$.

Hint: Find the line going through a point (x_0, y_0) with a gradient $-\frac{1}{a}$.

Then, equate that line to $y = ax$ to get a point (x_1, y_1).

Then, (x_1, y_1) is the midpoint of (x_0, y_0) and $(x_0', y_0') = T(x_0, y_0)$.

We follow the hint.

A line going through the point (x_0, y_0) with a gradient $-\frac{1}{a}$ has equation

$$y = \frac{-1}{a}(x - x_0) + y_0$$

There point of intersection (x_1, y_1) lies on both that line and the line $y = ax$, hence,

$$ax_1 = \frac{-1}{a}(x_1 - x_0) + y_0 \implies a^2x_1 + x_1 = x_0 + ay_0 \implies x_1 = \frac{ay_0 + x_0}{a^2 + 1}$$

and $y_1 = ax_1 = \dfrac{a^2y_0 + ax_0}{a^2 + 1}$

Since (x_1, y_1) is the midpoint of (x_0, y_0) and (x_0', y_0') we see that,

$$(x_0', y_0') + (x_0, y_0) = 2(x_1, y_1) \implies (x_0', y_0') = 2(x_1, y_1) - (x_0, y_0)$$

Hence

$$x_0' = 2\frac{ay_0 + x_0}{a^2 + 1} - x_0 = \frac{2ay_0 + 2x_0 - a^2x_0 - x_0}{a^2 + 1} = \frac{x_0(1 - a^2) + 2ay_0}{1 + a^2}$$

and

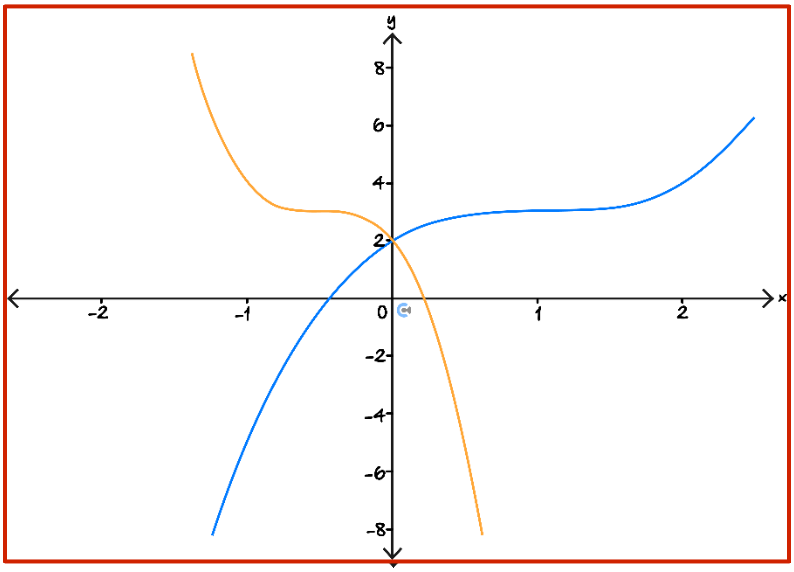
$$y_0' = 2\frac{a^2y_0 + ax_0}{a^2 + 1} - y_0 = \frac{2a^2y_0 + 2ax_0 - a^2y_0 - y_0}{a^2 + 1} = \frac{y_0(a^2 - 1) + 2ax_0}{a^2 + 1}$$

This transformation sends the point (x_0, y_0) to $T(x_0, y_0)$. Hence as (x_0, y_0) is arbitrary, T describes a reflection in the line $y = ax$.

Sub-Section [1.3.2]: Transforming Graphs of Functions.

Question 15

a. The graph of $f(x)$ is shown below.



On the same axes, sketch the graph of $g(x) = f(-2x)$.

b. Let $f(x) = e^x$. The transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, $T(x, y) = (x - 1, y + 2)$ maps the graph of $f(x)$ onto the graph of $g(x)$. Find the rule for $g(x)$.

Consider some points (x', y') on the graph of $g(x)$.

We observe that $(x', y') = T(x, y) = (x - 1, y + 2)$ for some point (x, y) on the graph of $f(x)$. To relate x' with y' we express x in terms of x' and y in terms of y', specifically,

$$x' = x - 1 \implies x = x' + 1 \quad \text{and} \quad y' = y + 2 \implies y = y' - 2$$

We substitute the above two into $y = e^x$ to relate x' with y'. Hence

$$y' - 2 = e^{x+1} \implies y' = e^{x+1} + 2$$

Thus the rule for $g(x)$ is, $g(x) = e^{x+1} + 2$

c. Find the rule for the image of the graph of $y = \cos(x)$ under the transformation,

$$S = \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = \left(-3x, \frac{1}{2}y\right).$$

We apply the same logic as in part *b*.

Observe that $x' = -3x \implies x = -\frac{x'}{3}x$, and $y' = \frac{1}{2}y \implies y = 2y'$. Substituting these into $y = \sin(x)$ yields,

$$2y' = \cos\left(\frac{-x'}{3}\right) \implies y' = \frac{1}{2}\cos\left(\frac{x'}{3}\right)$$

Thus the rule for the image of the graph of $y = \cos(x)$ under S is, $y = \frac{1}{2}\cos\left(\frac{x}{3}\right)$

Question 16

a. Let $f(x) = 5\sqrt{x} - 3$. The transformation $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x,y) = (4x, 3-y)$ maps the graph of $f(x)$ onto the graph of $g(x)$. Find the rule for $g(x)$.

Consider some points (x', y') on the graph of $g(x)$.

We observe that $(x', y') = T(x,y) = (4x, 3-y)$ for some point (x,y) on the graph of $f(x)$. To relate x' with y' we express x in terms of x' and y in terms of y', specifically,

$$x' = 4x \implies x = \frac{x'}{4} \quad \text{and} \quad y' = 3 - y \implies y = 3 - y'$$

We substitute the above two into f to relate x' with y'. Hence

$$3 - y' = 5\sqrt{\frac{x'}{4}} - 3 \implies y' = 6 - \frac{5}{2}\sqrt{x'}$$

Thus the rule for $g(x)$ is, $g(x) = 6 - \frac{5}{2}\sqrt{x}$

b. Find the rule for the image of the graph of $y = e^{x+2} - \log_e(-2x)$ under the transformation,

$$S: \mathbb{R}^2 \to \mathbb{R}^2, T(x,y) = (-2x-1, y+3).$$

We apply the same logic as in part a.

Observe that $x' = -2x - 1 \implies x = -\frac{x'+1}{2}$ and $y' = y + 3 \implies y = y' - 3$. We substitute the following two values into $y = -e^{x+2} - \log_e(-2x)$ to get,

$$y' - 3 = e^{2-\frac{x'+1}{2}} - \log_e(x'+1) \implies y' = e^{2-\frac{x'+1}{2}} - \log_e(x'+1) + 3$$

Thus the rule for the image of the graph of $y = e^{x+2} - \log_e(-2x)$ under S is, $e^{2-\frac{x'+1}{2}} - \log_e(x'+1) + 3$

c. Let $f(x) = (x-1)(x+2)(x-3)$, and let $g(x) = 4f(2-x) + 5$.

Solve $g(x) = 5$.

$g(x) = 5 \implies 4f(2-x) + 5 = 5 \implies 4f(2-x) = 0 \implies f(2-x) = 0.$

Hence $2 - x = -2, 1, 3 \implies x = -1, 1, 4.$

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Question 17

a. Consider the transformation, $T : \mathbb{R}^2 \to \mathbb{R}^2$ which can be described by the following sequence of transformations:

- A translation is 3 units up and 2 units left, followed by,
- A dilation by a factor of 3 from the x-axis and $\frac{1}{2}$ from the y-axis followed by,
- A reflection in the x-axis.

T maps the graph of $f(x)$ onto the graph of $g(x) = \log_e(x)$. Find the rule of $f(x)$.

We see that under T,

$$(x, y) \mapsto (x-2, y+3) \mapsto \left(\frac{x-2}{2}, 3(y+3)\right) \mapsto \left(\frac{x-2}{2}, -3(y+3)\right) = (x', y')$$

For any point (x, y) on the graph of $y = f(x)$, we know that $y' = g(x') = \log_e(x')$. Substituting $x' = \frac{x-2}{2}$ and $y' = -3(y+3)$ into this equation yields,

$$-3(y+3) = \log_e\left(\frac{x-2}{2}\right) \implies y = -\frac{1}{3}\log_e\left(\frac{x-2}{2}\right) - 3$$

Hence, $f(x) = -\frac{1}{3}\log_e\left(\frac{x-2}{2}\right) - 3$.

b. Consider the transformation $S : \mathbb{R}^2 \rightarrow \mathbb{R}^2$, which the following sequence of transformations can describe,

- A dilation by a factor of 2 from the x-axis and 5 from the y-axis, followed by,
- A translation 1 unit down and 4 units right.

Find the rule for the image of the graph of $y = 25x^2 + 5x - 1$ under S.

We see that under S,

$$(x, y) \mapsto (5x, 2y) \mapsto (5x + 4, 2y - 1) = (x', y')$$

Hence $x = \frac{x' - 4}{5}$ and $y = \frac{y' + 1}{2}$.
Substituting these equations into $y = 25x^2 + 5x - 1$ yields

$$\frac{y' + 1}{2} = (x' - 4)^2 + (x' - 4) - 1 = x'^2 - 7x' + 11 \implies y' = 2x'^2 - 14x' + 21$$

Thus the rule for the image of the graph of $y = 25x^2 + 5x - 1$ under S is,

$$y = 2x^2 - 14x + 21$$

c. A transformation $U : \mathbb{R}^2 \rightarrow \mathbb{R}^2$, $U(x, y) = (2x + 5, 3 - 2y)$ maps the graph of $y = af(x) + b$ onto the graph of $y = f(cx + d)$. Find the values of a, b, c and d.

As $x' = 2x + 5$ we see that $x = \frac{x' - 5}{2}$, and as $y' = 3 - 2y$ we see that $y = \frac{3 - y'}{2}$.

We note that if a pair (x, y) lies on the graph $y = af(x) + b$, then their image under U, (x', y') lies on the graph of $y = f(cx + d)$. Hence,

$$\frac{3 - y'}{2} = af\left(\frac{x' - 5}{2}\right) + b \implies y' = -2af\left(\frac{x' - 5}{2}\right) + 3 - 2b = f(cx' + d)$$

Equation coefficients yields,

$$-2a = 1 \implies a = -\frac{1}{2} \quad \text{and} \quad 3 - 2b = 0 \implies b = \frac{3}{2} \quad \text{and} \quad c = \frac{1}{2} \quad \text{and} \quad d = -\frac{5}{2}$$

Space for Personal Notes

Question 18

Consider the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, which is described by the following sequence of transformations.

- A translation of 3 units upwards and 5 units left, followed by,
- A reflection in the line $y = x$, followed by,
- A dilation by a factor of $\frac{1}{2}$ from the x-axis and $\frac{1}{4}$ from the y-axis, followed by,
- A reflection in the x-axis.

T maps the graph of $f : (-\infty, 2], f(x) = 3x^2 + 12x + 5$ onto the graph of g.

Find the rule of g.

We see that under T,

$$(x, y) \mapsto (x-5, y+3) \mapsto (y+3, x-5) \mapsto \left(\frac{y+3}{4}, \frac{x-5}{2}\right) \mapsto \left(\frac{y+3}{4}, -\frac{x-5}{2}\right) = (x', y')$$

Hence $x' = \frac{y+3}{4} \implies y = 4x' - 3$ and $y' = -\frac{x-5}{2} \implies x = -2y' + 5$

Since our transformation will be inverting f, let us express x as a function of y.

$$\begin{aligned} y &= 3x^2 + 12x + 5 \\ \implies y &= 3(x+2)^2 - 7 \\ \implies \frac{y+7}{3} &= (x+2)^2 \\ \implies x &= -\sqrt{\frac{y+7}{3}} - 2 \end{aligned}$$

Since $x \le -2$. Now we substitute x' and y' into our equation to get.

$$\begin{aligned} -2y' + 5 &= -\sqrt{\frac{4x'+4}{3}} - 2 \\ \implies -2y' &= -\frac{2\sqrt{x'+1}}{\sqrt{3}} - 7 \\ \implies y' &= \sqrt{\frac{x'+1}{3}} + \frac{7}{2} \end{aligned}$$

Hence the rule for g is, $g(x) = \sqrt{\frac{x+1}{3}} + \frac{7}{2}$

Sub-Section [1.3.3]: Find Transformations From Transformed Function

Question 19

a. Let $f(x) = x^2$ and $g(x) = 3x^2 - 2$.

Describe a transformation that maps the graph of f onto the graph of g.

Choose some point (x', y') on the graph of $g(x)$. Then $\frac{y'+2}{3} = f(x')$, hence there is some point (x, y) on the graph of $f(x)$ such that,

$$\left(x', \frac{y'+2}{3}\right) = (x, y) \implies (x', y') = (x, 3y-2)$$

This gives us our transformation, $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x,y) = (x, 3y-2)$

b. The transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$, $(x, y) \mapsto (ax + b, cx + d)$ maps the graph of $y = \log_e(x)$ to the graph of $y = 5 - \log_e(2x+3)$.

Find the values of a, b, c and d.

We first apply T to the graph of $y = \log_e(x)$.

Let (x', y') be a point on the image of $y = \log_e(x)$ under T. Then there is some pair (x, y) on the graph of $y = \log_e(x)$ such that,

$$(x', y') = (ax+b, cx+d) \implies (x, y) = \left(\frac{x'-b}{a}, \frac{y'-d}{c}\right).$$

We substitute this into the equation $y = \log_e(x)$ to get,

$$y' = c\log_e\left(\frac{x'-b}{a}\right) + d = 5 - \log_e(2x'+3)$$

By comparing coefficients, we see that, $a = \frac{1}{2}, b = -\frac{3}{2}, c = -1$ and $d = 5$.

c. A transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ maps the graph of $f(x) = \sqrt{x}$ onto the graph of $g(x) = 3\sqrt{x-1} + 5$.

- A dilation by a factor of 3 from the x-axis, followed by
- A translation of 1 unit(s) in the positive direction of the x-axis, followed by
- A translation of 5 units in the positive direction of the y-axis.

- A translation of ______
- A translation of ______

Fill in the blanks.

We observe that $g(x) = 3f(x-1) + 5$. Thus any pair (x', y') on the graph of $g(x)$ satisfies,

$$\frac{y'-5}{3} = f(x'-1)$$

Hence we can relate some pair (x, y) on the graph of $f(x)$, to (x', y') by,

$$(x, y) = \left(x'-1, \frac{y'-5}{3}\right) \implies (x', y') = (x+1, 3y+5)$$

We can then describe our transformation as above.

Question 20

a. Let $f(x) = 4(x - 5)^2$.

The transformations:

$$S: \mathbb{R}^2 \to \mathbb{R}^2, (x, y) \mapsto (x + b, ay),$$
and
$$T: \mathbb{R}^2 \to \mathbb{R}^2, (x, y) \mapsto (cx + d, y).$$

Both map the graph of $y = x^2$ onto the graph of f.

Find the values of a, b, c and d.

We first apply S onto the graph of $y = x^2$, this yields the graph of,

$$y = a(x - b)^2.$$

Comparing coefficients to $f(x) = 4(x - 5)^2$ we see that $a = 4$ and $b = 5$.

Now we apply T onto the graph of $y = x^2$, this yields the graph of,

$$y = \left(\frac{x - d}{c}\right)^2.$$

Comparing coefficients to $f(x) = 4(x - 5)^2$ we see that $c = \frac{1}{2}$ and $d = 5$.

b. Consider a function $f : [0, \infty) \to \mathbb{R}, f(x) = 100 - 4x$.

A different function g has the property, that g decreases at half the rate of f at any point in time and that $g(0) = f(0)$. State a single transformation that maps the graph of f onto the graph of g.

Since g decreases at half the rate of f at any point in time and $g(0) = f(0)$, we know that $g(x) = \frac{f(x)}{2} + 50 = 100 - 2x = f\left(\frac{x}{2}\right)$.

Hence a dilation by a factor of 2 from the y-axis will transform the graph of f onto the graph of g.

c. Let $g(x) = -\frac{f(4x+12)}{5} - 20$.

F… s map the graph of $f(x)$ onto the g…

- A dilation by a factor of $\frac{1}{5}$ from the x-axis, followed by,
- A translation of -12 units in the positive direction of the x-axis, followed by,
- A translation of 20 units in the positive direction of the y-axis, followed by,
- A dilation by a factor of $\frac{1}{4}$ from the y-axis, followed by,

➤ A translation of ______

➤ A translation of ______

➤ A dilation by a factor

➤ A reflection in the x-a

Choose a point (x', y') on the graph of g. We observe that,

$$-5(y' + 20) = f(4x' + 12)$$

Hence there is some point (x, y) on the graph of f such that,

$$(x, y) = (4x' + 12, -5y' - 100)) \implies (x', y') = \left(\frac{x-12}{4}, -\frac{y+100}{5}\right) = \left(\frac{x-12}{4}, -\left(20 + \frac{y}{5}\right)\right)$$

The last equation is useful for us since we are first dilating then translating then reflecting y, but we are first translating then dilating x. Hence we can read off the required transformations from the last equation.

Question 21

a. Describe a sequence of three transformations that map the graph of $f(x) = \sqrt{7 - 6x - x^2}$ onto the graph of $g(x) = \sqrt{4 - x^2}$.

By completing the square, we observe that $f(x) = \sqrt{4(x-2)^2}$.

For $x \geq 2$ this can be simplified down to $f(x) = 2(x - 2)$.

As $f\left(\frac{x-b}{a}\right) = x$ we observe that $a = 2$ and $b = -4$.

b. Let $f : [2, \infty) \to \mathbb{R}, f(x) = \sqrt{4x^2 - 16x + 16}$.

A transformation $T : \mathbb{R}^2 \to \mathbb{R}^2, (x, y) \mapsto (ax + b, y)$ maps the graph of $f(x)$ onto the graph of $g : [0, \infty) \to \mathbb{R}, g(x) = x$.

Find the values of a and b.

By completing the square, we observe that $f(x) = \sqrt{4(x-2)^2}$.

For $x \geq 2$ this can be simplified down to $f(x) = 2(x-2)$.

As $f\left(\frac{x-b}{a}\right) = x$ we observe that $a = 2$ and $b = -2$.

c. A function f has its only stationary point at $(2, 3)$ and its only x-axis intercept at $(-5, 0)$.

A function g has its only stationary point at $(6, -2)$ and only x-axis intercept at $(-8, 0)$.

A transformation $T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (ax + b, cy)$ maps the graph of f onto the graph of g.

Find a, b and c.

We know that T maps stationary points to stationary points, hence $T(2, 3) = (2a + b, 3c) = (6, -2)$. This implies that $c = \frac{-2}{3}$

Since T has no vertical translations, it maps x-axis intercepts to x-axis intercepts. Hence $T(-5, 0) = (-5a + b, 0) = (-8, 0)$.

We solve $-5a + b = -8$ and $2a + b = 6$ simultaneously to find a and b.

Subtracting the first equation from the second yields $7a = 14 \implies a = 2$. Substituting this back into the first equation implies that $b = 5a - 8 = 10 - 8 = 2$.

Question 22 Tech-Active.

Let $f(x) = x^4 + x^3 + x^2 + x + 1$ and $g(x) = x^4 + 2x^3 + 4x^2 + 8x + 11$.

A transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2, T(x, y) = (ax + b, cx + d)$ maps the graph of f onto the graph of g.

Find a, b, c, d and show that they are unique.

We observe that the rule of g is,

$$g(x) = cf\left(\frac{x-b}{a}\right) + d = x^4 + 2x^3 + 4x^2 + 8x + 11$$

We expand out f and compare the coefficients to get the following simultaneous equations.

$$\begin{aligned} c - \frac{bc}{a} + \frac{b^2c}{a^2} - \frac{b^3c}{a^3} + \frac{b^4c}{a^4} + d &= 11 \\ \frac{c}{a} - \frac{2bc}{a^2} + \frac{3b^2c}{a^3} - \frac{4b^3c}{a^4} &= 8 \\ \frac{c}{a^2} - \frac{3bc}{a^3} + \frac{6b^2c}{a^4} &= 4 \\ \frac{c}{a^3} - \frac{4bc}{a^4} &= 2 \\ \frac{c}{a^4} &= 1 \end{aligned}$$

We solve these equations to get $a = 2, b = 0, c = 16$ and $d = -5$.

Any other such values of a, b, c, d must satisfy those simultaneous equations. As those equations only have one solution, our values of a, b, c and d are unique.

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