Website: contoureducation.com.au | **Phone:** 1800 888 300
Email: hello@contoureducation.com.au

VCE Mathematical Methods ¾
Functions & Relations Exam Skills [1.2]
Homework

Homework Outline:

Compulsory Questions	Pg 2 – Pg 20
Supplementary Questions	Pg 21 – Pg 42

Section A: Compulsory Questions

Sub-Section: [1.2.1] – Finding a new domain to fix composite functions

Question 1

Consider the following functions defined over their maximal domains,

$$f(x) = x^2 - 1 \text{ and } g(x) = \sqrt{x}$$

a. Show that $g(f(x))$ does not exist.

b. Find the maximal domain of f such that $g(f(x))$ exists.

Space for Personal Notes

Question 2

Consider the functions,

$$f: [0, \infty) \to \mathbb{R}, f(x) = -3\sqrt{x} \text{ and } g: (-\infty, -3) \to \mathbb{R}, g(x) = \log_e(x^2 - 9)$$

a. Show that $g(f(x))$ does not exist.

b. Find the maximal domain of f such that $g(f(x))$ exists.

Space for Personal Notes

Question 3

Consider the following functions defined over their maximal domains,

$$f(x) = \frac{1}{x-1} \text{ and } g(x) = \sqrt{x^2 - 1}$$

a. Show that $g(f(x))$ does not exist.

b. Find the maximal domain of f such that $g(f(x))$ exists.

Space for Personal Notes

Sub-Section: [1.2.2] - Finding the range of complex composite functions

Question 4

Find the range of $f(x) = \log_3(x^2 - 1)$, where f is defined on its maximal domain.

Question 5

Find the range of $f(x) = \log_2(x^2 + 16)$.

Space for Personal Notes

Question 6

The functions f and g are defined over their maximal domains. Let,

$$f(x) = \frac{x+1}{x-1} \text{ and } g(x) = x^2 - 2x$$

Find the range of $h(x) = g(f(x))$.

Space for Personal Notes

Sub-Section: [1.2.3] - Finding the gradient of inverse functions

Question 7

Consider the one-to-one function f with the following properties:

$$f(3) = 4, f(2) = 3, f'(3) = 1 \text{ and } f'(2) = 6$$

Let g be the inverse function of f. Find the gradient of g when $x = 3$.

Question 8

Consider the one-to-one function f with the following properties:

$$f(a) = 3, f(1) = a, f'(1) = c \text{ and } f'(a) = d$$

Let g be the inverse function of f. Find the gradient of g when $x = a$.

Question 9

Let g be the inverse function of f. It is known that:

$$g'(a) = b \text{ and } f'(c) = \frac{1}{b}$$

where $f'(x)$ and $g'(x)$ are one-to-one functions.

Find $g(a)$.

Space for Personal Notes

Sub-Section: Exam 1 Questions

Question 10

Find the maximal domain of the following functions:

a. $f(x) = \sqrt{4-x} + \log_e(x^2 + 4x + 3)$.

b. $g(x) = 2x + \sqrt{\frac{1}{-x^2+x+12}}$.

Space for Personal Notes

Question 11

Let $f: (0, \infty) \to \mathbb{R}$, where $f(x) = \log_2(x)$ and $g: \mathbb{R} \to \mathbb{R}$, where $g(x) = x^2 + 4$.

a.

i. Find the rule for h, where $h(x) = f(g(x))$.

ii. State the value of x for which h is minimised.

iii. State the domain and range of h.

b. Let $k: (-\infty, a] \to \mathbb{R}$, where $k(x) = \log_2(x^2 + 4)$.

i. Find the largest value of a such that k^{-1}, the inverse function of k, exists.

ii. Find the rule for k^{-1}.

iii. State the domain and range of k^{-1}.

Space for Personal Notes

Question 12

Let $f: (-\infty, 1] \to \mathbb{R}, f(x) = \sqrt{1-x}$.

a. State the range of f.

b. Define the inverse function, f^{-1}, of f. Use functional notation.

c. Find all points of intersection of f and f^{-1}.

Space for Personal Notes

Sub-Section: Exam 2 Questions

Question 13

The function f defined by $f: A \to \mathbb{R}, f(x) = (x - 3)^2 + 2$ will have an inverse function if its domain A is:

A. $\mathbb{R}$

B. $\mathbb{R}^+ \cup \{0\}$

C. $x \geq 2$

D. $x \leq 3$

Question 14

The function $f(x) = \frac{\sqrt{x^2-9}}{2x}$ has a maximal domain and range.

A. Domain $= \mathbb{R}\backslash(-3, 3)$ and Range $= [0, \infty)$.

B. Domain $=$ R$\backslash(-3,3)$ and Range $= (-\frac{1}{2}, \frac{1}{2})$

C. Domain $= [-3, 0] \cup [3, \infty)$ and Range $= [1, \infty)$.

D. Domain $= [-\infty, 0) \cup [3, \infty)$ and Range $= [1, \infty)$.

Space for Personal Notes

Question 15

Let f be a one-to-one differentiable function, and the following values are known,

$$f(2) = 5, f(3) = 9, f'(2) = 3 \text{ and } f'(3) = 8$$

Let $g(x) = f^{-1}(x)$, the value of $g'(5)$ is:

A. $\frac{1}{8}$

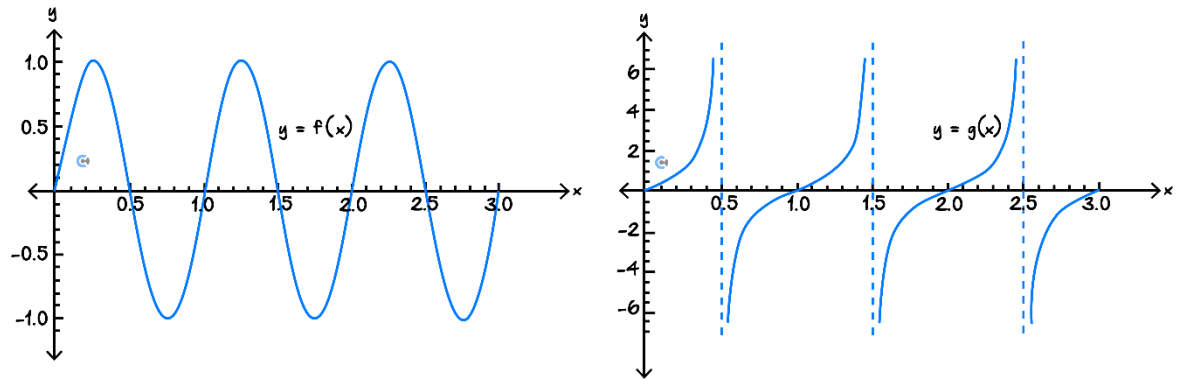
B. $\frac{1}{9}$

C. $\frac{1}{3}$

D. $\frac{1}{5}$

Question 16

Consider the functions f and g graphed below.



How many x-intercepts do the function $h(x) = f(x)g(x)$ have for $x \in [0, 3]$?

A. 4

B. 5

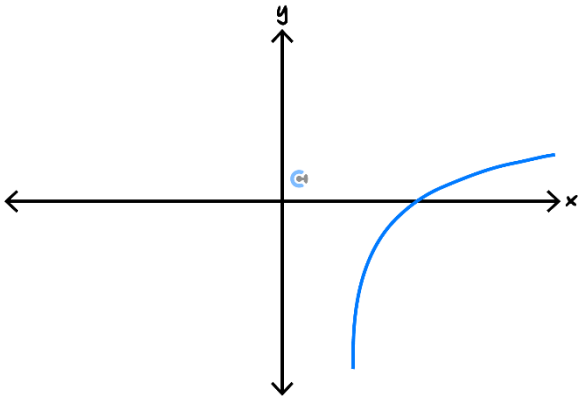
C. 6

D. 7

Space for Personal Notes

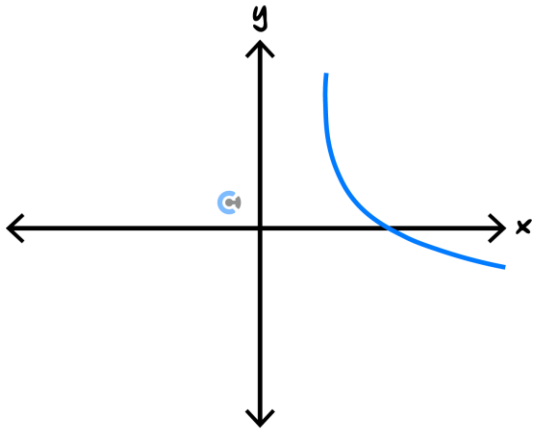
Question 17

Part of the graph of $y = f(x)$ is shown below.

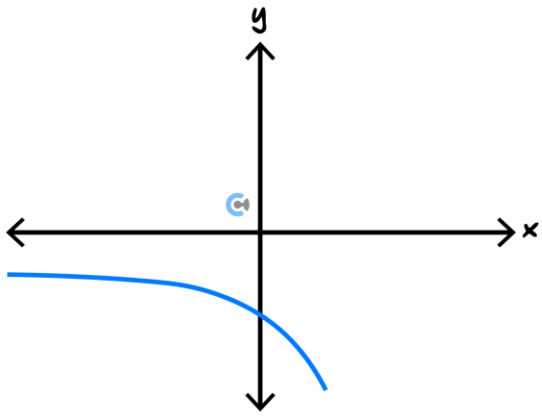


The inverse function f^{-1} is best represented by:

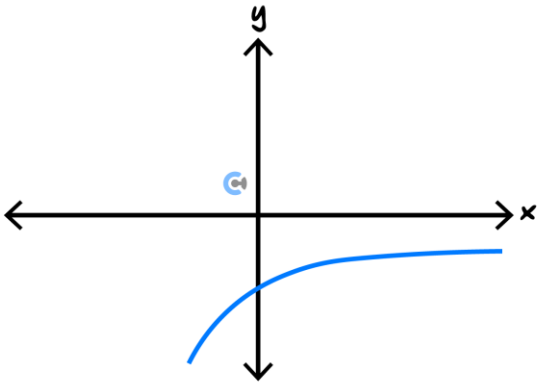
A.



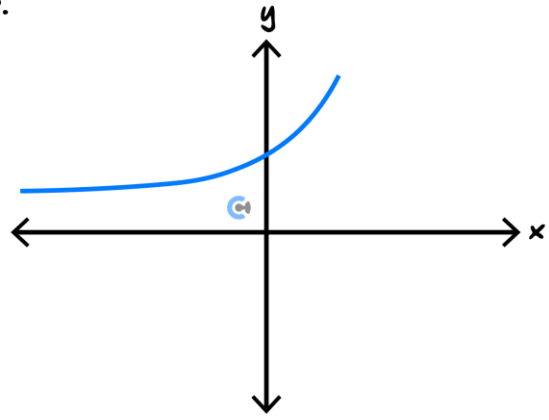
B.



C.



D.



Space for Personal Notes

Question 18

Consider the following functions defined on their maximal domains.

$$f(x) = \sqrt{3 - x}$$

$$g(x) = \log_e\left(\frac{1}{x}\right)$$

a. Find the maximal domain and range of $\frac{1}{f(x)} - g(x)$.

b. Show that $f(g(x))$ does not exist.

c. Show that $g(f(x))$ does not exist.

d. Restrict the domains of f and g to be as large as possible so that both $f(g(x))$ and $g(f(x))$ are defined.

Space for Personal Notes

Question 19

Consider the function:

$$f:(-\infty, a] \to \mathbb{R}, f(x) = -\frac{1}{2}x^2 + 6x + \frac{3}{2}$$

a. Find the smallest value of a such that the inverse function f^{-1} exists.

b. Define the inverse function, f^{-1}.

c. Find all points of intersection between f and f^{-1}.

d. Find the rule and domain for $f(f^{-1}(x))$.

Space for Personal Notes

Section B: Supplementary Questions

Sub-Section: [1.2.1] - Finding a new domain to fix composite functions

Question 20

Consider the functions the following functions defined over their maximal domains,

$$f(x) = \log_e(x) \text{ and } g(x) = e^x - 1$$

a. Show that $f(g(x))$ does not exist.

b. Find the maximal domain of g such that $f(g(x))$ exists.

Space for Personal Notes

Question 21

Consider the following functions defined over their maximal domains,

$$f(x) = (x^2 - 2)^2 \text{ and } g(x) = \sqrt{x - 1}$$

Find the maximal domain of f such that $g(f(x))$ exists.

Space for Personal Notes

Question 22

Consider the following functions defined over their maximal domains,

$$f(x) = \frac{1}{1+x} \text{ and } g(x) = \sqrt{16-(x-1)^2}$$

Find the maximal domain of f such that $g(f(x))$ exists.

Space for Personal Notes

Question 23

Consider the following functions,

$f: [0, 2) \to \mathbb{R}, f(x) = \log_2(4 - x^2)$ and $g: (-\infty, 2) \to \mathbb{R}, g(x) = 3(x - 1)^2 - 1$.

Find the largest interval of x values for which $f(g(x))$ and $g(f(x))$ both exist.

Space for Personal Notes

Sub-Section: [1.2.2] - Finding the range of complex composite functions

Question 24

Find the range of $f(x) = e^{x^2+1}$.

Question 25

Find the range of $f: [0, \infty) \to \mathbb{R}, f(x) = \log_3(3^x + 8)$.

Space for Personal Notes

Question 26

Find the range of $f(x) = \sqrt{\frac{x}{x+1}}$ where f is defined on its maximal domain.

Space for Personal Notes

Question 27

Consider the following functions defined on all real numbers,

$$f(x) = \sin(x) \text{ and } g(x) = \log_3(4x^2 - 4x + 2)$$

Find the range of $g(f(x))$.

Space for Personal Notes

Sub-Section: [1.2.3] - Finding the gradient of inverse functions

Question 28

Consider the function $f: [0, \infty) \to \mathbb{R}, f(x) = x^2$.

The gradient of f at $x = a$ is $2a$.

Let g be the inverse function of f. Find the gradient of g when $x = 2$.

Question 29

Consider the one-to-one function f with the following properties.

$$f(2) = 5, f(5) = 7, f'(2) = 3 \text{ and } f'(5) = 1$$

Let g be the inverse function of f. Find the gradient of g when $x = 5$.

Space for Personal Notes

Question 30

Consider the function $f(x)$, the gradient of f at $x = a$ is $2f(a) + 2a$, and $f(0) = 1$.

From this information, we can tell that the gradient of f^{-1} at $x = b$ is c. Find b and c.

Question 31

Consider the differentiable, one-to-one, function $f: (0, 1) \to \mathbb{R}$. It is known that:

1. $f'(x) = -[f(x)]^2$, for all $x \in (0, 1)$.
2. $\text{ran } f = (1, \infty)$.

If g is the inverse function of f, find the domain and range of $g'(x)$.

Hint: $g'(a)$ denotes the gradient of g at $x = a$.

Sub-Section: Exam 1 Questions

Question 32

Let $f: [0, \infty) \to \mathbb{R}, f(x) = \sqrt{x + 4}$.

a. State the range of f.

b. Let $g: (-\infty, c] \to \mathbb{R}, g(x) = x^2 + 6x + 7$, where $c < 0$.

Find the largest possible value of c such that the range of g is a subset of the domain of f.

c. For the value of c found in **part b.**, state the range of $f(g(x))$.

d. Let $h: \mathbb{R} \to \mathbb{R}, h(x) = x^2 + 5$.

State the range of $f(h(x))$.

Question 33

Let $f: (-2, \infty) \to \mathbb{R}, f(x) = 3 - \frac{4}{(x+2)^2}$.

State the rule and domain of f^{-1}.

Space for Personal Notes

Question 34

a. Let $f: \mathbb{R}\backslash\{3\} \to \mathbb{R}$, $f(x) = \frac{1}{x-3}$. Find the rule for f^{-1}.

b. State the domain of f^{-1}.

c. Let $g(x) = f(x - c) + d$ for $c, d \in \mathbb{R}$.

Find the values of c and d, given that $g = f^{-1}$.

d. Given that $f'(1) = -\frac{1}{4}$ and $f'(4) = -1$, find the value of $g'(1)$.

Space for Personal Notes

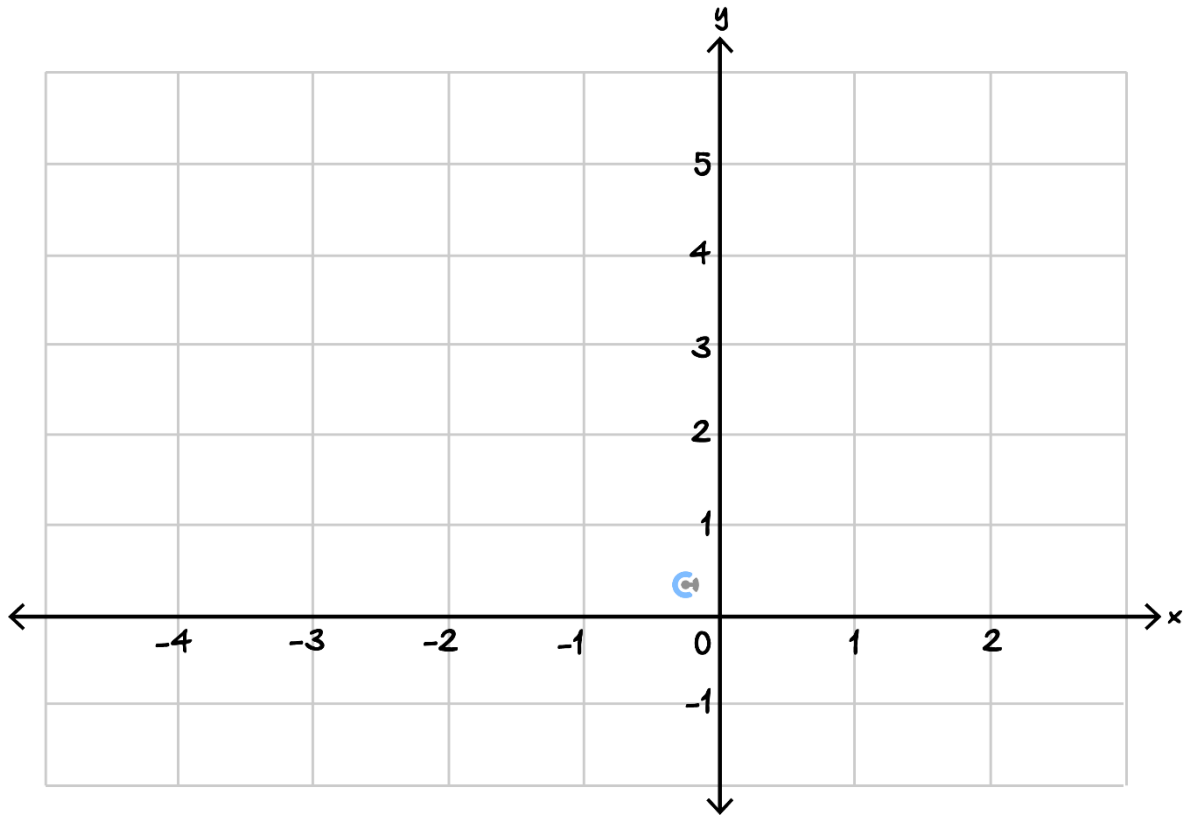
Question 35

Find the maximal domain of f, where $f(x) = \frac{1}{\sqrt{x^2-6x+5}}$.

Space for Personal Notes

Question 36

a. Sketch the graph of $f(x) = 3 + \frac{1}{x+1}$ on the axes below, labelling all asymptotes with their equations and axial intercepts with their coordinates.



b. Find the values of x for which $f(x) \in (2, 4)$.

Space for Personal Notes

Sub-Section: Exam 2 Questions

Question 37

Which one of the following is the inverse function of $g: (-\infty, 2] \to \mathbb{R}, g(x) = 4(x - 2)^2 + 3$?

A. $f: [3, \infty) \to \mathbb{R}, f(x) = 2 + \frac{\sqrt{x-3}}{2}$

B. $f: [3, \infty) \to \mathbb{R}, f(x) = 2 - \frac{\sqrt{x-3}}{2}$

C. $f: [3, \infty) \to \mathbb{R}, f(x) = 4 + \frac{\sqrt{x-3}}{4}$

D. $f: [3, \infty) \to \mathbb{R}, f(x) = 4 - \frac{\sqrt{x-3}}{4}$

Question 38

The maximal domain of the function f is $\left(-\infty, 1 - \sqrt{5}\right] \cup [1 + \sqrt{5}, \infty)$.

A possible rule of f is:

A. $f(x) = \sqrt{5 - (x-1)^2}$

B. $f(x) = \log_e(5 - (x-1)^2)$

C. $f(x) = \frac{1}{\sqrt{5}-(x-1)^2}$

D. $f(x) = \frac{1}{\log_e(5-(x-1)^2)}$

Space for Personal Notes

Question 39

Let f be a one-to-one differentiable function and the following values are known,

$$f(-1) = 3, f(3) = 7, f'(-1) = 5 \text{ and } f'(3) = 2$$

Let $g(x) = f^{-1}(x)$, the value of $g'(3)$ is:

A. 5

B. 2

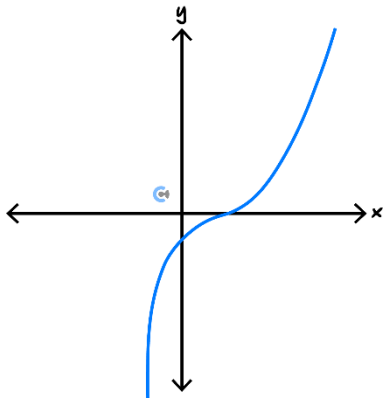
C. $\frac{1}{5}$

D. $\frac{1}{2}$

Space for Personal Notes

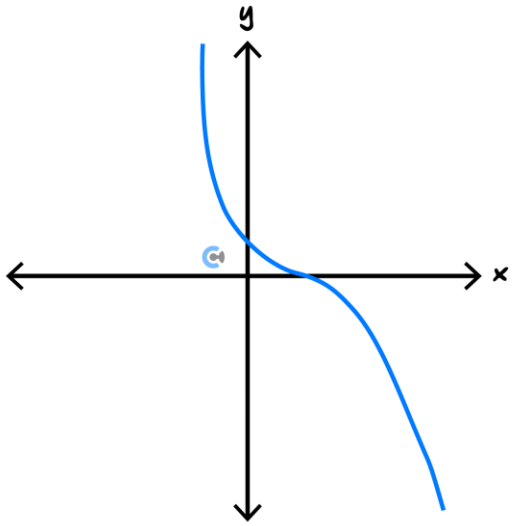
Question 40

Part of the graph of the function f is shown below. The same scale has been used on both axes.

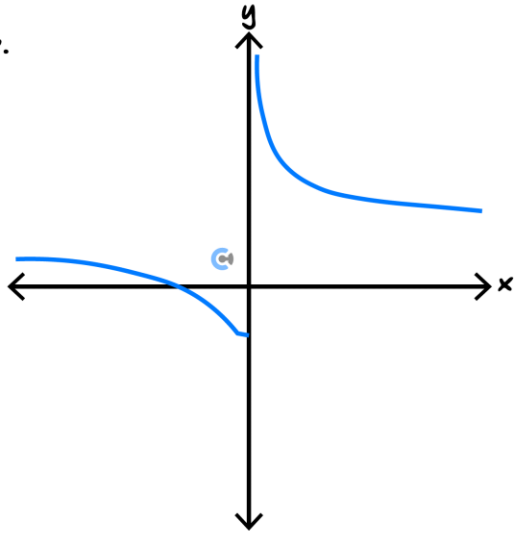


The corresponding part of the graph of the inverse function f^{-1} is best represented by:

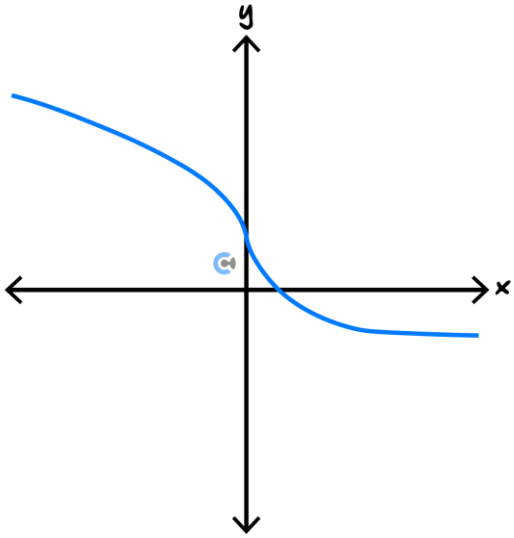
A.



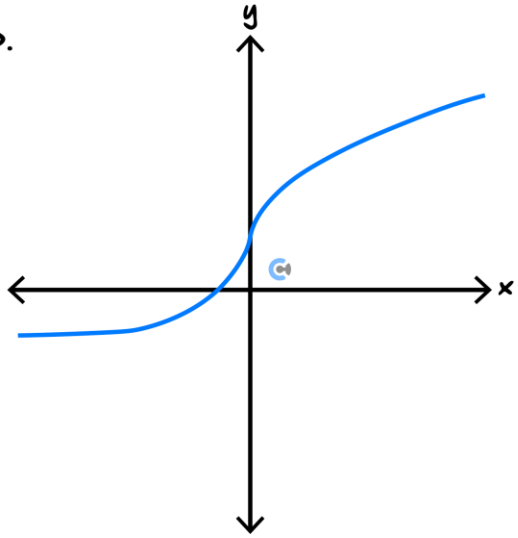
B.



C.



D.



Question 41

Consider the following functions,

$$f:\left(-\frac{\sqrt{3}}{2},\infty\right)\to\mathbb{R}, f(x)=\log_e\left(x+\frac{\sqrt{3}}{2}\right)$$

$$g:(-\infty,3)\to\mathbb{R}, g(x)=\cos(x)$$

The largest interval of x values for which $f(g(x))$ and $g(f(x))$ both exist is:

A. $\left[-\frac{\sqrt{3}}{2},\frac{\sqrt{3}}{2}\right]$

B. $\left(-\frac{\sqrt{3}}{2},\frac{5\pi}{6}\right)$

C. $\left(-\frac{5\pi}{6},\frac{5\pi}{6}\right)$

D. $\left(-\frac{\sqrt{3}}{2},\frac{\sqrt{3}}{2}\right)$

Space for Personal Notes

Question 42

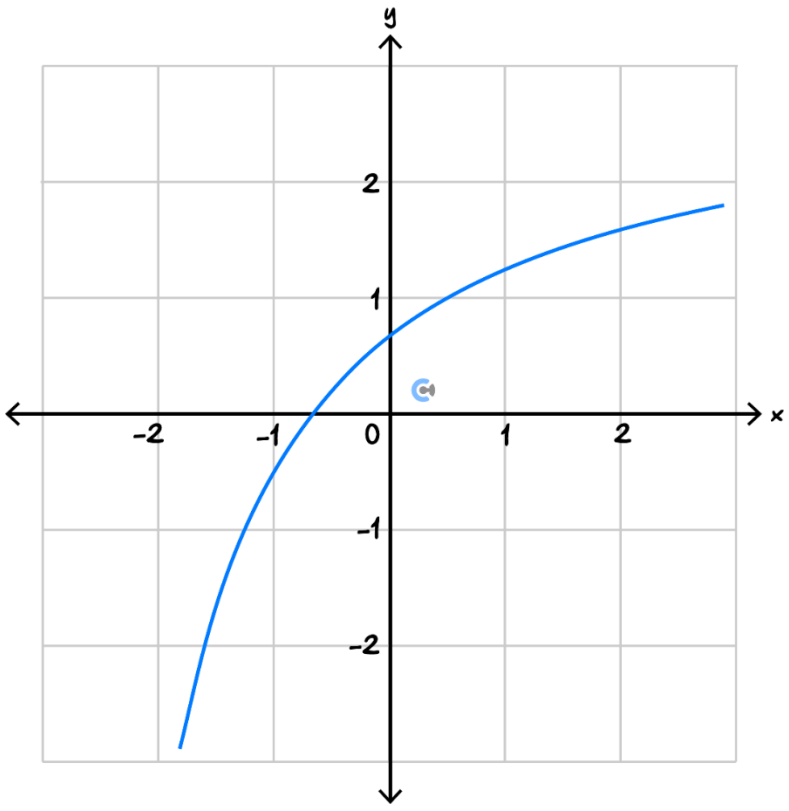
a. Express $\frac{3x+2}{x+3}$ in the form of $a + \frac{b}{x+2}$, where a and b are non-zero integers.

b. Let $f : \mathbb{R}\backslash\{-3\} \to \mathbb{R}, f(x) = \frac{3x+2}{x+3}$.

i. Find the rule and domain of f^{-1} and the inverse function of f.

ii. Part of the graph of f is shown in the diagram below.

Sketch the graph of $y = f^{-1}$, labelling all points of intersection with their coordinates.



c. Let $g(x) = -\sqrt{16 - x^2}$.

i. Show that both $f(g(x))$ and $g(f(x))$ do not exist.

ii. Find the largest interval on which both $f(g(x))$ and $g(f(x))$ are defined on.

Space for Personal Notes

Question 43

Let $f(x) = 2^{-x}$ and $g(x) = 4x^2 - 4x + 3$.

a.

i. State the rule of $f(g(x))$.

ii. State the range of $f(g(x))$.

b. Let $h: [a, \infty) \to \mathbb{R}, h(x) = g(f(x))$. Find the smallest value of a such that h is a one-to-one function.

c. For the value of a found in **part b.**, state the rule and domain for h^{-1}.

d. How many solutions does the equation $f(g(x)) + g(f(x)) = 0$ have?

Space for Personal Notes

CONTOUREDUCATION

Website: contoureducation.com.au | **Phone:** 1800 888 300 | **Email:** hello@contoureducation.com.au

VCE Mathematical Methods ¾

Free 1-on-1 Support

Be Sure to Make The Most of These (Free) Services!

- Experienced Contour tutors (45+ raw scores, 99+ ATARs).
- For fully enrolled Contour students with up-to-date fees.
- After school weekdays and all-day weekends.

1-on-1 Video Consults	Text-Based Support
➢ Book via bit.ly/contour-methods-consult-2025 (or QR code below). ➢ One active booking at a time (must attend before booking the next).	➢ Message +61 440 138 726 with questions. ➢ Save the contact as "Contour Methods".

Booking Link for Consults
bit.ly/contour-methods-consult-2025

Number for Text-Based Support
+61 440 138 726