CONTOUREDUCTION

Website: contoureducation.com.au | Phone: 1800 888 300
Email: hello@contoureducation.com.au

VCE Mathematical Methods ¾
Functions & Relations Exam Skills [1.2]
Workbook

Outline:

Recap of [1.1] Functions and Relations Pg 02-18	**Functions and Relations Exam Skills** Pg 19-28
- Maximal Domains - Domain of Sum, Difference, and Product of Functions - Basics of Composition - Validity of Composite Functions - Domain of Composite Functions - Range of Composite Functions - Basics of Inverses - Swapping x and y - Symmetry Around $y = x$ - Validity of Inverse Function - Intersection Between Inverses - Composition of Inverses	- Find a New Domain to Fix Composite Functions - Find the Range of Complex Composite Functions - Find the Gradient of Inverse Functions **Exam 1 Questions** Pg 29-33 **Tech Active Exam Skills** Pg 34-36 **Exam 2 Questions** Pg 37-42

5

Section A: Recap of [1.1] Functions and Relations

Sub-Section: Maximal Domains

Starting with a domain!

Definition

Maximal Domain

- **Definition**: The largest possible set of input values (elements of the domain) for which the function is well-defined.

- **Three Important Rules:**

Functions	Maximal Domain
$\sqrt{z}$	$z \geq 0$
$\log(z)$	$z > 0$
$\frac{1}{z}$	$z \neq 0$

Steps

1. Find the restriction of the inside.
2. Sketch the graph if needed.
3. Solve for domain.

Space for Personal Notes

Sub-Section: Domain of Sum, Difference, and Product of Functions

What about a domain of the sum of two functions?

Sums, Differences, and Products of Functions

- Rules:

$$(f+g)(x) = \underline{f(x)+g(x)}$$

$$(f-g)(x) = \underline{f(x)-g(x)}$$

$$(f \times g)(x) = \underline{f(x) \times g(x)}$$

- Idea:

$$\textbf{Domain of sum or product of two functions} = \underline{\text{intersection}} \textbf{ of the two domains}$$

- Steps:

1. Find the domain of each function.
2. Find the intersection (draw a number line if needed).

Question 1 Walkthrough.

Find the maximal domain of the following function:

$$g(x) = \sqrt{x-2} + \log_3(12-2x)$$

Question 2

Find the maximal domain of each of the following functions.

$$\log_3(x^2 - 4) + \frac{3}{x^2 - 1}$$

Question 3 Extension.

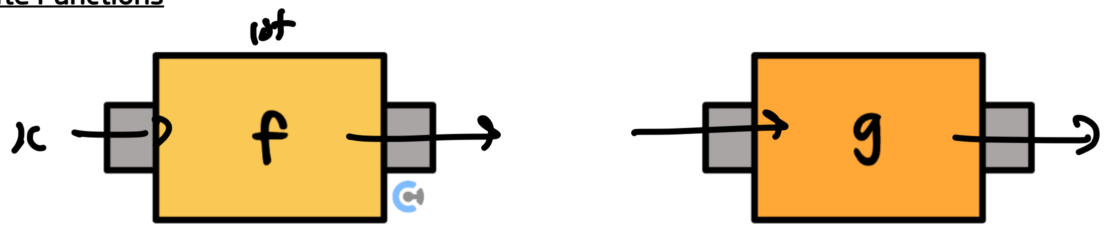
State the maximal domain of the following function.

$$y = \sqrt{5 - x} - \log_3\left(\frac{2}{x + 3}\right)$$

Sub-Section: Basics of Composition

What was the "composition" of functions?

Composite Functions



- Definition: A series of functions.
- Representation of the Above:

$$y = g(f(x)) = g \circ f(x)$$

Space for Personal Notes

Sub-Section: Validity of Composite Functions

Did composite functions work all the time?

Validity of Composite Functions

- Output of $f(x)$: Range of Inside *(Label Above)*
- Input of $g(x)$: Dom of Outside *(Label Above)*
- Composite Function is only valid if:

Range of Ins $\subseteq$ Dom of Out

- Acronym:

RIDO.

Question 4

Consider the functions $f(x) = \sqrt{x+2}$ and $g(x) = x^2 - 4$ defined over their maximal domain.

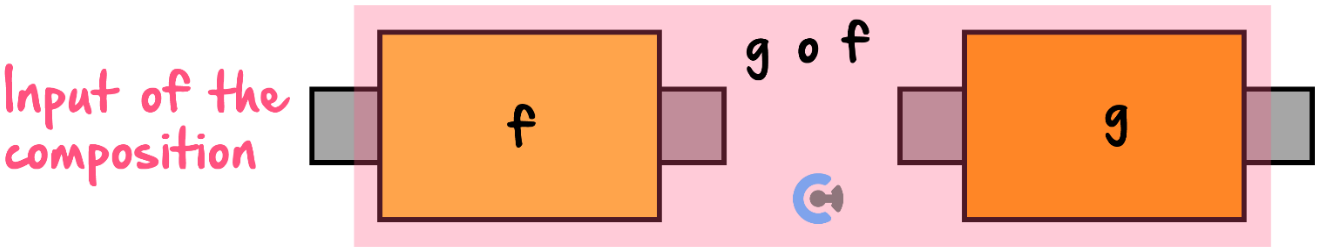
Explain why the composition $f\big(g(x)\big)$ is not valid.

Range g $\subseteq$ Dom f

Sub-Section: Domain of Composite Functions

How did we find the domain of a composite function?

Domain of Composite Functions



$$\textit{Domain of Composite} = \textit{Domain of Inside}$$

Question 5

Consider the functions $f(x) = \sqrt{x+4}$ and $g(x) = x^2 + 2$ defined over their maximal domain.

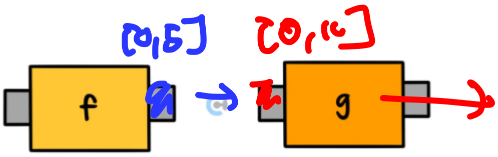
State the domain of the composite function $g(f(x))$.

Dom = Dom f

$= [-4, \infty)$

Sub-Section: Range of Composite Functions

Range of the Composite Functions



Range of Composite* $\subseteq$ *Range of the Outside

- **Finding the range of composition function**: Use the domain and the rule, just like another function.

Question 6 Walkthrough

Consider the functions:

$$f: R \to R, f(x) = x^2 - 4 \quad \to [-4, \infty)$$
$$g: [-9, \infty) \to R, g(x) = \sqrt{x+9}$$

a. For the composite function $g(f(x))$, state the rule and domain.

b. State the range of $g\big(f(x)\big)$.

c. State the range of $g(x)$.

d. Explain why the range of g is not the same as the range of $g \circ f$.

Your turn!

Question 7

Consider the functions:

$$f: [1, \infty) \to R, f(x) = x^2 + 6$$
$$g: R \to R, g(x) = x + 2$$

a. For the composite function $g(f(x))$, state the rule and domain.

$g(x^2+6) = x^2+6+2$
$= x^2+8$

Dom = Dom f = $[1, \infty)$

b. State the range of $g(f(x))$.

$y = x^2+8, \quad x \in [1, \infty)$

$\therefore$ Range $g(f(x)) = [9, \infty)$

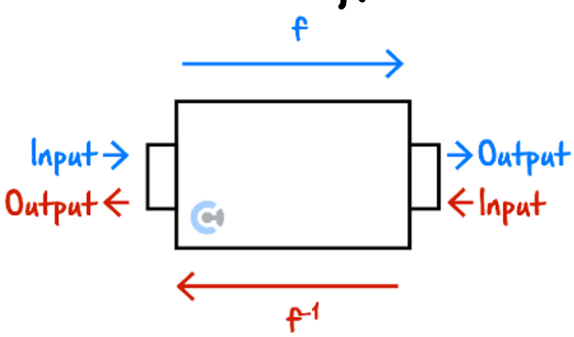
Space for Personal Notes

Sub-Section: Basics of Inverses

What did "Inverse" mean?

Inverse Relation

- **Definition**: Inverse is a relation which does the opposite.



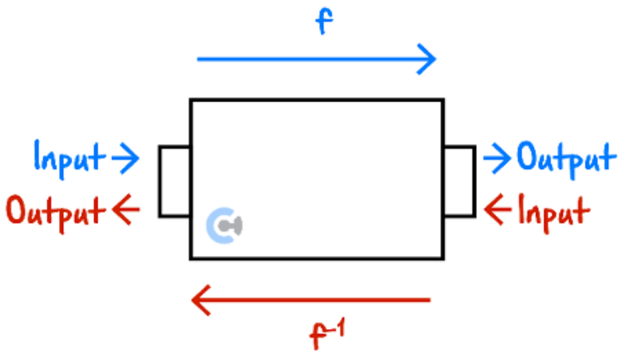
Space for Personal Notes

Sub-Section: Swapping x and y

Is there a better way of solving for an inverse relation?

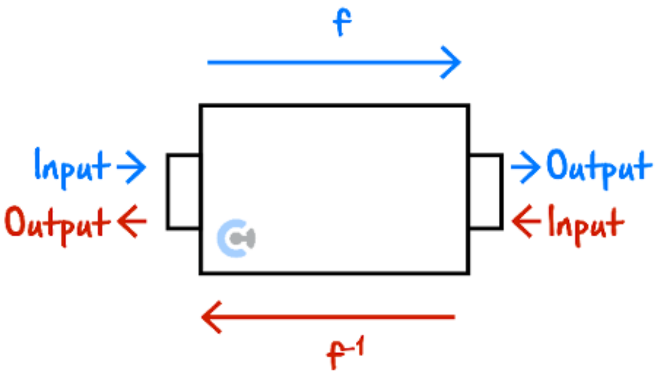
Solving for an Inverse Relation

- Swap x and y.



NOTE: $f(x) = y$.

Domain and Range of Inverse Functions



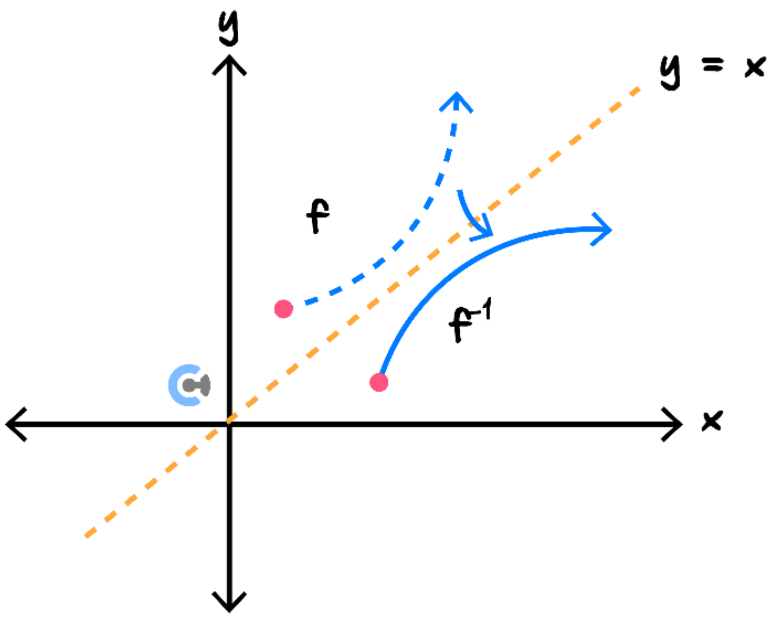
$Dom\ f^{-1} =$ Range f

$Ran\ f^{-1} =$ Dom f

Sub-Section: Symmetry Around $y = x$

Why does this happen?

Symmetry of Inverse Functions



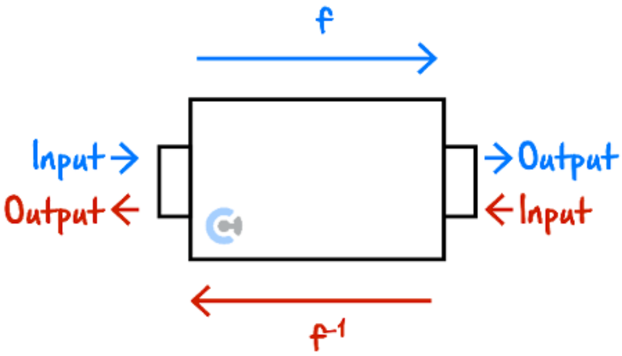
- Inverse functions are always symmetrical around $y = x$.

Space for Personal Notes

Sub-Section: Validity of Inverse Function

Does an inverse function always exist?

Validity of Inverse Functions



- Requirement for Inverse Function:

$$f \text{ needs to be } \underline{\ 1:1\ }$$

Question 8 Walkthrough.

Consider the function $f: (-\infty, a] \to \mathbb{R}, f(x) = 2(x-4)^2 - 8$.

a. Find the largest possible value of a such that the inverse function f^{-1} exists.

b. Find the inverse function and its range.

NOTE: Finding function means to find the rule AND the domain.

TIP: Always try sketching the function to find the domain such that an inverse function can exist!

Your turn!

Question 9

Consider the function $g: (-\infty, b] \rightarrow \mathbb{R}, g(x) = -x^2 - 8x - 14$.

a. Find the largest possible value of b such that the inverse function g^{-1} exists.

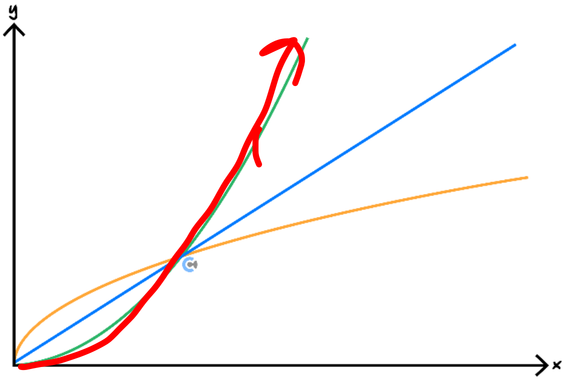
b. Find the inverse function and its range.

Space for Personal Notes

Sub-Section: Intersection Between Inverses

Where do inverses meet?

Intersection Between a Function and its Inverse



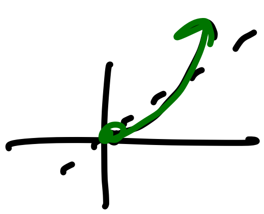
- Equate with y=x instead.

$$f(x) = x \text{ OR } f^{-1}(x) = x$$

- We cannot do this when the function is decreasing function.

Question 10

Find the intersection between $f: [0, \infty) \to R, f(x) = x^2$ and its inverse, without finding the inverse.



$x^2 = x$

Increasing.

$x^2 - x = 0$

$x(x-1) = 0$

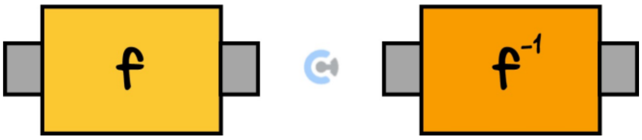
NOTE: This only works for an increasing function.

$x = 0, 1$

$(0,0), (1,1)$

Sub-Section: Composition of Inverses

Composition of Inverse Functions



$f \circ f^{-1}(x) =$ x, for all $x \in$ Dom f^{-1}

$f^{-1} \circ f(x) =$ x, for all $x \in$ Dom f

NOTE: Domain = Domain of Inside.

Question 11 (4 marks)

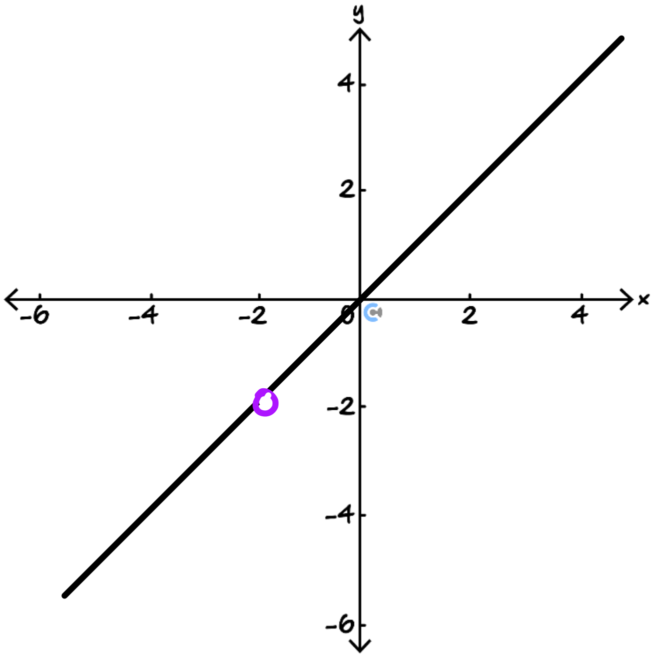
Consider the function $f(x) = \frac{1}{x+2} - 3$.

a. Find the rule and domain for $f^{-1}(f(x))$. (2 marks)

$= x$, Dom = Dom f

$= \mathbb{R} \setminus \{-2\}$

b. Sketch the graph of $y = f^{-1}(f(x))$ on the axes below. (2 marks)



Space for Personal Notes

Section B: Functions and Relations Exam Skills

Context: Exam Skills

- We will go through specific skills that are common in the exams!
- It will be slightly harder so get ready!

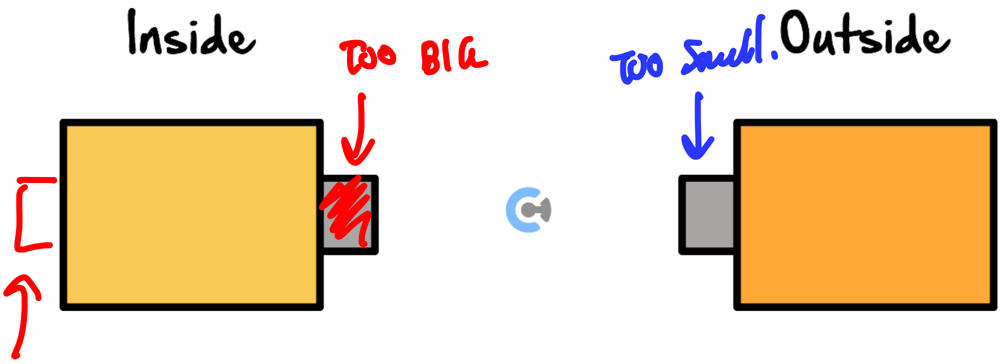
Space for Personal Notes

Sub-Section: Find a New Domain to Fix Composite Functions

How can we go about fixing a broken composite function?

Exploration: Fixing Broken Function

- Consider the following, where the range of the inside is larger than the domain of the outside:



- Is it easier to decrease the range of the inside function, or increase the domain of the outside function? *(Label Above)*

[decrease range of inside function] / [increase domain of outside function]

- How can this be done? *(Label Above)* Restrict the domain.

Fixing Broken Composite Functions

- **Aim**: Restrict the domain of the inside function so that the range of the inside function fits inside the domain of the outside.

- **Steps:**

1. **Write down the RIDO statement** with the domain of the outside (as it is fixed).

2. **Sketch the inside function** to see what domain is needed.

Let's look at some questions together!

Question 12 Walkthrough.

Consider $f(x) = \sqrt{x}$ and $g(x) = 2x - 2$, both defined over their maximal domains.

a. Is $f(g(x))$ defined?

Range $g \nsubseteq$ Dom f

$\mathbb{R} \nsubseteq [0, \infty)$

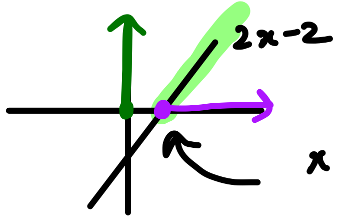
No

b. Find the largest domain of g such that "$f(g(x))$ is defined."

1) $\boxed{\text{Range } g \subseteq \text{Dom } f}$: "Rewrite the aim!"

Range $g \subseteq [0, \infty)$

2.



x int: $2x - 2 = 0$

$x = 1$

Dom $g \subseteq [1, \infty)$

TIP: Always start with the RIDO statement!

Active Recall

- [x] To restrict the domain of inside function so that the range of inside function fits inside the domain of outside.

1. **Write down** ____RIDO____ **statement** with the domain of the outside (as it is fixed).
2. ____Sketch____ **the inside function** to see what domain is needed.

Your Turn!

Question 13

Consider $f(x) = \frac{1}{x}$ and $g(x) = \log_e(x)$, both defined over their maximal domains.

a. Is $g(f(x))$ defined?

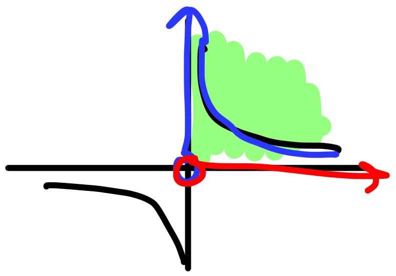
Range $f \subseteq$ Dom g.

$\mathbb{R} \setminus \{0\} \subseteq (0, \infty)$.

b. Find the largest domain of f such that $g(f(x))$ is defined.

$\boxed{\text{Range } f \subseteq \text{Dom } g}$

Range $f \subseteq (0, \infty)$



$\therefore$ Dom $f = (0, \infty)$

Question 14 Extension.

Consider $f(x) = x^2 - 1$ and $g(x) = \sqrt{(x+2)(x-3)}$, both defined over their maximal domains.

a. Is $g(f(x))$ defined?

$(x+2)(x-3) \geq 0$

Range $f \subseteq$ Dom g

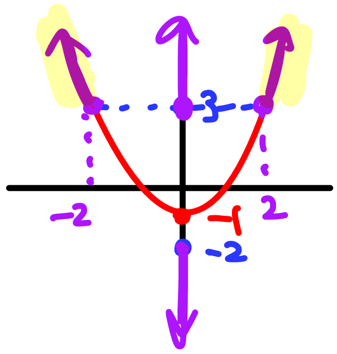
$[-1, \infty) \subseteq (-\infty, -2] \cup [3, \infty)$

b. Find the largest domain of f such that $g(f(x))$ is defined.

1) Range $f \subseteq$ Dom g

Range $f \subseteq (-\infty, -2] \cup [3, \infty)$

2)



$x^2 - 1 = 3$

$x^2 = 4$

$x = \pm 2$

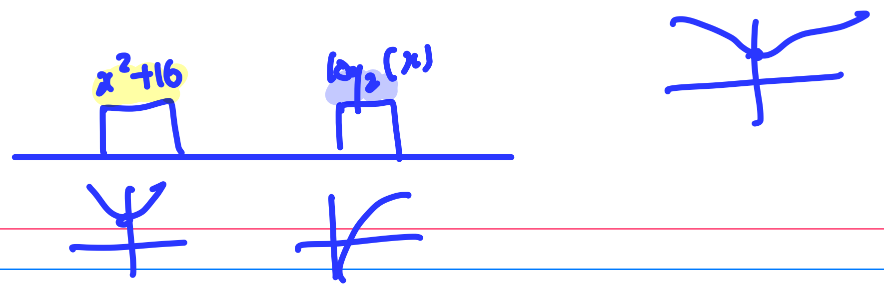
$\boxed{\text{Dom } f = (-\infty, -2] \cup [2, \infty)}$

Key Takeaways

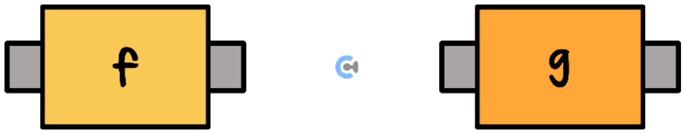
- ☑ The range of the inside function must be a subset of the domain of the outside function.
- ☑ We restrict the domain of the inside function so its range fits in the domain of the outside function.

Sub-Section: Find the range of complex composite functions

Discussion: How do we find a range of a complicated function? Eg: $\underbrace{\log_2(x^2+16)}$



Finding Range of Complex Composite Functions

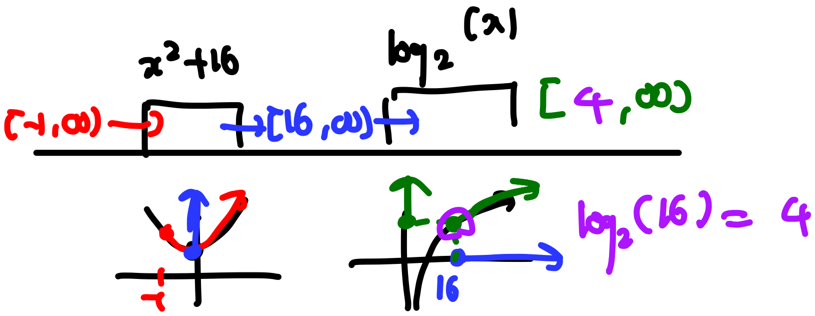


- **Aim**: Find the range of complicated functions.
- **Steps:**
  1. Break the function into composition of two simple functions.
  2. Follow the box diagram to find the range.

Question 15 Walkthrough.

Find the range of $f(x) = \log_2(x^2 + 16)$ where $x \geq -1$.

Range $f = [4, \infty)$

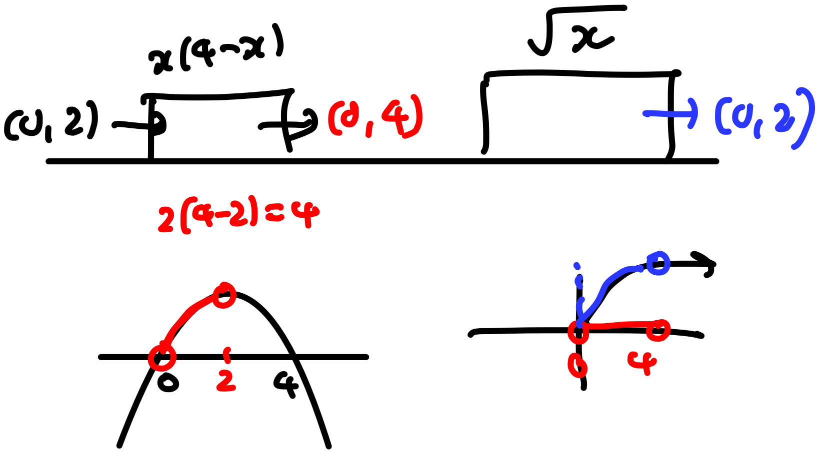


Question 16

Find the range of $f(x) = \sqrt{x(4-x)}$ where $0 < x < 2$.

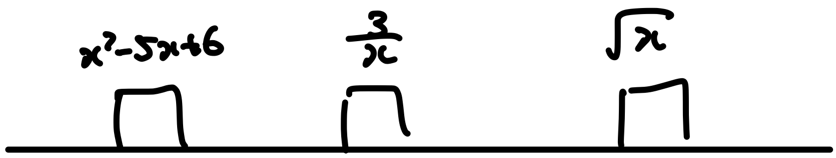
$\frac{\sqrt{3}}{\sqrt{x}}$

$\frac{\sqrt{x}}{\sqrt{3}}$



Question 17 Extension

Find the range of $f(x) = \sqrt{\frac{3}{x^2-5x+6}}$ where $0 < x < 2$.



<u>Key Takeaways</u>

☑ To find the range of a complicated function we can break the function into a composition of two simpler functions.

Sub-Section: Find the Gradient of Inverse Functions

This is a fun application of inverse with calculus!

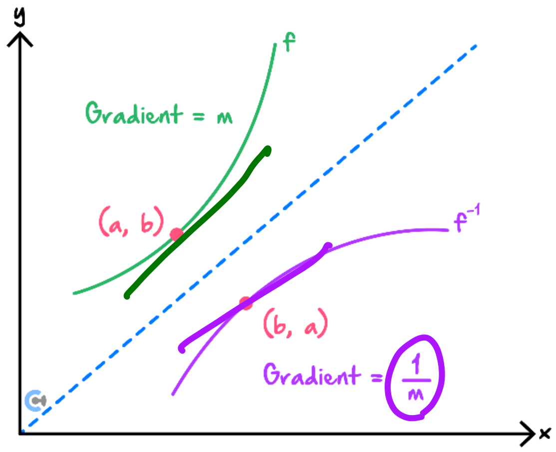
REMINDER: Gradient of a Point

$$\text{Gradient at a point} = \frac{dy}{dx}$$

Discussion: What would happen to the gradient when we inverse the function? (Inverse: Swap x and y.)

$\frac{dy}{dx} \rightarrow \frac{dx}{dy}$

Gradient of an inverse



$$If\ Gradient\ of\ f\ at\ (a, f(a)) = m$$

$$Gradient\ of\ f^{-1}\ at\ \underline{(f(a), a) = \frac{1}{m}}$$

NOTE: The x-value of the inverse is the y-value of the original function.

Question 18 Walkthrough.

Consider the one-to-one function f with the following properties:

$f(3) = 5$ and $f'(3) = 2$. Find the gradient of f^{-1} at $x = 5$ $= \frac{1}{2}$

$f: (3, 5) \quad m = 2.$

$f^{-1}: (5, 3) \quad m = \frac{1}{2}$

TIP: Try sketching the function roughly to see which point it corresponds to.

Question 19

Consider the one-to-one function f with the following properties:

$f(1) = 2, f(3) = 10,\ f'(1) = 4$ and $f'(3) = 6$. Find the gradient of f^{-1} at $x = 2$.

$m = 4 \quad m = 6$

$f: (1, 2),\ \cancel{(3, 10)}$

$f^{-1}: (2, 1) \quad \cancel{(10, 3)}$

$m = \frac{1}{4}$

$m = \frac{1}{4}$

$$\boxed{\begin{gathered} f'(x) \times f^{-1\prime}(f(x)) = 1 \\ f'(f^{-1}(x)) \times f^{-1\prime}(x) = 1 \end{gathered}}$$

Question 20 Extension.

Consider the one-to-one function f with the following properties:

$f(a) = 5, f(4) = a,\ f'(4) = c$ and $f'(a) = d.$ Find the gradient of f^{-1} at $x = a$.

Key Takeaways

- ☑ If the gradient of f at $(a, f(a)) = m$, then the gradient of f^{-1} at $(f(a), a) = \frac{1}{m}$.

NOTE: There are so many ways to link inverse functions to other topics we will see throughout the year!

Space for Personal Notes

Section C: Exam 1 Questions (19 Marks)

INSTRUCTION: 19 Marks. 19 Minutes Writing.

Question 21 (6 marks)

The rule for a function f is given by $f(x) = \sqrt{2x+4} - 1$, where f is defined on its maximal domain.

a. State the domain of f. (1 mark)

$[-2, \infty)$

b. Find the domain and rule of the inverse function f^{-1}. (2 marks)

let $y = f^{-1}(x)$

$x = \sqrt{2y+4} - 1$

$x+1 = \sqrt{2y+4}$

$(x+1)^2 = 2y+4$

$(x+1)^2 - 4 = 2y$

$f^{-1}(x) = \frac{1}{2}(x+1)^2 - 2$

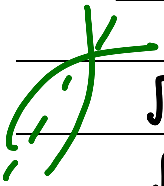
Dom = Range f

$= [-1, \infty)$

c. State the range of f^{-1}. (1 mark)

$[-2, \infty)$

d. Find the point of the intersection between f and f^{-1}. (2 marks)



$\sqrt{2x+4} - 1 = x$

$\sqrt{2x+4} = x+1$

$2x+4 = x^2+2x+1$

$3 = x^2$

$x = \pm\sqrt{3}$

Dom $f = [-2, \infty)$

Dom $f^{-1} = [-1, \infty)$

as $-\sqrt{3} \notin [-1, \infty)$

$(\sqrt{3}, \sqrt{3})$

Question 22 (8 marks)

Let $f: \mathbb{R} \to \mathbb{R}$, where $f(x) = 2^x$ and g: $\mathbb{R} \to \mathbb{R}$ where, $g(x) = x^2 - 4$.

a.

i. Find the rule for h, where $h(x) = f(g(x))$. (1 mark)

$f(x^2-4) = 2^{x^2-4}$

ii. State the domain and range of h. (2 marks)

Dom = Dom $g = \mathbb{R}$

$y = 2^{x^2-4}$

$\mathbb{R} \to \boxed{x^2-4} \to [-4, \infty) \to \boxed{2^x} \to [\frac{1}{16}, \infty)$

$2^{-4} = \frac{1}{2^4} = \frac{1}{16}$

Range $h = [\frac{1}{16}, \infty)$

b. Let $k: (-\infty, a] \to \mathbb{R}$, where $k(x) = 2^{x^2-4}$. It is known that k has a turning point at $x = 0$.

i. Find the largest value of a such that k^{-1}, the inverse function of k exists. (1 mark)

$a = 0$

ii. Find the rule for k^{-1}. (2 marks)

Let $y = k^{-1}(x)$ (Swap x & y)

$x = 2^{y^2-4}$

$\log_2(x) = y^2 - 4$

$\log_2(x) + 4 = y^2$

$f^{-1}(x) = -\sqrt{\log_2(x) + 4}$

as range k^{-1}

$= \text{Dom } k$

$= (-\infty, 0]$

iii. State the domain of k^{-1}. (2 marks)

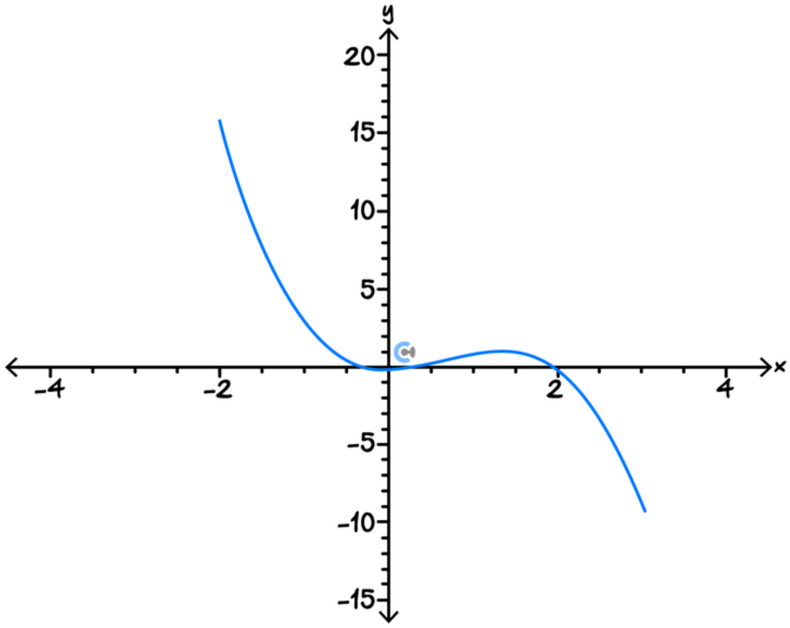
$\text{Dom } k^{-1} = \text{Range } k$

$= [\frac{1}{16}, \infty)$

Space for Personal Notes

Question 23 (5 marks)

Consider the graph of $f(x)$ and the function, below.



$$f: [-2,3] \to \mathbb{R}, f(x) = 2x^2 - x^3 = x^2(2-x)$$

$$g: (0, \infty) \to \mathbb{R}, g(x) = \log_e(x)$$

a. Find the range of f. (2 marks)

$f(-2) = 4(2+2) = 16$

$f(3) = 9 \cdot (2-3) = -9$ $\therefore$ $[-9, 16]$

b. Explain why $g(f(x))$ does not exist. (1 mark)

Range f $\not\subseteq$ Dom g

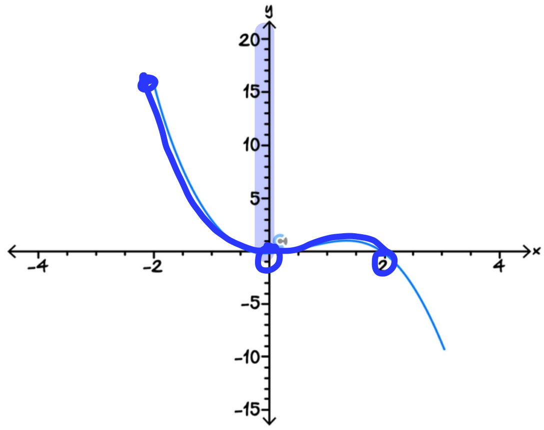
$[-9, 16] \not\subseteq (0, \infty)$

c. Restrict the domain of f to be as large as possible and such that $\boxed{h(x) = g(f(x)) \text{ is defined.}}$ (2 marks)

RIDO

Range $f \subseteq$ Dom g

Range $f \subseteq (0, \infty)$



$\therefore$ Dom $f = [-2, 0) \cup (0, 2)$

Space for Personal Notes

$[-2, 2) \setminus \{0\}$

Section D: Tech Active Exam Skills

Calculator Commands: Finding the domain and range

TI

Sketch

- domain $(f(x), x)$, $f\ Min$ 47. and $Fmax$ 48

Define $f(x)=\sqrt{9-x^2}$	Done
domain$(f(x),x)$	$-3\leq x\leq 3$
fMin$(f(x),x)$	$x=-3$ or $x=3$
fMax$(f(x),x)$	$x=0$
$f(3)$	0
$f(0)$	3

TI-UDF

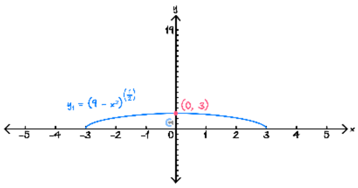
Analyse a Function: Find intercepts, critical points and their nature, maximal domain, asymptote.	analyse(<function>, <variable>) analysed(<function>, <variable>, <lower bound>, <upper bound>)

$$analysed\left(\frac{x^4-2\cdot x^3-3\cdot x^2+3\cdot x+1}{-3\cdot x^3-6\cdot x^2-x+1}, x, -5, 5\right)$$

- Start Point: $\left[-5 \quad \frac{262}{77}\right]$
- End Point: $\left[5 \quad \frac{-316}{529}\right]$
- Maximal Domain:
  $x\neq-1.68469$ and
  $x\neq-0.629579$ and
  $x\neq0.314273$ and
  $-5\leq x\leq 5$

Casio Classpad

- Graph the function and use G-Solve to find min and max values for the range.



Mathematica

```
In[127]:= f[x_] := √(9 - x^2)

In[128]:= FunctionDomain[f[x], x]
Out[128]= -3 ≤ x ≤ 3

In[129]:= FunctionRange[f[x], x, y]
Out[129]= 0 ≤ y ≤ 3
```

Mathematica UDF :

- FInfo [f [x], $\{x, x\ min, x\ max\}, y$]

  Returns useful information about a function, including derivative, domain, range, period,
  horizontal intercepts, vertical intercepts, stationary points, inflexion points, left and sided asymptotes, oblique asymptotes, and vertical asymptotes.

$$\text{FInfo}\left[\frac{x^2-1}{x\,(x^2-3)}, \{x, -\text{Infinity}, \text{Infinity}\}, y\right]$$

The function is $\frac{x^2-1}{x\,(x^2-3)}$

The derivative is $-\frac{x^4+3}{x^2\,(x^2-3)^2}$

Domain: $x<-\sqrt{3} \lor -\sqrt{3}<x<0 \lor 0<x<\sqrt{3} \lor x>\sqrt{3}$

Range: $y\in\mathbb{R}$

Period: 0

Horizontal Intercepts: $\{-1, 1\}$

Vertical Intercepts: None

Stationary points: $\{\}$

Inflexion points: $\{\{-0.871\ldots, -0.123\ldots\}, \{0.871\ldots, 0.123\ldots\}\}$

Left sided asymtote: $y=0$

Right sided asymtote: $y=0$

Oblique asymtote: $\{0\}$

Vertical asymtote: $\{x=0, x=-\sqrt{3}, x=\sqrt{3}\}$

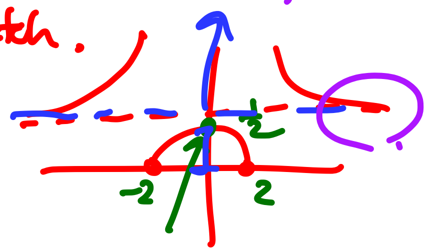
Question 24 Tech-Active.

Find the domain and range of $f(x) = \sqrt{\frac{x^2-1}{x^2-4}}$.

$x \in (-\infty, -2) \cup [-1, 1] \cup (2, \infty)$

$\lim_{x \to \infty} f(x) = 1$

Sketch.



fMax(f(x), x, -2, -2)

$x = 0$

$f(x) = \frac{1}{2}$

$[0, \frac{1}{2}] \cup (1, \infty)$

Calculator Commands: Finding the composite function

TI

Define $f(x)=\ln(x)$ *Done*

Define $g(x)=x^2+3$ *Done*

$f(g(x))$ $\ln(x^2+3)$

CASIO:

define f(x) = ln(x)
done
define g(x) = x^2+3
done
f(g(x))
$\ln(x^2+3)$

Mathematica

```
In[141]:= f[x_] := Log[x]
In[142]:= g[x_] := x^2 + 3
In[143]:= f[g[x]]
Out[143]= Log[3 + x^2]
```

Question 25 Tech-Active.

Let $f(x) = \sqrt{x-1}$ and $g(x) = 3x + 2$ be defined on their maximal domains.

Consider the function $h(x) = f(g(x))$.

a. Find the rule for $h(x)$.

b. Find the domain of $h(x)$.

c. Find the range of $h(x)$.

Calculator Commands: Finding the inverse function

TI

Define $\boxed{f(x)=x^2+4\cdot x+9}$ *Done*

solve$(f(y)=x,y)$ $y=-(\sqrt{x-5}+2)$ or $y=\sqrt{x-5}-2$

solve (f(y) = x, y)

f(x) = y

CASIO:

define f(x) = x^2+4x+9

done

solve(f(y)=x, y)

$\{y=-\sqrt{x-5}-2,\ y=\sqrt{x-5}-2\}$

Mathematica

In[154]:= f[x_] := x^2 + 4 x + 9

In[155]:= Solve[f[y] == x, y]

Out[155]= $\{\{y \to -2 - \sqrt{-5+x}\}, \{y \to -2 + \sqrt{-5+x}\}\}$

NOTE: It doesn't tell us which branch is correct.

Question 26 Tech-Active.

Find the inverse function of $f: (-\infty, 3] \to R, f(x) = x^2 - 6x + 5$.

$f^{-1}(x) = 3 \pm \sqrt{4+x}$

$= 3 - \sqrt{4+x}$.

Section E: Exam 2 Questions (21 Marks)

INSTRUCTION: 21 Marks. 5 Minutes Reading. 26 Minutes Writing.

Question 27 (1 mark)

Consider the functions $f(x) = \frac{1}{(x-2)}$ and $g(x) = \sqrt{x+3}$, defined on their maximal domians. The domain of $f(x)g(x)$ is:

A. $[-3, \infty)$

domain $\left(\frac{1}{x-2} \times \sqrt{x+3}, x\right)$

B. $[-3,2) \cup (2, \infty)$

C. $(-\infty, 2) \cup (2, \infty)$

D. $[-3, \infty) \cup \{2\}$

Question 28 (1 mark)

The function f defined by, $f: A \to \mathbb{R}, f(x) = (x-2)^2 + 3$ will have an inverse function if its domain A is:

A. $\mathbb{R}$

B. $\mathbb{R}^+ \cup \{0\}$

C. $x \geq 2$

D. $x \leq 3$

Question 29 (1 mark)

The function $f(x) = \sqrt{\frac{x^2-4}{x^2-9}}$ has maximal domain:

A. $\mathbb{R} \setminus (-2,2)$

dond (,)

B. $(-\infty, -3) \cup [-2,2] \cup (3, \infty)$

C. $(-\infty, -3) \cup (3, \infty)$

D. $(-2,2)$

Question 30 (1 mark)

The function $f : [-3,3] \to \mathbb{R}, f(x) = \log_4(x^2 + 16)$ has range:

A. $[2, \log_4(25)]$

B. $[0, \log_4(25)]$

C. $[-\log_4(25), \log_4(25)]$

D. $[0,2]$

Question 31 (1 mark)

Let f be a one-to-one differentiable function, and the following values are known:

$$f(2) = 5,\ \ f(3) = 7,\ \ f'(2) = 4 \text{ and } f'(3) = 6$$

Let $g(x) = f^{-1}(x)$, the values of $g'(5)$ is :

f: (2,5) ~~(3,7)~~

m = 4

f^{-1}: (5,2) ~~(7,3)~~

$m = \frac{1}{4}$

A. $\frac{1}{6}$

B. $\frac{1}{7}$

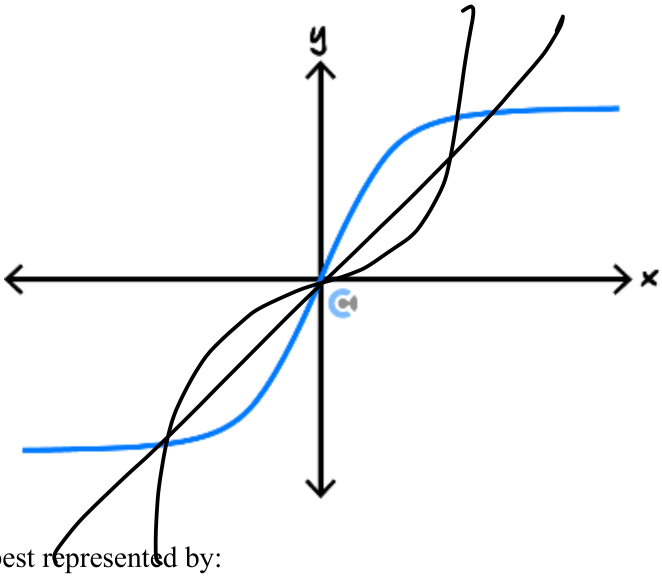
C. $\frac{1}{4}$

D. $\frac{1}{5}$

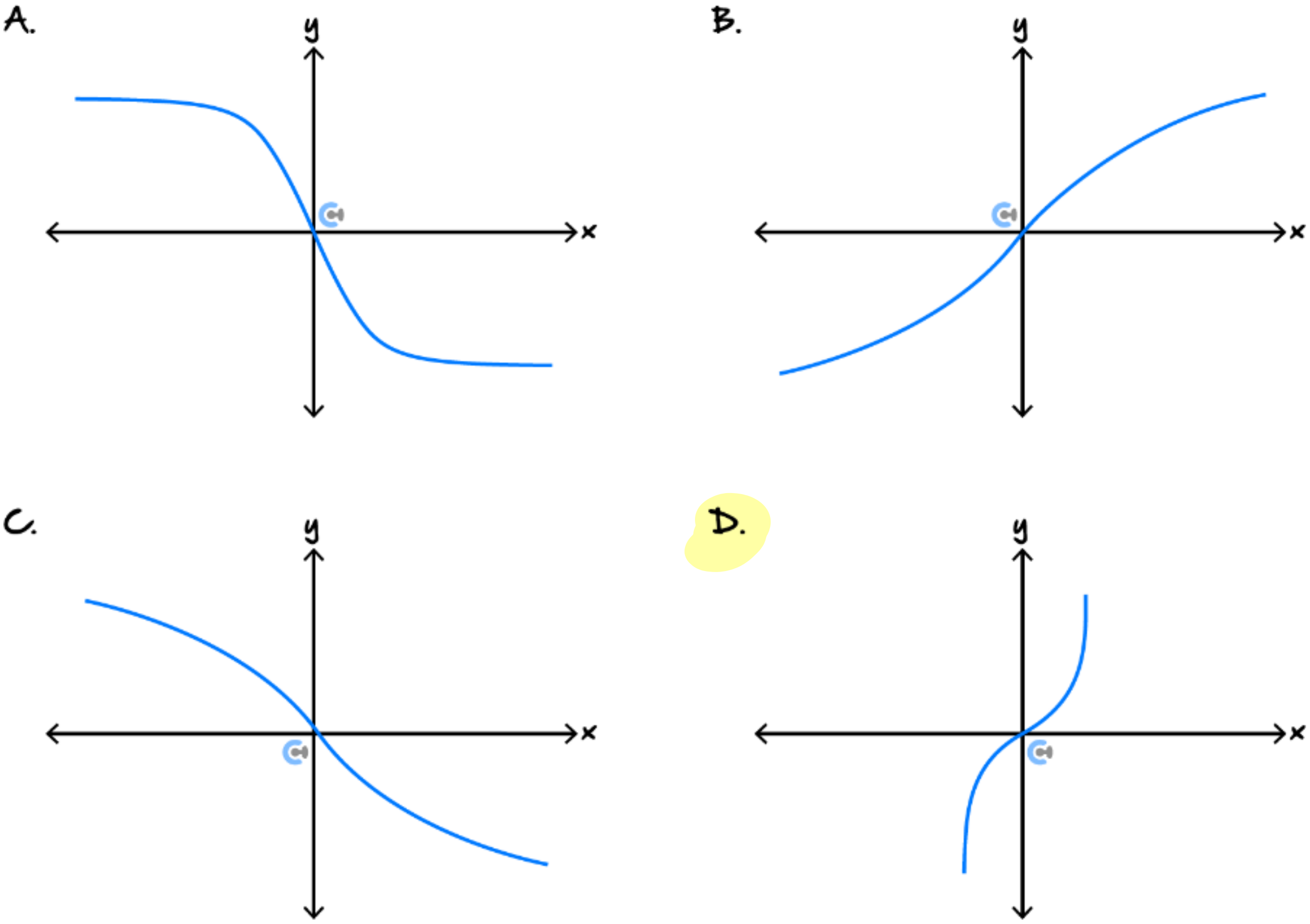
Space for Personal Notes

Question 32 (1 mark)

Part of the graph of $y = f(x)$ is shown below.



The inverse function f^{-1} is best represented by:



Question 33 (14 marks)

Let $f: \mathbb{R} \to \mathbb{R}$, where $f(x) = 4 - 2^x$.

a. Let $g(x) = 1 - 2^x$ and $h(x) = x - 2$.

i. Find the rule $g(h(x))$. (1 mark)

$g(h(x)) = 1 - 2^{x-2} = 1 - \frac{1}{4} 2^x$

ii. Find the real number a such that $f(x) = a \times g(h(x))$. (1 mark)

$a = 4$

b. Define f^{-1}, the inverse function of f. (2 marks)

$f^{-1}(x) = \log_2(4 - x)$

$\text{Dom} = \text{Rge } f^{-1} = (-\infty, 4)$.

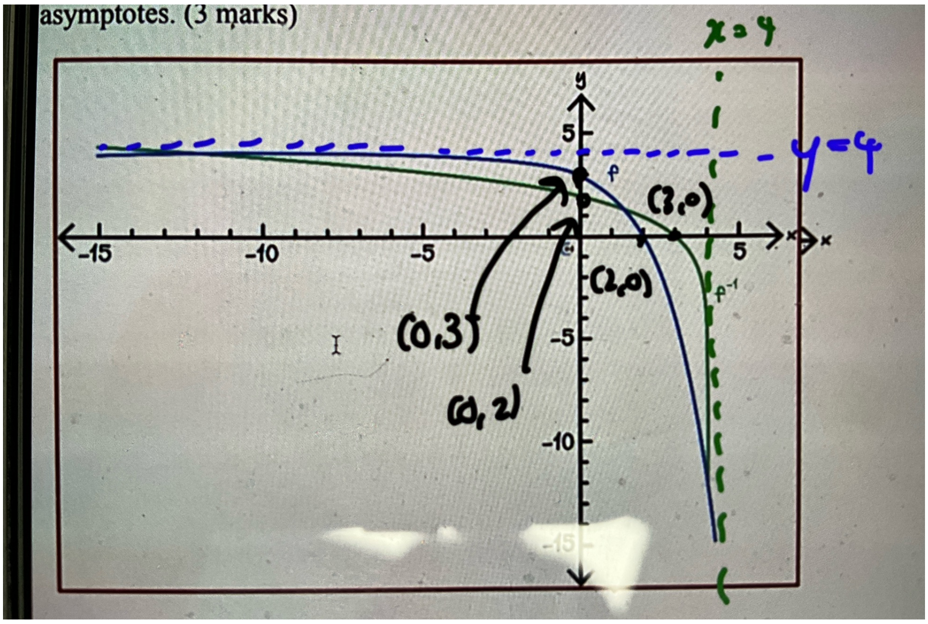
c. It is given that f has a gradient of a when $x = 2$. Find the gradient of f^{-1}, in terms of a, where $x = 0$. (1 mark)

$f: (2, f(2)) \quad m = a$

$f^{-1}: (\underline{f(2)}, 2) \quad m = \boxed{\frac{1}{a}}$

$f(2) = \underline{0}$ $\quad = 0$

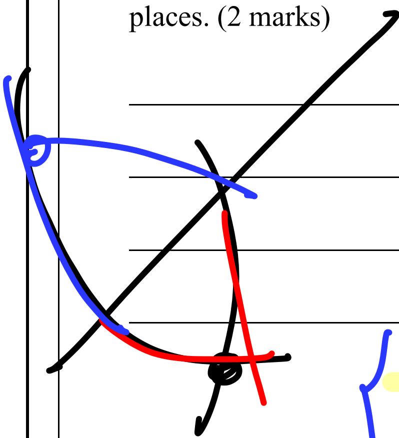
d. Sketch the graphs $y = f(x)$ and $y = f^{-1}(x)$ on the axes below. Label all axes intercepts and give the equations of any asymptotes. (3 marks)



e. Solve the equation $4 - 2^x = x$ for x correct to two decimal places. (1 mark)

$x = 1.39$

f. Find the coordinates of all points of intersection of f and f^{-1}. Give your answers correct to two decimal places. (2 marks)



(1.39, 1.39)

$f(x) = f^{-1}(x)$

decr fn

$x = -12, 4$

$(-12, 4)$ $f(-12) = 4$

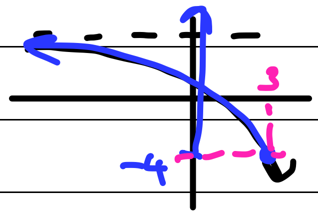
$(4, -12)$ $f(4) = -12$

g. Consider the function $d: [-4, \infty) \to \mathbb{R}, d(x) = \sqrt{x+4}$.

i. Find the largest domain of f such that $d(f(x))$ is defined. (1 mark)

$f(x) \subseteq [-4, \infty)$

Dom: $(-\infty, 3]$



$4 - 2^x = -4$

$8 = 2^x$

$x = 3$

ii. State the rule of $d(f(x))$. (1 mark)

$\sqrt{8 - 2^x}$

iii. Let $c(x) = d(f(x))$. Find the rule and domain for $c^{-1}(c(x))$. (1 mark)

$= x.$

Dom = Dom c = Dom f

$= (-\infty, 3]$

which allows $d(f(x))$ to exist

Space for Personal Notes

Contour Check

Learning Objective: [1.1.1] – Find Maximal Domain and Range

Key Takeaways

- Inside of a $\log$ must be ________________________.
- Inside of a root must be ____.
- Denominator __________________________.
- Domain of sum or product of two functions is equal to________________________ of the two domains.

Learning Objective: [1.1.2] – Find the Rule, Domain and Range of a Composite Function (Range Does Not Require Splitting to Find as the Function is Easy to Draw)

Key Takeaways

- $f(g(x)) =$ ____ $\circ$ ____ (x).
- For composite function to exist, ______________________ $\subseteq$ ______________________.
- The domain of composite is equal to the domain of ___________________ function.
- Range of composite is a ___________ of the range of the outside.

Learning Objective: [1.1.3] – Find the Rule, Domain, and Range of Inverse Functions

Key Takeaways

- f needs to be ________ $for\ f^{-1}$ to exist.
- Domain of the inverse function equals to ____ and vice versa.
- Symmetrical around __________.
- For intersections of inverses, we can equate the function to _______.

Learning Objective: [1.1.4] – Find the Composite Function of Inverse Function

Key Takeaways

- The composite function of inverses is always equal to __________.

Learning Objective: [1.2.1] – Find a new domain to fix composite functions

Key Takeaways

- The range of the ________function must be a subset of the __________of the outside function.
- We restrict the ____________ of the inside function so its __________ fits in the domain of the outside function.

Learning Objective: [1.2.2] - Find the range of complex composite functions

Key Takeaways

- To find the range of a complicated function, we can break the function into a ____________ of two simpler functions.

Learning Objective: [1.2.3] - Find the gradient of inverse functions

Key Takeaways

- If the gradient of f at $(a, f(a)) = m$, then the gradient of f^{-1} at ________