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Website: contoureducation.com.au | **Phone:** 1800 888 300
Email: hello@contoureducation.com.au

VCE Mathematical Methods ¾
Functions & Relations
Homework Solutions

Homework Outline:

Compulsory	Pg 2 – Pg 15
Supplementary	Pg 16 – Pg 28
Solutions	Pg 2 – Pg 28

Section A: Compulsory

Sub-Section [1.1.1]: Find the Maximal Domain and Range of Functions

Question 1

Find the maximal domain of the following functions.

a. $f(x) = \sqrt{x+3}$

Need $x + 3 \geq 0$, therefore domain $= [-3, \infty)$

b. $f(x) = \frac{1}{x-2} + 1$

Need $x - 2 \neq 0$ therefore domain $= \mathbb{R} \setminus \{2\}$

c. $f(x) = \log_e(4 - x)$

Need $4 - x > 0$, therefore domain $= (-\infty, 4)$

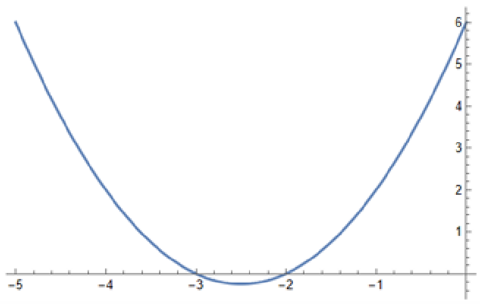
Question 2

Find the maximal domain of the following functions.

a. $f(x) = -\sqrt{x^2 + 5x + 6}$

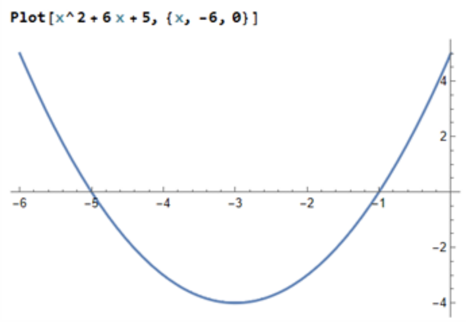
We need $x^2 + 5x + 6 \geq 0 \implies (x+2)(x+3) \geq 0$, therefore domain $= (-\infty, -3] \cup [-2, \infty)$

Plot[x^2+5x+6, {x, -5, 0}]



b. $f(x) = \log_e(x^2 + 6x + 5)$

We require $x^2 + 6x + 5 > 0 \implies (x+5)(x+1) > 0$ therefore domain $= \mathbb{R} \setminus [-5, -1]$.



c. $f(x) = \frac{1}{x^2+2x-3}$

We require that $x^2 + 2x - 3 \neq 0 \implies (x-1)(x+3) \neq 0$, therefore domain $= \mathbb{R} \setminus \{-3, 1\}$.

Question 3

Find the maximal domain of the following functions.

a. $f(x) = \log_e(5 - x) + \sqrt{2x - 7} + 1$

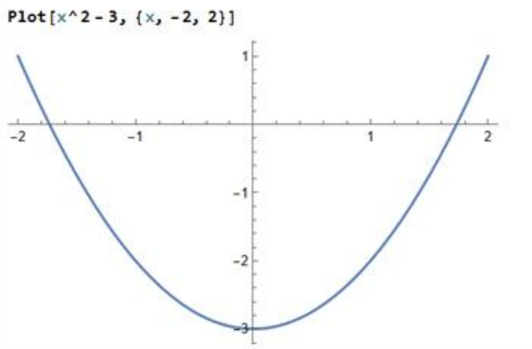
We require both that $5 - x > 0 \implies x < 5$ and $2x - 7 \geq 0 \implies x \geq \frac{7}{2}$. Therefore the domain is $\left[\frac{7}{2}, 5\right)$.

b. $f(x) = \frac{1}{x} - \frac{1}{x^2-5x+4}$

We require that both $x \neq 0$ and $x^2 - 5x + 4 \neq 0 \implies (x-4)(x-1) \neq 0 \implies x \neq 1, 4$. Therefore domain $= \mathbb{R} \setminus \{0, 1, 4\}$.

c. $f(x) = \frac{1}{x-4} \times \sqrt{x^2 - 3}$

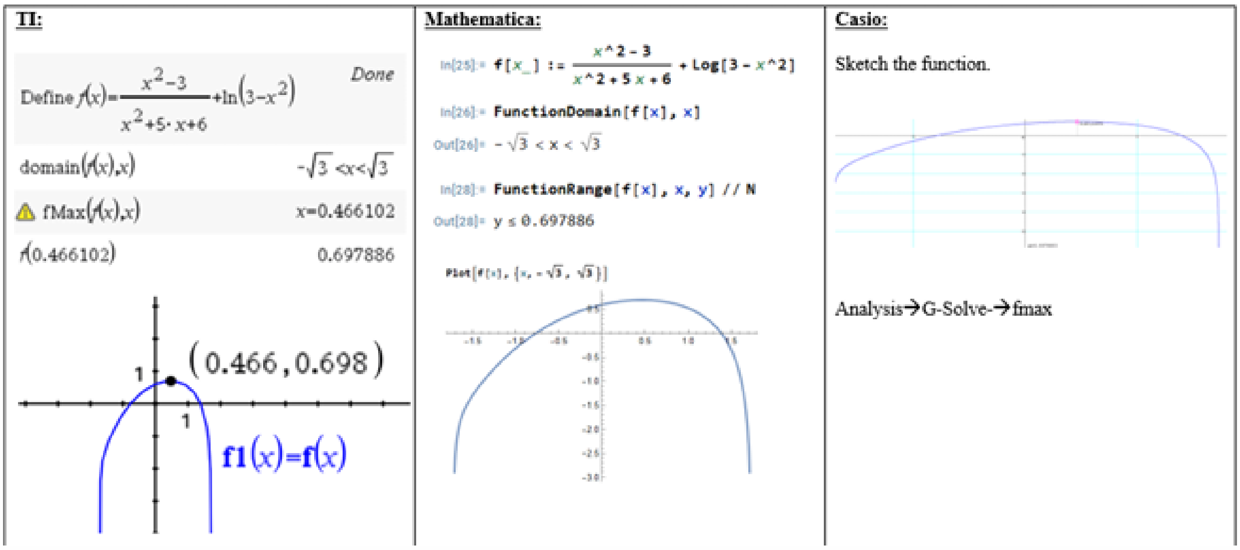
We require both that $x \neq 4$ and $x^2 - 3 \geq 0 \implies x > \sqrt{3}$ or $x < -\sqrt{3}$. Therefore the domain is $(-\infty, -\sqrt{3}) \cup (\sqrt{3}, 4) \cup (4, \infty)$.



Question 4 Tech-Active.

Find the maximal domain and range of $f(x) = \frac{x^2-3}{x^2+5x+6} + \log_e(3 - x^2)$. Give the range correct to three decimal places.

Domain $= (-\sqrt{3}, \sqrt{3})$. Range $= (-\infty, 0.698]$.



Sub-Section [1.1.2]: Existence, Rule, Domain, and Range of Composite Functions

Question 5

The following functions are defined over their maximal domain.

$$f(x) = \sqrt{x} \text{ and } g(x) = x - 3$$

a. Determine whether $f(g(x))$ and $g(f(x))$ exist.

$f(g(x))$ does not exist since ran $g(x) = \mathbb{R} \nsubseteq$ dom $f = [0, \infty)$.
$g(f(x))$ does exist since ran $f = [0, \infty) \subseteq$ dom $g = \mathbb{R}$.

b. Find the rule of any composition that exists.

$$g(f(x)) = f(x) - 3 = \sqrt{x} - 3$$

c. State the domain of any composition that exists.

$$\text{dom } g \circ f = \text{dom } f = [0, \infty)$$

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Question 6

The following functions are defined over their maximal domain.

$$f(x) = \frac{1}{x-1} \text{ and } g(x) = \frac{1}{x}$$

a. Determine whether $f(g(x))$ and $g(f(x))$ exist.

$f(g(x))$ does not exist since $\text{ran } g = \mathbb{R} \setminus \{0\} \not\subseteq \text{dom } f = \mathbb{R} \setminus \{1\}$.
$g(f(x))$ does exist since $\text{ran } f = \mathbb{R} \setminus \{0\} = \text{dom } g$.

b. Find the rule of any composition that exists.

$$g(f(x)) = \frac{1}{f(x)} = \frac{1}{\frac{1}{x-1}} = x - 1.$$

c. State the domain of any composition that exists.

$$\text{dom } g \circ f = \text{dom } f = \mathbb{R} \setminus \{1\}.$$

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Question 7

For the following functions:

$$f : [0, 6] \to \mathbb{R}, f(x) = x^3 \text{ and } g(x) = \sqrt{x+4}.$$

a. Determine whether $f(g(x))$ and $g(f(x))$ exist.

$f(g(x))$ does not exist since ran $g = [0, \infty) \not\subseteq$ dom $f = [0, 6]$.
$g(f(x))$ does exist since ran $f = [0, 216] \subseteq$ dom $g = [-4, \infty)$.

b. Find the rule of any composition that exists.

$$g(f(x)) = \sqrt{f(x)+4} = \sqrt{x^3+4}.$$

c. State the domain of any composition that exists.

dom $g \circ f =$ dom $f = [0, 6]$.

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Sub-Section [1.1.3]: Finding the Rule, Domain, and Range of Inverse Functions

Question 8

For the function:

$$f:(5,\infty)\to\mathbb{R}, f(x)=\frac{1}{5-x}$$

a. Fully define the inverse function.

Swap x and y. $x=\frac{1}{5-y} \implies 5-y=\frac{1}{x} \implies y=5-\frac{1}{x}$.
Then dom f^{-1} = ran $f=(-\infty,0)$

$$f^{-1}:(-\infty,0)\to\mathbb{R},\ f^{-1}(x)=5-\frac{1}{x}.$$

b. Find the range of the inverse function.

ran f^{-1} = dom $f=(5,\infty)$.

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Question 9

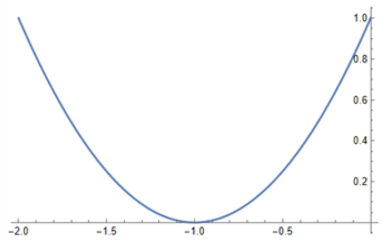
For the function:

$$f : (-\infty, k] \to \mathbb{R}, f(x) = x^2 + 2x + 1$$

a. Find the largest value of k such that the inverse function exists.

$f(x) = (x+1)^2$. The function must be one-to-one for the inverse to exist. f has a local minimum at $(-1, 0)$. Therefore the largest value of k is -1.

`Plot[x^2 + 2 x + 1, {x, -2, 0}]`



b. Fully define the inverse function.

Swap x and y. $x = (y+1)^2 \implies y + 1 = \pm\sqrt{x} \implies y = \pm\sqrt{x} - 1$.
Since $\text{dom } f = (-\infty, -1] = \text{ran } f^{-1}$ we take the negative and $\text{ran } f = \text{dom } f^{-1} = [0, \infty)$

$$f^{-1} : [0, \infty) \to \mathbb{R},\ f^{-1}(x) = -1 - \sqrt{x}.$$

c. Find the range of the inverse function.

$\text{ran } f^{-1} = \text{dom } f = (-\infty, -1]$

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Question 10

For the following functions:

$$f : [b, \infty) \to \mathbb{R}, f(x) = -\sqrt{x+2}.$$

a. Find the smallest value of b such that the inverse function exists.

f is one-to-one on its maximal domain. Therefore $b = -2$.

b. Fully define the inverse function.

Swap x and y. $x = -\sqrt{y+2} \implies x^2 = y + 2 \implies y = x^2 - 2$.
Now dom f^{-1} = ran $f = (-\infty, 0]$. Therefore,

$$f^{-1} : (-\infty, 0] \to \mathbb{R},\ f^{-1}(x) = x^2 - 2.$$

c. Find the range of the inverse function.

ran f^{-1} = dom $f = [-2, \infty)$.

d. Find the point of intersection between f and f^{-1}.

The functions intersect on the line $y = x$. Therefore solve

$$\begin{aligned} -\sqrt{x+2} &= x \\ x + 2 &= x^2 \\ x^2 - x - 2 &= 0 \\ (x-2)(x+1) &= 0 \end{aligned}$$

$x = -1, 2$. But only $x = -1$ is in the domain for both f and f^{-1}. Therefore intersection at $(-1, -1)$.

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Question 11 Tech-Active.

Fully define the inverse and state its range for:

$$f : (-\infty, 3] \rightarrow \mathbb{R}, f(x) = -x^2 + 6x - 12$$

$f^{-1} : (-\infty, -3] \rightarrow \mathbb{R},\ f^{-1}(x) = 3 - \sqrt{-x-3}$

$\text{ran } f^{-1} = (-\infty, 3]$

TI:	Mathematica:	Casio:
Define $f(x) = -x^2 + 6 \cdot x - 12$ *Done*	In[52]:= f[x_] := -x^2 + 6 x - 12	define f(x) = -x^2+6x-12 done
solve$(f(y) = x, y)$	In[53]:= FunctionRange[{f[x], -Infinity < x ≤ 3}, x, y]	Solve(f(y)=x, y)
$y = -(\sqrt{-x-3} - 3)$ or $y = \sqrt{-x-3} + 3$	Out[53]= $y \leq -3$	$\{y = -\sqrt{-x-3} + 3, y = \sqrt{-x-3} + 3\}$
	In[54]:= Solve[f[y] == x, y]	
	Out[54]= $\{\{y \to 3 - \sqrt{-3-x}\}, \{y \to 3 + \sqrt{-3-x}\}\}$	

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Sub-Section [1.1.4]: Finding the Composition of Inverse Functions

Question 12

Let $f: \mathbb{R} \setminus \{3\} \to \mathbb{R}, f(x) = \frac{2}{x-3} + 1$.

Find the rule and domain for $f^{-1}(f(x))$.

$f^{-1}(f(x)) = x$ for $x \in \mathbb{R} \setminus \{3\}$, since dom $f^{-1} \circ f =$ dom $f = \mathbb{R} \setminus \{3\}$.

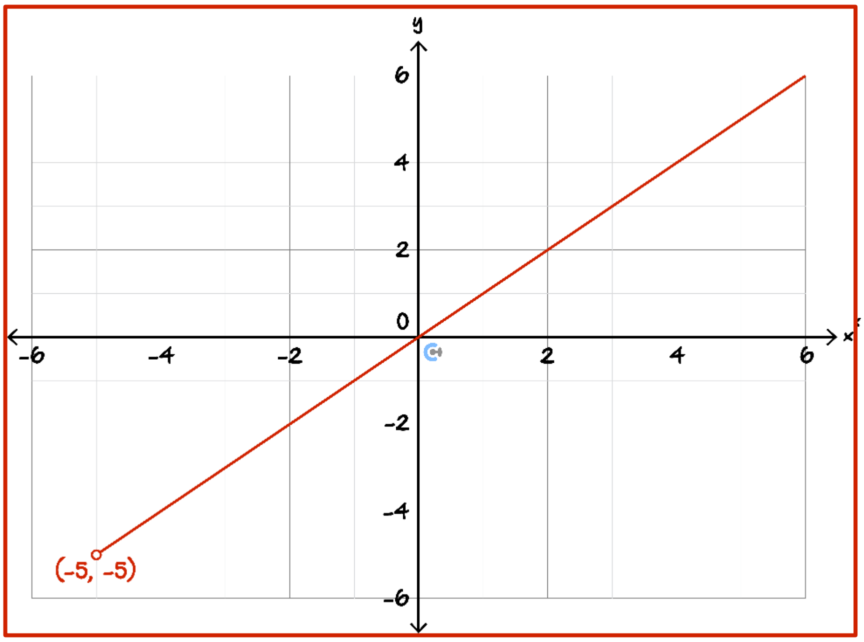
Question 13

Let $f : (-5, \infty) \to \mathbb{R}, f(x) = -(x+5)^2$

a. Find the rule and domain for $f^{-1}(f(x))$.

$f^{-1}(f(x)) = x$ for $x \in (-5, \infty)$, since dom $f^{-1} \circ f =$ dom $f = (-5, \infty)$.

b. Sketch the graph of $f^{-1}(f(x))$ on the axis below.



Question 14

Let $f(x) = x^2 - 4kx + 6$, where $x \geq 0$ and $k \geq 0$.

The function $f^{-1} \circ f$ is defined on its maximal domain.

Find the rule and domain for $f^{-1}(f(x))$.

$f(x) = (x - 2k)^2 + 6 - 4k^2$. So f has a turning point at $(2k, 6 - 4k^2)$.
We must have that f is one-to-one on a domain that is as large as possible. Therefore f must be defined on $[2k, \infty)$,

$$f^{-1}(f(x)) = x \quad \text{for } x \in [2k, \infty), \text{ since dom } f^{-1} \circ f = \text{dom } f = [2k, \infty).$$

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Sub-Section: Final Boss

Question 15 (13 marks)

Consider the functions f and g, defined over their maximal domains where:

$$f(x) = -\sqrt{x+3}$$

$$g(x) = \log_e(2-x)$$

a. Find the maximal domain of $f(x) + \frac{1}{g(x)}$. (2 marks)

We require that $x > -3$ (for $f(x)$) and $x < 2$ (for $g(x)$) and $\log_e(2-x) \neq 0 \implies 2-x \neq 1 \implies x \neq 1$. Therefore domain is

$$x \in (-3, 2) \setminus \{1\}$$

b. Show that only $g(f(x))$ is defined. (2 marks)

dom $f = [-3, \infty)$, ran $f = (-\infty, 0]$.
dom $g = (-\infty, 2)$, ran $g = \mathbb{R}$
$f(g(x))$ is not defined since ran $g \not\subseteq$ dom f.
But $g(f(x))$ is defined since ran $f \subseteq$ dom g.

c. Find the rule, domain, and range of $g(f(x))$. (2 marks)

$g(f(x)) = \log_e(2 - f(x)) = \log_e\left(2 + \sqrt{x+3}\right)$.
dom $g(f(x)) = [-3, \infty) =$ dom f and ran $g(f(x)) = [\log_e(2), \infty)$

d. Restrict the domain of g so that $f(g(x))$ is defined and the domain of g is as large as possible. (2 marks)

We want $\log_e(2-x)+3 \geq 0 \implies \log_e(2-x) \geq -3$
Solve $-3 = \log_e(2-x) \implies 2-x = e^{-3} \implies x = 2-e^{-3}$. Therefore

$$\text{dom } g = [2-e^{-3}, \infty).$$

Check by noting on this domain ran $g = [-3, \infty) = \text{dom } f$.

e. Fully define the inverse function, f^{-1}, of f. (2 marks)

Swap x and y. $x = -\sqrt{y+3} \implies y = x^2 - 3$. Now dom $f^{-1} = \text{ran } f = (-\infty, 0)$ therefore,

$$f^{-1} : (-\infty, 0] \to \mathbb{R},\ f^{-1}(x) = x^2 - 3.$$

f. Find all points of intersection between f and f^{-1}. (2 marks)

Intersections will be on the line $y = x$ therefore we may solve,
$-\sqrt{x+3} = x \implies x^2 = x+3 \implies x^2 - x - 3 = 0$

$$\left(x - \frac{1}{2}\right)^2 - \frac{13}{4} = 0$$
$$x - \frac{1}{2} = \frac{\sqrt{13}}{2}$$
$$x = \frac{1 \pm \sqrt{13}}{2}$$

By considering the domains where both f and f^{-1} are defined, the only point of intersection is at $\left(\frac{1-\sqrt{13}}{2}, \frac{1-\sqrt{13}}{2}\right)$

g. Find the rule and domain of $f(f^{-1}(x))$. (1 mark)

$f(f^{-1}(x)) = x$ for $x \leq 0$ since dom $f \circ f^{-1} = \text{dom } f^{-1}$.

Section B: Supplementary

Sub-Section [1.1.1]: Find the Maximal Domain and Range of Functions

Question 16

Find the maximal domain of the following functions.

a. $f(x) = \sqrt{x^2 + 1}$

Need $x^2 + 1 \geq 0$. This holds for all $\mathbb{R}$ since $x^2 \geq 0$ for all $x \in \mathbb{R}$. Therefore domain $= \mathbb{R}$

b. $f(x) = \log_e(x + 4)$

Need $x + 4 > 0$ therefore domain $= (-4, \infty)$

c. $f(x) = \frac{1}{x+2} - 3$

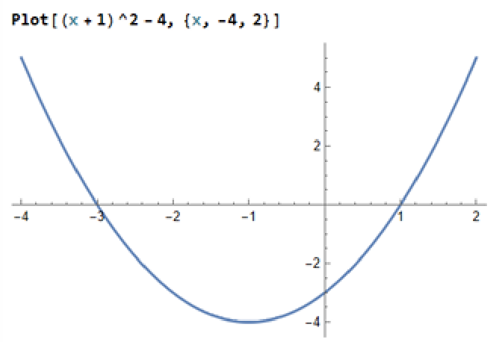
Need $x + 2 \neq 0$, therefore domain $= \mathbb{R} \setminus \{-2\}$

Question 17

Find the maximal domain of the following functions.

a. $f(x) = \sqrt{(x + 1)^2 - 4}$

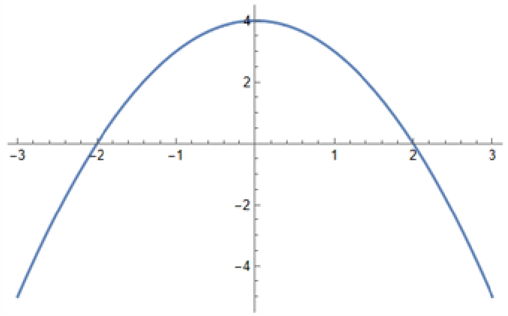
We need $x^2 + 2x - 3 \geq 0 \implies (x + 3)(x - 1) \geq 0$, therefore domain $= (-\infty, -3] \cup [1, \infty)$



b. $f(x) = \log_e(4 - x^2)$

We require $4 - x^2 > 0 \implies (2 - x)(2 + x) > 0$ therefore domain $= (-2, 2)$.

Plot[4 - x^2, {x, -3, 3}]



c. $f(x) = \frac{3+x^2}{x^2+5x+6}$

We require that $x^2 + 5x + 6 \neq 0 \implies (x + 2)(x + 3) \neq 0$, therefore domain $= \mathbb{R} \setminus \{-3, -2\}$.

Question 18

Find the maximal domain of the following functions.

a. $f(x) = \cos(x)\log_e(2x) + \frac{1}{x^2-5}$

cos is defined for all $\mathbb{R}$ but for the log we require $2x > 0 \implies x > 0$ and for the fraction we require $x^2 - 5 \neq 0 \implies x \neq \pm\sqrt{5}$. Therefore the domain is $(0, \sqrt{5}) \cup (\sqrt{5}, \infty)$.

b. $f(x) = \sqrt{\frac{x-3}{x+1}}$

We require that $\dfrac{x-3}{x+1} \geq 0$ and that $x \neq -1$.
If $x \geq 3$ then numerator ≥ 0 and denominator > 0 therefore $f(x)$ defined.
If $x \in (-1, 3)$ then numerator < 0 and denominator > 0 therefore $f(x)$ not defined.
If $x = -1$ then division by zero so $f(x)$ not defined.
If $x < -1$ then both numerator and denominator < 0 so $f(x)$ is defined.
Therefore domain $= (-\infty, -1) \cup [3, \infty)$.

c. $f(x) = \frac{1}{2-x} \times \sqrt{x^2 - 4}\log_e(x^2 - 1)$

From the fraction we require that $x \neq 2$
from the square root we require that $x^2 - 4 \geq 0 \implies x \in \mathbb{R} \setminus (-2, 2)$
from the log we require that $x^2 - 1 > 0 \implies x \in \mathbb{R} \setminus [-1, 1]$.
Therefore domain $= \mathbb{R} \setminus (-2, 2] = (-\infty, -2] \cup (2, \infty)$.

Question 19

Find the maximal domain and range of $f(x) = \frac{e^{2x}-1}{e^{2x}+1}$.

The denominator is never zero so dom $f = \mathbb{R}$.
To find the range consider what happens as $x \to \pm\infty$.
$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \dfrac{e^{2x}}{e^{2x}} = 1$
$\lim_{x\to-\infty} f(x) = \dfrac{0-1}{0+1} = -1$
The range is $(-1, 1)$.

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Plot[(Exp[2 x] - 1)/(Exp[2 x] + 1), {x, -10, 10}]
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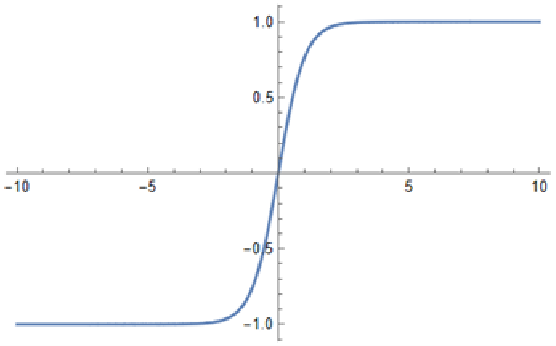

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Sub-Section [1.1.2]: Existence, Rule, Domain, and Range of Composite Functions

Question 20

The following functions are defined over their maximal domain:

$$f(x) = x^2 \text{ and } g(x) = 3 - x$$

a. Determine whether $f(g(x))$ and $g(f(x))$ exist.

Both compositions exist since both functions have domain $= \mathbb{R}$.

b. Find the rule of any composition that exists.

$$f(g(x)) = (g(x))^2 = (3 - x)^2$$
$$g(f(x)) = 3 - f(x) = 3 - x^2$$

c. State the domain of any composition that exists.

$$\text{dom } f \circ g = \text{dom } g = \mathbb{R}$$
$$\text{dom } g \circ f = \text{dom } f = \mathbb{R}$$

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Question 21

The following functions are defined over their maximal domain.

$$f(x) = e^{2x} \text{ and } g(x) = \log_e(2x)$$

a. Determine whether $f(g(x))$ and $g(f(x))$ exist.

dom $f = \mathbb{R}$ and ran $f = (0, \infty)$
dom $g = (0, \infty)$ and ran $g = \mathbb{R}$
Therefore, both compositions exist.

b. Find the rule of any composition that exists.

$f(g(x)) = e^{2g(x)} = e^{2\log_e(2x)} = (2x)^2 = 4x^2$
$g(f(x)) = \log_e(2f(x)) = \log_e(2e^{2x}) = 2x + \log_e(2)$

c. State the domain of any composition that exists.

dom $f \circ g =$ dom $g = \mathbb{R}^+$
dom $g \circ f =$ dom $f = \mathbb{R}$.

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Question 22

For the following functions:

$$f(x) = x^2 + 1 \text{ and } g(x) = \frac{1}{x^2-4}$$

a. Determine whether $f(g(x))$ and $g(f(x))$ exist.

$f(g(x))$ exists since ran $g = \left(-\infty, -\frac{1}{4}\right] \cup (0, \infty) \subseteq$ dom $f = \mathbb{R}$.
$g(f(x))$ does not exist since ran $f = [1, \infty) \nsubseteq$ dom $g = \mathbb{R} \setminus \{-2, 2\}$.

b. Find the rule of any composition that exists.

$$f(g(x)) = (g(x))^2 + 1 = \frac{1}{(x^2-4)^2} + 1.$$

c. State the domain of any composition that exists.

dom $f \circ g =$ dom $g = \mathbb{R} \setminus \{-2, 2\}$.

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Question 23

Functions are defined over their maximal domain unless specified otherwise.

For the functions f and g, determine whether $f(g(x))$ and $g(f(x))$ exist. State the rule and the domain of the composite function that do exist.

$$f(x) = e^x - e^{-x}$$

$$g(x) = \frac{1}{x(x-2)}$$

dom $f = \mathbb{R}$ and ran $f = \mathbb{R}$
dom $g = \mathbb{R} \setminus \{0, 2\}$ and ran $g = (-\infty, -1] \cup (0, \infty)$
Therefore, $f(g(x))$ does exist since ran $g \subseteq$ dom f.
$g(f(x))$ does not exist since ran $f \nsubseteq$ dom g.

$f(g(x)) = e^{\frac{1}{x^2-2x}} - e^{\frac{1}{2x-x^2}}$
$f(g(1)) = \frac{1}{e} - e$
dom $f(g(x))$ = dom $g = \mathbb{R} \setminus \{0, 2\}$ and ran $f(g(x)) = \left(-\infty, \frac{1}{e} - e\right] \cup (0, \infty)$.

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Plot[g[x], {x, -2, 3}]
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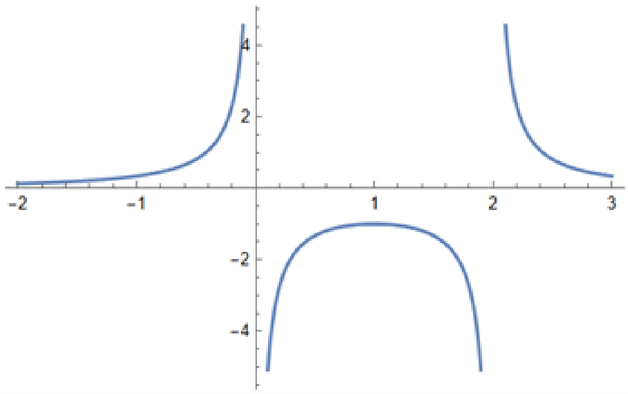


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Plot[f[g[x]], {x, -5, 5}]
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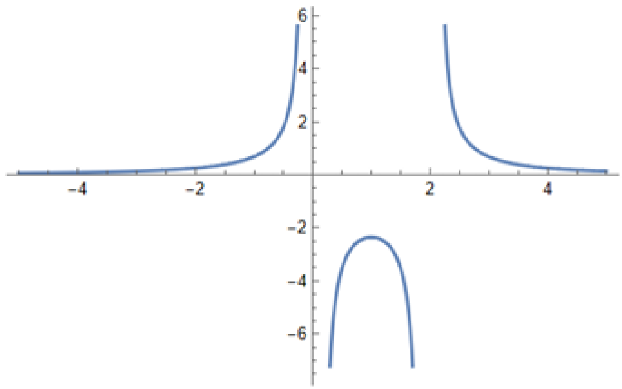

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Sub-Section [1.1.3]: Finding the Rule, Domain, and Range of Inverse Functions

Question 24

For the function:

$$f : (0, \infty) \to \mathbb{R}, f(x) = \log_e(3x)$$

a. Fully define the inverse function.

Swap x and y. $x = \log_e(3y) \implies 3y = e^x \implies y = \frac{1}{3}e^x$.
Now dom $f^{-1} = $ ran $f = \mathbb{R}$

$$f^{-1} : \mathbb{R} \to \mathbb{R},\ f^{-1}(x) = \frac{e^x}{3}.$$

b. Find the range of the inverse function.

ran $f^{-1} = $ dom $f = (0, \infty)$.

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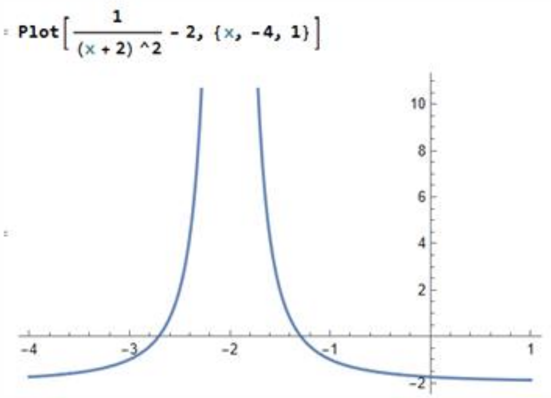
Question 25

For the function:

$$f : (b, -\infty) \to \mathbb{R}, f(x) = \frac{1}{(x+2)^2} - 2$$

a. Find the largest value of b such that the inverse function exists.

$f(x) = (x+1)^2$. The function must be one-to-one for the inverse to exist. f is a truncus with an asymptote at $x = -2$. Therefore the smallest value of b is -2.



b. Fully define the inverse function.

Swap x and y. $x = \frac{1}{(y+2)^2} - 2 \implies (y+2)^2 = \frac{1}{x+2} \implies y = \pm\frac{1}{\sqrt{x+2}} - 2.$
Since $\text{dom } f = (-2, \infty) = \text{ran } f^{-1}$ and $\text{ran } f^{-1} = \text{dom } f = (-2, \infty)$ we must have

$$f^{-1} : (-2, \infty) \to \mathbb{R},\ f^{-1}(x) = \frac{1}{\sqrt{x+2}} - 2.$$

c. Find the range of the inverse function.

$\text{ran } f^{-1} = \text{dom } f = (-2, \infty)$

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Question 26

For the following functions:

$$f : (-\infty, k] \to \mathbb{R}, f(x) = 2x^2 - 8x + 4.$$

a. Find the largest value of k such that the inverse function exists.

$f(x) = 2(x-2)^2 - 4$. Therefore f has a turning point at $(2, -4)$ so it is one-to-one for $x \in (-\infty, 2]$. Therefore $k = 2$.

b. Fully define the inverse function.

Swap x and y. $x = 2(y-2)^2 - 4 \implies \dfrac{x+4}{2} = (y-2)^2 \implies y = \pm\sqrt{\dfrac{x+4}{2}} + 2$. Now dom f^{-1} = ran $f = [-4, \infty)$ and ran f^{-1} = dom $f = (-\infty, 2]$. Therefore,

$$f^{-1} : [-4, \infty) \to \mathbb{R},\ f^{-1}(x) = 2 - \sqrt{\frac{x+4}{2}}.$$

c. Find the range of the inverse function.

ran f^{-1} = dom $f = (-\infty, 2]$.

d. Find the point of intersection between f and f^{-1}.

The functions intersect on the line $y = x$. Therefore solve

$$2x^2 - 8x + 4 = x$$
$$2x^2 - 7x + 4 = 0$$
$$(2x-1)(x-4) = 0$$

$x = \frac{1}{2}, 4$. But only $x = \frac{1}{2}$ is in the domain for both f and f^{-1}. Therefore intersection at $\left(\frac{1}{2}, \frac{1}{2}\right)$.

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Question 27

Find the inverse function of:

$$f(x) = e^{2x} + 4e^{x} + 1$$

And determine whether f and f^{-1} have any points of intersection.

$f(x) = (e^x + 2)^2 - 3$. Swap x and y.

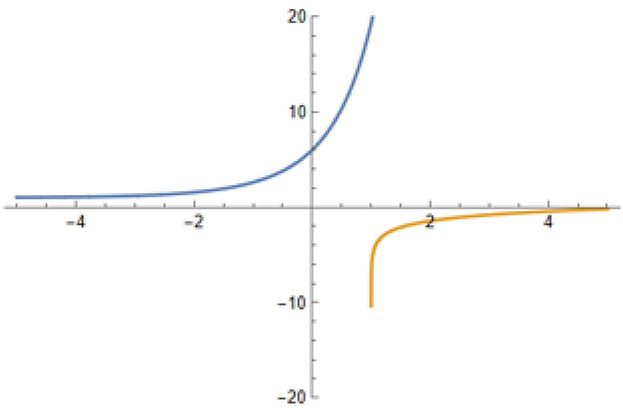
$$\begin{aligned} x &= (e^y + 2)^2 - 3 \\ e^y &= \sqrt{x+3} - 2 \\ y &= \log_e(-2 + \sqrt{x+3}) \end{aligned}$$

now dom $f^{-1} = \text{ran } f = (1, \infty)$

$$f^{-1} : (1, \infty) \to \mathbb{R},\ f^{-1}(x) = \log_e(-2 + \sqrt{x+3})$$

A rough sketch of the functions will show that there is no intersection between f and f^{-1}.

```
Plot[{Exp[2 x] + 4 Exp[x] + 1, Log[-2 + √(x + 3)]}, {x, -5, 5},
 PlotRange → {-20, 20}]
```



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Sub-Section [1.1.4]: Finding the Composition of Inverse Functions

Question 28

Let $f: (3, \infty) \to \mathbb{R}, f(x) = x^2 - 4x + 7$.

Find the rule and domain for $f^{-1}(f(x))$.

$f^{-1}(f(x)) = x$ for $x \in (3, \infty)$, since dom $f^{-1} \circ f =$ dom $f = (3, \infty)$.

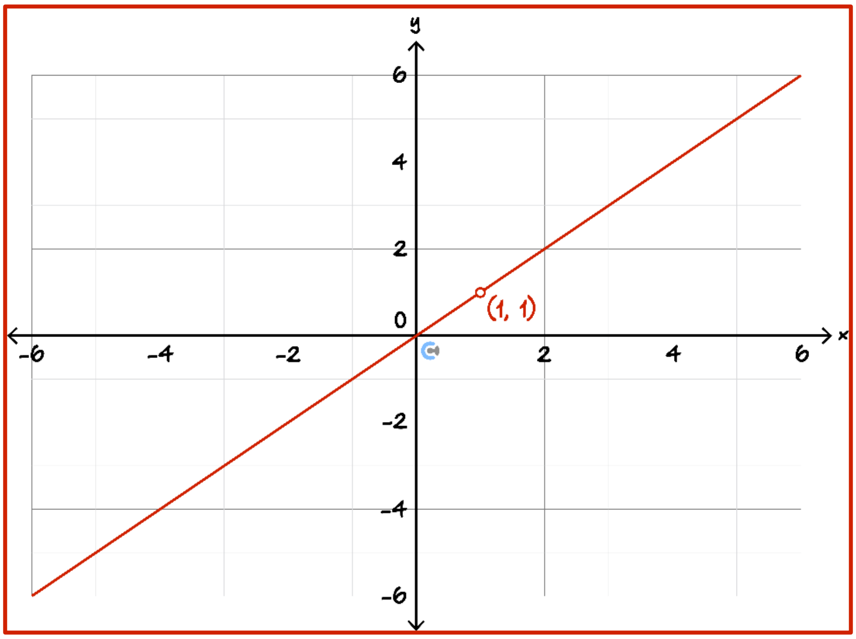
Question 29

Let $f: \mathbb{R} \setminus \{1\} \to \mathbb{R}, f(x) = \frac{5}{x-1} + 3$.

a. Find the rule and domain for $f^{-1}(f(x))$.

$f^{-1}(f(x)) = x$ for $x \in \mathbb{R} \setminus \{1\}$, since dom $f^{-1} \circ f =$ dom $f = \mathbb{R} \setminus \{1\}$.

b. Sketch the graph of $f^{-1}(f(x))$ on the axis below.



Question 30

Let $f(x) = x^2 - 2kx + 9$, where $x \geq 0$ and $k \geq 0$.

The function $f^{-1} \circ f$ is defined on its maximal domain.

Find the rule and domain for $f^{-1}(f(x))$.

$f(x) = (x-k)^2 + 9 - k^2$. We want f to be one-to-one on as large of a domain as possible. Therefore, the domain of f is $[k, \infty)$,

$$f^{-1}(f(x)) = x \quad \text{for } x \in [k, \infty), \text{ since dom } f^{-1} \circ f = \text{dom } f = [k, \infty).$$

Question 31

Let $f^{-1}: \left[\frac{\pi}{2}, \pi\right] \to \mathbb{R}, f^{-1}(x) = \sin(x)$.

Define the function f and find the rule and domain for $f^{-1}(f(x))$.

dom f = ran $f^{-1} = [0, 1]$ and ran f = dom $f^{-1} = \left[\frac{\pi}{2}, \pi\right]$

Now $f^{-1}(\pi) = 0 \implies f(0) = \pi$. Therefore,

$$f : [0, 1] \to \mathbb{R},\ f(x) = \pi - \sin^{-1}(x)$$

$$f^{-1}(f(x)) = x \text{ for } 0 \leq x \leq 1.$$

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