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VCE Mathematical Methods ¾
Functions & Relations [1.1]
Test Solutions

17.5 Marks. 1 Minute Reading. 21 Minutes Writing.

Results:

Test Questions	________ / 17.5
Extension	________ / 5

Section A: Test Questions (17.5 Marks)

INSTRUCTION: 17.5 Marks. 21 Minutes Writing.

Question 1 (2.5 marks)

Tick whether the following statements are **true** or **false.**

	True	False
a. Inside of the log can be all non negative numbers.		✓
b. To calculate the maximal domain of sum of two functions, you find the union of the two domain.		✓
c. Composite function is only defined if the range of the inside is a subset of the domain of the outside.	✓	
d. Composite function's range is always the range of the outside function.		✓
e. The composition of inverse functions is always equal to $y = x$ with a domain equal to domain of inside.	✓	

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Question 2 (2 marks)

Consider the following functions both defined on their maximal domains.

$$f(x) = -\sqrt{x+15}$$

$$g(x) = \log_2(-x+4)$$

Find the maximal domain of $f(x) + g(x)$.

Solution: We require that both $f(x)$ and $g(x)$ are defined.
$f(x)$ is defined for $x \geq -15$ and $g(x)$ is defined for $4 - x > 0 \implies x < 4$.
Therefore maximal domain is $-15 \leq x < 4$.

Question 3 (5 marks)

Consider the following functions both defined on their maximal domains.

$$f(x) = \sqrt{4-x^2}$$

$$g(x) = x^2 + 3$$

a. State whether $f(g(x))$ or $g(f(x))$ is defined. (1 mark)

Only $g(f(x))$ is defined.

b. Find the rule and domain of the composite function which is defined from **part. a**. (2 marks)

$g(f(x)) = (\sqrt{4-x^2})^2 + 3 = 7 - x^2$. dom $g(f(x)) =$ dom $f(x)$. Therefore,

$$g(f(x)) = 7 - x^2 \quad \text{with domain } = [-2, 2].$$

c. Find the range of the composite function which is defined from **part. a**. (2 marks)

ran $g(f(x)) = [3, 7]$.

Question 4 (8 marks)

Consider the following function:

$$f: (-\infty, a] \to R, f(x) = x^2 - 4x - 6$$

a. Solve for the largest value of a such that, the inverse function f^{-1} exists. (1 mark)

$f(x) = (x-2)^2 - 10$. Therefore f has a local minimum at $(2, 10)$. So $a = 2$.

b. Define the function $f^{-1}(x)$. (3 marks)

Solution: Let $y = (x-2)^2 - 10$. Swap x and y.

$$x = (y-2)^2 - 10$$
$$y - 2 = \pm\sqrt{x+10}$$
$$y = 2 \pm \sqrt{x+10}$$

Now $ran\ f^{-1} = Dom\ f = (-\infty, 2]$

$\therefore f^{-1}: (-\infty, 2] \to \mathbb{R}$,

$f^{-1}(x) = 2 - \sqrt{x+10}$

c. Find the point of intersection between $f(x)$ and $f^{-1}(x)$. (2 marks)

Solution: Solve $f(x) = x$

$$x^2 - 4x - 6 = x$$
$$x^2 - 5x - 6 = 0$$
$$(x-6)(x+1) = 0$$
$$x = -1, 6$$

By considering domains the only point of intersection is $(-1, -1)$.

d. Define the function $f^{-1}\left(f(x)\right)$. (2 marks)

$f^{-1}(f(x)) = x \quad$ for $x \in (-\infty, 2]$.

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Section B: Extension Test Questions (5 Marks)

INSTRUCTION: 5 Marks. 3 Minutes Writing.

Question 5 (5 marks)

Consider the following functions defined on their maximal domains.

$$f(x) = x^2 - 4$$

$$g(x) = \frac{1}{\sqrt{x-5}}$$

a. Restrict the domain of f so that the composite function $g \circ f$ is defined. (2 marks)

Solution: For the composition to be defined we require ran $f > 5$.
Therefore $x^2 - 4 > 5 \implies x^2 > 9$.
Restrict the domain of f to $x \in (-\infty, -3) \cup (3, \infty)$.

b. Hence, define the function $g \circ f$. (1 mark)

$g \circ f : (-\infty, -3) \cup (3, \infty) \to \mathbb{R}, \ (g \circ f)(x) = \dfrac{1}{\sqrt{x^2 - 9}}$.

Let $h(x) = x^4 - 2kx^2 + 21$ where $k > 0$.

c. Find the range of values of k for which the function $g \circ h$ exists. (2 mark)

Solution: We require that $x^4 - 2kx^2 + 16 > 0$. Let $a = x^2$, then consider

$$a^2 - 2ka + 16 > 0$$

Consider the discriminant to find the values of k for which this always holds.

$$\Delta = 4k^2 - 64 < 0$$
$$k^2 < 16$$

Therefore $0 < k < 4$.

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