**Website:** contoureducation.com.au | **Phone:** 1800 888 300
**Email:** hello@contoureducation.com.au

# VCE Mathematical Methods ¾
## Transformations [1.3]
## Homework

### Homework Outline:

| | |
|---|---|
| **Compulsory Questions** | Pg 2 – Pg 19 |
| **Supplementary Questions** | Pg 20 – Pg 34 |

## Section A: Compulsory Questions

### Sub-Section [1.3.1]: Applying Transformations to Points

**Question 1**

Consider the following transformations of the plane.

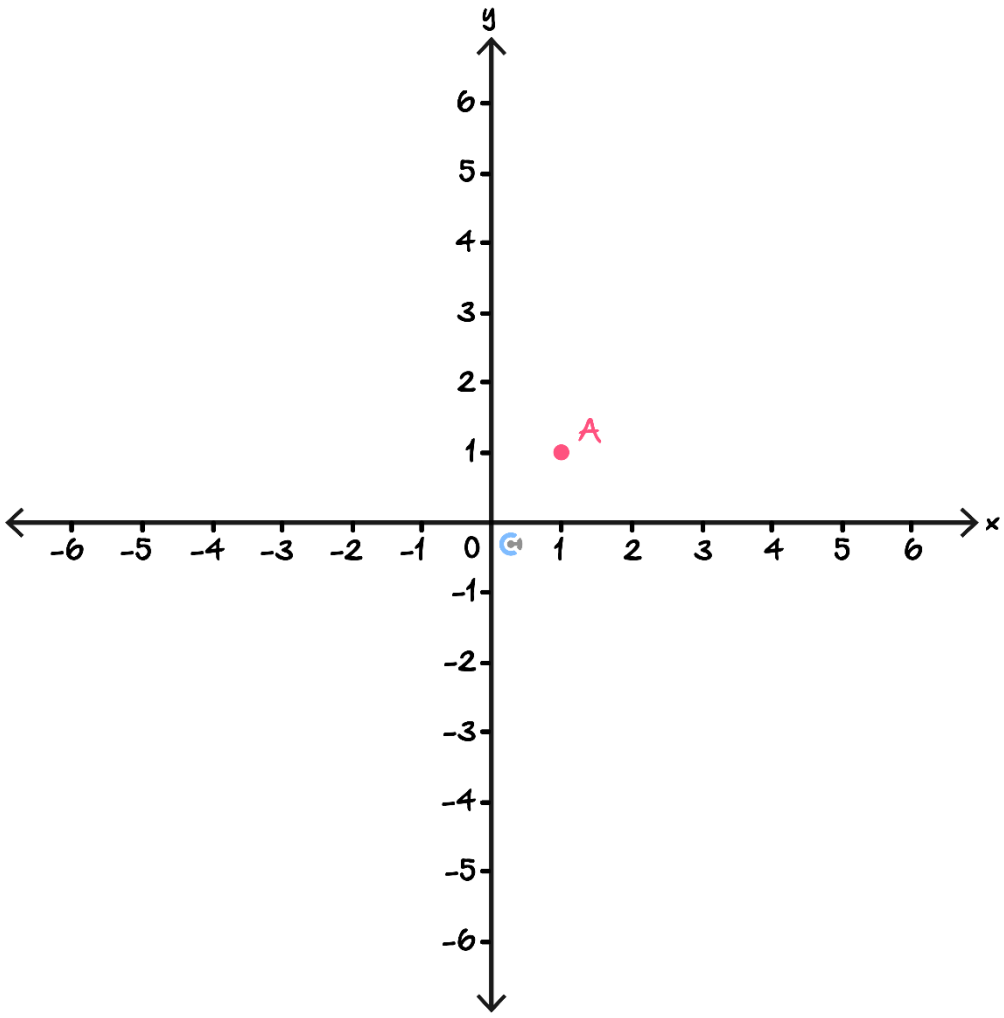
- $S$, a dilation by a factor of 2 from the $x$-axis.
- $T$, a translation of 2 units in the positive direction of the $x$-axis, and 3 units in the negative direction of the $y$-axis.
- $W$, a reflection in the $y$-axis, followed by a dilation by a factor of 2 from the $y$-axis.

**a.** Find $S(x, y) = (x', y')$.

**b.** Find $T(x, y)$.

**c.** Find $W(x, y)$.

**d.** The point $A(1, 1)$ is drawn on the axis below.




Label the following points on the axis above.

**i.** $B$ which is the image of $A$ after having $S$ and then $W$ applied to it.

**ii.** $C$ which is the image of $A$ after having $W$ and then $S$ applied to it.

**iii.** $D$ which is the image of $A$ after having $T$ and then $W$ applied to it.

**iv.** $E$ which is the image of $A$ after having $W$ and then $T$ applied to it.

Space for Personal Notes

**Question 2**

Consider the following transformation, $T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (3x + 6, -4y + 4)$.

$T$ can be described using the following sequence of transformations,

- A dilation by a factor of $a$ from the $x$-axis, followed by,
- A dilation by a factor of $b$ from the $y$-axis, followed by,
- A reflection in the $x$ -axis, followed by,
- A translation of $c$ unit in the positive direction of the $x$-axis, followed by,
- A translation of $d$ unit in the positive direction of the $y$-axis.

**a.** Find $a$, $b$, $c$ and $d$.

**b.** Describe $T$ as a sequence of two translations, followed by two dilations, and a reflection.

**c.** Find the pre-image of $(3, -8)$ under $T$.

**Question 3**

Consider the transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ described by the following sequence of transformations.

- A reflection in the line $y = x$, followed by,
- A translation of 6 units in the negative direction of the $x$-axis, followed by,
- A dilation by a factor of $\frac{1}{3}$ from the $y$-axis, followed by,
- A dilation by a factor of 5 from the $x$-axis, followed by,
- A translation of 7 units in the positive direction of the $y$-axis, followed by,
- A reflection in the $x$-axis.

**a.** Let $(x', y')$ be the image of $(x, y)$ under $T$.

Express $x$ and $y$ in terms of $x'$ and $y'$.

**b.** The transformation $T$ can also be described using the following sequence of transformations.

- A dilation by a factor of _____ from the $x$-axis, followed by,
- A dilation by a factor of _____ from the $y$-axis, followed by,
- A reflection in the _____ axis, followed by,
- A reflection in the line $y = x$, followed by,
- A translation of _____ units in the positive direction of the $x$-axis, followed by,
- A translation of _____ units in the positive direction of the $y$-axis

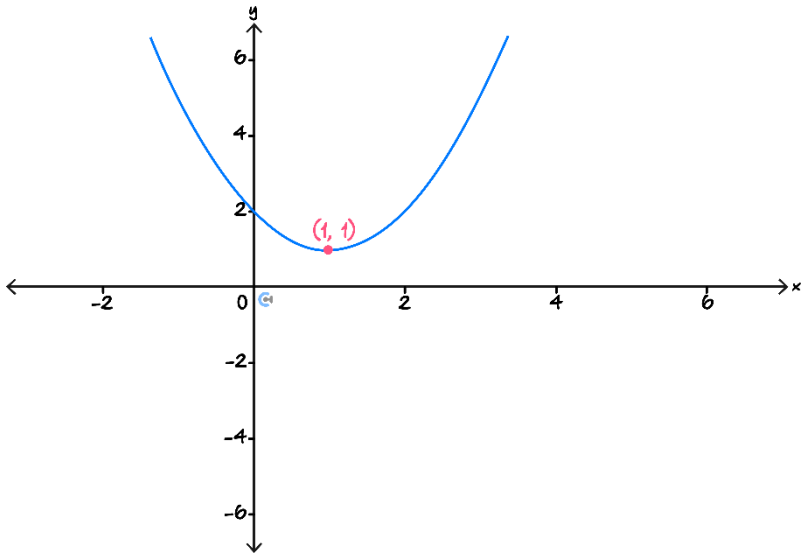
Fill in the blanks.

Space for Personal Notes

## Sub-Section [1.3.2]: Transforming Graphs of Functions.

**Question 4**

**a.** The graph of $f(x)$ is shown below.




On the same axes, sketch the graph of $g(x) = -f\left(\frac{x}{2}\right)$.

**b.** Let $f(x) = \log_e(x)$. The transformation $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (x + 1, y + 3)$ maps the graph of $f(x)$ onto the graph of $g(x)$. Find the rule for $g(x)$.

**c.** Find the rule for the image of the graph of $y = \sin(x)$ under the transformation,

$$S: \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = \left(\frac{x}{2}, -y\right).$$

**Question 5**

**a.** Let $f(x) = 2x^2 + 4$. The transformation $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (4x + 2, -y)$ maps the graph of $f(x)$ onto the graph of $g(x)$. Find the rule for $g(x)$.

**b.** Find the rule for the image of the graph of $y = -\sqrt{x+1} + 3x$ under the transformation,

$$S: \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (-x + 5, y + 1).$$

**c.** Let $f(x) = x^2 + 5$, and let $g(x) = 3(f(x + 2) - 6)$.

**i.** Find and simplify $g(x)$.

**ii.** Solve $g(x) = 0$.

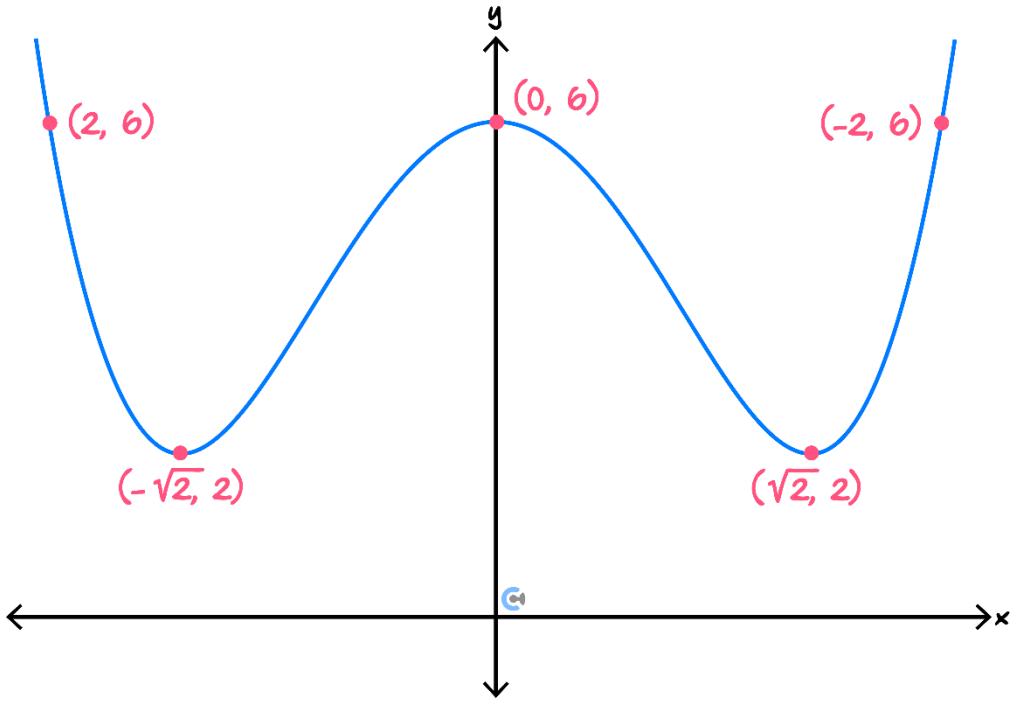
**Question 6**

**a.** Consider the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that the following sequence of transformations can describe.

- A dilation by a factor of 3 from the $x$-axis, followed by,
- A reflection in the $y$-axis, followed by,
- A translation of 2 units up and 4 units left.

Find the rule for the image of the graph of $y = e^{2x+3}$ under $T$.

**b.** The graph of $f(x)$ is shown below.




The function $g(x)$ has a rule, $g(x) = -f(x) + a$.

For what values of $a$ does the equation $g(x) = f(x)$ have 4 solutions?

**c.** The transformation $S(x, y) = (-5x + 3, 3y - 2)$ maps the graph of $f(x)$ onto the graph of $g(x)$.

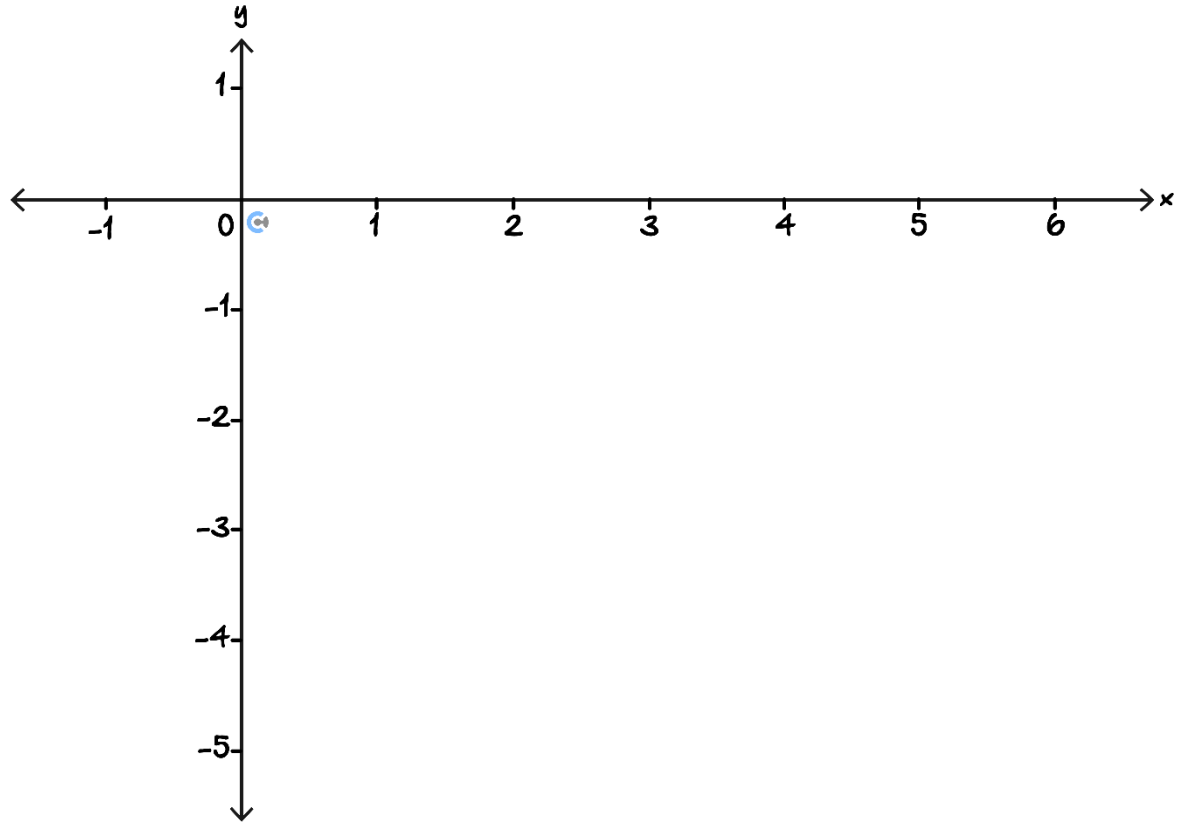
If the rule for $g(x) = \sqrt{x}$, find the rule for $f(x)$.

**d. (Tech-Active.)**

Let $f: [0, \infty) \to \mathbb{R}, f(x) = e^x + e^{-x}$.

The transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$, $(x, y) \mapsto (y - 3, -2x)$ maps the graph of $f(x)$ onto the graph of $g(x)$.

Sketch the graph of $g(x)$ on the axis below, labelling endpoints and axis intercepts with their coordinates, correct to 3 decimal places.




Space for Personal Notes

## Sub-Section [1.3.3]: Find Transformations From Transformed Function

**Question 7**

**a.** Let $f(x) = x^2$ and $g(x) = 2x^2 + 1$.

Describe a transformation that maps the graph of $f$ onto the graph of $g$.

_____

_____

_____

_____

**b.** The transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, $(x, y) \mapsto (ax + b, cy + d)$ maps the graph of $y = e^x$ to the graph of $y = 2e^{x-4} + 3$.

Find the values of $a$, $b$, $c$ and $d$.

_____

_____

_____

_____

_____

**c.** A transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ maps the graph of $f(x) = x^2$ onto the graph of $g(x) = 2(x + 1)^2 + 3$.

$T$ can be described by the following sequence of transformations,

- A dilation by a factor of _____ from the $x$-axis, followed by,
- A translation of _____ unit(s) in the positive direction of the $x$-axis, followed by,
- A translation of _____ units in the positive direction of the $y$-axis.

Fill in the blanks.

**Question 8**

**a.** Let $f(x) = \frac{1}{2x+2}$.

The transformations:

$$S : \mathbb{R}^2 \to \mathbb{R}^2, (x, y) \mapsto (x + a, by),$$
and
$$T : \mathbb{R}^2 \to \mathbb{R}^2, (x, y) \mapsto (c(x + d), y).$$

Both map the graph of $y = \frac{1}{x}$ onto the graph of $f$.

Find the values of $a$, $b$, $c$ and $d$.

**b.** The function $s : [0, 365] \to \mathbb{R}$, $s(t) = \frac{200}{t+1}$ models the number of minutes per day James smiles $t$ days after the start of the school year.

A new function $s_1(t)$ models the number of minutes Sam smiles. It is known that $s_1(0) = s(0)$, but $s_1$ decreases at half the rate of $s$ at any point in time.

State a sequence of two transformations that maps $s$ to this new model $s_1$.

**c.** Let $f(x) = \tan(x)$ and $g(x) = -2\tan(3x + 6) + 8$.

Fill in the blank lines to make the following sequences of transformations map the graph of $f(x)$ onto the graph of $g(x)$.

- A dilation by a factor of _____ from the $y$-axis, followed by,
- A translation of _____units in the positive direction of the $x$-axis, followed by,
- A translation of _____ units in the positive direction of the $y$-axis, followed by,
- A dilation by a factor of _____ from the $x$-axis, followed by,
- A reflection in the $x$ -axis.

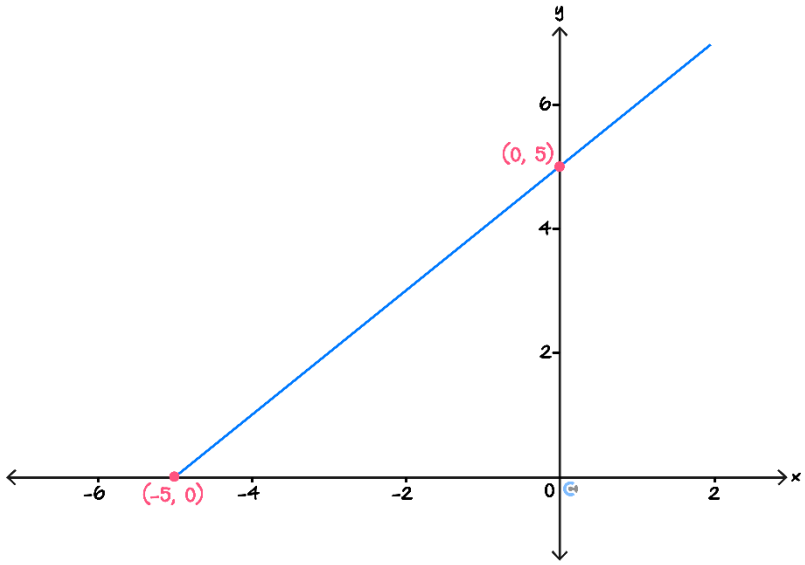
**Question 9**

**a.** Describe a sequence of three transformations that map the graph of $f(x) = \sqrt{4x - x^2}$ onto the graph of $g(x) = \sqrt{1 - x^2}$.

**b.** Let $f : (-\infty, -1] \rightarrow \mathbb{R}, f(x) = x^2 + 2x$.

A transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2, (x, y) \mapsto (ax + b, y + c)$ maps the graph of $f(x)$ onto the graph of $g(x)$.

The graph of $y = \sqrt{g(x)}$ is shown below.




Find the values of $a$, $b$ and $c$.

**c.** Describe 3 different transformations of the plane that map the graph of $y = x^3$ onto the graph of $y = 3(x - 1)^3 + 5$.

**d.** **(Tech-Active)**

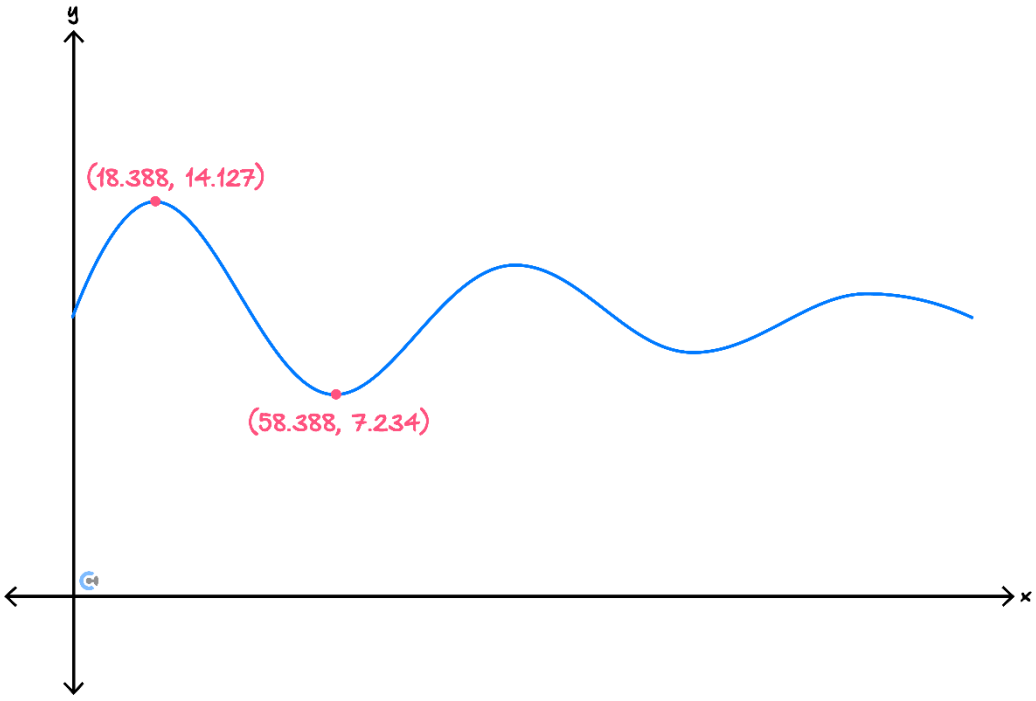
Let $f(x) = \cos(\pi(x^2 + 16x))$.

State a transformation that maps the graph of $y = f(x)$ onto the graph of $y = 2\cos(\pi x^2)$.

Space for Personal Notes

**Question 10 Tech-Active.**

Part of the graph of $f : [0, \infty) \to \mathbb{R}, f(x) = 4e^{-0.01x} \sin\left(\frac{\pi x}{40}\right) + 10$ is shown below.




Let $g(x) = 2f(5 - x) - 4$.

**a.** Complete a possible sequence of transformations to map $f$ to $g$.

**1.** A dilation by a factor of 2 from the $x$-axis.

**2.** _____

**3.** _____

**4.** A reflection in the $y$-axis.

**b.** Find the value of $x$ which,

**i.** Minimises $g$ correct to 3 decimal places.

**ii.** Maximises $g$ correct to 3 decimal places.

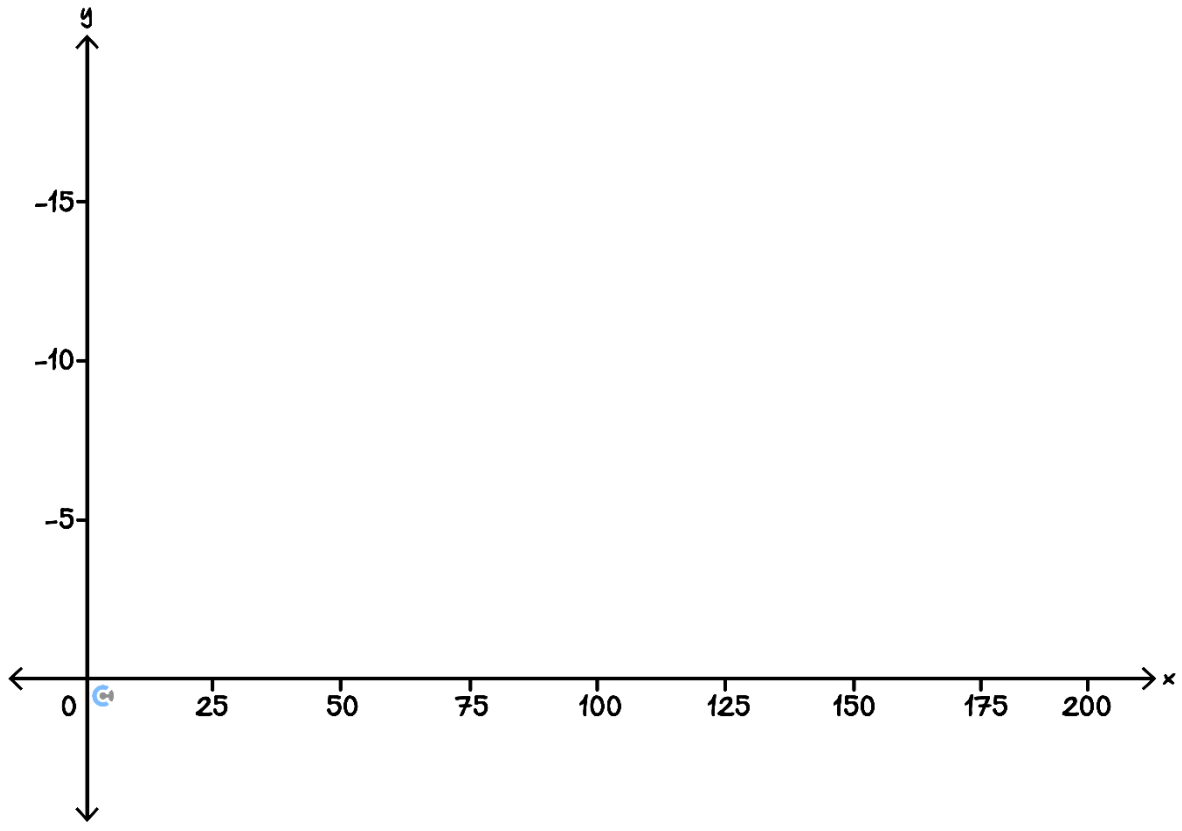
**c.** State the range of $g$ correct to 2 decimal places.

A transformation $T(x, y) = (x, cy + d)$ maps the graph of $f(x)$ onto the graph of $h(x)$. The graph of $h$ has the following properties:

- The global minimum of $h$ is at $(18.388, 7.937)$.
- The global maximum of $h$ is at $(58.388, 11.383)$.

**d.** Find the values of $a$ and $b$ correct to 1 decimal place.

**e.** Sketch the graph of $h$ on the axis below, labelling its global minimum and maximum.



## Section B: Supplementary Questions

### Sub-Section [1.3.1]: Applying Transformations to Points

**Question 11**

Consider the following transformations of the plane.

- $S$, a dilation by a factor of 2 from the $y$-axis, followed by a translation of 3 units up.
- $T$, a translation of 2 units left and 1 unit up.
- $W$, a reflection in the line $y = x$.

**a.** Find $S(x, y)$.

**b.** Find $T(x, y) = (x', y')$. Express $x$ and $y$ in terms of $x'$ and $y'$.

**c.** Find $W(x, y)$.

Space for Personal Notes

**Question 12**

Consider the following transformation, $T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (-2x + 4, 5(y + 3))$.

$T$ can be described using the following sequence of transformations:

- A dilation by a factor of $a$ from the $x$-axis, followed by,
- A dilation by a factor of $b$ from the $y$-axis, followed by,
- A reflection in the $y$-axis, followed by,
- A translation $c$ units in the positive direction of the $x$-axis, followed by,
- A translation of $d$ units in the positive direction of the $y$-axis.

**a.** Find $a$, $b$, $c$ and $d$.

**b.** Describe $T$ as a sequence of two translations, followed by two dilations, and a reflection.

**c.** The image of $(p, -5)$ under $T$ is $(2, q)$. Find $p$ and $q$.

**Question 13**

Consider the transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ described by the following sequence of transformations:

- A dilation by a factor of $\frac{1}{5}$ from the $x$-axis, followed by,
- A translation of 2 units in the positive direction of the $x$-axis, followed by,
- A reflection in the $y$-axis, followed by,
- A translation of 3 units in the positive direction of the $x$-axis, followed by,
- A translation of 5 units in the negative direction of the $y$-axis, followed by,
- A dilation by a factor of 5 from the $x$-axis, followed by,
- A reflection in the $x$-axis, followed by,
- A dilation by a factor of 3 from the $y$-axis.

**a.** Find $(x', y')$, the image of $(x, y)$ under $T$.

**b.** Express $x$ in terms of $x'$ and $y$ in terms of $y'$.

**c.** A transformation $S: \mathbb{R}^2 \to \mathbb{R}^2$ maps $T(x, y) = (x', y')$ to $(x, y)$.

Describe $S$ as a sequence of 2 translations followed by 2 reflections followed by a dilation.

**Question 14**

**a.** Describe a reflection in the line $y = x + b$ using elementary transformations.

A reflection in the line $y = ax$ can be described via the following transformation,

$$T(x, y) = \left(\frac{x(1-a^2)+2ay}{1+a^2}, \frac{y(a^2-1)+2ax}{1+a^2}\right).$$

**b.** Describe a reflection in the line $y = ax + b$ using elementary transformations and $T$.

**c.** Find the image of the point $(2, 4)$ when it is reflected in the line $y = 3x + 5$.

**d.** Show using coordinate geometry that $T$ describes a reflection in the line $y = ax$.

Hint: Find the line going through a point $(x_0, y_0)$ with a gradient $-\frac{1}{a}$.

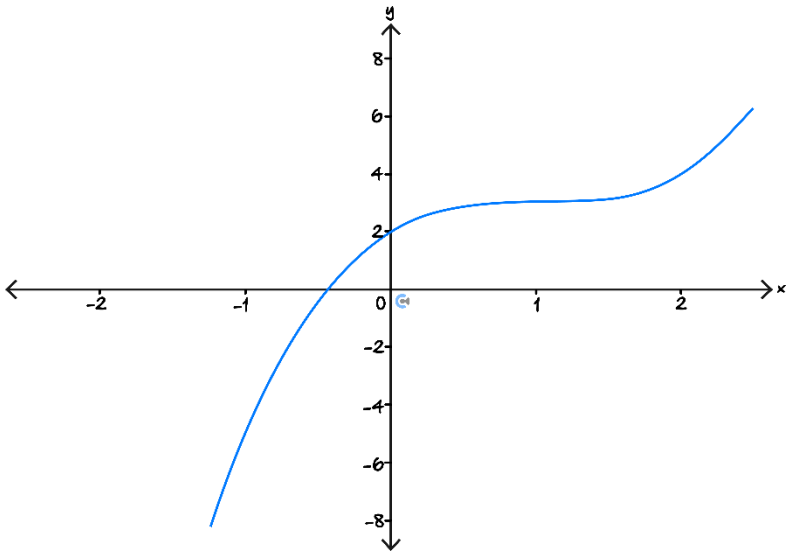
Then, equate that line to $y = ax$ to get a point $(x_1, y_1)$.

Then, $(x_1, y_1)$ is the midpoint of $(x_0, y_0)$ and $(x_0', y_0') = T(x_0, y_0)$.

## Sub-Section [1.3.2]: Transforming Graphs of Functions.

**Question 15**

**a.** The graph of $f(x)$ is shown below.




On the same axes, sketch the graph of $g(x) = f(-2x)$.

**b.** Let $f(x) = e^x$. The transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$, $T(x, y) = (x - 1, y + 2)$ maps the graph of $f(x)$ onto the graph of $g(x)$. Find the rule for $g(x)$.

**c.** Find the rule for the image of the graph of $y = \cos(x)$ under the transformation,

$$S = \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = \left(-3x, \frac{1}{2}y\right).$$

**Question 16**

**a.** Let $f(x) = 5\sqrt{x} - 3$. The transformation $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (4x, 3 - y)$ maps the graph of $f(x)$ onto the graph of $g(x)$. Find the rule for $g(x)$.

**b.** Find the rule for the image of the graph of $y = e^{x+2} - \log_e(-2x)$ under the transformation,

$$S: \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (-2x - 1, y + 3).$$

**c.** Let $f(x) = (x - 1)(x + 2)(x - 3)$, and let $g(x) = 4f(2 - x) + 5$.

Solve $g(x) = 5$.

Space for Personal Notes

**Question 17**

**a.** Consider the transformation, $T: \mathbb{R}^2 \to \mathbb{R}^2$ which can be described by the following sequence of transformations:

- A translation is 3 units up and 2 units left, followed by,
- A dilation by a factor of 3 from the $x$-axis and $\frac{1}{2}$ from the $y$-axis followed by,
- A reflection in the $x$-axis.

$T$ maps the graph of $f(x)$ onto the graph of $g(x) = \log_e(x)$. Find the rule of $f(x)$.

**b.** Consider the transformation $S: \mathbb{R}^2 \to \mathbb{R}^2$, which the following sequence of transformations can describe,

- A dilation by a factor of 2 from the $x$-axis and 5 from the $y$-axis, followed by,
- A translation 1 unit down and 4 units right.

Find the rule for the image of the graph of $y = 25x^2 + 5x - 1$ under $S$.

**c.** A transformation $U: \mathbb{R}^2 \to \mathbb{R}^2$, $U(x, y) = (2x + 5, 3 - 2y)$ maps the graph of $y = af(x) + b$ onto the graph of $y = f(cx + d)$. Find the values of $a$, $b$, $c$ and $d$.

Space for Personal Notes

**Question 18**

Consider the transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$, which is described by the following sequence of transformations.

- A translation of 3 units upwards and 5 units left, followed by,
- A reflection in the line $y = x$, followed by,
- A dilation by a factor of $\frac{1}{2}$ from the $x$-axis and $\frac{1}{4}$ from the $y$-axis, followed by,
- A reflection in the $x$-axis.

$T$ maps the graph of $f: (-\infty, 2], f(x) = 3x^2 + 12x + 5$ onto the graph of $g$.

Find the rule of $g$.

## Sub-Section [1.3.3]: Find Transformations From Transformed Function

**Question 19**

**a.** Let $f(x) = x^2$ and $g(x) = 3x^2 - 2$.

Describe a transformation that maps the graph of $f$ onto the graph of $g$.

_____

_____

_____

_____

**b.** The transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, $(x, y) \mapsto (ax + b, cx + d)$ maps the graph of $y = \log_e(x)$ to the graph of $y = 5 - \log_e(2x + 3)$.

Find the values of $a$, $b$, $c$ and $d$.

_____

_____

_____

_____

_____

**c.** A transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ maps the graph of $f(x) = \sqrt{x}$ onto the graph of $g(x) = 3\sqrt{x-1} + 5$.

$T$ can be described by the following sequence of transformations,

- A dilation by a factor of _____ from the $x$-axis, followed by
- A translation of _____ unit(s) in the positive direction of the $x$-axis, followed by
- A translation of _____ units in the positive direction of the $y$-axis.

Fill in the blanks.

**Question 20**

**a.** Let $f(x) = 4(x - 5)^2$.

The transformations:

$$S : \mathbb{R}^2 \to \mathbb{R}^2, (x, y) \mapsto (x + b, ay),$$
and
$$T : \mathbb{R}^2 \to \mathbb{R}^2, (x, y) \mapsto (cx + d, y).$$

Both map the graph of $y = x^2$ onto the graph of $f$.

Find the values of $a$, $b$, $c$ and $d$.

**b.** Consider a function $f : [0, \infty) \to \mathbb{R}, f(x) = 100 - 4x$.

A different function $g$ has the property, that $g$ decreases at half the rate of $f$ at any point in time and that $g(0) = f(0)$. State a single transformation that maps the graph of $f$ onto the graph of $g$.

**c.** Let $g(x) = -\frac{f(4x+12)}{5} - 20$.

Fill in the blank lines to make the following sequences of transformations map the graph of $f(x)$ onto the graph of $g(x)$.

- A dilation by a factor of _____ from the $x$-axis, followed by
- A translation of _____units in the positive direction of the $x$-axis, followed by,
- A translation of _____ units in the positive direction of the $y$-axis, followed by,
- A dilation by a factor of _____ from the $y$-axis, followed by
- A reflection in the $x$-axis.

**Question 21**

**a.** Describe a sequence of three transformations that map the graph of $f(x) = \sqrt{7 - 6x - x^2}$ onto the graph of $g(x) = \sqrt{4 - x^2}$.

**b.** Let $f : [2, \infty) \to \mathbb{R}, f(x) = \sqrt{4x^2 - 16x + 16}$.

A transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ , $(x, y) \mapsto (ax + b, y)$ maps the graph of $f(x)$ onto the graph of $g : [0, \infty) \to \mathbb{R}, g(x) = x$.

Find the values of $a$ and $b$.

**c.** A function $f$ has its only stationary point at $(2, 3)$ and its only $x$-axis intercept at $(-5, 0)$.

A function $g$ has its only stationary point at $(6, -2)$ and only $x$-axis intercept at $(-8, 0)$.

A transformation $T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (ax + b, cy)$ maps the graph of $f$ onto the graph of $g$.

Find $a$, $b$ and $c$.

**Question 22 Tech-Active.**

Let $f(x) = x^4 + x^3 + x^2 + x + 1$ and $g(x) = x^4 + 2x^3 + 4x^2 + 8x + 11$.

A transformation $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x,y) = (ax + b, cx + d)$ maps the graph of $f$ onto the graph of $g$.

Find $a$, $b$, $c$, $d$ and show that they are unique.

Space for Personal Notes

CONTOUREDUCATION

**Website:** contoureducation.com.au | **Phone:** 1800 888 300 | **Email:** hello@contoureducation.com.au

## VCE Mathematical Methods ¾

# Free 1-on-1 Support

**Be Sure to Make The Most of These (Free) Services!**

- Experienced Contour tutors (45+ raw scores, 99+ ATARs).
- For fully enrolled Contour students with up-to-date fees.
- After school weekdays and all-day weekends.

| 1-on-1 Video Consults | Text-Based Support |
|---|---|
| Book via bit.ly/contour-methods-consult-2025 (or QR code below).<br>One active booking at a time (must attend before booking the next). | Message +61 440 138 726 with questions.<br>Save the contact as "Contour Methods". |

## Booking Link for Consults

## bit.ly/contour-methods-consult-2025

## Number for Text-Based Support

## +61 440 138 726