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VCE Mathematical Methods ¾
AOS 1 Revision [1.0]
Contour Check (Part 3) Solutions

Contour Checklist

Section A: [1.1 - 1.8] - Exam 1 Overall (VCAA Qs) (77 Marks)

Question 178 (7 marks)

a. Consider the function p, where $p: [1, \infty) \to R, p(x) = x^4 - x^3 - x^2 + x + 1$.

i. Find the value of a when $p^{-1}(a) = 2$, where $a \in R$. (2 marks)

This question involved recognition of the inverse case.

If $p^{-1}(a) = 2 \Rightarrow p(2) = a$

$p(2) = 2^4 - 2^3 - 2^2 + 2 + 1 = a$,

$\therefore a = 7$

ii. Find the value of b when $p(b) = 1$, where $b > 0$. (2 marks)

$b^4 - b^3 - b^2 + b + 1 = 1$

$b^4 - b^3 - b^2 + b = 0$

$b(b^3 - b^2 - b + 1) = 0$

$b(b-1)(b^2-1) = 0$

$\therefore b = -1, 0, 1$ and $b > 0$ thus

$\therefore b = 1$

b. Find the rule and the domain of f^{-1}, the inverse of f, if $f: R\backslash\{2\} \to R, f(x) = \frac{x+3}{x-2}$. (3 marks)

Students are reminded to take care with their use of notation. Setting $y = f(x)$ and then later writing $y =$ the inverse function is a contradiction of ideas and should be avoided. Students need to be using $f^{-1}(x)$ as the preferred naming notation for the inverse function.

Let $y = f(x)$,

Swap x and y for inverse

$x = \frac{y+3}{y-2}$

$xy - 2x = y + 3$

$-2x = (1-x)y + 3$

$(1-x)y = -(2x+3)$

$\therefore y = -\frac{2x+3}{1-x} = \frac{2x+3}{x-1}$

$f^{-1}(x) = \frac{2x+3}{x-1}$

Domain $R\backslash\{1\}$

Alternative:

Let $y = f(x) = 1 + \frac{5}{x-2}$

Swap x and y for inverse

$x = 1 + \frac{5}{y-2}$

$f^{-1}(x) = \frac{5}{x-1} + 2$

Space for Personal Notes

Question 179 (5 marks)

Let $h: \left[-\frac{3}{2}, \infty\right) \to R, h(x) = \sqrt{2x+3} - 2$.

a. Find the value(s) of x such that $[h(x)]^2 = 1$. (2 marks)

$\left[(h(x))^2\right] = 1$ so $h(x) = 1$ or -1

$$\sqrt{2x+3} - 2 = 1, -1$$
$$\sqrt{2x+3} = 3, 1$$
$$2x + 3 = 9, 1$$
$$x = -1, 3$$

Both values are in the domain of h.

b. Find the domain and the rule of the inverse function h^{-1}. (3 marks)

Let $y = h^{-1}(x)$

$$x = \sqrt{2y+3} - 2$$
$$x + 2 = \sqrt{2y+3}$$
$$(x+2)^2 = 2y + 3$$
$$y = \frac{1}{2}(x+2)^2 - \frac{3}{2}$$

Hence $h^{-1}(x) = \frac{1}{2}(x+2)^2 - \frac{3}{2}$

Range: $[-2, \infty)$

Question 180 (4 marks)

Let $f(x) = -x^2 + x + 4$ and $g(x) = x^2 - 2$.

a. Find $g\big(f(3)\big)$. (2 marks)

$f(3) = -2$

$g(f(3)) = 2$

b. Express the rule for $f\left(g(x)\right)$ in the form $ax^4 + bx^2 + c$, where a, b, and c are non-zero integers. (2 marks)

$$\begin{aligned} f(g(x)) &= -\left(x^2-2\right)^2 + \left(x^2-2\right) + 4 \\ &= -x^4 + 5x^2 - 2 \end{aligned}$$

Question 181 (3 marks)

Let $h: R^+ \cup \{0\} \to R, h(x) = \frac{7}{x+2} - 3$.

a. State the range of h.

Range: $\left(-3, \frac{1}{2}\right]$

b. Find the rule for h^{-1}.

For inverse: $x = \dfrac{7}{h^{-1}(x)+2} - 3$

Thus $h^{-1}(x) = \dfrac{7}{x+3} - 2$

Question 182 (8 marks)

The rule for a function f is given by $f(x) = \sqrt{2x+3} - 1$, where f is defined on its maximal domain.

a. Find the domain and rule of the inverse function f^{-1}. (2 marks)

$f(x) = \sqrt{2x+3} - 1$

let $x = \sqrt{2y+3} - 1$

$x + 1 = \sqrt{2y+3}$

$y = \frac{(x+1)^2}{2} - \frac{3}{2}$

Hence $f^{-1}(x) = \frac{(x+1)^2 - 3}{2}$

Dom $f^{-1} = [-1, \infty)$

b. Solve $f(x) = f^{-1}(x)$. (2 marks)

Solve a suitable equation and check; for example,

$\sqrt{2x+3} - 1 = x$

$2x + 3 = (x+1)^2$

$x = \pm\sqrt{2}$

$x = \sqrt{2}$ as $x \geq -1$

c. Let $g: D \to R, g(x) = \sqrt{2x + c} - 1$ where D is the maximal domain of g and c is a real number.

i. For what value(s) of c, does $g(x) = g^{-1}(x)$ have no real solutions? (2 marks)

$$\sqrt{2x+c} - 1 = x$$
$$2x + c = (x+1)^2$$
$$x^2 = c - 1$$

No solutions if $c - 1 < 0$

$c < 1$

ii. For what value(s) of c, does $g(x) = g^{-1}(x)$ have exactly one real solution? (2 marks)

$x = \pm\sqrt{c-1}$ and must be in domain of g and g^{-1}. Exactly one solution for $c = 1$ or for $c > 2$.

Space for Personal Notes

Question 183 (4 marks)

a. Let $f: R\backslash\left\{\frac{1}{3}\right\} \to R, f(x) = \frac{1}{3x-1}$.

Find the rule of f^{-1}.

$$x = \frac{1}{3\left(f^{-1}(x)\right)-1}$$

Thus $f^{-1}(x) = \frac{1}{3x} + \frac{1}{3}$ or $f^{-1}(x) = \frac{1+x}{3x}$

b. State the domain of f^{-1}.

Domain = $R\backslash\{0\}$

c. Let g be the function obtained by applying the transformation T to the function f, where:

$$T(x,y) = (x+c, y+d)$$

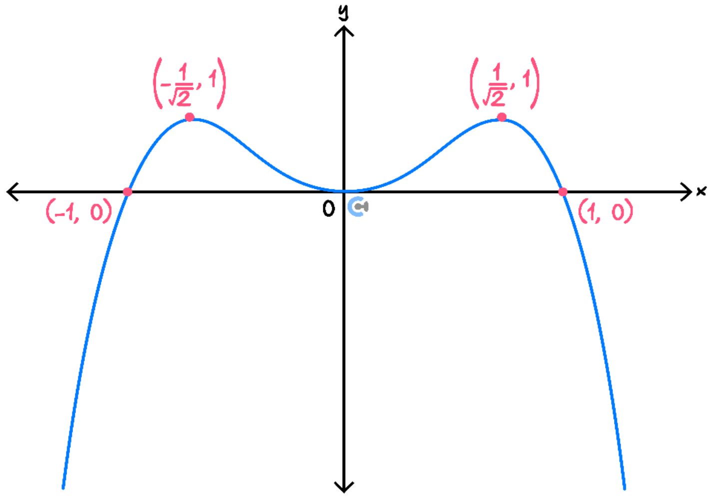
And $c, d \in R$.

Find the values of c and d given that, $g = f^{-1}$. (1 mark)

$c = -\frac{1}{3}$ and $d = \frac{1}{3}$

Question 184 (4 marks)

The function $f: R \to R, f(x)$ is a polynomial function of degree 4. Part of the graph of f is shown below. The graph of f touches the x-axis at the origin.



a. Find the rule of f. (1 mark)

$f(x) = -4x^2\left(x^2 - 1\right)$

Let g be a function with the same rule as f.

Let $h: D \to R, h(x) = \log_e\left(g(x)\right) - \log_e(x^3 + x^2)$, where D is the maximal domain of h.

b. State D. (1 mark)

Maximal Domain: $(-1,0) \cup (0,1)$ or $(-1,1)\backslash\{0\}$

Students who did this question well realised that the maximal domain could be obtained by considering the common domains for $f(x) > 0$ (observed from the graph given in part a.) and $\{x: x^3 + x^2 > 0\}$. Some students were not clear on how to express the interval.

c. State the range of h. (2 marks)

$h(x) = log_e(4(1 - x))$

$(-\infty, log_e(4)) \cup (log_e(4), log_e(8))$ or $(-\infty, log_e(8))\backslash\{log_e(4)\}$

Question 185 (5 marks)

Let $f: [0, \infty) \to R, f(x) = \sqrt{x+1}$.

a. State the range of f.

$\boxed{[1, \infty)}$

b. Let $g: (-\infty, c] \to R, g(x) = x^2 + 4x + 3$, where $c < 0$.

i. Find the largest possible value of c such that, the range of g is a subset of the domain of f.

$\boxed{c = -3}$

ii. For the value of c found in **part. b. i**, state the range of $f\big(g(x)\big)$.

$\boxed{[1, \infty)}$

c. Let $h: R \to R, h(x) = x^2 + 3$.

State the range of $f(h(x))$.

Range of $f(h(x)) = [2, \infty)$

Domain of $f(h(x)) = R$, and $f(h(x)) = \sqrt{x^2 + 4}$. Most students could identify the composite function but struggled with determining its range.

Space for Personal Notes

Question 186 (4 marks)

Let $f: [0, \infty) \to R, f(x) = \sqrt{x+1}$.

a. State the range of f. (1 mark)

$\boxed{[1, \infty)}$

b. Let $g: (-\infty, c] \to R, g(x) = x^2 + 4x + 3$, where $c < 0$.

i. Find the largest possible value of c such that, the range of g is a subset of the domain of f. (2 marks)

$\boxed{c = -3}$

ii. For the value of c found in **part.b.i.**, state the range of $f\left(g(x)\right)$. (1 mark)

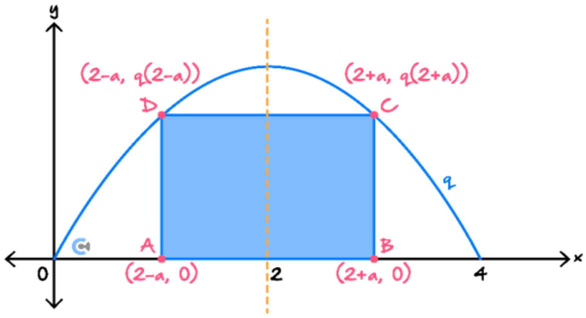
$\boxed{[1, \infty)}$

Space for Personal Notes

Question 187 (5 marks)

Let $q: [0,4] \to R, q(x) = x(4 - x)$.

A rectangle $ABCD$ is inscribed between the graph of the function q and the x-axis. Its vertices are a units, where $a > 0$, from the axis of symmetry, $x = 2$, as shown below.



a. Find the value of a when the rectangle is a square. Give your answer in the form $b + \sqrt{c}$, where b is an integer and c is a positive integer.

$2a = q(2 + a)$

$a^2 + 2a - 4 = 0$

$a = -1 + \sqrt{5}$

b. Find the maximum area of the rectangle $ABCD$. Give your answer in the form $\frac{m\sqrt{n}}{p}$, where m, n, and p are positive integers.

$Area = A = 2a \times q(2 + a) = 8a - 2a^3$

$\frac{dA}{da} = 8 - 6a^2 = 0$

Max when $a = \frac{2}{\sqrt{3}}$

$A\left(\frac{2}{\sqrt{3}}\right) = \frac{4}{\sqrt{3}} \times \frac{8}{3} = \frac{32\sqrt{3}}{9}$

Question 188 (5 marks)

Let P be a point on the straight line $y = 2x - 4$ such that the length of OP, the line segment from the origin O to P, is a minimum.

a. Find the coordinates of P. (3 marks)

Using distance formula:

$$OP = \sqrt{x^2 + (2x-4)^2}$$

$$d = \sqrt{5x^2 - 16x + 16}$$

$$\frac{d(OP)}{dx} = 0 \;\; when \quad 10x - 16 = 0,\; x = \frac{8}{5}$$

$$So\ P\ is\ \text{the point}\ \left(\frac{8}{5}, -\frac{4}{5}\right)$$

b. Find the distance OP. Express your answer in the form $\frac{a\sqrt{b}}{b}$, where a and b are positive integers. (2 marks)

$$d = \sqrt{\left(0 - \frac{8}{5}\right)^2 + \left(0 + \frac{4}{5}\right)^2}$$

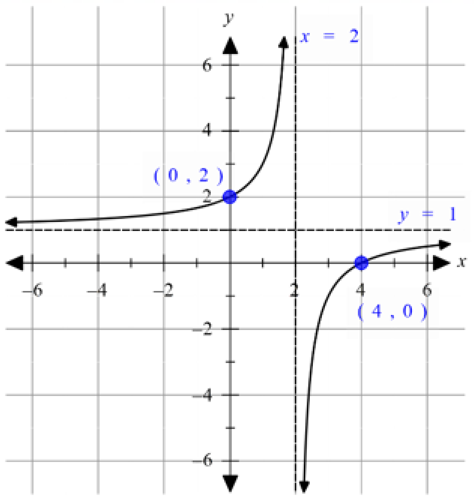
$$= \frac{4\sqrt{5}}{5}$$

Space for Personal Notes

Question 189 (4 marks)

a. Sketch the graph of $y = 1 - \frac{2}{x-2}$ on the axes below. Label asymptotes with their equations and axis intercepts with their coordinates. (3 marks)

Marks	0	1	2	3	Average
%	11	9	24	56	2.3



Most students recognised that the graph was a rectangular hyperbola and presented a neatly drawn curve with branches correctly positioned. Students generally paid attention to curvature and asymptotic behaviour. Asymptotes were sometimes correctly positioned but labelled inaccurately or not at all. The axial intercepts were generally given as coordinates with occasional errors seeing the x-intercept labelled (4,0) but positioned at (3,0) or the y-intercept given as (0,3).

b. Find the values of x for which $1 - \frac{2}{x-2} \geq 3$. (1 mark)

Marks	0	1	Average
%	69	32	0.3

$x \in [1, 2)$

This question, while well attempted, was not done well. Most students attempted to solve algebraically instead of using the graph, and only obtained the lower bound of inequality. Other errors saw students write the interval as $(2,1]$. Others had the values but incorrect brackets.

Space for Personal Notes

Question 190

Consider the simultaneous linear equations:

Marks	0	1	2	3	Average
%	43	22	10	25	1.2

The following are three different methods that can be used to identify and justify $k = -4$.

Method 1: Using gradient, *m*, and *y*-intercept, *c*.

$m_1 = \frac{k}{3}, c_1 = -\left(\frac{k+3}{3}\right)$

$m_2 = \frac{-4}{k+7}, c_2 = \frac{1}{k+7}$

from the gradient $k(k+7) = -12$ so

$k = -4$ *or* $k = -3$

from the intercepts $k = -3 \Rightarrow c_1 = 0$ and $c_2 = \frac{1}{4}$ so

$k = -4 \Rightarrow c_1 = c_2 = \frac{1}{3}$ so there are infinitely man

Answer: $k = -4$

Method 2: Using ratios

Comparison of coefficients as common ratios (all r

$\frac{k}{4} = \frac{-3}{k+7} = \frac{k+3}{1}$

Using $\frac{k}{4} = \frac{-3}{k+7}$

$k^2 + 7k = -12 \Rightarrow k = -4$ or $k = -3$

Using $\frac{-3}{k+7} = \frac{k+3}{1}$

$k^2 + 10k + 21 = -3 \Rightarrow k = -4$ or $k = -6$

Using $\frac{k}{4} = \frac{k+3}{1} \Rightarrow k = -4$

$k = -4$ satisfies all 3 ratios and the lines will be identical.

Answer: $k = -4$

Method 3: Using determinant

determinant $= k(k+7) + 12$

determinant $= 0$ for infinitely many solutions or no solutions

$k^2 + 7k + 12 = (k+4)(k+3) = 0$

$\Rightarrow k = -4$ or $k = -3$

when $k = -3$ the two equations become

$-3x - 3y = 0$ and $4x + 4y = 1$ so there are no solutions in this case

when $k = -4$ the two equations become

$-4x - 3y = -1$ and $4x + 3y = 1$ so there are infinitely many solutions in this case

Answer: $k = -4$

Many students who tried to solve simultaneous equations rarely succeeded. Other variations of these methods were seen. Whichever method was chosen, students had to justify their selection of k.

b. Find the values of k for which there is a unique solution.

Marks	0	1	Average
%	67	33	0.4

For unique solutions

$k^2 - 7k + 12 \neq 0$

$\therefore k \in R \setminus \{-4, -3\}$

Most students were aware of the two values of k that led to either no or infinite solutions. The most successful approach was using the determinant. Many students chose a value of k that was not selected as the answer to **part a**.

Space for Personal Notes

Question 191 (4 marks)

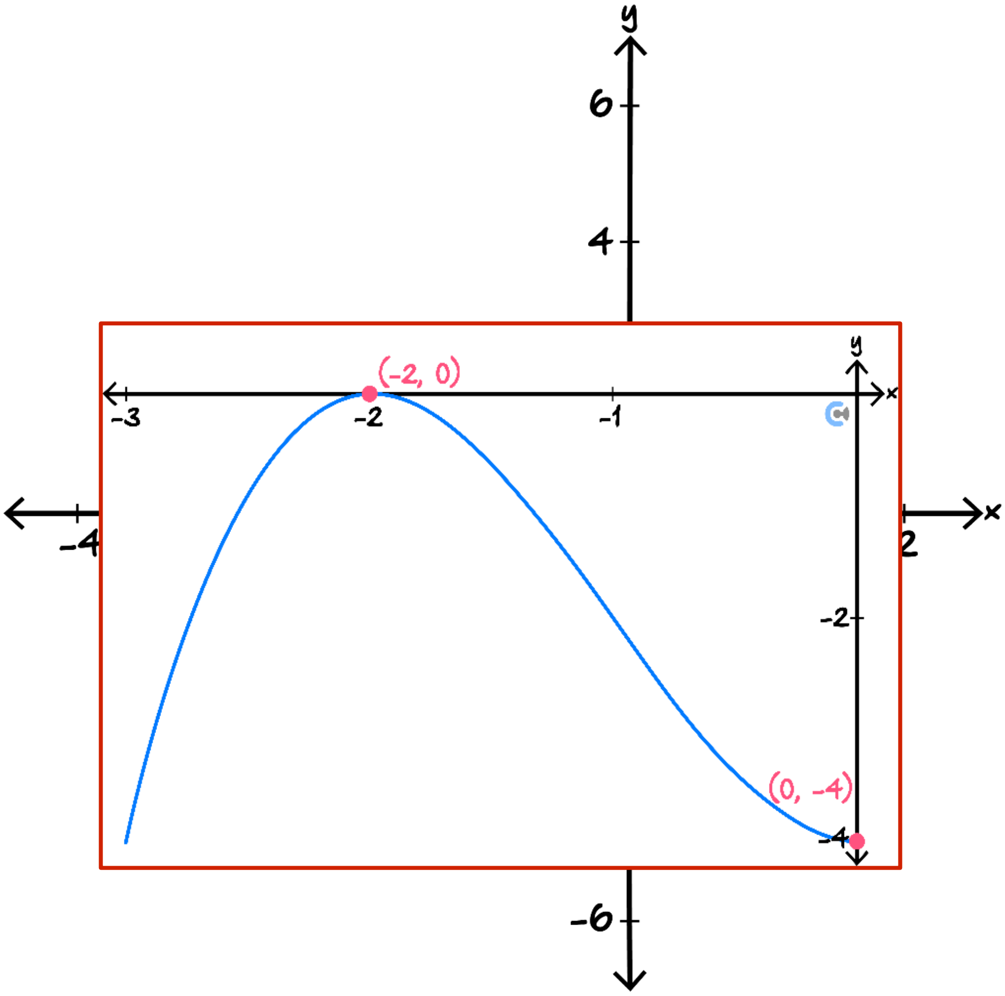
Let $f: [-3,0] \to R, f(x) = (x+2)^2(x-1)$.

a. Show that $(x+2)^2(x-1) = x^3 + 3x^2 - 4$.

$(x+2)^2(x-1)$
$= (x^2+4x+4)(x-1)$
$= x^3+4x^2+4x-x^2-4x-4$
$= x^3+3x^2-4$

This question was answered well, although some students either did not fully expand the cubic or made notational errors by omitting the brackets on the quadratic. It should be noted that $x^2+4x+4(x-1)$ is not equivalent to $x^3+4x^2+4x-x^2-4x-4$.

b. Sketch the graph of f on the axes below. Label the axis intercept and any stationary points with their coordinates.



Question 192 (6 marks)

Let $f: R \to R$, where $f(x) = 2x^3 + 1$, and let $g: R \to R$, where $g(x) = 4 - 2x$.

a.

i. Find $g(f(x))$. (1 mark)

$$g(f(x)) = 4 - 2(2x^3 + 1)$$
$$= 2 - 4x^3$$

ii. Find $f(g(x))$ and express it in the form $k - m(x - d)^3$, where m, k, and d are integers. (2 marks)

$$f(g(x)) = 2(4 - 2x)^3 + 1$$
$$= 2(2(2 - x))^3 + 1$$
$$= 1 - 16(x - 2)^3$$

b. The transformation $T: R^2 \to R^2$ with rule $T(x, y) = (x + b, ay + c)$, where a, b, and c are integers, maps the graph of $y = g(f(x))$ onto the graph of $y = f(g(x))$.

Find the values of a, b, and c. (3 marks)

$g(f(x)) = 2 - 4x^3$ to $f(g(x)) = 1 - 16(x-2)^3$

$x' = x + b,\ y' = ay + c$

$$\frac{y' - c}{a} = 2 - 4(x' - b)^3$$

$y = 2a - 4a(x' - b)^3 + c$ equate to $y = 1 - 16(x-2)^3$

$a = 4,\ b = 2,\ c = -7$

Alternatively, dilation of 4 from the x-axis: $4(2 - 4x^3) = 8 - 16x^3$, translation of 2 in the positive direction of the x-axis: $8 - 16(x-2)^3$,

translation of 7 in the negative y direction: $1 - 16(x-2)^3$

$$\text{Matrix} = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} 2 \\ -7 \end{bmatrix}$$

So $a = 4,\ b = 2,\ c = -7$

Space for Personal Notes

Question 193 (5 marks)

Let $f: R \to R, f(x) = 2e^x + 1$ and let $g: (-2, \infty) \to R, g(x) = \log_e(x+2)$.

a.

i. Find $f\big(g(x)\big)$ in the form $ax + b$, where $a, b \in R$. (1 mark)

$$f\big(g(x)\big) = 2x + 5$$

ii. State the range of $f\big(g(x)\big)$. (1 mark)

$$f\big(g(-2)\big) = 1$$

$$\therefore (1, \infty) = \text{range}$$

b. Let $T: R^2 \to R^2, T(x, y) = (x + c, y + d)$ and let the graph of the function h be the transformation of the graph of the function g under T.

If $h = f^{-1}$, then find the values of c and d. (3 marks)

$$x = 2e^y + 1$$

$$f^{-1}(x) = \log_e\left(\frac{x-1}{2}\right)$$

$$h(x) = \log_e\left(\frac{x-1}{2}\right) = g(x - c) + d = \log_e(x + 2 - c) + d$$

$$\log_e\left(\frac{x-1}{2}\right) = \log_e(x-1) - \log_e(2) = \log_e(x + 2 - 3) - \log_e(2)$$

$$c = 3, \quad d = -\log_e(2) \text{ or } d = \log_e\left(\frac{1}{2}\right)$$

Space for Personal Notes

Question 194 (4 marks)

a. Let $f: R\backslash\left\{\frac{1}{3}\right\} \to R, f(x) = \frac{1}{3x-1}$.

Find the rule of f^{-1}. (2 marks)

Marks	0	1	2	Average
%	6	37	57	1.5

$$x = \frac{1}{3\left(f^{-1}(x)\right)-1}$$

Thus $f^{-1}(x) = \frac{1}{3x} + \frac{1}{3}$ or $f^{-1}(x) = \frac{1+x}{3x}$

This question was well attempted and generally well done; however, in some cases progression to the correct answer was hindered by errors with algebraic manipulation (transposition) or poor use of notation.

b. State the domain of f^{-1}. (1 mark)

Marks	0	1	Average
%	36	64	0.7

Domain = $R\backslash\{0\}$

In general students knew that the domain of f^{-1} = range of f.

c. Let g be the function obtained by applying the transformation T to the function f, where:

$$T(x, y) = (x + c, y + d)$$

and $c, d \in R$.

Find the values of c and d given that $g = f^{-1}$. (1 mark)

Marks	0	1	Average
%	76	24	0.3

$c = -\frac{1}{3}$ and $d = \frac{1}{3}$

This question, while well attempted, was not done well. Some students had the incorrect sign for c and d. Other students attempted dilations rather than translations as specified by the question.

Space for Personal Notes

Section B: [1.1 - 1.8] - Exam 2 Overall (VCAA Qs) (179 Marks)

Question 195 (1 mark)

Let $h: (-1,1) \to R, h(x) = \frac{1}{x-1}$.

Which one of the following statements about h is **not** true?

A. $h(x)h(-x) = -h(x^2)$

B. $h(x) + h(-x) = 2h(x^2)$

C. $h(x) - h(0) = xh(x)$

D. $h(x) - h(-x) = 2xh(x^2)$

E. $(h(x))^2 = h(x^2)$

$$h(x) = \frac{1}{x-1}$$

$$(h(x))^2 \neq h(x^2)$$

$$\left(\frac{1}{x-1}\right)^2 = \frac{1}{x^2-2x+1} \neq \frac{1}{x^2-1}$$

Question 196 (1 mark)

The linear function $f: D \to R, f(x) = 5 - x$ has range $[-4,5)$.

The domain D is:

A. $(0,9]$

B. $(0,1]$

C. $[5,-4)$

D. $[-9,0)$

E. $[1,9)$

Space for Personal Notes

Question 197 (1 mark)

Let $f: R \to R, f(x) = 1 - 2\cos\left(\frac{\pi x}{2}\right)$.

The period and range of this function are respectively:

A. 4 and $[-2,2]$.

B. 4 and $[-1,3]$.

C. 1 and $[-1,3]$.

D. 4π and $[-1,3]$.

E. 4π and $[-2,2]$.

Question 198 (1 mark)

The function f has the property $f(x) - f(y) = (y - x)f(xy)$ for all non-zero real numbers x and y.

Which one of the following is a possible rule for the function?

A. $f(x) = x^2$

B. $f(x) = x^2 + x^4$

C. $f(x) = x\log_e(x)$

D. $f(x) = \frac{1}{x}$

E. $f(x) = \frac{1}{x^2}$

$$\frac{dy}{dx} = 2f(x) - f(y) = (y - x)f(xy)$$

$$f(x) = \frac{1}{x}$$

$$\text{LHS} = \frac{1}{x} - \frac{1}{y}$$

$$\text{RHS} = (y - x) \times \frac{1}{xy} = \frac{1}{x} - \frac{1}{y} = \text{LHS}$$

Space for Personal Notes

Question 199 (1 mark)

The linear function $f: D \to R, f(x) = 4 - x$ has range $[-2,6)$.

The domain D of the function is:

A. $[-2,6)$

B. $(-2,2]$

C. R

D. $(-2,6]$

E. $[-6,2]$

Question 200 (1 mark)

Which one of the following functions satisfies the functional equation $f\big(f(x)\big) = x$ for every real number x?

A. $f(x) = 2x$

B. $f(x) = x^2$

C. $f(x) = 2\sqrt{x}$

D. $f(x) = x - 2$

E. $f(x) = 2 - x$

Question 201 (1 mark)

If $f: (-\infty, 1) \to R, f(x) = 2\log_e(1 - x)$ and $g: [-1, \infty) \to R, g(x) = 3\sqrt{x + 1}$, then the maximal domain of the function $f + g$ is:

A. $[-1,1)$

B. $(1, \infty)$

C. $(-1,1]$

D. $(-\infty, -1]$

E. R

Question 202 (1 mark)

If the equation $f(2x) - 2f(x) = 0$ is true for all real values of x, then the rule for f could be:

A. $\frac{x^2}{2}$

B. $\sqrt{2x}$

C. $2x$

D. $\log_e\left(\frac{|x|}{2}\right)$

E. $x - 2$

Question 203 (1 mark)

The range of the function $f: [-2,3) \to R, f(x) = x^2 - 2x - 8$ is:

A. R

B. $(-9,-5]$

C. $(-5,0)$

D. $[-9,0]$

E. $[-9,-5)$

Question 204 (1 mark)

Let $f: R \to R, f(x) = x^2$.

Which one of the following is **not** true?

A. $f(xy) = f(x)f(y)$

B. $f(xy) - f(-x) = 0$

C. $f(2x) = 4f(x)$

D. $f(x-y) = f(x) - f(y)$

E. $f(x+y) + f(x-y) = 2(f(x) + f(y))$

$$f(x-y) = (x-y)^2 = x^2 - 2xy + y^2$$
$$f(x) - f(y) = x^2 - y^2$$
$$f(x-y) \neq f(x) - f(y)$$

Question 205 (1 mark)

The linear function $f: D \to R, f(x) = 6 - 2x$ has range $[-4,12]$.

The domain D is:

A. $[-3,5]$

B. $[-5,3]$

C. R

D. $[-14,18]$

E. $[-18,14]$

Question 206 (1 mark)

The range of the function $f: [-2,7) \to R, f(x) = 5 - x$ is:

A. $(-2,7]$

B. $[-2,7)$

C. $(-2,\infty)$

D. $(-2,7)$

E. R

Space for Personal Notes

Question 207 (1 mark)

The function f satisfies the functional equation $f\left(\frac{x+y}{2}\right) = \frac{f(x)+f(y)}{2}$ where x and y are any non-zero real numbers.

A possible rule for the function is:

A. $f(x) = \log_e|x|$

B. $f(x) = \frac{1}{x}$

C. $f(x) = 2^x$

D. $f(x) = 2x$

E. $f(x) = \sin(2x)$

If $f(x) = 2x$, then $f\left(\frac{x+y}{2}\right) = 2\left(\frac{x+y}{2}\right) = x+y$.

Moreover, $\frac{f(x)+f(y)}{2} = \frac{2x+2y}{2} = x+y$. Hence the function $f(x) = 2x$ satisfies the given functional equation.

Question 208 (1 mark)

If $3f(x) = f(3x)$ for $x > 0$, then the rule for f could be:

A. $f(x) = 3x$

B. $f(x) = \sqrt{3x}$

C. $f(x) = \frac{x^3}{3}$

D. $f(x) = \log_e\left(\frac{x}{3}\right)$

E. $f(x) = x - 3$

Space for Personal Notes

Question 209 (1 mark)

The range of the function $f: \left(\frac{-1}{\sqrt{2}}, \sqrt{2}\right] \to R, f(x) = 2x^3 - 3x + 4$ is:

A. $\left(4 - \sqrt{2}, 4 + \sqrt{2}\right)$

B. $\left(\frac{-1}{\sqrt{2}}, \sqrt{2}\right)$

C. $\left(4 - \sqrt{2}, 4 + \sqrt{2}\right]$

D. $\left(\frac{-2}{\sqrt{2}, \sqrt{2}}\right]$

E. $\left[4 - \sqrt{2}, 4 + \sqrt{2}\right]$

Question 210 (1 mark)

A function f satisfies the relation $f(x^2) = f(x) + f(x+2)$.

A possible rule for f is:

A. $f(x) = \sqrt{x+2}$

B. $f(x) = x + 2$

C. $f(x) = \log_{10}(x-1)$

D. $f(x) = \frac{1}{2}(x^2 - 1)$

E. $f(x) = \frac{1}{x-1}$

Space for Personal Notes

Question 211 (1 mark)

The function f and its inverse, f^{-1}, are one-to-one for all values of x.

If $f(1) = 5, f(3) = 7$, and $f(8) = 10$, then $f^{-1}(7)$ and $f^{-1}(5)$ respectively are equal to:

A. 5 and 7.

B. 3 and 1.

C. 7 and 5.

D. 8 and 5.

E. 5 and 8.

Question 212 (1 mark)

The function f with rule $f(x) = 2\log_e(16 - x)$ has a maximal domain given by:

A. $x \in (16, \infty)$

B. $x \in (-\infty, 4)$

C. $x \in (4, \infty)$

D. $x \in (-4, 4)$

E. $x \in (-\infty, 16)$

Question 213 (1 mark)

The range of the function with rule $y = \sqrt{4 - x^2} + \log_e(x + 2)$ is contained within the interval:

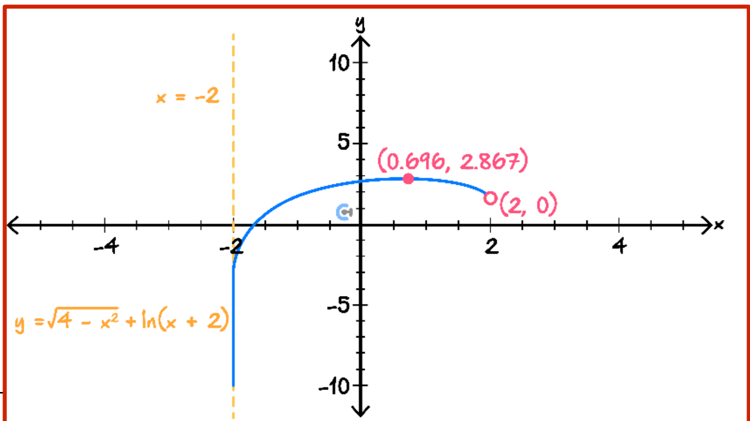
A. $[-4, 2.8]$

B. $(-\infty, 2.8]$

C. $(-4, 2.9)$

D. $(-\infty, 2.9)$

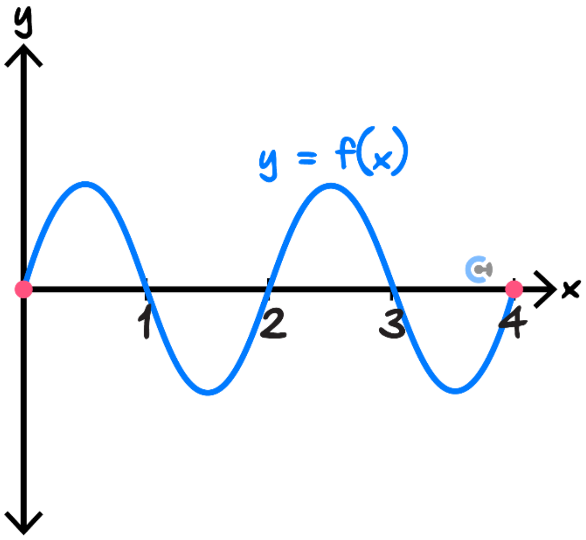
E. $[-4, 2.9)$

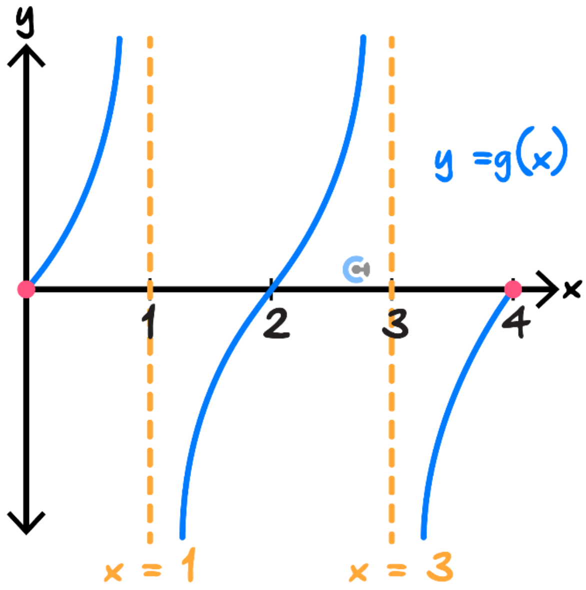


The range is $(-\infty, 2.866...]$. So it is contained within the interval $(-\infty, 2.9)$.

Question 214 (1 mark)

Consider the graphs of two circular functions, f and g, shown on the axes below.





On the interval $x \in [0, 4]$, the number of x-intercepts for the graph of the product function $h = f \times g$ is:

A. 3

B. 4

C. 5

D. 6

E. 7

A	$h = f \times g$ has three x-intercepts on the interval $x \in [0,4]$. The x-intercepts are at $x = 0$, $x = 2$ and $x = 4$.

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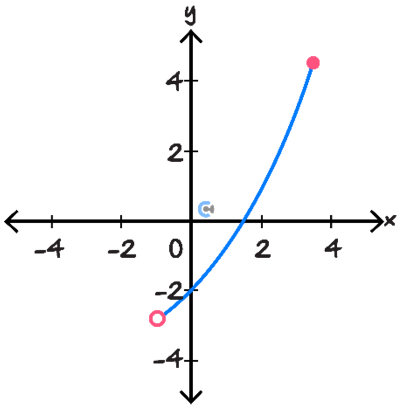
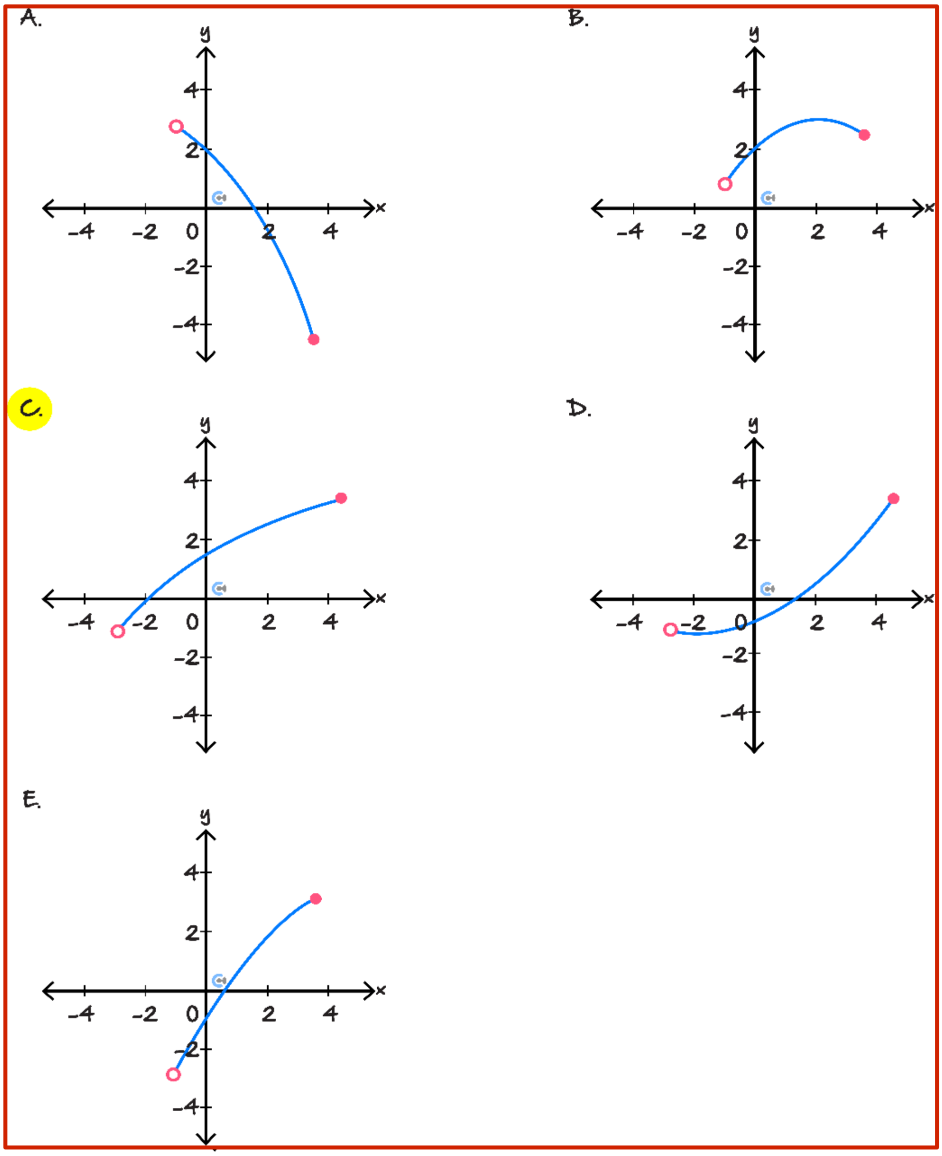
Question 215 (1 mark)

The graph of $y = f(x)$ is shown below.



The corresponding graph of the inverse of f, $y = f^{-1}(x)$, is best represented by:



Question 216 (1 mark)

If $3f(x) = f(3x)$ for $x > 0$, then the rule for f could be:

A. $f(x) = 3x$

B. $f(x) = \sqrt{3x}$

C. $f(x) = \frac{x^3}{3}$

D. $f(x) = \log_e\left(\frac{x}{3}\right)$

E. $f(x) = x - 3$

Question 217 (1 mark)

The function $f: D \to R, f(x) = \frac{x^4}{4} - \frac{x^3}{3} - \frac{9x^2}{2} + 9x$ will have an inverse function for:

A. $D = R$

B. $D = (-3,1)$

C. $D = (1, \infty)$

D. $D = (-\infty, 0)$

E. $D = (0, \infty)$

Question 218 (1 mark)

A cubic polynomial function $f: R \to R$ has roots at $x = 1$ and $x = 3$ only and its graph has a y-intercept at $y = 3$. Which one of the following statements **must** be true about the function g, where $g(x) = \sqrt{f(x)}$?

A. The function g has a local maximum at $x = 2$.

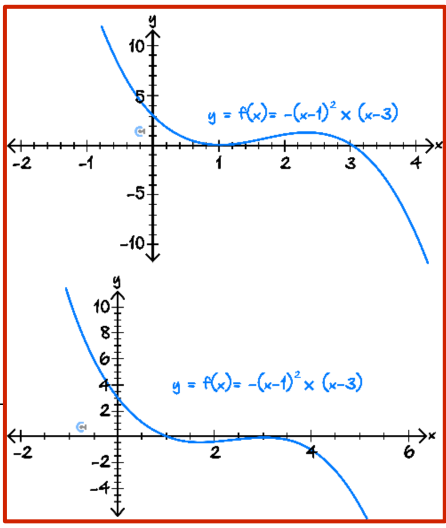
B. $g(2) = 1$.

C. The domain of g does not include the interval $(1, 3)$.

D. The domain of g includes the interval $(1, 3)$.

E. The domain of g does not include the interval $(3, \infty)$.

$g(x) = \sqrt{f(x)}$, $f(x) \geq 0$, so the domain of g does not include the interval $(3, \infty)$



Question 219 (1 mark)

The graph of the function $f: D \to R, f(x) = \frac{2x-3}{4+x}$, where D is the maximal domain, has asymptotes:

A. $x = -4, y = 2$

B. $x = \frac{3}{2}, y = -4$

C. $x = -4, y = \frac{3}{2}$

D. $x = \frac{3}{2}, y = 2$

E. $x = 2, y = 1$

A	$y = \frac{2x-3}{4+x} = 2 - \frac{11}{x+4}$, the asymptotes are $y = 2$ and $x = -4$

Question 220 (1 mark)

The function $f: D \to R, f(x) = 5x^3 + 10x^2 + 1$ will have an inverse function for:

A. $D = R$

B. $D = (-2, \infty)$

C. $D = \left(-\infty, \frac{1}{2}\right]$

D. $D = (-\infty, -1]$

E. $D = [0, \infty)$

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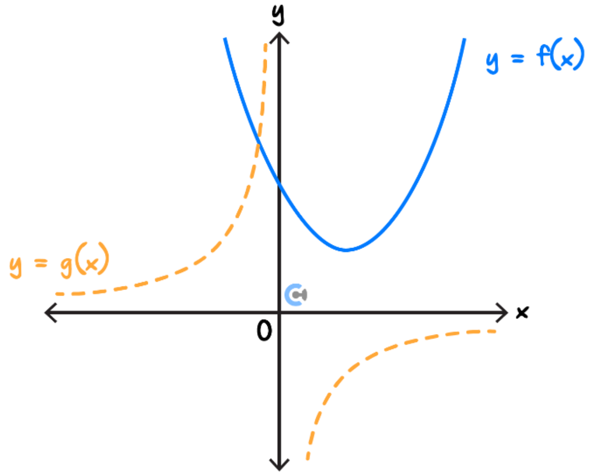
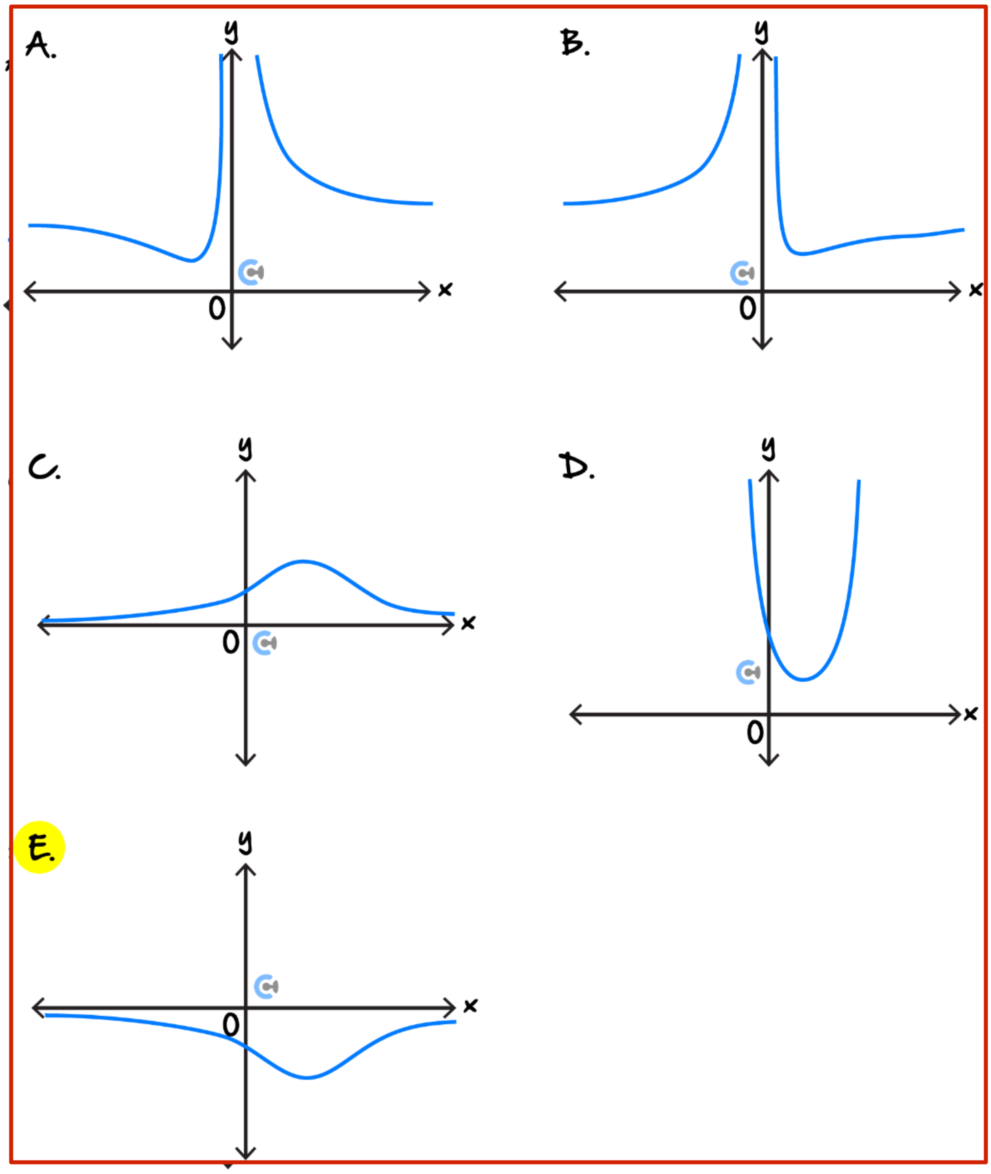
Question 221 (1 mark)

Part of the graphs of $y = f(x)$ and $y = g(x)$ are shown below.





Question 222 (1 mark)

The range of the function $f\colon \left(\frac{-1}{\sqrt{2}}, \sqrt{2}\right] \to R, f(x) = 2x^3 - 3x + 4$ is:

A. $\left(4-\sqrt{2}, 4+\sqrt{2}\right)$

B. $\left(\frac{-1}{\sqrt{2}}, \sqrt{2}\right)$

C. $\left(4-\sqrt{2}, 4+\sqrt{2}\right]$

D. $\left(\frac{-1}{\sqrt{2}}, \sqrt{2}\right]$

E. $\left[4-\sqrt{2}, 4+\sqrt{2}\right]$

Question 223 (1 mark)

The function f has the property $f(2x) = \left(f(x)\right)^2 - 2$ for all real numbers x.

A possible rule for the function $f(x)$ is:

A. $\frac{1}{x^2+4}$

B. $\cos(x)$

C. $2\log_e(x^2+1)$

D. $e^x + e^{-x}$

E. x^2

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Question 224 (1 mark)

Which one of the following is the inverse function of the function $f: (-\infty, 3) \to R, f(x) = \frac{2}{\sqrt{3-x}} + 1$?

A. $f^{-1}: (-\infty, 3) \to R, f^{-1}(x) = -\frac{4}{(x-1)^2} + 3$

B. $f^{-1}: (1, \infty) \to R, f^{-1}(x) = -\frac{4}{(x-3)^2} + 1$

C. $f^{-1}: (1, \infty) \to R, f^{-1}(x) = -\frac{4}{(x-1)^2} + 3$

D. $f^{-1}: (1, \infty) \to R, f^{-1}(x) = -\frac{4}{x^2} + 3$

E. $f^{-1}: R^+ \to R, f^{-1}(x) = -\frac{4}{(x-1)^2} + 3$

Question 225 (1 mark)

Let $f: D \to R, f(x) = \frac{3x-5}{2-x}$, where D is the maximal domain of f.

Which of the following are the equations of the asymptotes of the graph of f?

A. $x = 2$ and $y = \frac{5}{3}$.

B. $x = 2$ and $y = -3$.

C. $x = -2$ and $y = 3$.

D. $x = -3$ and $y = 2$.

E. $x = 2$ and $y = 3$.

Space for Personal Notes

Question 226 (1 mark)

A function f satisfies the relation $f(x^2) = f(x) + f(x+2)$.

A possible rule for f is:

A. $f(x) = \sqrt{x+2}$

B. $f(x) = x + 2$

C. $f(x) = \log_{10}(x-1)$

D. $f(x) = \frac{1}{2}(x^2 - 1)$

E. $f(x) = \frac{1}{x-1}$

Question 227 (1 mark)

Consider the function $f: [2, \infty) \to R, f(x) = x^4 + 2(a-4)x^2 - 8ax + 1$, where $a \in R$.

The maximal set of values of a for which the inverse function f^{-1} exists is:

A. $(-9, \infty)$

B. $(-\infty, 1)$

C. $[-9, 1]$

D. $[-8, \infty)$

E. $(-\infty, -8]$

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Question 228 (1 mark)

The range of the function with rule $y = \sqrt{4 - x^2} + \log_e(x + 2)$ is contained within the interval:

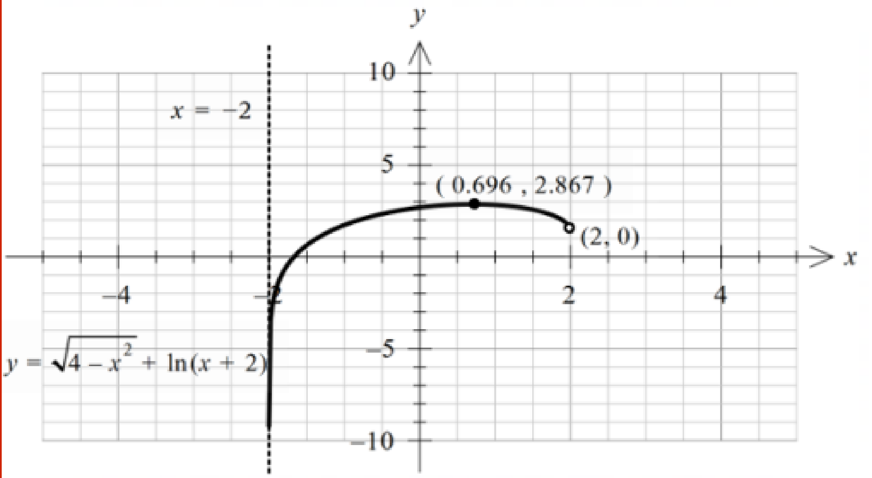
A. $[-4, 2.8]$

B. $(-\infty, 2.8]$

C. $(-4, 2.9)$

D. $(-\infty, 2.9)$

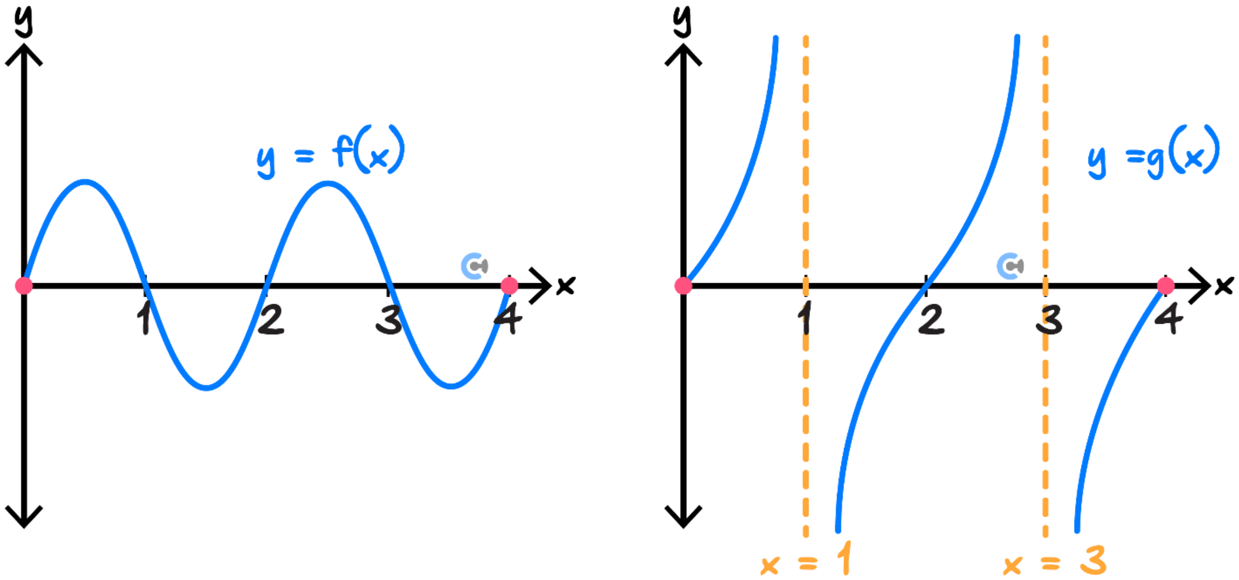
E. $[-4, 2.9)$



The range is $(-\infty, 2.866...]$. So it is contained within the interval $(-\infty, 2.9)$.

Question 229 (1 mark)

Consider the graphs of two circular functions, f and g, shown on the axes below.



On the interval $x \in [0, 4]$, the number of x-intercepts for the graph of the product function $h = f \times g$ is:

A. 3

B. 4

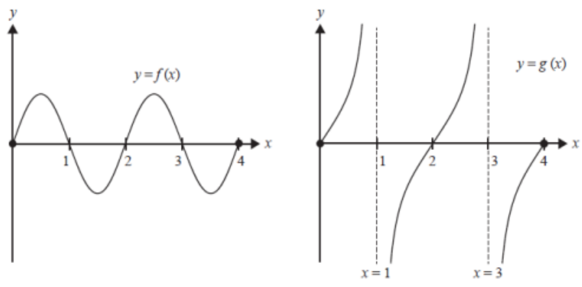
C. 5

D. 6

E. 7

A

$h = f \times g$ has three x-intercepts on the interval $x \in [0, 4]$. The x-intercepts are at $x = 0$, $x = 2$ and $x = 4$.



Question 230 (1 mark)

If $3f(x) = f(3x)$ for $x > 0$, then the rule for f could be:

A. $f(x) = 3x$

B. $f(x) = \sqrt{3x}$

C. $f(x) = \frac{x^3}{3}$

D. $f(x) = \log_e\left(\frac{x}{3}\right)$

E. $f(x) = x - 3$

Question 231 (1 mark)

The range of the function $f: \left(\frac{-1}{\sqrt{2}}, \sqrt{2}\right] \to R, f(x) = 2x^3 - 3x + 4$ is:

A. $(4 - \sqrt{2}, 4 + \sqrt{2})$

B. $\left(\frac{-1}{\sqrt{2}}, \sqrt{2}\right)$

C. $(4 - \sqrt{2}, 4 + \sqrt{2}]$

D. $\left(\frac{-1}{\sqrt{2}}, \sqrt{2}\right]$

E. $[4 - \sqrt{2}, 4 + \sqrt{2}]$

Space for Personal Notes

Question 232 (1 mark)

A function f satisfies the relation $f(x^2) = f(x) + f(x + 2)$.

A possible rule for f is:

A. $f(x) = \sqrt{x+2}$

B. $f(x) = x + 2$

C. $f(x) = \log_{10}(x - 1)$

D. $f(x) = \frac{1}{2}(x^2 - 1)$

E. $f(x) = \frac{1}{x-1}$

Question 233 (1 mark)

Consider the following four functional relations:

$$f(x) = f(-x) \qquad -f(x) = f(-x) \qquad f(x) = -f(x) \qquad \left(f(x)\right)^2 = f(x^2)$$

The number of these functional relations that are satisfied by the function $f: R \to R, f(x) = x$ is:

A. 0

B. 1

C. 2

D. 3

E. 4

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Question 234 (1 mark)

Let $g(x) = x + 2$ and $f(x) = x^2 - 4$.

If h is the composite function given by $h : [-5, -1) \to R, h(x) = f(g(x))$, then the range of h is:

A. $(-3,5]$

B. $[-3,5)$

C. $(-3,5)$

D. $(-4,5]$

E. $[-4,5]$

Question 235 (1 mark)

Consider the functions $f(x) = \sqrt{x+2}$ and $g(x) = \sqrt{1-2x}$, defined over their maximal domains.

The maximal domain of the function $h = f + g$ is:

A. $\left(-2, \frac{1}{2}\right)$

B. $[-2, \infty)$

C. $(-\infty, -2) \cup \left(\frac{1}{2}, \infty\right)$

D. $\left[-2, \frac{1}{2}\right]$

E. $[-2,1]$

Space for Personal Notes

Question 236 (1 mark)

Let f and g be functions such that $f(-1) = 4, f(2) = 5, g(-1) = 2$, $g(2) = 7$, and $g(4) = 6$.

The value of $g(f(-1))$ is:

A. 2

B. 4

C. 5

D. 6

E. 7

Question 237 (1 mark)

The graph of the function $f: D \to R, f(x) = \frac{3x+2}{5-x}$, where D is the maximal domain has asymptotes:

A. $x = -5, y = \frac{3}{2}$

B. $x = -3, y = 5$

C. $x = \frac{2}{3}, y = -3$

D. $x = 5, y = 3$

E. $x = 5, y = -3$

Space for Personal Notes

Question 238 (1 mark)

Let $a \in (0, \infty)$ and $b \in R$.

Consider the function $h: [-a, 0) \cup (0, a] \to R, h(x) = \frac{a}{x} + b$.

The range of h is:

A. $[b - a, b + 1]$

B. $(b - a, b + 1)$

C. $(-\infty, b - 1) \cup (b + 1, \infty)$

D. $(-\infty, b - 1] \cup [b + 1, \infty)$

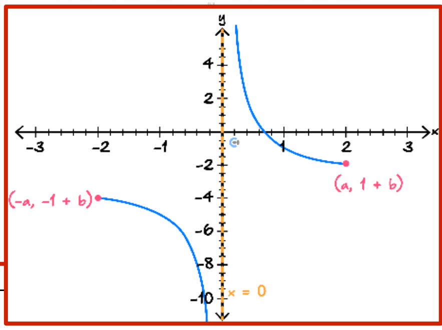
E. $[b - 1, \infty)$

$$h(x) = \frac{a}{x} + b$$

The coordinates of the endpoints are $(-a, -1+b)$ and $(a, 1+b)$.

The range is $(-\infty, -1+b] \cup [b+1, \infty)$.

An example, using the graph of $y = \frac{2}{x} - 3$ is shown below.



Question 239 (1 mark)

Let $f: R \to R, f(x) = \cos(ax)$, where $a \in R\backslash\{0\}$, be a function with the property:

$$f(x) = f(x + h), \text{ for all } h \in Z.$$

Let $g: D \to R, g(x) = \log_2(f(x))$ be a function where the range of g is $[-1,0]$.

A possible interval for D is:

A. $\left[\frac{1}{4}, \frac{5}{12}\right]$

B. $\left[1, \frac{7}{6}\right]$

C. $\left[\frac{5}{3}, 2\right]$

D. $\left[-\frac{1}{3}, 0\right]$

E. $\left[-\frac{1}{12}, \frac{1}{4}\right]$

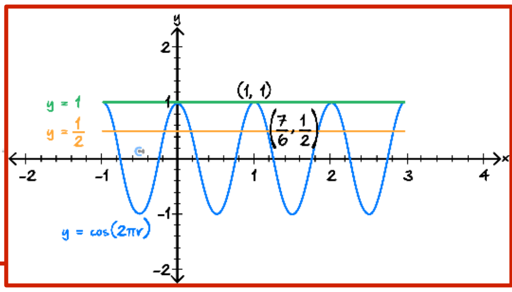
$f(x) = \cos(ax) = f(x+h) = \cos(a(x+h)),\ a = 2\pi$

$g(x) = \log_2(f(x)) = \log_2(f(x+h)) = \log_2(\cos(a(x+h)))$

$$-1 \le \log_2(\cos(2\pi x)) \le 0$$

$$\frac{1}{2} \le \cos(2\pi x) \le 1$$

Checking each of the options for a suitable domain gives $1 \le x \le \frac{7}{6}$.



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Question 240 (1 mark)

The graph of the function f passes through the point $(-2, 7)$.

If $h(x) = f\left(\frac{x}{2}\right) + 5$, then the graph of the function h must pass through the point:

A. $(-1, -12)$

B. $(-1,19)$

C. $(-4,12)$

D. $(-4, -14)$

E. $(3,3.5)$

Question 241 (1 mark)

The maximal domain of the function f is $R\backslash\{1\}$.

A possible rule for f is:

A. $f(x) = \frac{x^2-5}{x-1}$

B. $f(x) = \frac{x+4}{x-5}$

C. $f(x) = \frac{x^2+x+4}{x^2+1}$

D. $f(x) = \frac{5-x^2}{1+x}$

E. $f(x) = \sqrt{x-1}$

Space for Personal Notes

Question 242 (1 mark)

Consider the function $f: [a, b) \to R, f(x) = \frac{1}{x}$ where a and b are positive real numbers.

The range of f is:

A. $\left[\frac{1}{a}, \frac{1}{b}\right)$

B. $\left(\frac{1}{a}, \frac{1}{b}\right]$

C. $\left[\frac{1}{b}, \frac{1}{a}\right)$

D. $\left(\frac{1}{b}, \frac{1}{a}\right]$

E. $[a, b)$

$$f:[a, b) \to R, f(x) = \frac{1}{x},$$

$$f(a) = \frac{1}{a}, f(b) = \frac{1}{b}, f(a) > f(b)$$

Range $\left(\frac{1}{b}, \frac{1}{a}\right]$

Question 243 (1 mark)

Let f and g be two functions such that, $f(x) = 2x$ and $g(x+2) = 3x + 1$.

The function $f(g(x))$ is:

A. $6x - 5$

B. $6x + 1$

C. $6x^2 + 1$

D. $6x - 10$

E. $6x + 2$

Space for Personal Notes

Question 244 (1 mark)

The function f has the property $f\left(x + f(x)\right) = f(2x)$ for all non-zero real numbers x.

Which one of the following is a possible rule for the function?

A. $f(x) = 1 - x$

B. $f(x) = x - 1$

C. $f(x) = x$

D. $f(x) = \frac{x}{2}$

E. $f(x) = \frac{1-x}{2}$

Question 245 (1 mark)

Let f and g be functions such that, $f(2) = 5, f(3) = 4, g(2) = 5, g(3) = 2$, and $g(4) = 1$.

The value of $f\left(g(3)\right)$ is:

A. 1

B. 2

C. 3

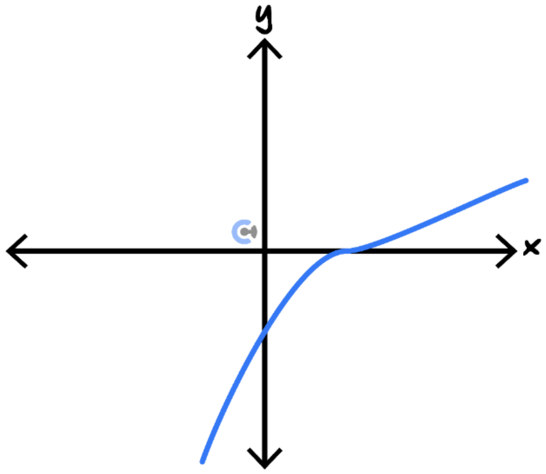
D. 4

E. 5

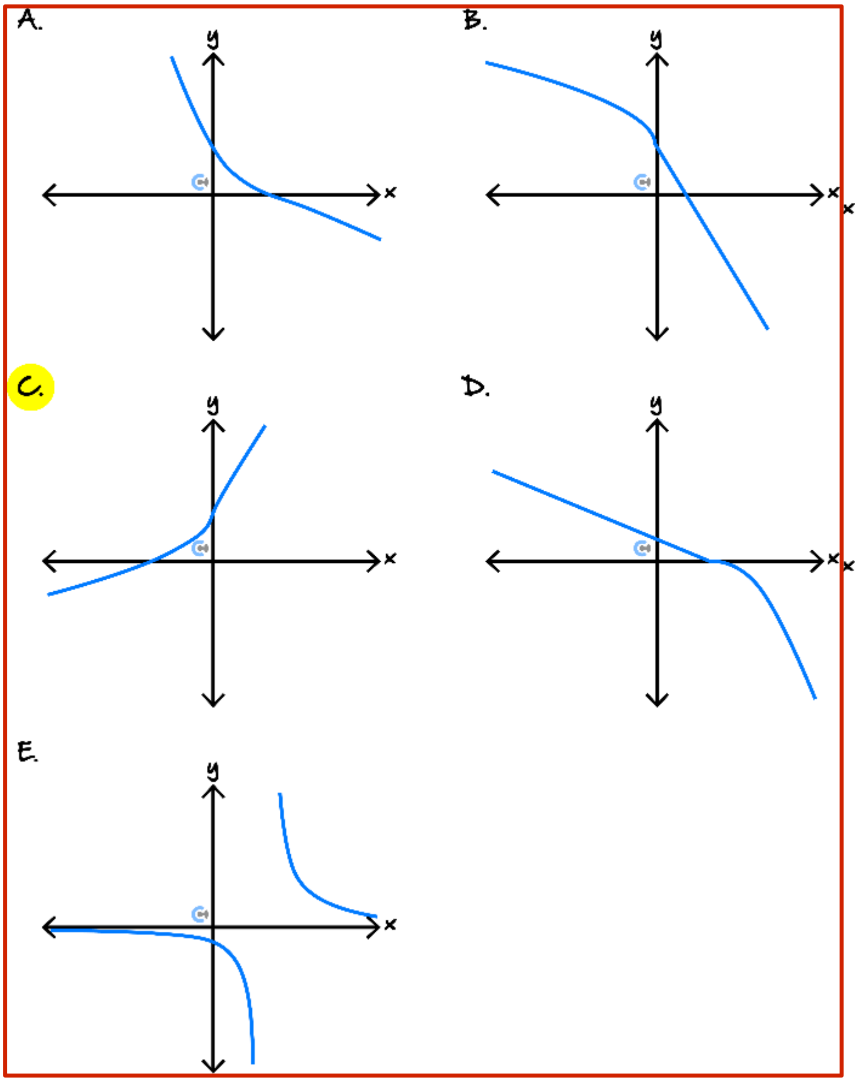
Space for Personal Notes

Question 246 (1 mark)

Part of the graph of the function f is shown below. The same scale has been used on both axes.



The corresponding part of the graph of the inverse function f^{-1} is best represented by:



Question 247 (1 mark)

Let $h: (-1,1) \to R, h(x) = \frac{1}{x-1}$.

Which one of the following statements about h is **not** true?

A. $h(x)h(-x) = -h(x^2)$

B. $h(x) + h(-x) = 2h(x^2)$

C. $h(x) - h(0) = xh(x)$

D. $h(x) - h(-x) = 2xh(x^2)$

E. $(h(x))^2 = h(x^2)$

$$h(x) = \frac{1}{x-1}$$

$$(h(x))^2 \neq h(x^2)$$

$$\left(\frac{1}{x-1}\right)^2 = \frac{1}{x^2-2x+1} \neq \frac{1}{x^2-1}$$

Question 248 (1 mark)

The linear function $f: D \to R, f(x) = 5 - x$ has range $[-4,5)$.

The domain D is:

A. $(0,9]$

B. $(0,1]$

C. $[5,-4)$

D. $[-9,0)$

E. $[1,9)$

Space for Personal Notes

Question 249 (1 mark)

Which one of the following is the inverse function of $g: [3, \infty) \to R, g(x) = \sqrt{2x - 6}$?

A. $g^{-1}: [3, \infty) \to R, g^{-1}(x) = \frac{x^2+6}{2}$

B. $g^{-1}: [0, \infty) \to R, g^{-1}(x) = (2x - 6)^2$

C. $g^{-1}: [0, \infty) \to R, g^{-1}(x) = \sqrt{\frac{x}{2}} + 6$

D. $g^{-1}: [0, \infty) \to R, g^{-1}(x) = \frac{x^2+6}{2}$

E. $g^{-1}: R \to R, g^{-1}(x) = \frac{x^2+6}{2}$

Question 250 (1 mark)

The function f has the property $f(x) - f(y) = (y - x)f(xy)$ for all non-zero real numbers x and y.

Which one of the following is a possible rule for the function?

A. $f(x) = x^2$

B. $f(x) = x^2 + x^4$

C. $f(x) = x \log_e(x)$

D. $f(x) = \frac{1}{x}$

E. $f(x) = \frac{1}{x^2}$

$$\frac{dy}{dx} = 2f(x) - f(y) = (y - x)f(xy)$$

$$f(x) = \frac{1}{x}$$

$$\text{LHS} = \frac{1}{x} - \frac{1}{y}$$

$$\text{RHS} = (y - x) \times \frac{1}{xy} = \frac{1}{x} - \frac{1}{y} = \text{LHS}$$

Space for Personal Notes

Question 251 (1 mark)

The function f has the property $f\big(x + f(x)\big) = f(2x)$ for all non-zero real numbers x.

Which one of the following is a possible rule for the function?

A. $f(x) = 1 - x$

B. $f(x) = x - 1$

C. $f(x) = x$

D. $f(x) = \frac{x}{2}$

E. $f(x) = \frac{1-x}{2}$

Question 252 (1 mark)

Let $h: (-1,1) \to R$, $h(x) = \frac{1}{x-1}$.

Which one of the following statements about h is **not** true?

A. $h(x)h(-x) = -h(x^2)$

B. $h(x) + h(-x) = 2h(x^2)$

C. $h(x) - h(0) = xh(x)$

D. $h(x) - h(-x) = 2xh(x^2)$

E. $(h(x))^2 = h(x^2)$

$$h(x) = \frac{1}{x-1}$$

$$\left(h(x)\right)^2 \neq h\left(x^2\right)$$

$$\left(\frac{1}{x-1}\right)^2 = \frac{1}{x^2 - 2x + 1} \neq \frac{1}{x^2 - 1}$$

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Question 253 (1 mark)

The linear function $f: D \to R, f(x) = 5 - x$ has a range $[-4, 5)$.

The domain D is:

A. $(0, 9]$

B. $(0, 1]$

C. $[5, -4)$

D. $[-9, 0)$

E. $[1, 9)$

Question 254 (1 mark)

Let $f: R \to R, f(x) = 1 - 2\cos\left(\frac{\pi x}{2}\right)$.

The period and range of this function are respectively:

A. 4 and $[-2, 2]$

B. 4 and $[-1, 3]$

C. 1 and $[-1, 3]$

D. 4π and $[-1, 3]$

E. 4π and $[-2, 2]$

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Question 255 (1 mark)

The function f has the property $f(x) - f(y) = (y - x)f(xy)$ for all non-zero real numbers x and y.

Which one of the following is a possible rule for the function?

A. $f(x) = x^2$

B. $f(x) = x^2 + x^4$

C. $f(x) = x\log_e(x)$

D. $f(x) = \frac{1}{x}$

E. $f(x) = \frac{1}{x^2}$

$$\frac{dy}{dx} = 2f(x) - f(y) = (y-x)f(xy)$$

$$f(x) = \frac{1}{x}$$

$$\text{LHS} = \frac{1}{x} - \frac{1}{y}$$

$$\text{RHS} = (y-x) \times \frac{1}{xy} = \frac{1}{x} - \frac{1}{y} = \text{LHS}$$

Question 256 (1 mark)

If the equation $f(2x) - 2f(x) = 0$ is true for all real values of x, then the rule for f could be:

A. $\frac{x^2}{2}$

B. $\sqrt{2x}$

C. $2x$

D. $\log_e\left(\frac{|x|}{2}\right)$

E. $x - 2$

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Question 257 (1 mark)

The range of the function $f: [-2, 3) \to R, f(x) = x^2 - 2x - 8$ is:

A. R

B. $(-9, -5]$

C. $(-5, 0)$

D. $[-9, 0]$

E. $[-9, -5)$

Question 258 (1 mark)

Let $f: R \to R, f(x) = x^2$.

Which one of the following is **not** true?

A. $f(xy) = f(x)f(y)$

B. $f(x) - f(-x) = 0$

C. $f(2x) = 4f(x)$

D. $f(x - y) = f(x) - f(y)$

E. $f(x + y) + f(x - y) = 2\big(f(x) + f(y)\big)$

$$f(x-y) = (x-y)^2 = x^2 - 2xy + y^2$$
$$f(x) - f(y) = x^2 - y^2$$
$$f(x-y) \neq f(x) - f(y)$$

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Question 259 (1 mark)

The linear function $f: D \to R, f(x) = 6 - 2x$ has a range $[-4, 12]$.

The domain D is:

A. $[-3, 5]$

B. $[-5, 3]$

C. R

D. $[-14, 18]$

E. $[-18, 14]$

Question 260 (1 mark)

The range of the function $f: [-2, 7) \to R, f(x) = 5 - x$ is:

A. $(-2, 7]$

B. $[-2, 7)$

C. $(-2, \infty)$

D. $(-2, 7)$

E. R

Question 261 (1 mark)

The function f satisfies the functional equation $f\left(\frac{x+y}{2}\right) = \frac{f(x)+f(y)}{2}$ where x and y are any non-zero real numbers. A possible rule for the function is:

A. $f(xy) = \log_e|x|$

B. $f(x) = \frac{1}{x}$

C. $f(x) = 2^x$

D. $f(x) = 2x$

E. $f(x) = \sin(2x)$

If $f(x) = 2x$, then $f\left(\frac{x+y}{2}\right) = 2\left(\frac{x+y}{2}\right) = x + y$.

Moreover, $\frac{f(x)+f(y)}{2} = \frac{2x+2y}{2} = x + y$. Hence the function $f(x) = 2x$ satisfies the given functional equation.

Question 262 (1 mark)

Inspired from VCAA Mathematics Exam 2007
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2007mmCAS2.pdf

Let $g(x) = x^2 + 2x - 3$ and $f(x) = e^{2x+3}$.

Then $f(g(x))$ is given by:

A. $e^{4x+6} + 2e^{2x+3} - 3$

B. $2x^2 + 4x - 6$

C. e^{2x^2+4x+9}

D. e^{2x^2+4x-3}

E. e^{2x^2+4x-6}

Question 263 (1 mark)

Inspired from VCAA Mathematics Exam 2007
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2007mmCAS2.pdf

The function f satisfies the functional equation $f(f(x)) = x$ for the maximal domain of f.

The rule for the function is:

A. $f(x) = x + 1$

B. $f(x) = x - 1$

C. $f(x) = \frac{x-1}{x+1}$

D. $f(x) = \log_e(x)$

E. $f(x) = \frac{x+1}{x-1}$

$f(f(x)) = x$ for $f(f(x)) = x$

Define $f(x)$ and evaluate $f(f(x))$.

Algebraically,

$$f(f(x)) = \frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}$$

$$= \frac{x+1+x-1}{x+1-x+1}$$

$$= \frac{2x}{2}$$

$$= x$$

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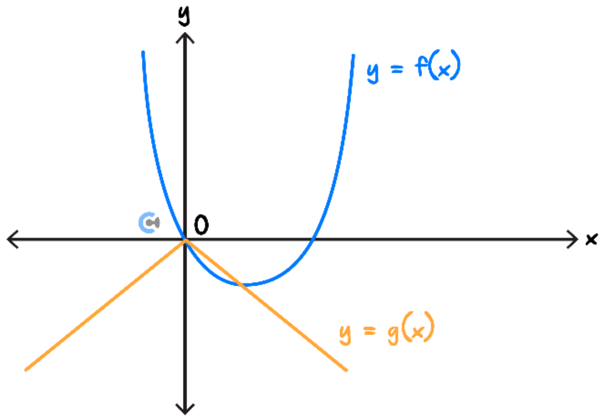
Question 264 (1 mark)

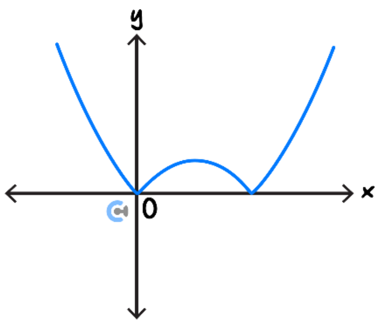
Inspired from VCAA Mathematics Exam 2007
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2007mmCAS2.pdf

The graphs of $y = f(x)$ and $y = g(x)$ are as shown.

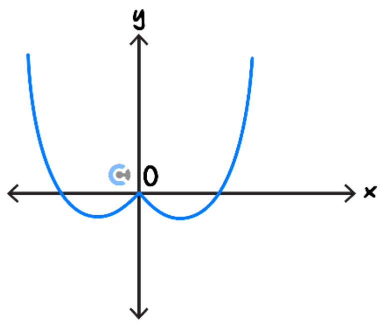


The graph of $y = f(g(x))$ is best represented by:

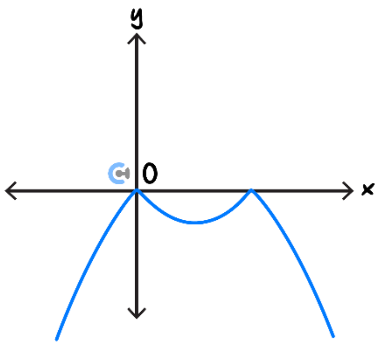
A.



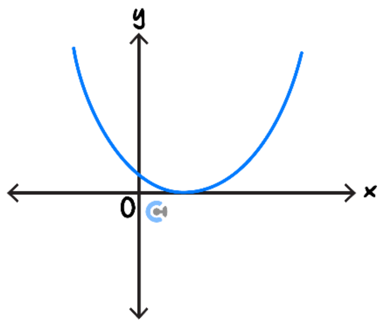
B.



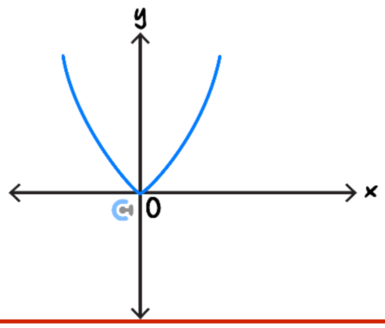
C.



D.



E.



$f(g(0)) = f(0) = 0$

The point $(0, 0)$ belongs to the curve.

$g(x) < 0$ elsewhere

$f(x) > 0$ for $x < 0$

Hence, $f(g(x)) > 0$ elsewhere

Question 265 (1 mark)

Inspired from VCAA Mathematics Exam 2022
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2022/NHT/2022mm2-nht-w.pdf

Let $g: R \to R, g(x) = 3x + a$, where a is a real constant.

Given that $g(g(2)) = 10$, the value of a is:

A. -1

B. -2

C. -3

D. -4

E. -5

Question 266 (1 mark)

Inspired from VCAA Mathematics Exam 2019
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2019/NHT/2019MM2-nht-w.pdf

Let $f: [0, \infty) \to R, f(x) = x^2 + 1$.

The equation $f(f(x)) = \frac{185}{16}$ has real solution(s):

A. $x = \pm\frac{\sqrt{13}}{4}$

B. $x = \frac{\sqrt{13}}{4}$

C. $x = \pm\frac{\sqrt{13}}{2}$

D	$f(x)=x^2+1$, solve $f(f(x))=\frac{185}{16}$ for x, $x=\pm\frac{3}{2}$, domain $x\geq 0$, $x=\frac{3}{2}$

D. $x = \frac{3}{2}$

E. $x = \pm\frac{3}{2}$

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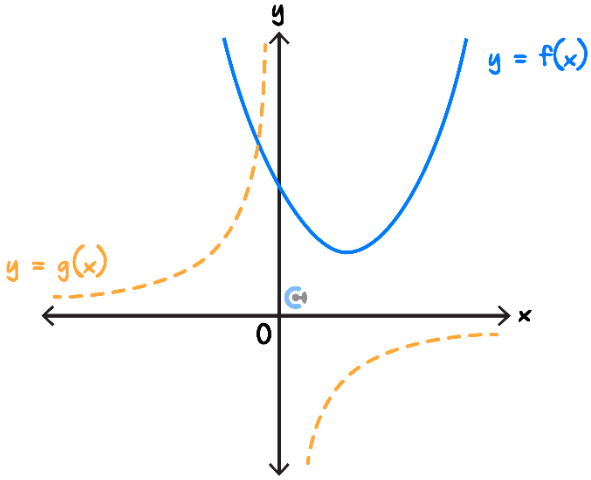
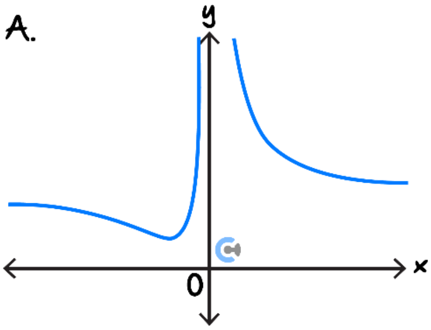
Question 267 (1 mark)

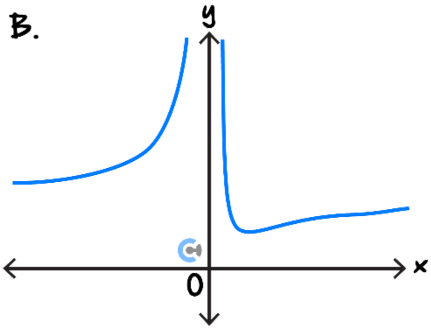
Inspired from VCAA Mathematics Exam 2019
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2019/NHT/2019MM2-nht-w.pdf

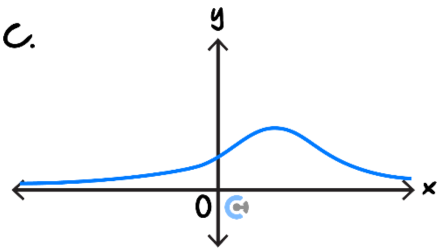
Parts of the graphs of $y = f(x)$ and $y = g(x)$ are as shown.

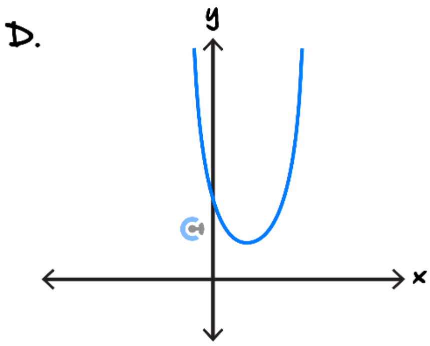


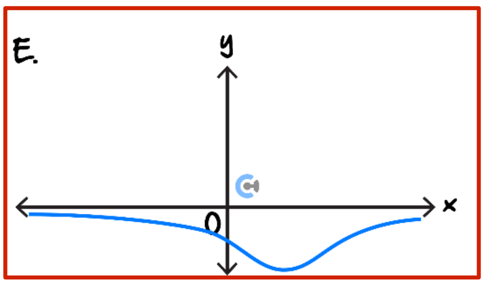
The corresponding part of the graph of $y = g(f(x))$ is best represented by:











Question 268 (1 mark)

Inspired from VCAA Mathematics Exam 2018
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2018/nht/2018MM2-nht-w.pdf

Let $f: R^+ \to R, f(x) = -\log_e(x)$ and $g: R \to R, g(x) = x^2 + 1$.

The domain and range of $f(g(x))$ are respectively:

A. R and $R^+ \cup \{0\}$

B. R and R^-

C. $[1, \infty)$ and $R^+ \cup \{0\}$

D. R^+ and $R^+ \cup \{0\}$

E. R and $R^- \cup \{0\}$

Question 269 (1 mark)

The midpoint of the line segment that joins $(1, -5)$ to $(d, 2)$ is:

A. $\left(\frac{d+1}{2}, -\frac{3}{2}\right)$

B. $\left(\frac{1-d}{2}, -\frac{7}{2}\right)$

C. $\left(\frac{d-4}{2}, 0\right)$

D. $\left(0, \frac{1-d}{3}\right)$

E. $\left(\frac{5+d}{2}, 2\right)$

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Question 270 (1 mark)

The midpoint of the line segment joining $(0, -5)$ to $(d, 0)$ is:

A. $\left(\frac{d}{2}, -\frac{5}{2}\right)$

B. $(0, 0)$

C. $\left(\frac{d-5}{2}, 0\right)$

D. $\left(0, \frac{5-d}{2}\right)$

E. $\left(\frac{5+d}{2}, 0\right)$

Question 271 (1 mark)

The gradient of a line **perpendicular** to the line which passes through $(-2, 0)$ and $(0, -4)$ is:

A. $\frac{1}{2}$

B. -2

C. $-\frac{1}{2}$

D. 4

E. 2

$$\boxed{m_{tangent} = \frac{-4-0}{0+2} = -2,\ m_{normal} = \frac{-1}{-2} = \frac{1}{2}}$$

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Question 272 (1 mark)

The coordinates of the point on a curve with the equation $y = \sqrt{x}$ that are closest to the point $(4, 0)$ are:

A. $(0, 0)$

B. $(3, \sqrt{3})$

C. $\left(\frac{7}{2}, \frac{\sqrt{14}}{2}\right)$

D. $\left(\frac{7}{2}, \frac{\sqrt{15}}{2}\right)$

E. $(4, 2)$

Question 273 (1 mark)

The set of values of p for which $x^3 - px + 2 = 0$ has three distinct, real solutions is:

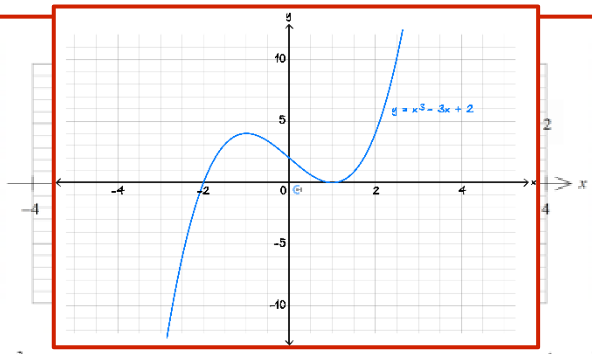
A. $(3, \infty)$

B. $(-\infty, -3)$

C. $(-3, 3)$

D. $(-\infty, 3]$

E. $[3, \infty)$

A	 $x^3 - px + 2 = 0$ has three distinct real solutions for $p \in (3, \infty)$. When $x = 3$, there are two distinct real solutions as shown. For values of p greater than three, the y-coordinate of the local maximum turning point is positive and the y-coordinate of the local minimum turning point is negative, which means there will be three distinct real solutions.

Question 274 (1 mark)

The simultaneous linear equations $2y + (m - 1)\,x = 2$ and $my + 3x = k$ have infinitely many solutions for:

A. $m = 3$ and $k = -2$

B. $m = 3$ and $k = 2$

C. $m = 3$ and $k = 4$

D. $m = -2$ and $k = -2$

E. $m = -2$ and $k = 3$

D	Solve $\frac{m-1}{3} = \frac{2}{m}$ for m for the lines to be parallel, $m = -2$ or $m = 3$, $\frac{2}{m} = \frac{2}{k}, m = k$ for an infinite number of solutions, $m = -2$ and $k = -2$

Question 275 (1 mark)

The gradient of a line perpendicular to the line that passes through $(3, 0)$ and $(0, -6)$ is:

A. $-\frac{1}{2}$

B. -2

C. $\frac{1}{2}$

D. 4

E. 2

Question 276 (1 mark)

The simultaneous linear equations $mx + 7y = 12$ and $7x + my = m$ have a unique solution only for:

A. $m = 7$ or $m = -7$

B. $m = 12$ or $m = 3$

C. $m \in R \setminus \{-7, 7\}$

D. $m = 4$ or $m = 3$

E. $m \in R \setminus \{12, 1\}$

Question 277 (1 mark)

The graph of $y = kx - 2$ will not intersect or touch the graph of $y = x^2 + 3x$ when:

A. $3 - 2\sqrt{2} < k < 3 + 2\sqrt{2}$

B. $\{k: k < 3 - 2\sqrt{2}\} \cup \{k: k > 3 + 2\sqrt{2}\}$

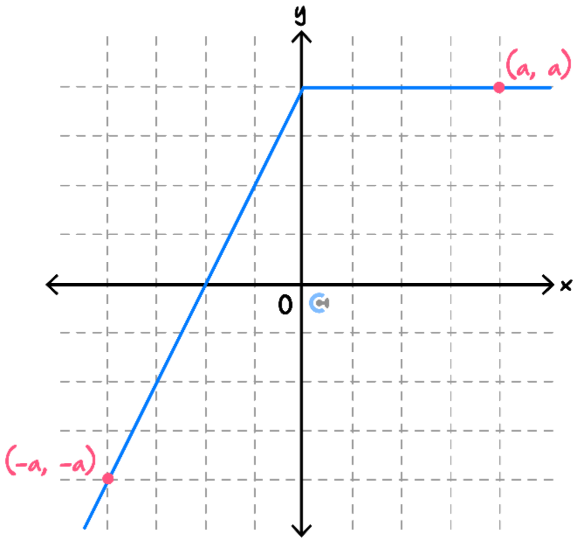
C. $-5 < k < 11$

D. $3 - 2\sqrt{2} \leq k \leq 3 + 2\sqrt{2}$

E. $k \in R^+$

Question 278 (1 mark)

Part of the graph of a function f is shown below.



Which one of the following is the average value of the function f over the interval $[-a, a]$?

A. 0

B. $\frac{3a}{4}$

C. $\frac{3a}{8}$

D. $\frac{a}{2}$

E. $\frac{a}{4}$

Question 279 (1 mark)

The simultaneous linear equations $ax - 3y = 5$ and $3x - ay = 8 - a$ have **no solution** for:

A. $a = 3$

B. $a = -3$

C. both $a = 3$ and $a = -3$

D. $a \in R \setminus \{3\}$

E. $a \in R \setminus [-3, 3]$

Question 280 (1 mark)

Let $p(x) = x^3 - 2ax^2 + x - 1$, where $a \in R$. When p is divided by $x + 2$, the remainder is 5.

The value of a is:

A. 2

B. $-\frac{7}{4}$

C. $\frac{1}{2}$

D. $-\frac{3}{2}$

E. -2

Question 281 (1 mark)

If $x + a$ is a factor of $8x^3 - 14x^2 - a^2x$, where $a \in R \setminus \{0\}$, then the value of a is:

A. 7

B. 4

C. 1

D. -2

E. -1

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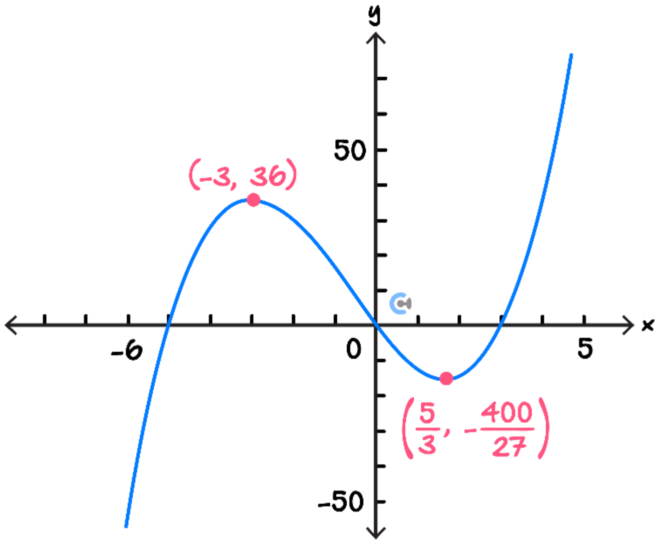
Question 282 (1 mark)

Part of the graph of a cubic polynomial function f and the coordinates of its stationary points are shown below.



$f'(x) < 0$ for the interval:

A. $(0, 3)$

B. $(-\infty, -5) \cup (0, 3)$

C. $(-\infty, -3) \cup \left(\frac{5}{3}, \infty\right)$

D. $\left(-3, \frac{5}{3}\right)$

E. $\left(\frac{-400}{27}, 36\right)$

Question 283 (1 mark)

The equation $(p-1)x^2 + 4x = 5 - p$ has no real roots when:

A. $p^2 - 6p + 6 < 0$

B. $p^2 - 6p + 1 > 0$

C. $p^2 - 6p - 6 < 0$

D. $p^2 - 6p + 1 < 0$

E. $p^2 - 6p + 6 > 0$

$(p-1)x^2 + 4x = 5 - p$

$(p-1)x^2 + 4x - 5 + p = 0$

The discriminant is negative for no real solutions.

$16 - 4(p-1)(p-5) < 0$

$-4p^2 + 24p - 4 < 0$

Divide by -4 and change the inequality.

$p^2 - 6p + 1 > 0$

Question 284 (1 mark)

The simultaneous linear equations $(m-1)x+5y=7$ and $3x+(m-3)y=0.7m$ have infinitely many solutions for:

A. $m \in R \setminus \{0, -2\}$

B. $m \in R \setminus \{0\}$

C. $m \in R \setminus \{6\}$

D. $m = 6$

E. $m = -2$

Question 285 (1 mark)

The simultaneous linear equations,

$$kx - 3y = 0$$

$$5x - (k+2)y = 0$$

Where k is a real constant, have a unique solution provided.

A. $k \in \{-5, 3\}$

B. $k \in R \setminus \{-5, 3\}$

C. $k \in \{-3, 5\}$

D. $k \in R \setminus \{-3, 5\}$

E. $k \in R \setminus \{0\}$

Using multiples of coefficients and combining equations, a unique solution exists when $k(k+2)-15$ is non zero, that is, $k \in R \setminus \{-5, 3\}$.

Alternatively, solve $\begin{vmatrix} k & -3 \\ 5 & -k-2 \end{vmatrix} = 0$

$k = -5$ or $k = 3$ (infinite number of solutions).
A unique solution will occur if $k \in R \setminus \{-5, 3\}$.

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Question 286 (1 mark)

The simultaneous linear equations,

$$ax + 3y = 0$$

$$2x + (a + 1)y = 0$$

Where a is a real constant, have infinitely many solutions for:

A. $a \in R$

B. $a \in \{-3, 2\}$

C. $a \in R \setminus \{-3, 2\}$

D. $a \in \{-2, 3\}$

E. $a \in R \setminus \{-2, 3\}$

Question 287 (1 mark)

The simultaneous linear equations,

$$mx + 12y = 24$$

$$3x + my = m$$

Have a unique solution only for:

A. $m = 6$ or $m = -6$

B. $m = 12$ or $m = 3$

C. $m \in R \setminus \{-6, 6\}$

D. $m = 2$ or $m = 1$

E. $m \in R \setminus \{-12, -3\}$

$mx + 12y = 24$

$3x + my = m$

There will be no solution if $\begin{vmatrix} m & 12 \\ 3 & m \end{vmatrix} = 0$, or equivalent. A unique solution exists for $m \in R \setminus \{-6, 6\}$.

Space for Personal Notes

Question 288 (1 mark)

The graph of $y = kx - 3$ intersects the graph of $y = x^2 + 8x$ at two distinct points for:

A. $k = 11$

B. $k > 8 + 2\sqrt{3}$ or $k < 8 - 2\sqrt{3}$

C. $5 \leq k \leq 6$

D. $8 - 2\sqrt{3} \leq k \leq 8 + 2\sqrt{3}$

E. $k = 5$

At the point(s) of intersection, $x^2 + 8x = kx - 3$ or $x^2 + (8-k)x + 3 = 0$

there will be two distinct solutions when the discriminant is greater than zero.

$b^2 - 4ac > 0$

$(8-k)^2 - 4 \times 1 \times 3 > 0$

$(8-k)^2 - 12 > 0$

Solving for k gives the result

$k < 8 - 2\sqrt{3}$ or $k > 8 + 2\sqrt{3}$

Question 289 (1 mark)

The solution set of the equation $e^{4x} - 5e^{2x} + 4 = 0$ over R is:

A. $\{1, 4\}$

B. $\{-4, -1\}$

C. $\{-2, -1, 1, 2\}$

D. $\{-\log_e(2), 0, \log_e(2)\}$

E. $\{0, \log_e(2)\}$

Question 290 (1 mark)

The simultaneous linear equations $(m-2)x + 3y = 6$ and $2x + (m-3)y = m - 1$ have **no solution** for:

A. $m \in R \setminus \{0, 5\}$

B. $m \in R \setminus \{0\}$

C. $m \in R \setminus \{6\}$

D. $m = 5$

E. $m = 0$

To have either no solutions, or infinitely many solutions, the ratio of the coefficients of the x and the y terms must be equal,

hence $\frac{m-2}{2} = \frac{3}{m-3}, m \neq 3$. (If $m = 3$, then the equations will have a unique solution $x = 1$ and $y = \frac{5}{3}$.) This can be rearranged to form the quadratic equation $(m-2)(m-3) = 6$, or $m^2 - 5m = 0$, which has solutions $m = 0$ or $m = 5$. If $m = 0$, the simultaneous equations are $-2x + 3y = 6$ and $2x - 3y = -1$ and they have no solution as the equations correspond to distinct parallel lines. If $m = 5$, the simultaneous equations are $3x + 3y = 6$ and $2x + 2y = 4$, and they have many solutions since each equation represents the same line.

Question 291 (1 mark)

The set of values of k for which $x^2 + 2x - k = 0$ has two real solutions is:

A. $\{-1, 1\}$

B. $(-1, \infty)$

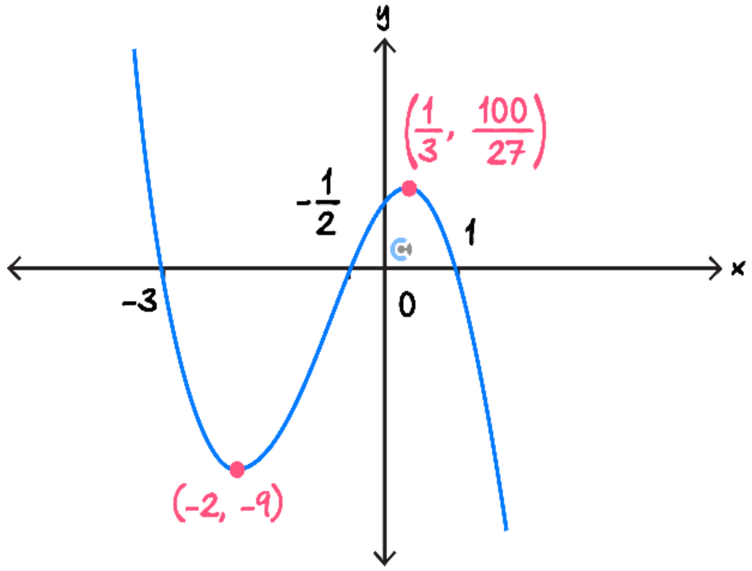
C. $(-\infty, -1)$

D. $\{-1\}$

E. $[-1, \infty)$

Question 292 (1 mark)

Part of the graph $y = f(x)$ of the polynomial function f is shown below.



$f'(x) < 0$ for

A. $x \in (-2,0) \cup \left(\frac{1}{3}, \infty\right)$

B. $x \in \left(-9, \frac{100}{27}\right)$

C. $x \in (-\infty, -2) \cup \left(\frac{1}{3}, \infty\right)$

D. $x \in \left(-2, \frac{1}{3}\right)$

E. $x \in (-\infty, -2] \cup (1, \infty)$

Question 293 (1 mark)

The line with equation $y = mx + 1$ and the curve with equation $y = 3x^2 + 2x + 4$ intersect at two distinct points. The values of m are:

A. $-4 < m < 8$

B. $m < -4$

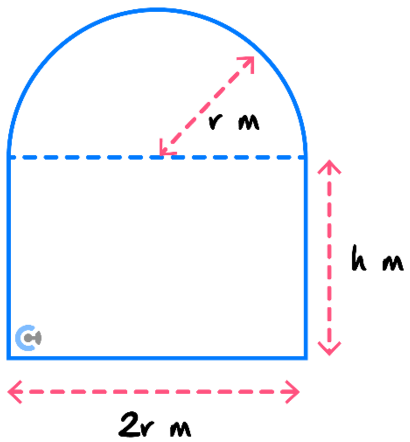
C. $m > 8$

D. $m < -4$ or $m > 8$

E. $m = -4$ or $m = 8$

Question 294 (1 mark)

The diagram below shows a glass window consisting of a rectangle of height h metres and width $2r$ metres, and a semicircle of radius r metres. The perimeter of the window is 8 m.



An expression for the area of the glass window, A, in terms of r is:

A. $A = 8r - 2r^2 - \frac{3\pi r^2}{2}$

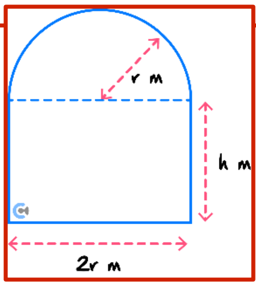
B. $A = 8r - 2r^2 + \frac{\pi r^2}{2}$

C. $A = 8r - 4r^2 - \frac{3\pi r^2}{2}$

D. $A = 8r - 4r^2 - \frac{\pi r^2}{2}$

E. $A = 8r - 2r^2 - \frac{\pi r^2}{2}$

E



$A = 2rh + \frac{\pi r^2}{2}$, $P = \pi r + 2h + 2r = 8$, $h = 4 - \frac{\pi r}{2} - r$, $A = 8r - 2r^2 - \frac{\pi r^2}{2}$

Question 295 (1 mark)

The simultaneous linear equations $2y + (m-1)x = 2$ and $my + 3x = k$ have infinitely many solutions for:

A. $m = 3$ and $k = -2$

B. $m = 3$ and $k = 2$

C. $m = 3$ and $k = 4$

D. $m = -2$ and $k = -2$

E. $m = -2$ and $k = 3$

Solve $\frac{m-1}{3} = \frac{2}{m}$ for m for the lines to be parallel, $m = -2$ or $m = 3$,
$\frac{2}{m} = \frac{2}{k}, m = k$ for an infinite number of solutions, $m = -2$ and $k = -2$

Question 296 (1 mark)

The simultaneous linear equations $mx + 7y = 12$ and $7x + my = m$ have a unique solution only for:

A. $m = 7$ and $m = -7$

B. $m = 12$ and $m = 3$

C. $m \in R \setminus \{-7, 7\}$

D. $m = 4$ and $m = 3$

E. $m \in R \setminus \{12, 1\}$

Space for Personal Notes

Question 297 (1 mark)

Let $f: [0, \infty) \to R, f(x) = x^2 + 1$.

The equation $f(f(x)) = \frac{185}{16}$ has real solution(s):

A. $x = \pm\frac{\sqrt{13}}{4}$

B. $x = \frac{\sqrt{13}}{4}$

C. $x = \pm\frac{\sqrt{13}}{2}$

D. $x = \frac{3}{2}$

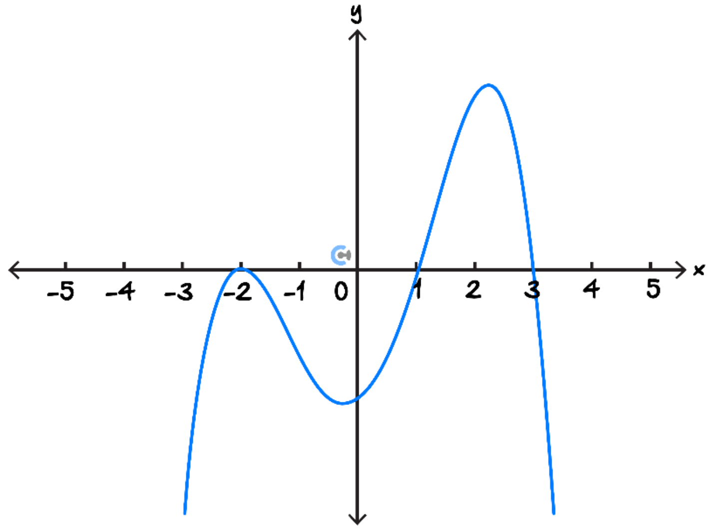
E. $x = \pm\frac{3}{2}$

D	$f(x) = x^2 + 1$, solve $f(f(x)) = \frac{185}{16}$ for x, $x = \pm\frac{3}{2}$, domain $x \geq 0$, $x = \frac{3}{2}$

Space for Personal Notes

Question 298 (1 mark)

The diagram below shows part of the graph of a polynomial function.



A possible rule for this function is:

A. $y = (x + 2)(x - 1)(x - 3)$

B. $y = (x + 2)^2(x - 1)(x - 3)$

C. $y = (x + 2)^2(x - 1)(3 - x)$

D. $y = -(x - 2)^2(x - 1)(3 - x)$

E. $y = -(x + 2)(x - 1)(x - 3)$

Space for Personal Notes

Question 299 (1 mark)

A set of three numbers that could be the solutions of $x^3 + ax^2 + 16x + 84 = 0$ is:

A. $\{3, 4, 7\}$

B. $\{-4, -3, 7\}$

C. $\{-2, -1, 21\}$

D. $\{-2, 6, 7\}$

E. $\{2, 6, 7\}$

Question 300 (1 mark)

The transformation $T: R^2 \to R^2$ maps the graph of $y = x^3 - x$ onto the graph of $y = 2(x-1)^3 - 2(x-1) + 4$. The transformation T could be given by:

A. $T(x, y) = (x + 1, 2y + 4)$

B. $T(x, y) = \left(x + 1, \frac{1}{2}y + 4\right)$

C. $T(x, y) = (2x + 1, y + 2)$

D. $T(x, y) = \left(\frac{1}{2}x + 1, y + 2\right)$

E. $T(x, y) = (x + 1, 2y + 2)$

Question 301 (1 mark)

The transformation $T: R^2 \to R^2$, which maps the graph of $y = -\sqrt{2x+1} - 3$ onto the graph of $y = \sqrt{x}$, has rules:

A. $T(x, y) = \left(\frac{1}{2}x - 1, -y - 3\right)$

B. $T(x, y) = \left(\frac{1}{2}x - 1, -y + 3\right)$

C. $T(x, y) = \left(\frac{1}{2}x + 1, -y - 3\right)$

D. $T(x, y) = (2x + 1, -y - 3)$

E. $T(x, y) = (2x - 1, -y + 3)$

Question 302 (1 mark)

The point (a, b) is transformed by:

$$T(x, y) = \left(\frac{1}{2}x - \frac{1}{2}, -2y - 2\right)$$

If the image of (a, b) is $(0, 0)$, then (a, b) is:

A. $(1, 1)$

B. $(-1, 1)$

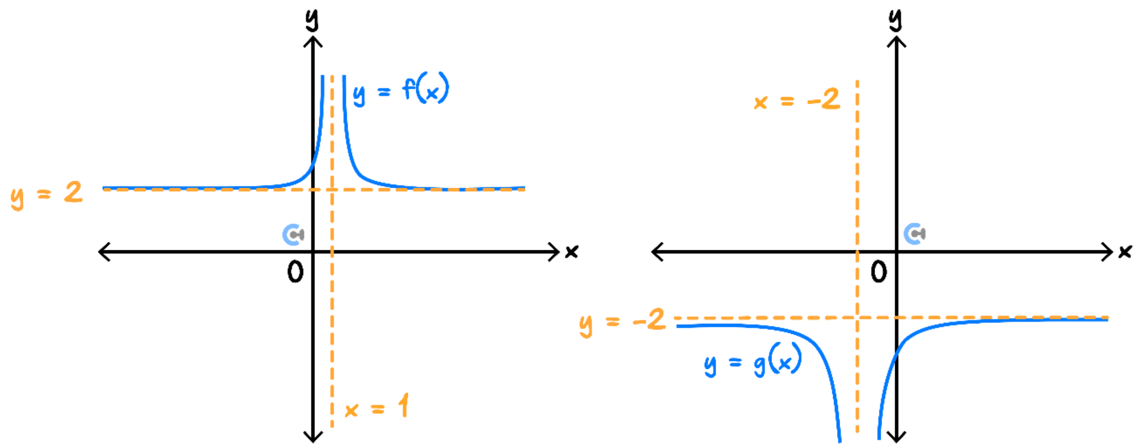
C. $(-1, 0)$

D. $(0, 1)$

E. $(1, -1)$

Question 303 (1 mark)

Consider the graphs of f and g below, which have the same scale.



If T transforms the graph of f onto the graph of g, then:

A. $T(x, y) = (x - 3, y - 4)$

B. $T(x, y) = (-x - 2$ ... $4)$

D	Dilate the graph of *f* by a factor of 2 from the *y*-axis and reflect the image in the *x* and *y* axes.

C. $T(x, y) = (x - 3,$ ... $y)$

D. $T(x, y) = (-2x, -y)$

E. $T(x, y) = (-x, -2y)$

Question 304 (1 mark)

The graph of the function $f: [0, \infty) \to R$, where $f(x) = 4x^{\frac{1}{3}}$, is reflected in the x-axis and then translated five units to the right and six units vertically down.

Which one of the following is the rule of the transformed graph?

A. $y = 4(x-5)^{\frac{1}{3}} + 6$

B. $y = -4(x+5)^{\frac{1}{3}} - 6$

C. $y = -4(x+5)^{\frac{1}{3}} + 6$

D. $y = -4(x-5)^{\frac{1}{3}} - 6$

E. $y = 4(x-5)^{\frac{1}{3}} + 1$

Question 305 (1 mark)

The point $A(3, 2)$ lies on the graph of the function f. A transformation maps the graph of f to the graph of g where $g(x) = \frac{1}{2}f(x-1)$. The same transformation maps the point A to the point P.

The coordinates of the point P are:

A. $(2, 1)$

B. $(2, 4)$

C. $(4, 1)$

D. $(4, 2)$

E. $(4, 4)$

A (3, 2), $g(x) = \frac{1}{2}f(x-1)$,

Dilate by a factor of a $\frac{1}{2}$ from the x-axis: $(3, 1)$

Translate 1 unit to the right: $(4, 1)$

Space for Personal Notes

Question 306 (1 mark)

The graph of a function f is obtained from the graph of the function g with rule $g(x) = \sqrt{2x - 5}$ by a reflection in the x-axis followed by a dilation from the y-axis by a factor of $\frac{1}{2}$.

Which one of the following is the rule for the function f?

A. $f(x) = \sqrt{5 - 4x}$

B. $f(x) = -\sqrt{x - 5}$

C. $f(x) = \sqrt{x + 5}$

D. $f(x) = -\sqrt{4x - 5}$

E. $f(x) = -\sqrt{4x - 10}$

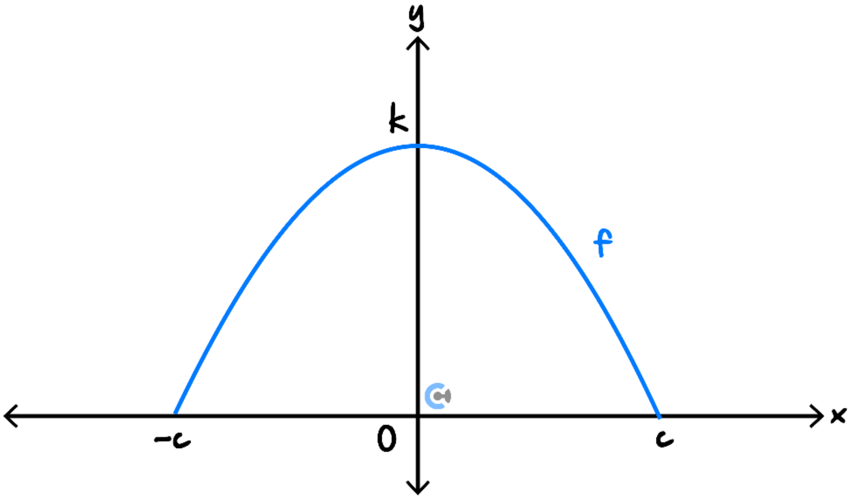
Space for Personal Notes

Question 307 (10 marks)

The parabolic arch of a tunnel is modelled by the function $f: [-c, c] \to R, f(x) = ax^2 + b$, where $a < 0$, $b \in R$ and $c > 0$.

Let x be the horizontal distance, in metres, from the origin and let y be the vertical distance, in metres, above the base of the arch.

The graph of f is shown below, where the coordinates of the y-intercept are $(0, k)$ and the coordinates of the x-intercepts are $(-c, 0)$ and $(c, 0)$.



a. Express a and b in terms of c and k. (2 marks)

$$\boxed{a = -\frac{k}{c^2} \text{ and } b = k}$$

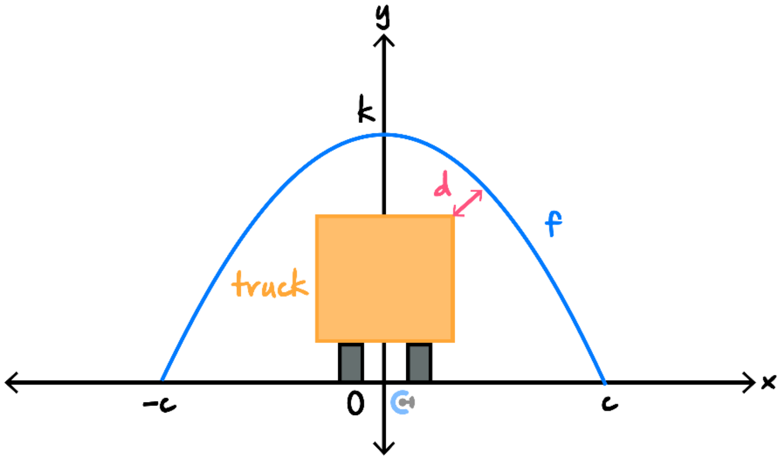
A particular tunnel has an arch modelled by f. It has a height of 6 m at the centre and a width of 8 m at the base.

b.

i. Find the rule for this arch. (1 mark)

$$\boxed{f(x) = -\frac{3}{8}x^2 + 6}$$

ii. A truck that has a height of 3.7 m and a width of 2.7 m will fit through the arch with the function f found in **part b. i.**



Assuming that the truck drives directly through the middle of the arch, let d be the minimum distance between the arch and the top corner of the truck.

Find d and the value of x for which this occurs, correct to three decimal places. (3 marks)

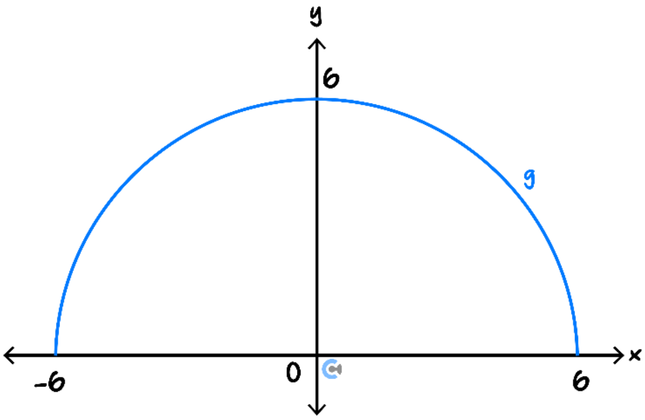
$$\text{Distance} = \sqrt{(x-1.35)^2 + \left(-\frac{3}{8}x^2 + 6 - 3.7\right)^2}$$

$x = 2.185$ $(2.18506 \ldots)$

distance $= 0.978$ $(0.9782556 \ldots)$

A different tunnel has a semicircular arch. This arch can be modelled by the function $g: [-6, 6] \to R$, $g(x) = \sqrt{r^2 - x^2}$, where $r > 0$.

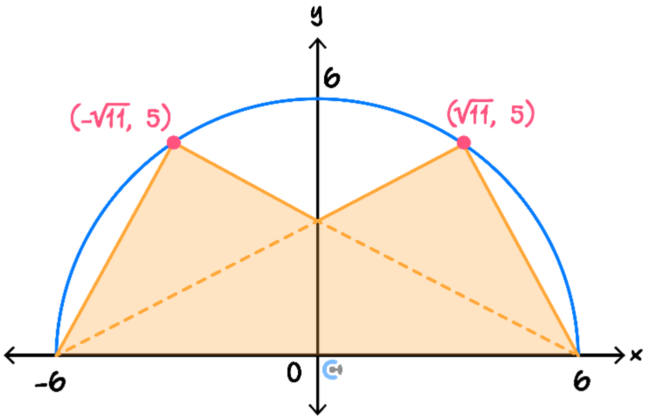
The graph of g is shown below.



c. State the value of r. (1 mark)

$\boxed{r = 6}$

d. Two lights have been placed on the arch to light the entrance of the tunnel. The positions of the lights are $(-\sqrt{11}, 5)$ and $(\sqrt{11}, 5)$. The area that is lit by these lights is shaded in the diagram below.



Determine the proportion of the cross-section of the tunnel entrance that is lit by the lights. Give your answer as a percentage, correct to the nearest integer. (3 marks)

y intercept of the light lines $= \dfrac{36 - 6\sqrt{11}}{5} = \dfrac{30}{6 + \sqrt{11}} \approx 3.22 \ldots$

Shaded area (using sum of 2 triangles subtract a third)

$$= 2\frac{12 \times 5}{2} - \frac{1}{2} \times 12 \times \frac{36 - 6\sqrt{11}}{5} = \frac{36\sqrt{11} + 84}{5} \approx 40.679698 \ldots$$

OR

Shaded area (using trapizum and triangle)

$$= 2\left(\frac{1}{2}\left(\frac{36 - 6\sqrt{11}}{5} + 5\right)\sqrt{11} + \frac{1}{2}(6 - \sqrt{11})5\right) = \frac{12(3\sqrt{11} + 7)}{5}$$

$\approx 40.679698 \ldots$

$$\% \text{ of area} = \frac{\frac{(36\sqrt{11} + 84)}{5}}{\frac{36\pi}{2}} \times 100 = \frac{2(3\sqrt{11} + 7)}{15\pi} \times 100 \approx 72\%$$

Space for Personal Notes

Question 308 (3 marks)

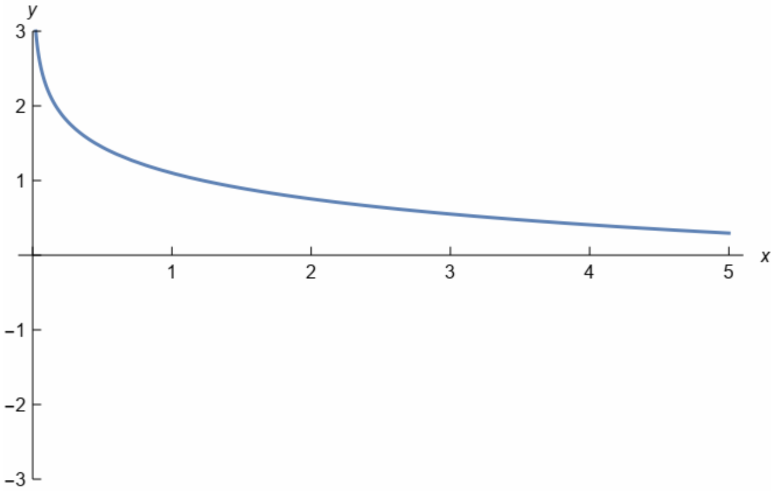
A well-designed computer screen display aims to make it quick and easy for a user to do tasks such as clicking on a screen button. Fitts' Law models the way in which the time taken to move to and click on a screen button depends on the distance the mouse is moved and the width of the screen button.

According to Fitts' Law, for a fixed distance travelled by the mouse, the time taken, in seconds, is given by $a - b\log_e(x)$, $0 \leq x \leq 5$, where $x\ cm$ is the button width and a and b are positive constants for a particular user.

a. Minnie discovers that, for her, $a = 1.1$ and $b = 0.5$.

i. Let $f(0, 5] \to R, f(x) = 1.1 - 0.5\log_e(x)$.

Sketch the graph of $y = f(x)$ on the axes below. Label any asymptote with its equation and any end-point with its exact coordinates. (3 marks)



1ai		Asymptote $x = 0$, endpoint $(5, 1.1 - \log_e(5))$
0/3	17	
1/3	27	
2/3	33	
3/3	23	
(Average mark 1.61)		

ii. Explain why f^{-1}, the inverse function of f, exists.

Function is 1:1

1aii		Function is one to one
0/1	19	
1/1	81	
(Average mark 0.81)		

iii. Find $f^{-1}(x)$, the rule for f^{-1}.

$e^{\left(\frac{1.1-x}{0.5}\right)}$

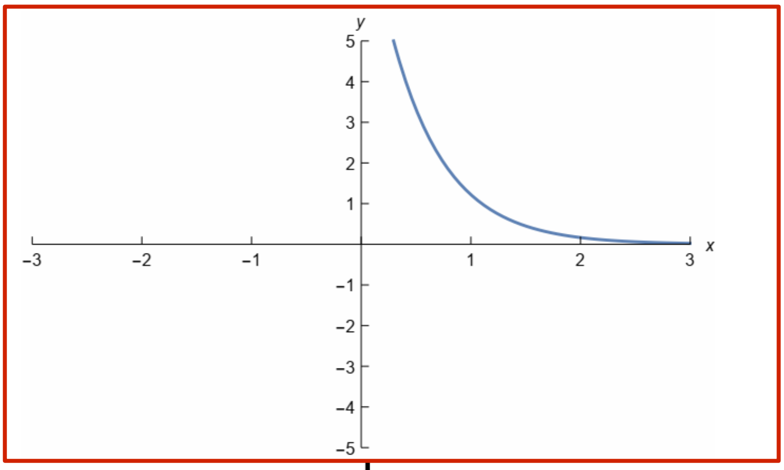
1aiii	
0/2	5
1/2	12
2/2	83
(Average mark 1.77)	

$[1.1-0.5\log_e(5),\infty)$

iv. State the domain of f^{-1}.

1aiv		$[1.1-\log_e(5),\infty)$
0/1	78	
1/1	22	
(Average mark 0.22)		

v. Sketch the graph of $y = f^{-1}(x)$ on the axes below. Label any asymptote with its equation and any end-point with its exact coordinates.



1av		Asymptote $y = 0$, endpoint $(1.1-\log_e(5),5)$
0/2	46	
1/2	36	
2/2	18	
(Average mark 0.71)		

b. Mickey decides to find the values of a and b for his use. He finds that when x is 1, his time is 0.5 seconds, and when x is 1.5, his time is 0.3 seconds.

Find the exact values of a and b for Mickey.

1b		
0/2	9	$a = 0.5,\ b = \dfrac{0.2}{\log_e(1.5)}$
1/2	24	
2/2	67	
(Average mark 1.57)		

c. Show that, when the button width is halved, the time taken by Minnie (for whom $a = 1.1$ and $b = 0.5$) is increased by $\log_2 \sqrt{2}$ seconds.

$$t_1 = 1.1 - 0.5\ \log_e(x)$$

$$t_2 = 1.1 - 0.5\ \log_e(0.5x)$$

$$t_2 - t_1 = -0.5\ \log_e(0.5x) + 0.5\log_e(x)$$

$$= 0.5\ \log_e\left(\frac{x}{0.5x}\right)$$

$$= 0.5\ \log_e(2) = \log_e(\sqrt{2})$$

Space for Personal Notes

Question 309 (12 marks)

Sally is using graph sketching software to design the landscape of the four hills shown in Figure 1 below.

She starts by using the square root functions h, h_1, and h_2 to model the shapes of three of the four hills, as shown in Figure 2 below.

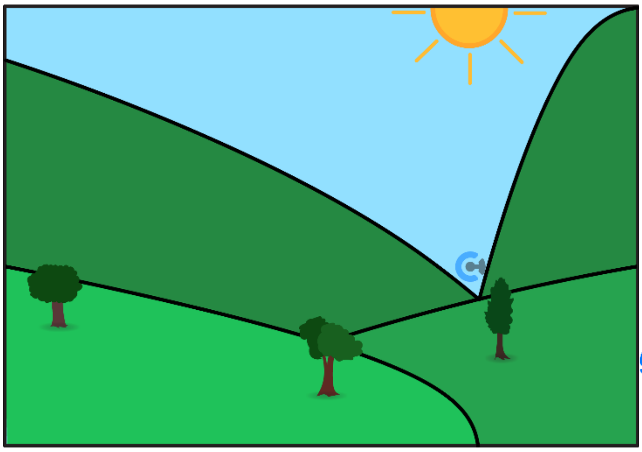


Figure 1

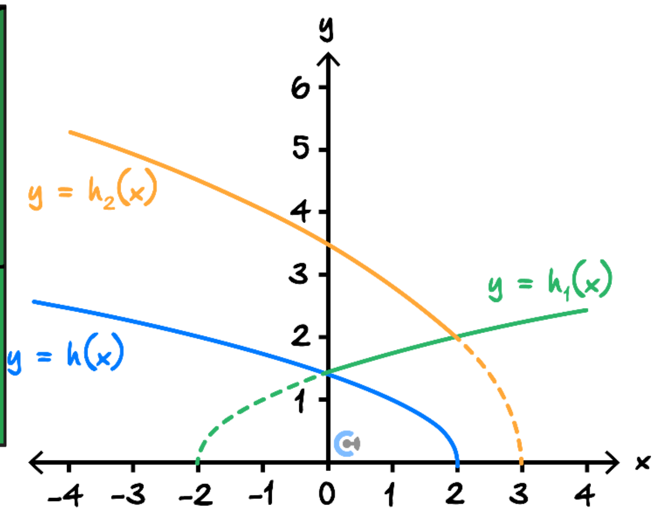


Figure 2

The rule for the function h is $h(x) = \sqrt{2 - x}$.

a.

i. State the maximal domain for h. (1 mark)

$\boxed{x \in (-\infty, 2]}$

ii. The rule for the function h_1 is obtained by reflecting the graph of h in the vertical axis.

State the rule for the function h_1. (1 mark)

$\boxed{h_1(x) = \sqrt{2 + x}}$

b. The rule for the function h_2 is $h_2(x) = 2\sqrt{3 - x}$.

i. Write a sequence of two transformations that map the graph of h onto the graph of h_2. (1 mark)

Dilation by a factor 2 from the x-axis, translation of 1 unit to the right, or

Dilation by a factor of $\frac{1}{4}$ from the y-axis, translation by $\frac{5}{2}$ units to the right.

ii. Let $T_1(x, y) = (ax + c, by + d)$ be a transformation that maps the graph of h onto the graph of h_2.

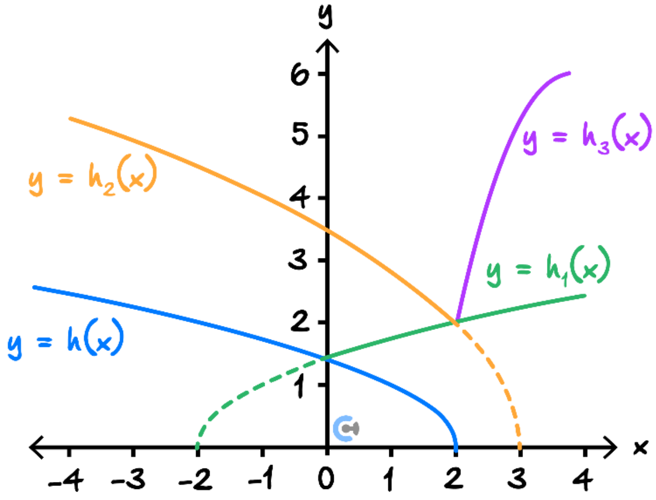
Find **one** set of possible values for a, b, c and d. (2 marks)

$a = 1,\ b = 2,\ c = 1,\ d = 0$ or $a = \frac{1}{4},\ b = 1,\ c = \frac{5}{2},\ d = 0$

iii. Find the value of x for which the slope of the hill defined by the function h is equal to the slope of the hill defined by the function h_2. (1 mark)

$x = \frac{5}{3}$

Sally decides to use a quadratic function, h_3, to model the shape of the fourth hill in her landscape.



c. Find the rule for h_3, a quadratic function with a stationary point at $(4, 6)$ and which passes through $(2, 2)$. (2 marks)

$$h_3(x) = a(x-4)^2 + 6,$$

$$h_3(x) = -(x-4)^2 + 6 \text{ or } h_3(x) = -x^2 + 8x - 10$$

Sally believes the function h_3 is closely related to the inverse of h.

d. Find the domain and the rule for the function h^{-1}, the inverse of $h(x) = \sqrt{2-x}$. (2 marks)

$$x \in [0, \infty),\ h^{-1}(x) = -x^2 + 2$$

e. Consider the transformation $T_2(x, y) = (y + 4, x + 4)$.

Does the transformation above map the function h onto the function h_3? Give a reason to justify your answer. (2 marks)

Method 1:

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ performs the inverse transformation to get $y = 2 - x^2$ for $x \geq 0$, adding $\begin{bmatrix} 4 \\ 4 \end{bmatrix}$ translates the graph to $y = -(x-4)^2 + 6$ for $x \geq 4$, although the rule is correct, because the maximal domain of the image is $x \geq 4$, the transformation cannot give h_3.

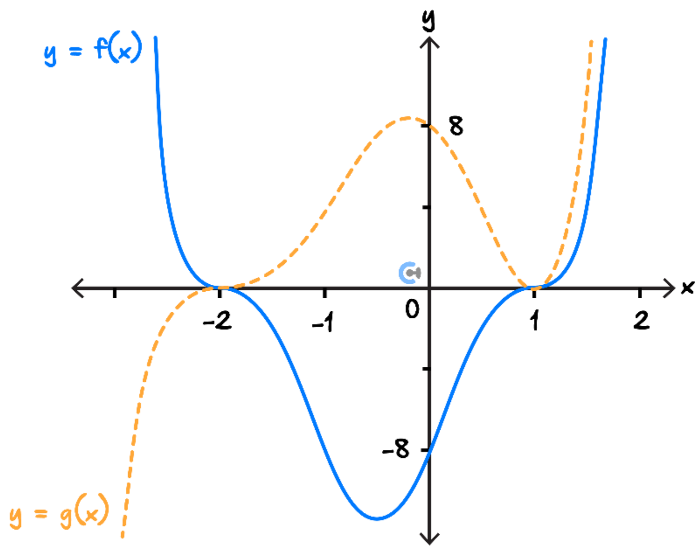
Method 2:

Let $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 4 \\ 4 \end{bmatrix}$, $x' = y + 4, y = x' - 4$, $y' = x + 4, x = y' - 4$, substitute into $y = \sqrt{2 - x}$,

$y = -(x-4)^2 + 6$ for $x \geq 4$, although the rule is correct, because the maximal domain of the image is $x \geq 4$, the transformation cannot give h_3.

Question 310 (9 marks)

Parts of the graphs of $f(x) = (x-1)^3(x+2)^3$ and $g(x) = (x-1)^2(x+2)^3$ are shown on the axes below.



The two graphs intersect at three points, $(-2, 0)$, $(1, 0)$ and (c, d). The point (c, d) is not shown in the diagram above.

a. Find the values of c and d. (2 marks)

$$\boxed{c = 2,\ d = 64}$$

b. Find the values of x such that $f(x) > g(x)$. (1 mark)

$$\boxed{(-\infty, -2) \cup (2, \infty)}$$

c. State the values of x for which:

i. $f'(x) > 0$. (1 mark)

$$\boxed{\left(-\frac{1}{2}, 1\right) \cup (1, \infty)}$$

ii. $g'(x) > 0$. (1 mark)

$$\boxed{(-\infty, -2) \cup \left(-2, -\frac{1}{5}\right) \cup (1, \infty)}$$

d. Show that $f(1+m) = f(-2-m)$ for all m. (1 mark)

$$\boxed{f(1+m) = m^3(m+3)^3,\ f(-2-m) = (-m-3)^3(-m)^3 = m^3(m+3)^3,\ \text{so } f(1+m) = f(-2-m)}$$

e. Find the values of h such that $g(x+h) = 0$ has exactly one negative solution. (2 marks)

$$\boxed{-2 < h \leq 1}$$

f. Find the values of k such that $f(x) + k = 0$ has no solutions. (1 mark)

$$\boxed{k > \frac{729}{64}}$$

Question 311 (8 marks)

Inspired from VCAA Mathematics Exam 2019
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2019/NHT/mm2nht_examrep19.pdf

Let $f: R \to R, f(x) = e^{\left(\frac{x}{2}\right)}$ and $g: R^+ \to R, g(x) = 2\log_e(x)$.

a. Find $g^{-1}(x)$. (1 mark)

$$\boxed{g^{-1}(x) = e^{\frac{x}{2}}}$$

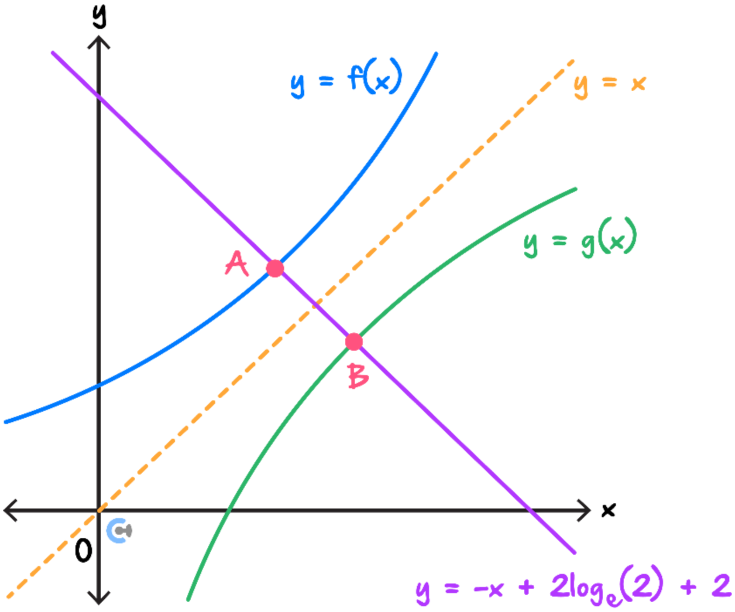
b. Find the coordinates of point A, where the tangent to the graph of f at A is parallel to the graph of $y = x$. (2 marks)

$$\boxed{f'(x) = 1, \ \left(2\log_e(2), \ 2\right)}$$

c. Show that the equation of the line that is perpendicular to the graph of $y = x$ and goes through point A is $y = -x + 2\log_e(2) + 2$. (1 mark)

$$\boxed{\left(2\log_e(2), \ 2\right), \ m = -1, \ y - 2 = -(x - 2\log_e(2)), \ y = -x + 2\log_e(2) + 2}$$

Let B be the point of intersection of the graphs of g and $y = -x + 2\log_e(2) + 2$, as shown in the diagram below.

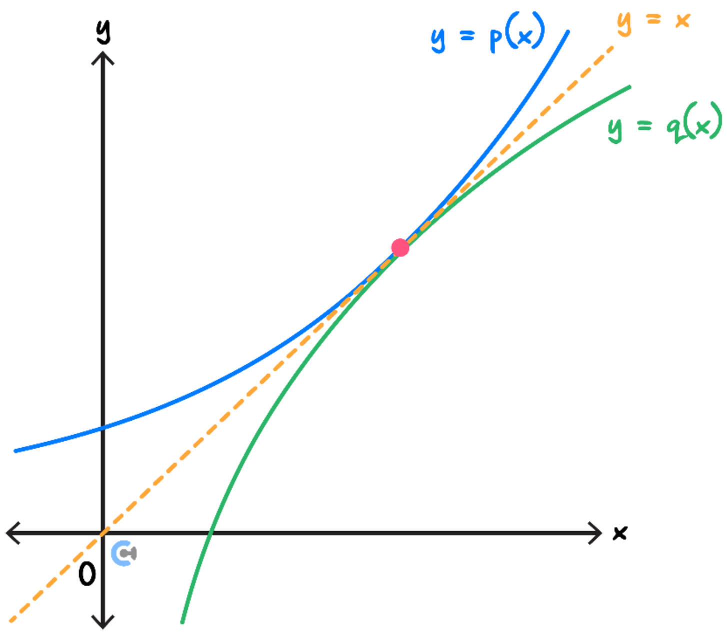


d. Determine the coordinates of point B. (1 mark)

$\boxed{(2,\ 2\log_e(2))}$

Let $p: R \to R, p(x) = e^{kx}$ and $q: R^+ \to R, q(x) = \frac{1}{k}\log_e(x)$.

e. The graphs of p, q and $y = x$ are shown in the diagram below. The graphs of p and q touch but do not cross.



Find the value of k. (2 marks)

$\boxed{p(x) = q(x) = x,\ p'(x) = q'(x) = 1,\ k = \frac{1}{e}}$

f. Find the value of k, $k > 0$, for which the tangent to the graph of p at its y-intercept and the tangent to the graph of q at its x-intercept are parallel. (1 mark)

$\boxed{k = 1}$

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Question 312 (16 marks)

Inspired from VCAA Mathematics Exam 2023
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2023/NHT/2023mathsmethods2-NHT-report.docx

Let $g: R \to R, g(x) = (x+2)^2 - 1$.

a. Express the rule for g in the form $g(x) = ax^2 + bx + c$, where $a, b, c \in R$. (1 mark)

$g(x) = x^2 + 4x + 3$

b. The function g can also be written in the form $g(x) = (x-p)(x-q)$, where $p, q \in Z$. Give the values of p and q. (1 mark)

$p = -1,\ q = -3$ or $p = -3,\ q = -1$

c. Find the value of k for which the graph of $y = g(x) + k$ passes through the origin. (2 marks)

Method 1:

Solving $g(0) + k = 0$ for k

$k = -3$

Method 2:

$g(x)$ has a y-intercept at (0, 3)

k is a vertical translation, so for $y = g(x) + k$ to pass through the origin $k = -3$

d. Using algebra, find the value(s) of d such that the graph of $y = g(x-d)$ will pass through the origin. (2 marks)

Method 1:

$g(x)$ has x-intercepts at $(-1, 0)$ and $(-3, 0)$

d is a horizontal translation, so for $y = g(x-d)$ to pass through the origin

$d = 1$ or $d = 3$

Method 2:

Solving $g(0-d) = 0$ for d gives

$d = 1$ or $d = 3$

e. Describe the transformation from the graph of $y = g(x)$ to the graph of $y = g(3x)$. (1 mark)

Dilation by a factor of $\frac{1}{3}$ from the y-axis (in the direction of the x-axis)

Let $h: R \to R, h(x) = mx + n$, where m and n are real numbers.

f. Find the value of m, such that the graph of the sum function $y = g(x) + h(x)$ has a turning point on the y-axis. (2 marks)

$y = g(x) + h(x) = x^2 + 4x + 3 + mx + n = x^2 + (4+m)x + 3 + n$

At $x = 0$, $\frac{dy}{dx} = 2x + 4 + m = 0$

$m = -4$

g. Find n in terms of m, such that the graph of the sum function $y = g(x) + h(x)$ has a turning point on the x-axis. (2 marks)

Method 1:

Using the discriminant condition $\Delta = 0$

$(m+4)^2 - 4(1)(n+3) = 0$

$n = \frac{m^2 + 8m + 4}{4}$

Method 2:

$x_{TP} = \frac{-4-m}{2}$, Solving $y(x_{TP}) = 0$

$n = \frac{m^2 + 8m + 4}{4}$

h. Find **two** pairs of values for m and n, such that the graph of the product function $y = g(x)h(x)$ has exactly two x-intercepts. (3 marks)

$y = g(x)h(x) = (x+3)(x+1)(mx+n)$

$x = -3$, $x = -1$, $x = -\frac{n}{m}$

Solving $-1 = -\frac{n}{m}$ or $-3 = -\frac{n}{m}$

$m = n$ or $n = 3m$

Two pairs (examples)

$m = 1$, $n = 1$

$m = 1$, $n = 3$

$m = 0$, $n \in R \backslash \{0\}$

i. Find the coordinates of the turning point of the graph of $y = g(h(x))$, giving your answer in terms of m and n. (2 marks)

$$\boxed{\begin{array}{l} g(h(x)) = (mx+n+2)^2 - 1 \\ \left(-\dfrac{n+2}{m}, -1\right) \end{array}}$$

Question 313 (9 marks)

Inspired from VCAA Mathematics Exam 2018
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2018/nht/mathsmethods2nht_examrep18.pdf

Let $f: R \to R, f(x) = x^4 - 4x - 8$.

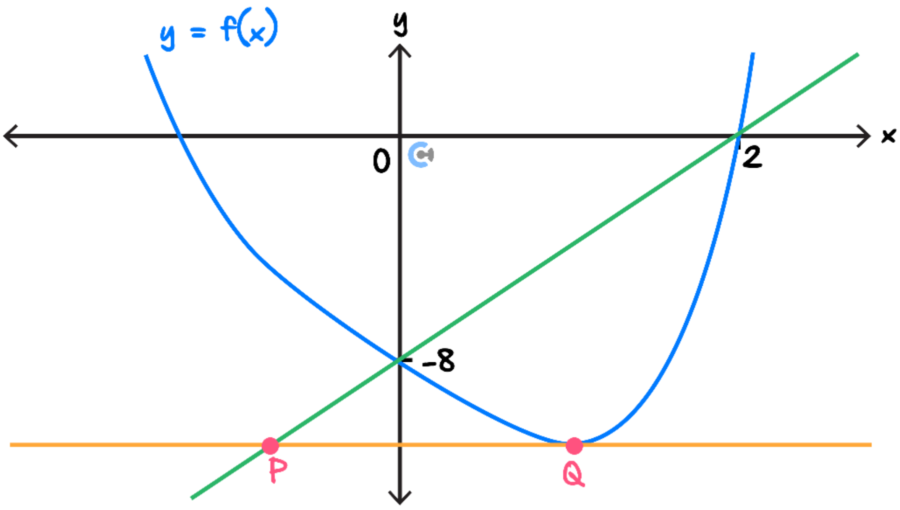
a. Given $f(x) = (x-2)(x^3 + ax^2 + bx + c)$, find a, b and c. (1 mark)

$$\boxed{a = 2,\ b = 4,\ c = 4}$$

b. Find two consecutive integers m and n such that a solution to $f(x) = 0$ is in the interval (m, n), where $m < n < 0$. (2 marks)

$$\boxed{x = -1.29...,\ m = -2,\ n = -1}$$

The diagram below shows part of the graph of f and a straight line drawn through the points $(0,-8)$ and $(2,0)$. A second straight line is drawn parallel to the horizontal axis and it touches the graph off at the point Q. The two straight lines intersect at the point P.



c.

i. Find the equation of the line through $(0,-8)$ and $(2,0)$. (1 mark)

$\boxed{y = 4x - 8}$

ii. State the equation of the line through the points P and Q. (1 mark)

$\boxed{y = -11}$

iii. State the coordinates of the points P and Q. (2 marks)

$\boxed{P\left(-\frac{3}{4}, -11\right),\ Q(1, -11)}$

d. A transformation $T: R^2 \rightarrow R^2, T(x, y) = (x + d, y)$ is applied to the graph of f.

i. Find the value of d for which P is the image of Q. (1 mark)

$$\boxed{d = -\frac{7}{4}}$$

ii. Let $(m', 0)$ and $(n', 0)$ be the images of $(m, 0)$ and $(1, 0)$ respectively, under the transformation T, where m and n are defined in **part b**.

Find the values of m' and n'. (1 mark)

$$\boxed{m' = -\frac{15}{4}, n' = -\frac{11}{4}}$$

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