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VCE Mathematical Methods ¾
Functions & Relations
Homework

Homework Outline:

Compulsory	Pg 2 – Pg 15
Supplementary	Pg 16 – Pg 28
Solutions	Pg 2 – Pg 28

Section A: Compulsory

Sub-Section [1.1.1]: Find the Maximal Domain and Range of Functions

Question 1

Find the maximal domain of the following functions.

a. $f(x) = \sqrt{x+3}$

b. $f(x) = \frac{1}{x-2} + 1$

c. $f(x) = \log_e(4-x)$

Question 2

Find the maximal domain of the following functions.

a. $f(x) = -\sqrt{x^2+5x+6}$

b. $f(x) = \log_e(x^2 + 6x + 5)$

c. $f(x) = \frac{1}{x^2+2x-3}$

Question 3

Find the maximal domain of the following functions.

a. $f(x) = \log_e(5 - x) + \sqrt{2x - 7} + 1$

b. $f(x) = \frac{1}{x} - \frac{1}{x^2-5x+4}$

c. $f(x) = \frac{1}{x-4} \times \sqrt{x^2 - 3}$

Question 4 Tech-Active.

Find the maximal domain and range of $f(x) = \frac{x^2-3}{x^2+5x+6} + \log_e(3 - x^2)$. Give the range correct to three decimal places.

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Sub-Section [1.1.2]: Existence, Rule, Domain, and Range of Composite Functions

Question 5

The following functions are defined over their maximal domain.

$$f(x) = \sqrt{x} \text{ and } g(x) = x - 3$$

a. Determine whether $f(g(x))$ and $g(f(x))$ exist.

b. Find the rule of any composition that exists.

c. State the domain of any composition that exists.

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Question 6

The following functions are defined over their maximal domain.

$$f(x) = \frac{1}{x-1} \text{ and } g(x) = \frac{1}{x}$$

a. Determine whether $f(g(x))$ and $g(f(x))$ exist.

b. Find the rule of any composition that exists.

c. State the domain of any composition that exists.

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Question 7

For the following functions:

$$f : [0, 6] \to \mathbb{R}, f(x) = x^3 \text{ and } g(x) = \sqrt{x + 4}.$$

a. Determine whether $f(g(x))$ and $g(f(x))$ exist.

b. Find the rule of any composition that exists.

c. State the domain of any composition that exists.

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Sub-Section [1.1.3]: Finding the Rule, Domain, and Range of Inverse Functions

Question 8

For the function:

$$f:(5,\infty)\to\mathbb{R}, f(x) = \frac{1}{5-x}$$

a. Fully define the inverse function.

b. Find the range of the inverse function.

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Question 9

For the function:

$$f : (-\infty, k] \to \mathbb{R}, f(x) = x^2 + 2x + 1$$

a. Find the largest value of k such that the inverse function exists.

b. Fully define the inverse function.

c. Find the range of the inverse function.

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Question 10

For the following functions:

$$f : [b, \infty) \to \mathbb{R}, f(x) = -\sqrt{x + 2}.$$

a. Find the smallest value of b such that the inverse function exists.

b. Fully define the inverse function.

c. Find the range of the inverse function.

d. Find the point of intersection between f and f^{-1}.

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Question 11 Tech-Active.

Fully define the inverse and state its range for:

$$f : (-\infty, 3] \to \mathbb{R}, f(x) = -x^2 + 6x - 12$$

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Sub-Section [1.1.4]: Finding the Composition of Inverse Functions

Question 12

Let $f: \mathbb{R} \setminus \{3\} \to \mathbb{R}, f(x) = \frac{2}{x-3} + 1$.

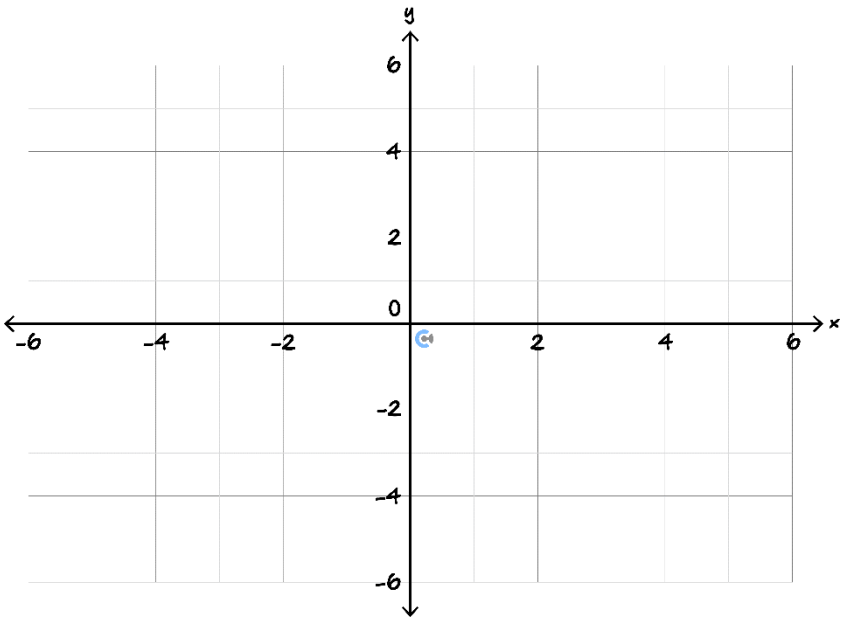
Find the rule and domain for $f^{-1}(f(x))$.

Question 13

Let $f: (-5, \infty) \to \mathbb{R}, f(x) = -(x+5)^2$

a. Find the rule and domain for $f^{-1}(f(x))$.

b. Sketch the graph of $f^{-1}(f(x))$ on the axis below.



Question 14

Let $f(x) = x^2 - 4kx + 6$, where $x \geq 0$ and $k \geq 0$.

The function $f^{-1} \circ f$ is defined on its maximal domain.

Find the rule and domain for $f^{-1}(f(x))$.

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Sub-Section: Final Boss

Question 15 (13 marks)

Consider the functions f and g, defined over their maximal domains where:

$$f(x) = -\sqrt{x+3}$$

$$g(x) = \log_e(2-x)$$

a. Find the maximal domain of $f(x) + \frac{1}{g(x)}$. (2 marks)

b. Show that only $g(f(x))$ is defined. (2 marks)

c. Find the rule, domain, and range of $g(f(x))$. (2 marks)

d. Restrict the domain of g so that $f(g(x))$ is defined and the domain of g is as large as possible. (2 marks)

e. Fully define the inverse function, f^{-1}, of f. (2 marks)

f. Find all points of intersection between f and f^{-1}. (2 marks)

g. Find the rule and domain of $f(f^{-1}(x))$. (1 mark)

Section B: Supplementary

Sub-Section [1.1.1]: Find the Maximal Domain and Range of Functions

Question 16

Find the maximal domain of the following functions.

a. $f(x) = \sqrt{x^2 + 1}$

b. $f(x) = \log_e(x + 4)$

c. $f(x) = \frac{1}{x+2} - 3$

Question 17

Find the maximal domain of the following functions.

a. $f(x) = \sqrt{(x + 1)^2 - 4}$

b. $f(x) = \log_e(4 - x^2)$

c. $f(x) = \frac{3+x^2}{x^2+5x+6}$

Question 18

Find the maximal domain of the following functions.

a. $f(x) = \cos(x)\log_e(2x) + \frac{1}{x^2-5}$

b. $f(x) = \sqrt{\frac{x-3}{x+1}}$

c. $f(x) = \frac{1}{2-x} \times \sqrt{x^2-4}\log_e(x^2-1)$

Question 19

Find the maximal domain and range of $f(x) = \frac{e^{2x}-1}{e^{2x}+1}$.

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Sub-Section [1.1.2]: Existence, Rule, Domain, and Range of Composite Functions

Question 20

The following functions are defined over their maximal domain:

$$f(x) = x^2 \text{ and } g(x) = 3 - x$$

a. Determine whether $f(g(x))$ and $g(f(x))$ exist.

b. Find the rule of any composition that exists.

c. State the domain of any composition that exists.

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Question 21

The following functions are defined over their maximal domain.

$$f(x) = e^{2x} \text{ and } g(x) = \log_e(2x)$$

a. Determine whether $f(g(x))$ and $g(f(x))$ exist.

b. Find the rule of any composition that exists.

c. State the domain of any composition that exists.

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Question 22

For the following functions:

$$f(x) = x^2 + 1 \text{ and } g(x) = \frac{1}{x^2-4}$$

a. Determine whether $f(g(x))$ and $g(f(x))$ exist.

b. Find the rule of any composition that exists.

c. State the domain of any composition that exists.

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Question 23

Functions are defined over their maximal domain unless specified otherwise.

For the functions f and g, determine whether $f(g(x))$ and $g(f(x))$ exist. State the rule and the domain of the composite function that do exist.

$$f(x) = e^x - e^{-x}$$

$$g(x) = \frac{1}{x(x-2)}$$

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Sub-Section [1.1.3]: Finding the Rule, Domain, and Range of Inverse Functions

Question 24

For the function:

$$f : (0, \infty) \to \mathbb{R}, f(x) = \log_e(3x)$$

a. Fully define the inverse function.

b. Find the range of the inverse function.

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Question 25

For the function:

$$f:(b,-\infty)\to\mathbb{R}, f(x)=\frac{1}{(x+2)^2}-2$$

a. Find the largest value of b such that the inverse function exists.

b. Fully define the inverse function.

c. Find the range of the inverse function.

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Question 26

For the following functions:

$$f: (-\infty, k] \to \mathbb{R}, f(x) = 2x^2 - 8x + 4.$$

a. Find the largest value of k such that the inverse function exists.

b. Fully define the inverse function.

c. Find the range of the inverse function.

d. Find the point of intersection between f and f^{-1}.

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Question 27

Find the inverse function of:

$$f(x) = e^{2x} + 4e^{x} + 1$$

And determine whether f and f^{-1} have any points of intersection.

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Sub-Section [1.1.4]: Finding the Composition of Inverse Functions

Question 28

Let $f: (3, \infty) \to \mathbb{R}, f(x) = x^2 - 4x + 7$.

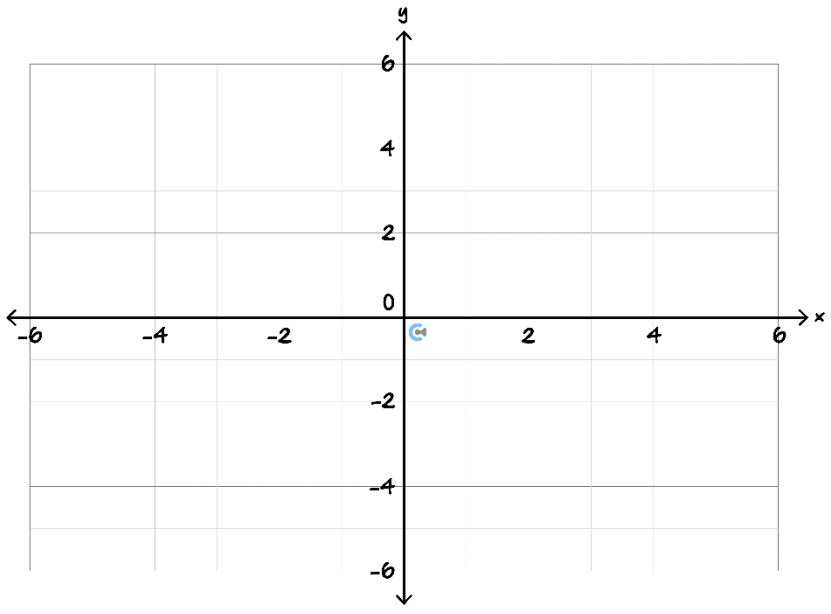
Find the rule and domain for $f^{-1}(f(x))$.

Question 29

Let $f: \mathbb{R} \setminus \{1\} \to \mathbb{R}, f(x) = \frac{5}{x-1} + 3$.

a. Find the rule and domain for $f^{-1}(f(x))$.

b. Sketch the graph of $f^{-1}(f(x))$ on the axis below.



Question 30

Let $f(x) = x^2 - 2kx + 9$, where $x \geq 0$ and $k \geq 0$.

The function $f^{-1} \circ f$ is defined on its maximal domain.

Find the rule and domain for $f^{-1}(f(x))$.

Question 31

Let $f^{-1}: \left[\frac{\pi}{2}, \pi\right] \to \mathbb{R}, f^{-1}(x) = \sin(x)$.

Define the function f and find the rule and domain for $f^{-1}(f(x))$.

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