CONTOUR EDUCATION

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VCE Mathematical Methods ¾
AOS 1 Revision [1.0]
Contour Check (Part 3)

Contour Checklist

Section A: [1.1 - 1.8] - Exam 1 Overall (VCAA Qs) (77 Marks)

Question 178 (7 marks)

a. Consider the function p, where $p: [1, \infty) \to R, p(x) = x^4 - x^3 - x^2 + x + 1$.

i. Find the value of a when $p^{-1}(a) = 2$, where $a \in R$. (2 marks)

ii. Find the value of b when $p(b) = 1$, where $b > 0$. (2 marks)

b. Find the rule and the domain of f^{-1}, the inverse of f, if $f: R\backslash\{2\} \to R, f(x) = \frac{x+3}{x-2}$. (3 marks)

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Question 179 (5 marks)

Let $h: \left[-\frac{3}{2}, \infty\right) \to R, h(x) = \sqrt{2x+3} - 2$.

a. Find the value(s) of x such that $[h(x)]^2 = 1$. (2 marks)

b. Find the domain and the rule of the inverse function h^{-1}. (3 marks)

Question 180 (4 marks)

Let $f(x) = -x^2 + x + 4$ and $g(x) = x^2 - 2$.

a. Find $g\big(f(3)\big)$. (2 marks)

b. Express the rule for $f(g(x))$ in the form $ax^4 + bx^2 + c$, where a, b, and c are non-zero integers. (2 marks)

Question 181 (3 marks)

Let $h: R^+ \cup \{0\} \to R, h(x) = \frac{7}{x+2} - 3$.

a. State the range of h.

b. Find the rule for h^{-1}.

Question 182 (8 marks)

The rule for a function f is given by $f(x) = \sqrt{2x+3} - 1$, where f is defined on its maximal domain.

a. Find the domain and rule of the inverse function f^{-1}. (2 marks)

b. Solve $f(x) = f^{-1}(x)$. (2 marks)

c. Let $g: D \to R, g(x) = \sqrt{2x + c} - 1$ where D is the maximal domain of g and c is a real number.

i. For what value(s) of c, does $g(x) = g^{-1}(x)$ have no real solutions? (2 marks)

ii. For what value(s) of c, does $g(x) = g^{-1}(x)$ have exactly one real solution? (2 marks)

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Question 183 (4 marks)

a. Let $f: R\backslash\left\{\frac{1}{3}\right\} \to R, f(x) = \frac{1}{3x-1}$.

Find the rule of f^{-1}.

b. State the domain of f^{-1}.

c. Let g be the function obtained by applying the transformation T to the function f, where:

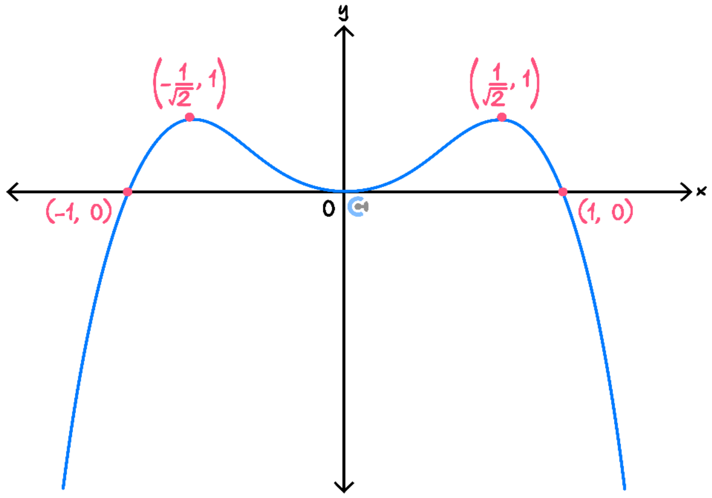
$$T(x, y) = (x + c, y + d)$$

And $c, d \in R$.

Find the values of c and d given that, $g = f^{-1}$. (1 mark)

Question 184 (4 marks)

The function $f: R \to R, f(x)$ is a polynomial function of degree 4. Part of the graph of f is shown below.
The graph of f touches the x-axis at the origin.



a. Find the rule of f. (1 mark)

Let g be a function with the same rule as f.

Let $h: D \to R, h(x) = \log_e(g(x)) - \log_e(x^3 + x^2)$, where D is the maximal domain of h.

b. State D. (1 mark)

c. State the range of h. (2 marks)

Question 185 (5 marks)

Let $f: [0, \infty) \to R, f(x) = \sqrt{x+1}$.

a. State the range of f.

b. Let $g: (-\infty, c] \to R, g(x) = x^2 + 4x + 3$, where $c < 0$.

i. Find the largest possible value of c such that, the range of g is a subset of the domain of f.

ii. For the value of c found in **part. b. i**, state the range of $f\big(g(x)\big)$.

c. Let $h: R \to R, h(x) = x^2 + 3$.

State the range of $f\big(h(x)\big)$.

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Question 186 (4 marks)

Let $f: [0, \infty) \to R, f(x) = \sqrt{x+1}$.

a. State the range of f. (1 mark)

b. Let $g: (-\infty, c] \to R, g(x) = x^2 + 4x + 3$, where $c < 0$.

i. Find the largest possible value of c such that, the range of g is a subset of the domain of f. (2 marks)

ii. For the value of c found in **part.b.i.**, state the range of $f(g(x))$. (1 mark)

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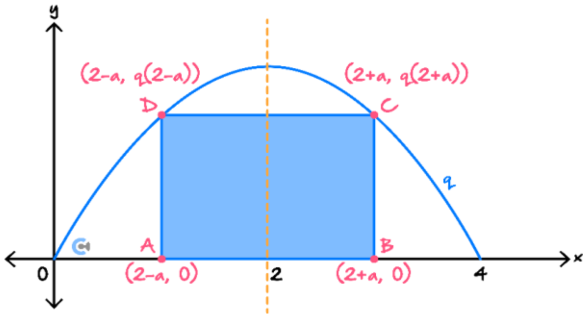
Question 187 (5 marks)

Let $q: [0,4] \to R, q(x) = x(4 - x)$.

A rectangle $ABCD$ is inscribed between the graph of the function q and the x-axis. Its vertices are a units, where $a > 0$, from the axis of symmetry, $x = 2$, as shown below.



a. Find the value of a when the rectangle is a square. Give your answer in the form $b + \sqrt{c}$, where b is an integer and c is a positive integer.

b. Find the maximum area of the rectangle $ABCD$. Give your answer in the form $\frac{m\sqrt{n}}{p}$, where m, n, and p are positive integers.

Question 188 (5 marks)

Let P be a point on the straight line $y = 2x - 4$ such that the length of OP, the line segment from the origin O to P, is a minimum.

a. Find the coordinates of P. (3 marks)

b. Find the distance OP. Express your answer in the form $\frac{a\sqrt{b}}{b}$, where a and b are positive integers. (2 marks)

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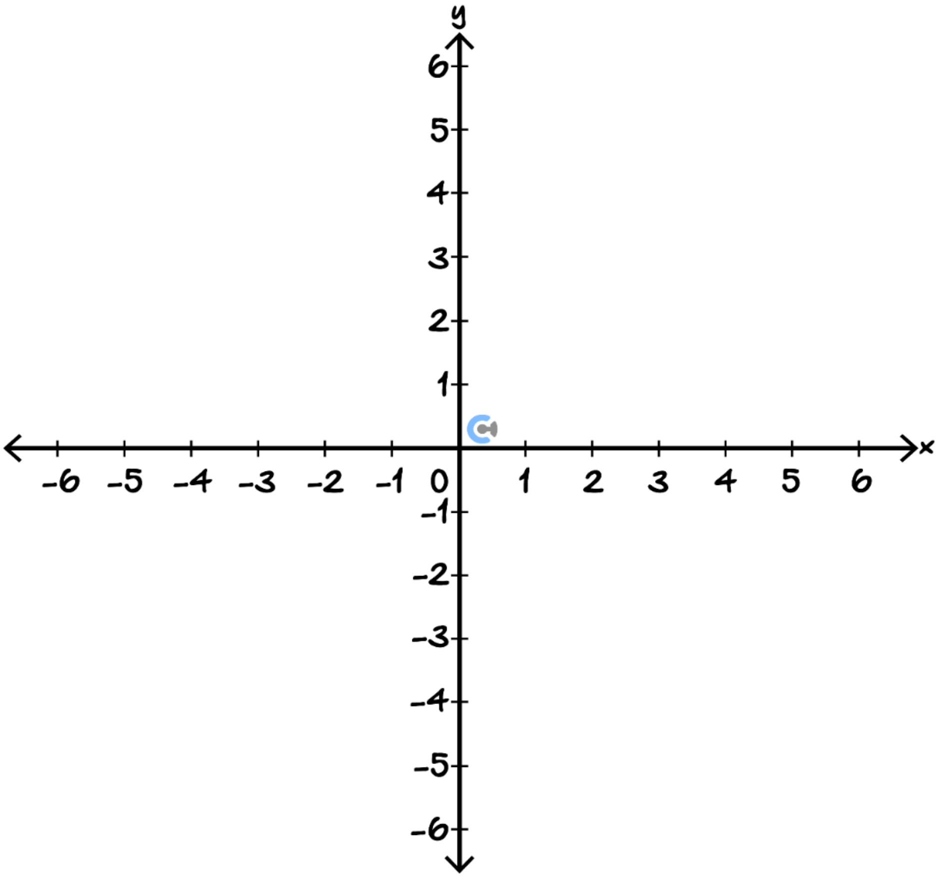
Question 189 (4 marks)

a. Sketch the graph of $y = 1 - \frac{2}{x-2}$ on the axes below. Label asymptotes with their equations and axis intercepts with their coordinates. (3 marks)



b. Find the values of x for which $1 - \frac{2}{x-2} \geq 3$. (1 mark)

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Question 190

Consider the simultaneous linear equations:

$$kx - 3y = k + 3$$

$$4x + (k + 7)y = 1$$

Where k is a real constant.

a. Find the value of k for which there are infinitely many solutions.

b. Find the values of k for which there is a unique solution.

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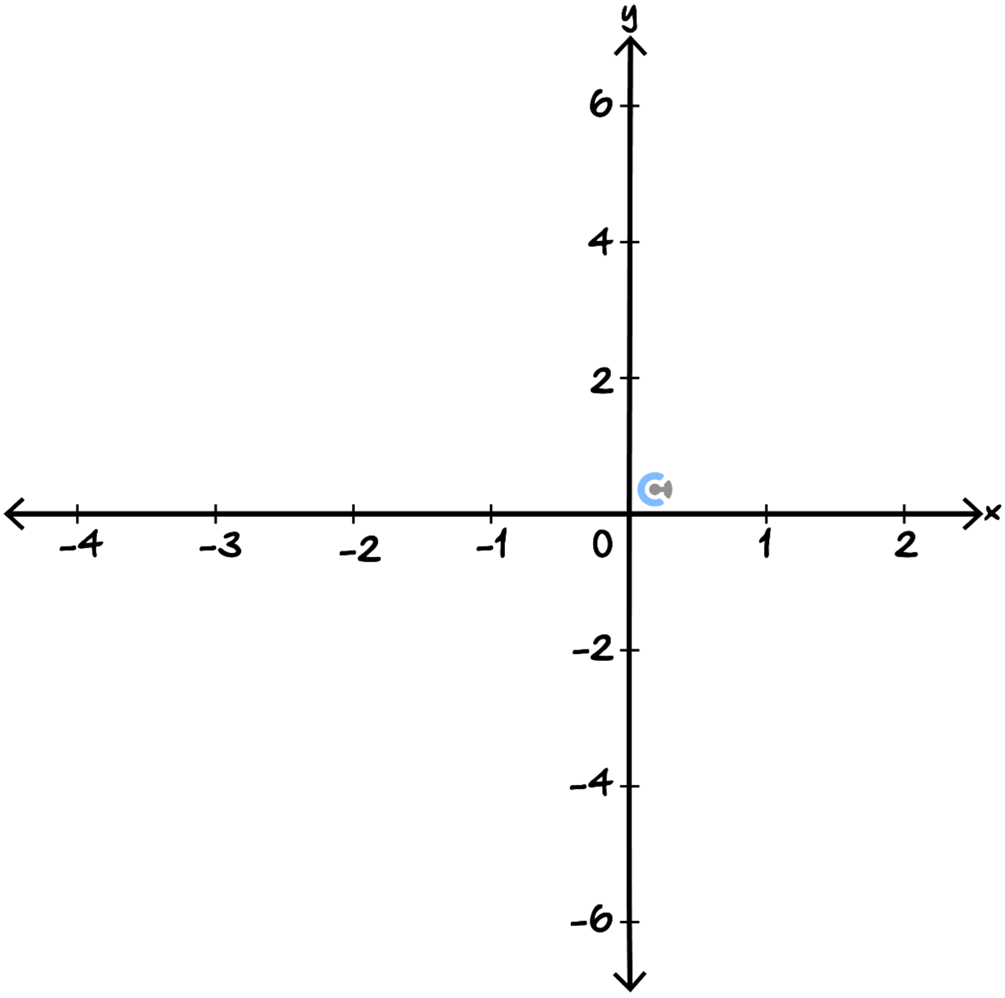
Question 191 (4 marks)

Let $f: [-3,0] \to R, f(x) = (x+2)^2(x-1)$.

a. Show that $(x+2)^2(x-1) = x^3 + 3x^2 - 4$.

b. Sketch the graph of f on the axes below. Label the axis intercept and any stationary points with their coordinates.



Question 192 (6 marks)

Let $f: R \to R$, where $f(x) = 2x^3 + 1$, and let $g: R \to R$, where $g(x) = 4 - 2x$.

a.

i. Find $g\big(f(x)\big)$. (1 mark)

ii. Find $f\big(g(x)\big)$ and express it in the form $k - m(x - d)^3$, where m, k, and d are integers. (2 marks)

b. The transformation $T: R^2 \to R^2$ with rule $T(x, y) = (x + b, ay + c)$, where a, b, and c are integers, maps the graph of $y = g\big(f(x)\big)$ onto the graph of $y = f\big(g(x)\big)$.

Find the values of a, b, and c. (3 marks)

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Question 193 (5 marks)

Let $f: R \to R, f(x) = 2e^x + 1$ and let $g: (-2, \infty) \to R, g(x) = \log_e(x + 2)$.

a.

i. Find $f\big(g(x)\big)$ in the form $ax + b$, where $a, b \in R$. (1 mark)

ii. State the range of $f\big(g(x)\big)$. (1 mark)

b. Let $T: R^2 \to R^2, T(x, y) = (x + c, y + d)$ and let the graph of the function h be the transformation of the graph of the function g under T.

If $h = f^{-1}$, then find the values of c and d. (3 marks)

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Question 194 (4 marks)

a. Let $f: R\backslash\left\{\frac{1}{3}\right\} \to R, f(x) = \frac{1}{3x-1}$.

Find the rule of f^{-1}. (2 marks)

b. State the domain of f^{-1}. (1 mark)

c. Let g be the function obtained by applying the transformation T to the function f, where:

$$T(x, y) = (x + c, y + d)$$

and $c, d \in R$.

Find the values of c and d given that $g = f^{-1}$. (1 mark)

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Section B: [1.1 - 1.8] - Exam 2 Overall (VCAA Qs) (179 Marks)

Question 195 (1 mark)

Let $h: (-1,1) \to R, h(x) = \frac{1}{x-1}$.

Which one of the following statements about h is **not** true?

A. $h(x)h(-x) = -h(x^2)$

B. $h(x) + h(-x) = 2h(x^2)$

C. $h(x) - h(0) = xh(x)$

D. $h(x) - h(-x) = 2xh(x^2)$

E. $\left(h(x)\right)^2 = h(x^2)$

Question 196 (1 mark)

The linear function $f: D \to R, f(x) = 5 - x$ has range $[-4,5)$.

The domain D is:

A. $(0,9]$

B. $(0,1]$

C. $[5,-4)$

D. $[-9,0)$

E. $[1,9)$

Space for Personal Notes

Question 197 (1 mark)

Let $f: R \to R, f(x) = 1 - 2\cos\left(\frac{\pi x}{2}\right)$.

The period and range of this function are respectively:

A. 4 and $[-2,2]$.

B. 4 and $[-1,3]$.

C. 1 and $[-1,3]$.

D. 4π and $[-1,3]$.

E. 4π and $[-2,2]$.

Question 198 (1 mark)

The function f has the property $f(x) - f(y) = (y - x)f(xy)$ for all non-zero real numbers x and y.

Which one of the following is a possible rule for the function?

A. $f(x) = x^2$

B. $f(x) = x^2 + x^4$

C. $f(x) = x\log_e(x)$

D. $f(x) = \frac{1}{x}$

E. $f(x) = \frac{1}{x^2}$

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Question 199 (1 mark)

The linear function $f: D \to R, f(x) = 4 - x$ has range $[-2, 6)$.

The domain D of the function is:

A. $[-2, 6)$

B. $(-2, 2]$

C. R

D. $(-2, 6]$

E. $[-6, 2]$

Question 200 (1 mark)

Which one of the following functions satisfies the functional equation $f\big(f(x)\big) = x$ for every real number x?

A. $f(x) = 2x$

B. $f(x) = x^2$

C. $f(x) = 2\sqrt{x}$

D. $f(x) = x - 2$

E. $f(x) = 2 - x$

Question 201 (1 mark)

If $f: (-\infty, 1) \to R, f(x) = 2\log_e(1 - x)$ and $g: [-1, \infty) \to R, g(x) = 3\sqrt{x + 1}$, then the maximal domain of the function $f + g$ is:

A. $[-1, 1)$

B. $(1, \infty)$

C. $(-1, 1]$

D. $(-\infty, -1]$

E. R

Question 202 (1 mark)

If the equation $f(2x) - 2f(x) = 0$ is true for all real values of x, then the rule for f could be:

A. $\frac{x^2}{2}$

B. $\sqrt{2x}$

C. $2x$

D. $\log_e\left(\frac{|x|}{2}\right)$

E. $x - 2$

Question 203 (1 mark)

The range of the function $f: [-2,3) \to R, f(x) = x^2 - 2x - 8$ is:

A. R

B. $(-9, -5]$

C. $(-5,0)$

D. $[-9,0]$

E. $[-9, -5)$

Question 204 (1 mark)

Let $f: R \to R, f(x) = x^2$.

Which one of the following is **not** true?

A. $f(xy) = f(x)f(y)$

B. $f(xy) - f(-x) = 0$

C. $f(2x) = 4f(x)$

D. $f(x - y) = f(x) - f(y)$

E. $f(x + y) + f(x - y) = 2\big(f(x) + f(y)\big)$

Question 205 (1 mark)

The linear function $f: D \to R, f(x) = 6 - 2x$ has range $[-4,12]$.

The domain D is:

A. $[-3,5]$

B. $[-5,3]$

C. R

D. $[-14,18]$

E. $[-18,14]$

Question 206 (1 mark)

The range of the function $f: [-2,7) \to R, f(x) = 5 - x$ is:

A. $(-2,7]$

B. $[-2,7)$

C. $(-2,\infty)$

D. $(-2,7)$

E. R

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Question 207 (1 mark)

The function f satisfies the functional equation $f\left(\frac{x+y}{2}\right) = \frac{f(x)+f(y)}{2}$ where x and y are any non-zero real numbers.

A possible rule for the function is:

A. $f(x) = \log_e|x|$

B. $f(x) = \frac{1}{x}$

C. $f(x) = 2^x$

D. $f(x) = 2x$

E. $f(x) = \sin(2x)$

Question 208 (1 mark)

If $3f(x) = f(3x)$ for $x > 0$, then the rule for f could be:

A. $f(x) = 3x$

B. $f(x) = \sqrt{3x}$

C. $f(x) = \frac{x^3}{3}$

D. $f(x) = \log_e\left(\frac{x}{3}\right)$

E. $f(x) = x - 3$

Space for Personal Notes

Question 209 (1 mark)

The range of the function $f: \left(\frac{-1}{\sqrt{2}}, \sqrt{2}\right] \to R, f(x) = 2x^3 - 3x + 4$ is:

A. $\left(4 - \sqrt{2}, 4 + \sqrt{2}\right)$

B. $\left(\frac{-1}{\sqrt{2}}, \sqrt{2}\right)$

C. $\left(4 - \sqrt{2}, 4 + \sqrt{2}\right]$

D. $\left(\frac{-2}{\sqrt{2}, \sqrt{2}}\right]$

E. $\left[4 - \sqrt{2}, 4 + \sqrt{2}\right]$

Question 210 (1 mark)

A function f satisfies the relation $f(x^2) = f(x) + f(x + 2)$.

A possible rule for f is:

A. $f(x) = \sqrt{x + 2}$

B. $f(x) = x + 2$

C. $f(x) = \log_{10}(x - 1)$

D. $f(x) = \frac{1}{2}(x^2 - 1)$

E. $f(x) = \frac{1}{x-1}$

Space for Personal Notes

Question 211 (1 mark)

The function f and its inverse, f^{-1}, are one-to-one for all values of x.

If $f(1) = 5, f(3) = 7$, and $f(8) = 10$, then $f^{-1}(7)$ and $f^{-1}(5)$ respectively are equal to:

A. 5 and 7.

B. 3 and 1.

C. 7 and 5.

D. 8 and 5.

E. 5 and 8.

Question 212 (1 mark)

The function f with rule $f(x) = 2\log_e(16 - x)$ has a maximal domain given by:

A. $x \in (16, \infty)$

B. $x \in (-\infty, 4)$

C. $x \in (4, \infty)$

D. $x \in (-4,4)$

E. $x \in (-\infty, 16)$

Question 213 (1 mark)

The range of the function with rule $y = \sqrt{4 - x^2} + \log_e(x + 2)$ is contained within the interval:

A. $[-4,2.8]$

B. $(-\infty, 2.8]$

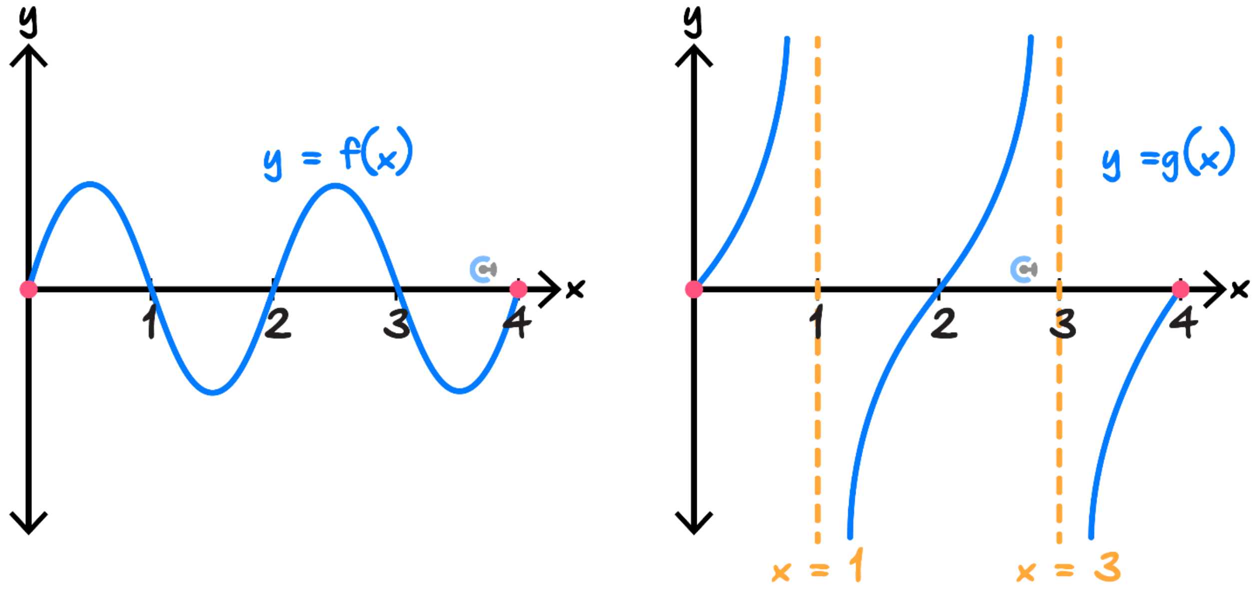
C. $(-4,2.9)$

D. $(-\infty, 2.9)$

E. $[-4,2.9)$

Question 214 (1 mark)

Consider the graphs of two circular functions, f and g, shown on the axes below.



On the interval $x \in [0, 4]$, the number of x-intercepts for the graph of the product function $h = f \times g$ is:

A. 3

B. 4

C. 5

D. 6

E. 7

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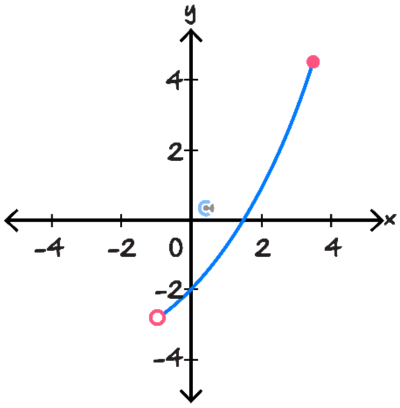
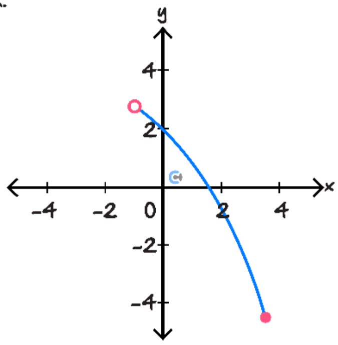
Question 215 (1 mark)

The graph of $y = f(x)$ is shown below.



The corresponding graph of the inverse of f, $y = f^{-1}(x)$, is best represented by:

A.



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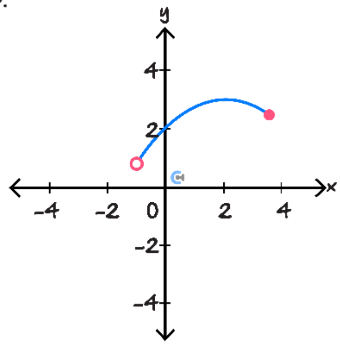
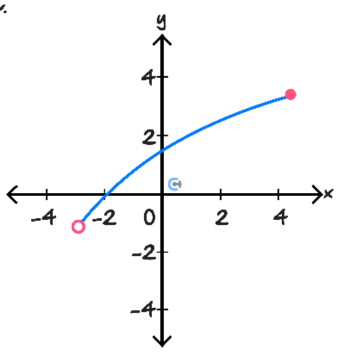
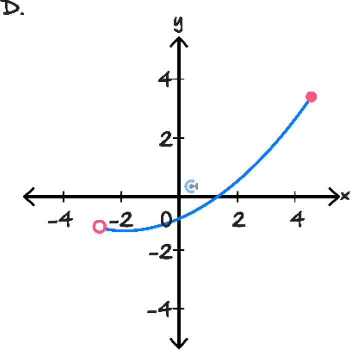


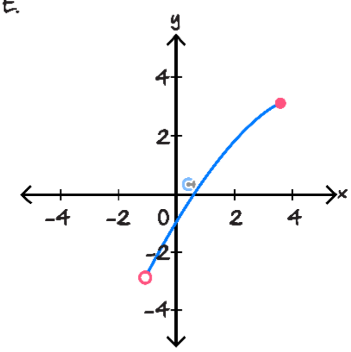
C.



D.



E.



Question 216 (1 mark)

If $3f(x) = f(3x)$ for $x > 0$, then the rule for f could be:

A. $f(x) = 3x$

B. $f(x) = \sqrt{3x}$

C. $f(x) = \frac{x^3}{3}$

D. $f(x) = \log_e\left(\frac{x}{3}\right)$

E. $f(x) = x - 3$

Question 217 (1 mark)

The function $f: D \to R, f(x) = \frac{x^4}{4} - \frac{x^3}{3} - \frac{9x^2}{2} + 9x$ will have an inverse function for:

A. $D = R$

B. $D = (-3,1)$

C. $D = (1, \infty)$

D. $D = (-\infty, 0)$

E. $D = (0, \infty)$

Question 218 (1 mark)

A cubic polynomial function $f: R \to R$ has roots at $x = 1$ and $x = 3$ only and its graph has a y-intercept at $y = 3$. Which one of the following statements **must** be true about the function g, where $g(x) = \sqrt{f(x)}$?

A. The function g has a local maximum at $x = 2$.

B. $g(2) = 1$.

C. The domain of g does not include the interval $(1, 3)$.

D. The domain of g includes the interval $(1, 3)$.

E. The domain of g does not include the interval $(3, \infty)$.

Question 219 (1 mark)

The graph of the function $f: D \to R, f(x) = \frac{2x-3}{4+x}$, where D is the maximal domain, has asymptotes:

A. $x = -4, y = 2$

B. $x = \frac{3}{2}, y = -4$

C. $x = -4, y = \frac{3}{2}$

D. $x = \frac{3}{2}, y = 2$

E. $x = 2, y = 1$

Question 220 (1 mark)

The function $f: D \to R, f(x) = 5x^3 + 10x^2 + 1$ will have an inverse function for:

A. $D = R$

B. $D = (-2, \infty)$

C. $D = \left(-\infty, \frac{1}{2}\right]$

D. $D = (-\infty, -1]$

E. $D = [0, \infty)$

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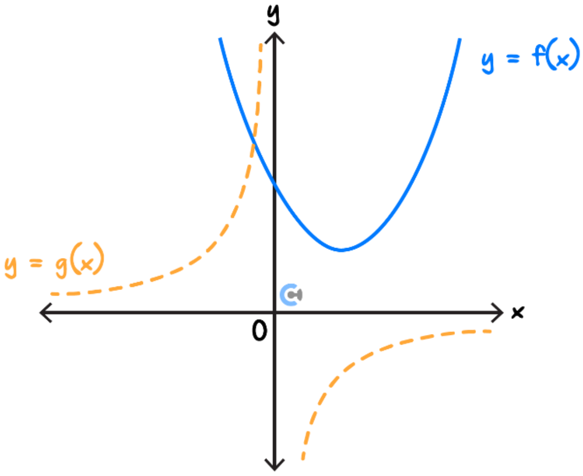
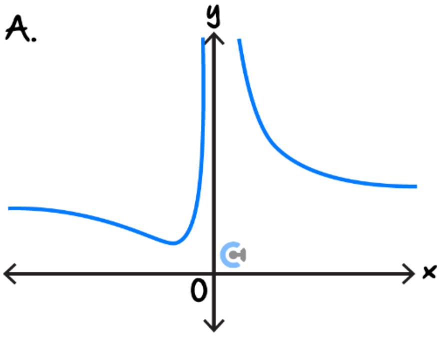
Question 221 (1 mark)

Part of the graphs of $y = f(x)$ and $y = g(x)$ are shown below.



A.



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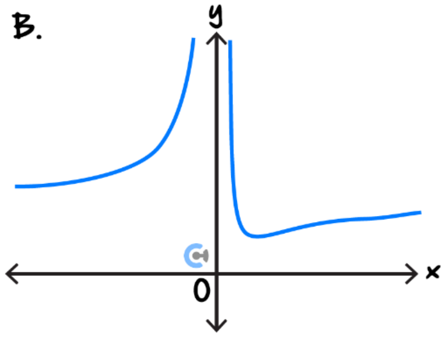

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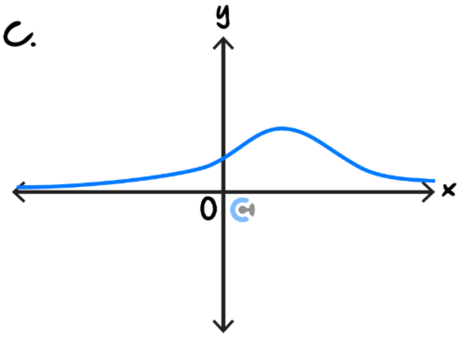

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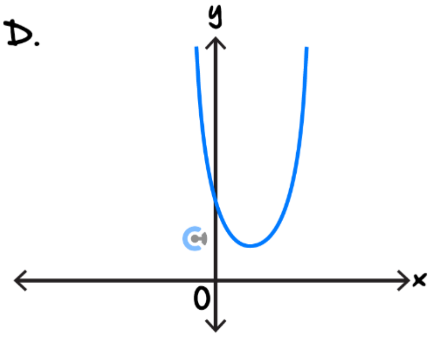
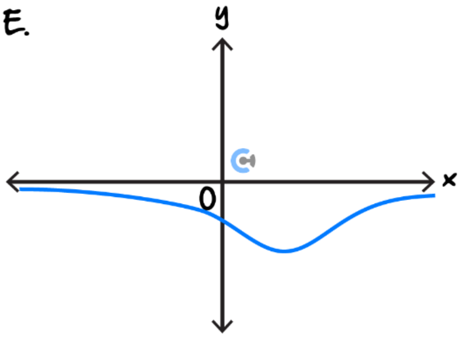


E.



Question 222 (1 mark)

The range of the function $f: \left(\frac{-1}{\sqrt{2}}, \sqrt{2}\right] \to R, f(x) = 2x^3 - 3x + 4$ is:

A. $\left(4 - \sqrt{2}, 4 + \sqrt{2}\right)$

B. $\left(\frac{-1}{\sqrt{2}}, \sqrt{2}\right)$

C. $\left(4 - \sqrt{2}, 4 + \sqrt{2}\right]$

D. $\left(\frac{-1}{\sqrt{2}}, \sqrt{2}\right]$

E. $\left[4 - \sqrt{2}, 4 + \sqrt{2}\right]$

Question 223 (1 mark)

The function f has the property $f(2x) = \left(f(x)\right)^2 - 2$ for all real numbers x.

A possible rule for the function $f(x)$ is:

A. $\frac{1}{x^2+4}$

B. $\cos(x)$

C. $2\log_e(x^2 + 1)$

D. $e^x + e^{-x}$

E. x^2

Space for Personal Notes

Question 224 (1 mark)

Which one of the following is the inverse function of the function $f: (-\infty, 3) \to R, f(x) = \frac{2}{\sqrt{3-x}} + 1$?

A. $f^{-1}: (-\infty, 3) \to R, f^{-1}(x) = -\frac{4}{(x-1)^2} + 3$

B. $f^{-1}: (1, \infty) \to R, f^{-1}(x) = -\frac{4}{(x-3)^2} + 1$

C. $f^{-1}: (1, \infty) \to R, f^{-1}(x) = -\frac{4}{(x-1)^2} + 3$

D. $f^{-1}: (1, \infty) \to R, f^{-1}(x) = -\frac{4}{x^2} + 3$

E. $f^{-1}: R^+ \to R, f^{-1}(x) = -\frac{4}{(x-1)^2} + 3$

Question 225 (1 mark)

Let $f: D \to R, f(x) = \frac{3x-5}{2-x}$, where D is the maximal domain of f.

Which of the following are the equations of the asymptotes of the graph of f?

A. $x = 2$ and $y = \frac{5}{3}$.

B. $x = 2$ and $y = -3$.

C. $x = -2$ and $y = 3$.

D. $x = -3$ and $y = 2$.

E. $x = 2$ and $y = 3$.

Space for Personal Notes

Question 226 (1 mark)

A function f satisfies the relation $f(x^2) = f(x) + f(x+2)$.

A possible rule for f is:

A. $f(x) = \sqrt{x+2}$

B. $f(x) = x + 2$

C. $f(x) = \log_{10}(x-1)$

D. $f(x) = \frac{1}{2}(x^2 - 1)$

E. $f(x) = \frac{1}{x-1}$

Question 227 (1 mark)

Consider the function $f: [2, \infty) \to R, f(x) = x^4 + 2(a-4)x^2 - 8ax + 1$, where $a \in R$.

The maximal set of values of a for which the inverse function f^{-1} exists is:

A. $(-9, \infty)$

B. $(-\infty, 1)$

C. $[-9, 1]$

D. $[-8, \infty)$

E. $(-\infty, -8]$

Space for Personal Notes

Question 228 (1 mark)

The range of the function with rule $y = \sqrt{4 - x^2} + \log_e(x + 2)$ is contained within the interval:

A. $[-4, 2.8]$

B. $(-\infty, 2.8]$

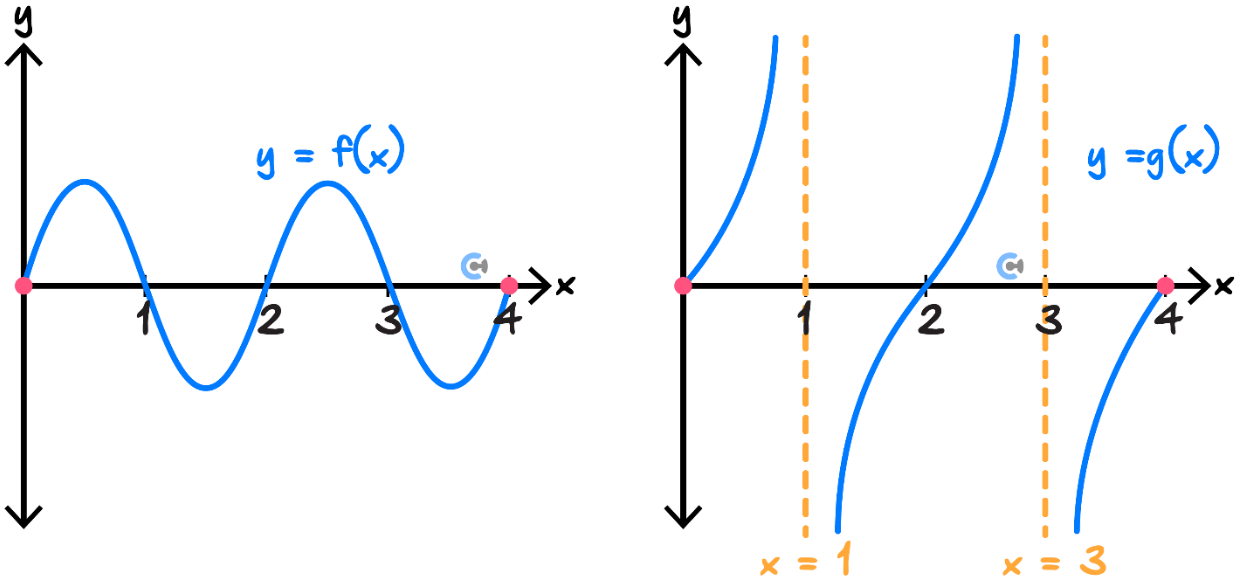
C. $(-4, 2.9)$

D. $(-\infty, 2.9)$

E. $[-4, 2.9)$

Question 229 (1 mark)

Consider the graphs of two circular functions, f and g, shown on the axes below.



On the interval $x \in [0, 4]$, the number of x-intercepts for the graph of the product function $h = f \times g$ is:

A. 3

B. 4

C. 5

D. 6

E. 7

Question 230 (1 mark)

If $3f(x) = f(3x)$ for $x > 0$, then the rule for f could be:

A. $f(x) = 3x$

B. $f(x) = \sqrt{3x}$

C. $f(x) = \frac{x^3}{3}$

D. $f(x) = \log_e\left(\frac{x}{3}\right)$

E. $f(x) = x - 3$

Question 231 (1 mark)

The range of the function $f: \left(\frac{-1}{\sqrt{2}}, \sqrt{2}\right] \to R, f(x) = 2x^3 - 3x + 4$ is:

A. $(4 - \sqrt{2}, 4 + \sqrt{2})$

B. $\left(\frac{-1}{\sqrt{2}}, \sqrt{2}\right)$

C. $(4 - \sqrt{2}, 4 + \sqrt{2}]$

D. $\left(\frac{-1}{\sqrt{2}}, \sqrt{2}\right]$

E. $[4 - \sqrt{2}, 4 + \sqrt{2}]$

Space for Personal Notes

Question 232 (1 mark)

A function f satisfies the relation $f(x^2) = f(x) + f(x + 2)$.

A possible rule for f is:

A. $f(x) = \sqrt{x+2}$

B. $f(x) = x + 2$

C. $f(x) = \log_{10}(x - 1)$

D. $f(x) = \frac{1}{2}(x^2 - 1)$

E. $f(x) = \frac{1}{x-1}$

Question 233 (1 mark)

Consider the following four functional relations:

$$f(x) = f(-x) \qquad -f(x) = f(-x) \qquad f(x) = -f(x) \qquad \left(f(x)\right)^2 = f(x^2)$$

The number of these functional relations that are satisfied by the function $f: R \to R, f(x) = x$ is:

A. 0

B. 1

C. 2

D. 3

E. 4

Space for Personal Notes

Question 234 (1 mark)

Let $g(x) = x + 2$ and $f(x) = x^2 - 4$.

If h is the composite function given by $h: [-5, -1) \to R, h(x) = f\big(g(x)\big)$, then the range of h is:

A. $(-3,5]$

B. $[-3,5)$

C. $(-3,5)$

D. $(-4,5]$

E. $[-4,5]$

Question 235 (1 mark)

Consider the functions $f(x) = \sqrt{x+2}$ and $g(x) = \sqrt{1-2x}$, defined over their maximal domains.

The maximal domain of the function $h = f + g$ is:

A. $\left(-2, \frac{1}{2}\right)$

B. $[-2, \infty)$

C. $(-\infty, -2) \cup \left(\frac{1}{2}, \infty\right)$

D. $\left[-2, \frac{1}{2}\right]$

E. $[-2,1]$

Space for Personal Notes

Question 236 (1 mark)

Let f and g be functions such that $f(-1) = 4, f(2) = 5, g(-1) = 2$, $g(2) = 7$, and $g(4) = 6$.

The value of $g\big(f(-1)\big)$ is:

A. 2

B. 4

C. 5

D. 6

E. 7

Question 237 (1 mark)

The graph of the function $f: D \to R, f(x) = \frac{3x+2}{5-x}$, where D is the maximal domain has asymptotes:

A. $x = -5, y = \frac{3}{2}$

B. $x = -3, y = 5$

C. $x = \frac{2}{3}, y = -3$

D. $x = 5, y = 3$

E. $x = 5, y = -3$

Space for Personal Notes

Question 238 (1 mark)

Let $a \in (0, \infty)$ and $b \in R$.

Consider the function $h: [-a, 0) \cup (0, a] \to R, h(x) = \frac{a}{x} + b$.

The range of h is:

A. $[b - a, b + 1]$

B. $(b - a, b + 1)$

C. $(-\infty, b - 1) \cup (b + 1, \infty)$

D. $(-\infty, b - 1] \cup [b + 1, \infty)$

E. $[b - 1, \infty)$

Question 239 (1 mark)

Let $f: R \to R, f(x) = \cos(ax)$, where $a \in R \backslash \{0\}$, be a function with the property:

$$f(x) = f(x + h), \text{ for all } h \in Z.$$

Let $g: D \to R, g(x) = \log_2(f(x))$ be a function where the range of g is $[-1,0]$.

A possible interval for D is:

A. $\left[\frac{1}{4}, \frac{5}{12}\right]$

B. $\left[1, \frac{7}{6}\right]$

C. $\left[\frac{5}{3}, 2\right]$

D. $\left[-\frac{1}{3}, 0\right]$

E. $\left[-\frac{1}{12}, \frac{1}{4}\right]$

Space for Personal Notes

Question 240 (1 mark)

The graph of the function f passes through the point $(-2, 7)$.

If $h(x) = f\left(\frac{x}{2}\right) + 5$, then the graph of the function h must pass through the point:

A. $(-1, -12)$

B. $(-1,19)$

C. $(-4,12)$

D. $(-4, -14)$

E. $(3,3.5)$

Question 241 (1 mark)

The maximal domain of the function f is $R\backslash\{1\}$.

A possible rule for f is:

A. $f(x) = \frac{x^2-5}{x-1}$

B. $f(x) = \frac{x+4}{x-5}$

C. $f(x) = \frac{x^2+x+4}{x^2+1}$

D. $f(x) = \frac{5-x^2}{1+x}$

E. $f(x) = \sqrt{x-1}$

Space for Personal Notes

Question 242 (1 mark)

Consider the function $f : [a, b) \to R, f(x) = \frac{1}{x}$ where a and b are positive real numbers.

The range of f is:

A. $\left[\frac{1}{a}, \frac{1}{b}\right)$

B. $\left(\frac{1}{a}, \frac{1}{b}\right]$

C. $\left[\frac{1}{b}, \frac{1}{a}\right)$

D. $\left(\frac{1}{b}, \frac{1}{a}\right]$

E. $[a, b)$

Question 243 (1 mark)

Let f and g be two functions such that, $f(x) = 2x$ and $g(x + 2) = 3x + 1$.

The function $f\big(g(x)\big)$ is:

A. $6x - 5$

B. $6x + 1$

C. $6x^2 + 1$

D. $6x - 10$

E. $6x + 2$

Space for Personal Notes

Question 244 (1 mark)

The function f has the property $f(x + f(x)) = f(2x)$ for all non-zero real numbers x.

Which one of the following is a possible rule for the function?

A. $f(x) = 1 - x$

B. $f(x) = x - 1$

C. $f(x) = x$

D. $f(x) = \frac{x}{2}$

E. $f(x) = \frac{1-x}{2}$

Question 245 (1 mark)

Let f and g be functions such that, $f(2) = 5, f(3) = 4, g(2) = 5, g(3) = 2$, and $g(4) = 1$.

The value of $f(g(3))$ is:

A. 1

B. 2

C. 3

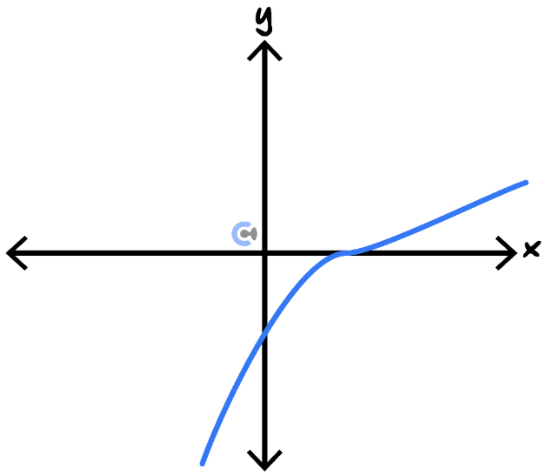
D. 4

E. 5

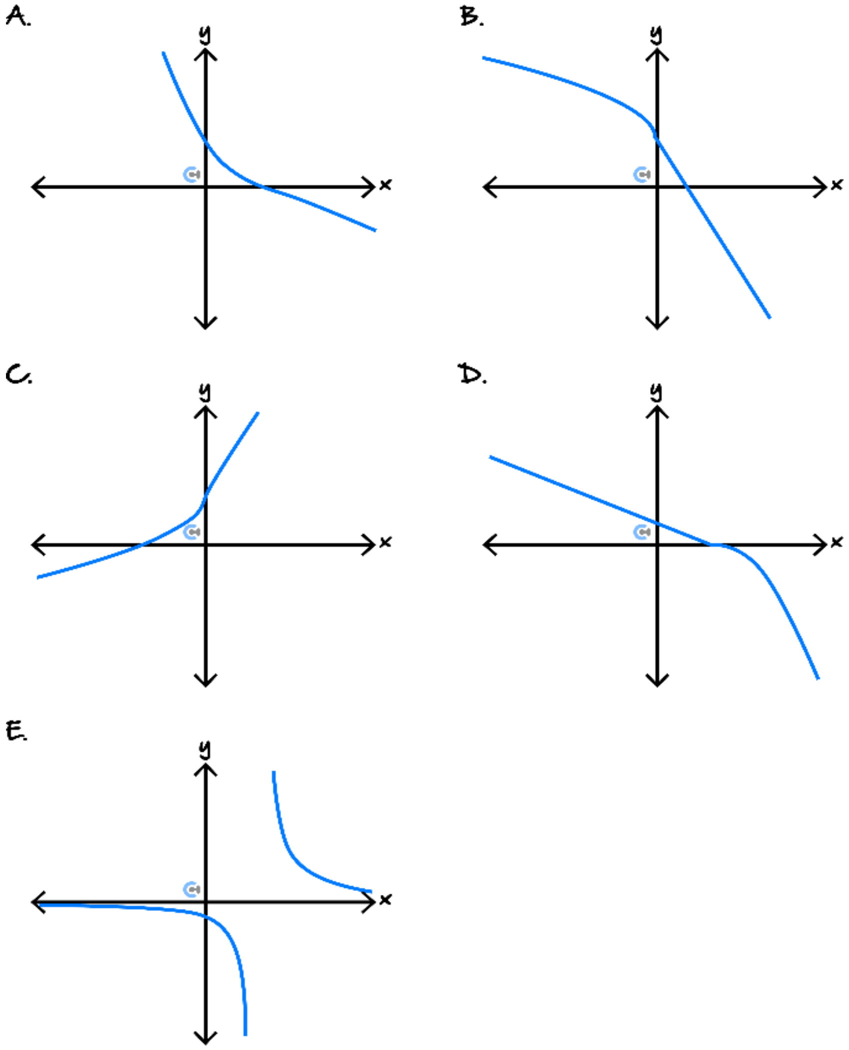
Space for Personal Notes

Question 246 (1 mark)

Part of the graph of the function f is shown below. The same scale has been used on both axes.



The corresponding part of the graph of the inverse function f^{-1} is best represented by:



Question 247 (1 mark)

Let $h: (-1,1) \to R, h(x) = \frac{1}{x-1}$.

Which one of the following statements about h is **not** true?

A. $h(x)h(-x) = -h(x^2)$

B. $h(x) + h(-x) = 2h(x^2)$

C. $h(x) - h(0) = xh(x)$

D. $h(x) - h(-x) = 2xh(x^2)$

E. $\left(h(x)\right)^2 = h(x^2)$

Question 248 (1 mark)

The linear function $f: D \to R, f(x) = 5 - x$ has range $[-4,5)$.

The domain D is:

A. $(0,9]$

B. $(0,1]$

C. $[5,-4)$

D. $[-9,0)$

E. $[1,9)$

Space for Personal Notes

Question 249 (1 mark)

Which one of the following is the inverse function of $g: [3, \infty) \to R, g(x) = \sqrt{2x - 6}$?

A. $g^{-1}: [3, \infty) \to R, g^{-1}(x) = \frac{x^2+6}{2}$

B. $g^{-1}: [0, \infty) \to R, g^{-1}(x) = (2x - 6)^2$

C. $g^{-1}: [0, \infty) \to R, g^{-1}(x) = \sqrt{\frac{x}{2}} + 6$

D. $g^{-1}: [0, \infty) \to R, g^{-1}(x) = \frac{x^2+6}{2}$

E. $g^{-1}: R \to R, g^{-1}(x) = \frac{x^2+6}{2}$

Question 250 (1 mark)

The function f has the property $f(x) - f(y) = (y - x)f(xy)$ for all non-zero real numbers x and y.

Which one of the following is a possible rule for the function?

A. $f(x) = x^2$

B. $f(x) = x^2 + x^4$

C. $f(x) = x\log_e(x)$

D. $f(x) = \frac{1}{x}$

E. $f(x) = \frac{1}{x^2}$

Space for Personal Notes

Question 251 (1 mark)

The function f has the property $f\big(x + f(x)\big) = f(2x)$ for all non-zero real numbers x.

Which one of the following is a possible rule for the function?

A. $f(x) = 1 - x$

B. $f(x) = x - 1$

C. $f(x) = x$

D. $f(x) = \frac{x}{2}$

E. $f(x) = \frac{1-x}{2}$

Question 252 (1 mark)

Let $h: (-1,1) \to R$, $h(x) = \frac{1}{x-1}$.

Which one of the following statements about h is **not** true?

A. $h(x)h(-x) = -h(x^2)$

B. $h(x) + h(-x) = 2h(x^2)$

C. $h(x) - h(0) = xh(x)$

D. $h(x) - h(-x) = 2xh(x^2)$

E. $(h(x))^2 = h(x^2)$

Space for Personal Notes

Question 253 (1 mark)

The linear function $f: D \to R, f(x) = 5 - x$ has a range $[-4, 5)$.

The domain D is:

A. $(0, 9]$

B. $(0, 1]$

C. $[5, -4)$

D. $[-9, 0)$

E. $[1, 9)$

Question 254 (1 mark)

Let $f: R \to R, f(x) = 1 - 2\cos\left(\frac{\pi x}{2}\right)$.

The period and range of this function are respectively:

A. 4 and $[-2, 2]$

B. 4 and $[-1, 3]$

C. 1 and $[-1, 3]$

D. 4π and $[-1, 3]$

E. 4π and $[-2, 2]$

Space for Personal Notes

Question 255 (1 mark)

The function f has the property $f(x) - f(y) = (y - x)f(xy)$ for all non-zero real numbers x and y.

Which one of the following is a possible rule for the function?

A. $f(x) = x^2$

B. $f(x) = x^2 + x^4$

C. $f(x) = x\log_e(x)$

D. $f(x) = \frac{1}{x}$

E. $f(x) = \frac{1}{x^2}$

Question 256 (1 mark)

If the equation $f(2x) - 2f(x) = 0$ is true for all real values of x, then the rule for f could be:

A. $\frac{x^2}{2}$

B. $\sqrt{2x}$

C. $2x$

D. $\log_e\left(\frac{|x|}{2}\right)$

E. $x - 2$

Space for Personal Notes

Question 257 (1 mark)

The range of the function $f: [-2, 3) \to R, f(x) = x^2 - 2x - 8$ is:

A. R

B. $(-9, -5]$

C. $(-5, 0)$

D. $[-9, 0]$

E. $[-9, -5)$

Question 258 (1 mark)

Let $f: R \to R, f(x) = x^2$.

Which one of the following is **not** true?

A. $f(xy) = f(x)f(y)$

B. $f(x) - f(-x) = 0$

C. $f(2x) = 4f(x)$

D. $f(x - y) = f(x) - f(y)$

E. $f(x + y) + f(x - y) = 2\big(f(x) + f(y)\big)$

Space for Personal Notes

Question 259 (1 mark)

The linear function $f: D \to R, f(x) = 6 - 2x$ has a range $[-4, 12]$.

The domain D is:

A. $[-3, 5]$

B. $[-5, 3]$

C. R

D. $[-14, 18]$

E. $[-18, 14]$

Question 260 (1 mark)

The range of the function $f: [-2, 7) \to R, f(x) = 5 - x$ is:

A. $(-2, 7]$

B. $[-2, 7)$

C. $(-2, \infty)$

D. $(-2, 7)$

E. R

Question 261 (1 mark)

The function f satisfies the functional equation $f\left(\frac{x+y}{2}\right) = \frac{f(x)+f(y)}{2}$ where x and y are any non-zero real numbers. A possible rule for the function is:

A. $f(xy) = \log_e|x|$

B. $f(x) = \frac{1}{x}$

C. $f(x) = 2^x$

D. $f(x) = 2x$

E. $f(x) = \sin(2x)$

Question 262 (1 mark)

Inspired from VCAA Mathematics Exam 2007
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2007mmCAS2.pdf

Let $g(x) = x^2 + 2x - 3$ and $f(x) = e^{2x+3}$.

Then $f(g(x))$ is given by:

A. $e^{4x+6} + 2e^{2x+3} - 3$

B. $2x^2 + 4x - 6$

C. e^{2x^2+4x+9}

D. e^{2x^2+4x-3}

E. e^{2x^2+4x-6}

Question 263 (1 mark)

Inspired from VCAA Mathematics Exam 2007
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2007mmCAS2.pdf

The function f satisfies the functional equation $f(f(x)) = x$ for the maximal domain of f.

The rule for the function is:

A. $f(x) = x + 1$

B. $f(x) = x - 1$

C. $f(x) = \frac{x-1}{x+1}$

D. $f(x) = \log_e(x)$

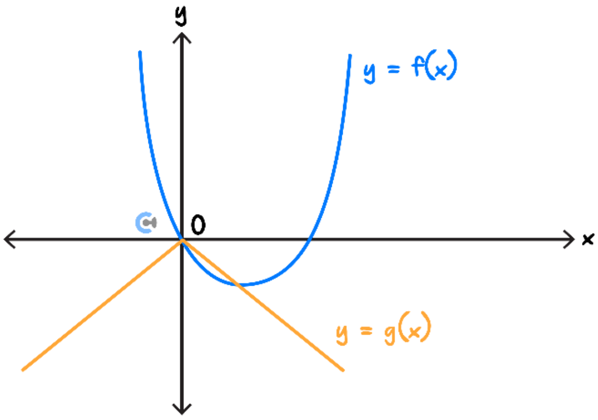
E. $f(x) = \frac{x+1}{x-1}$

Space for Personal Notes

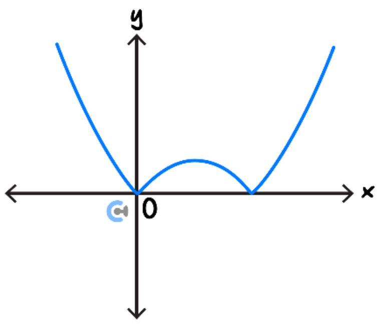
Question 264 (1 mark)

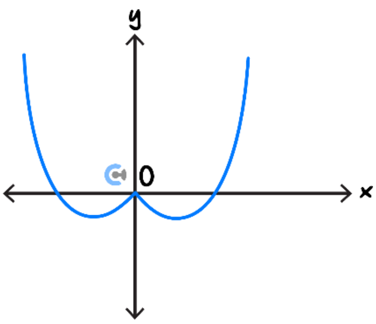
Inspired from VCAA Mathematics Exam 2007
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2007mmCAS2.pdf

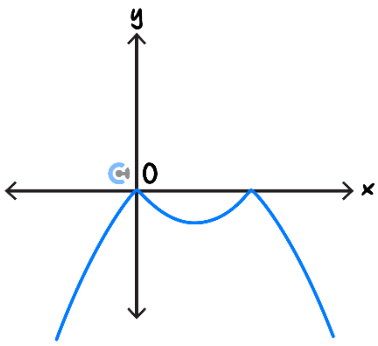
The graphs of $y = f(x)$ and $y = g(x)$ are as shown.

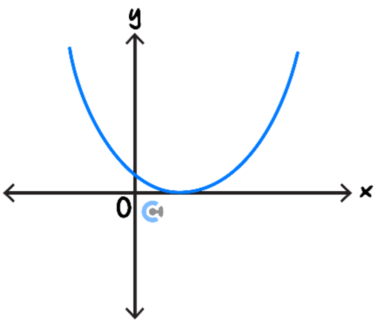


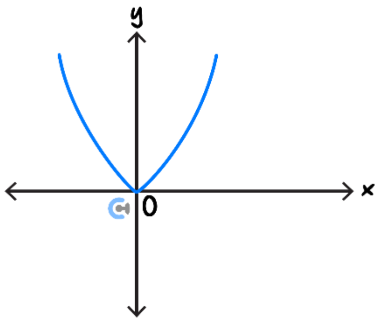
The graph of $y = f(g(x))$ is best represented by:

A.


B.


C.


D.


E.


Question 265 (1 mark)

Inspired from VCAA Mathematics Exam 2022
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2022/NHT/2022mm2-nht-w.pdf

Let $g: R \to R, g(x) = 3x + a$, where a is a real constant.

Given that $g(g(2)) = 10$, the value of a is:

A. -1

B. -2

C. -3

D. -4

E. -5

Question 266 (1 mark)

Inspired from VCAA Mathematics Exam 2019
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2019/NHT/2019MM2-nht-w.pdf

Let $f: [0, \infty) \to R, f(x) = x^2 + 1$.

The equation $f(f(x)) = \frac{185}{16}$ has real solution(s):

A. $x = \pm\frac{\sqrt{13}}{4}$

B. $x = \frac{\sqrt{13}}{4}$

C. $x = \pm\frac{\sqrt{13}}{2}$

D. $x = \frac{3}{2}$

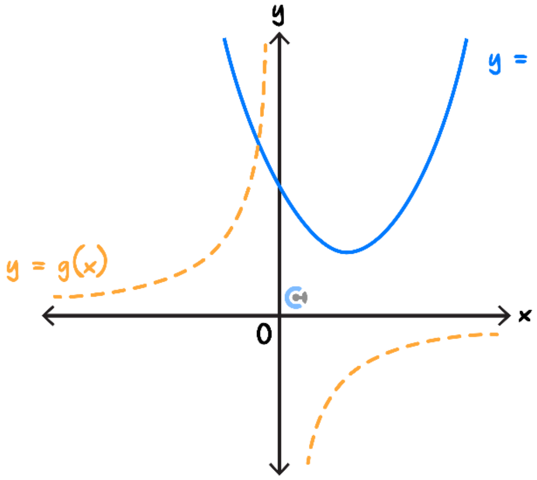
E. $x = \pm\frac{3}{2}$

Space for Personal Notes

Question 267 (1 mark)

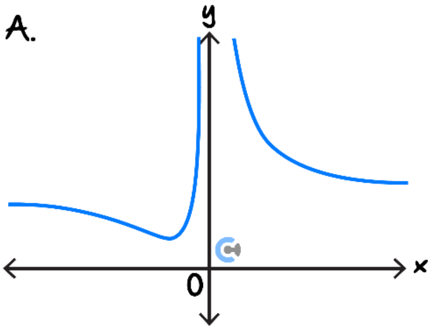
Inspired from VCAA Mathematics Exam 2019
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2019/NHT/2019MM2-nht-w.pdf

Parts of the graphs of $y = f(x)$ and $y = g(x)$ are as shown.

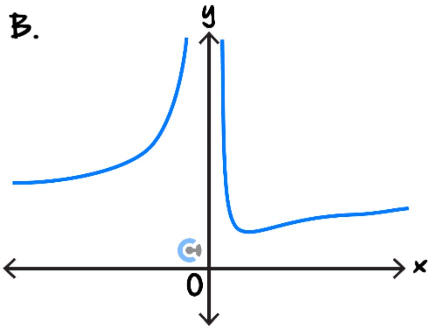


The corresponding part of the graph of $y = g(f(x))$ is best represented by:

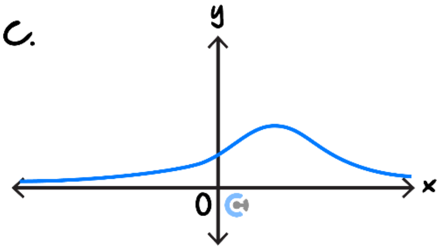
A.



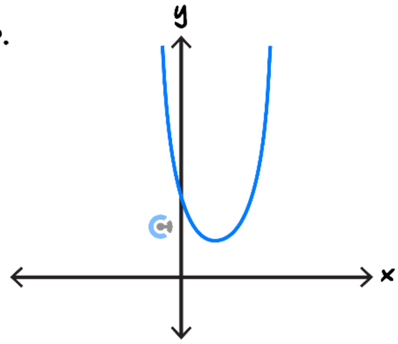
B.



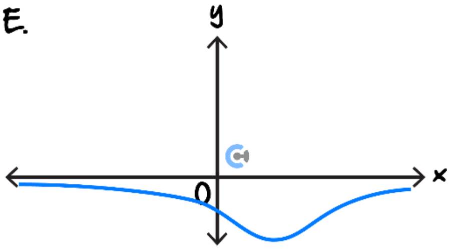
C.



D.



E.



Question 268 (1 mark)

Inspired from VCAA Mathematics Exam 2018
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2018/nht/2018MM2-nht-w.pdf

Let $f: R^+ \to R, f(x) = -\log_e(x)$ and $g: R \to R, g(x) = x^2 + 1$.

The domain and range of $f(g(x))$ are respectively:

A. R and $R^+ \cup \{0\}$

B. R and R^-

C. $[1, \infty)$ and $R^+ \cup \{0\}$

D. R^+ and $R^+ \cup \{0\}$

E. R and $R^- \cup \{0\}$

Question 269 (1 mark)

The midpoint of the line segment that joins $(1, -5)$ to $(d, 2)$ is:

A. $\left(\frac{d+1}{2}, -\frac{3}{2}\right)$

B. $\left(\frac{1-d}{2}, -\frac{7}{2}\right)$

C. $\left(\frac{d-4}{2}, 0\right)$

D. $\left(0, \frac{1-d}{3}\right)$

E. $\left(\frac{5+d}{2}, 2\right)$

Space for Personal Notes

Question 270 (1 mark)

The midpoint of the line segment joining $(0, -5)$ to $(d, 0)$ is:

A. $\left(\frac{d}{2}, -\frac{5}{2}\right)$

B. $(0, 0)$

C. $\left(\frac{d-5}{2}, 0\right)$

D. $\left(0, \frac{5-d}{2}\right)$

E. $\left(\frac{5+d}{2}, 0\right)$

Question 271 (1 mark)

The gradient of a line **perpendicular** to the line which passes through $(-2, 0)$ and $(0, -4)$ is:

A. $\frac{1}{2}$

B. -2

C. $-\frac{1}{2}$

D. 4

E. 2

Space for Personal Notes

Question 272 (1 mark)

The coordinates of the point on a curve with the equation $y = \sqrt{x}$ that are closest to the point $(4, 0)$ are:

A. $(0, 0)$

B. $(3, \sqrt{3})$

C. $\left(\frac{7}{2}, \frac{\sqrt{14}}{2}\right)$

D. $\left(\frac{7}{2}, \frac{\sqrt{15}}{2}\right)$

E. $(4, 2)$

Question 273 (1 mark)

The set of values of p for which $x^3 - px + 2 = 0$ has three distinct, real solutions is:

A. $(3, \infty)$

B. $(-\infty, -3)$

C. $(-3, 3)$

D. $(-\infty, 3]$

E. $[3, \infty)$

Question 274 (1 mark)

The simultaneous linear equations $2y + (m - 1)\,x = 2$ and $my + 3x = k$ have infinitely many solutions for:

A. $m = 3$ and $k = -2$

B. $m = 3$ and $k = 2$

C. $m = 3$ and $k = 4$

D. $m = -2$ and $k = -2$

E. $m = -2$ and $k = 3$

Question 275 (1 mark)

The gradient of a line perpendicular to the line that passes through $(3, 0)$ and $(0, -6)$ is:

A. $-\frac{1}{2}$

B. -2

C. $\frac{1}{2}$

D. 4

E. 2

Question 276 (1 mark)

The simultaneous linear equations $mx + 7y = 12$ and $7x + my = m$ have a unique solution only for:

A. $m = 7$ or $m = -7$

B. $m = 12$ or $m = 3$

C. $m \in R \setminus \{-7, 7\}$

D. $m = 4$ or $m = 3$

E. $m \in R \setminus \{12, 1\}$

Question 277 (1 mark)

The graph of $y = kx - 2$ will not intersect or touch the graph of $y = x^2 + 3x$ when:

A. $3 - 2\sqrt{2} < k < 3 + 2\sqrt{2}$

B. $\{k: k < 3 - 2\sqrt{2}\} \cup \{k: k > 3 + 2\sqrt{2}\}$

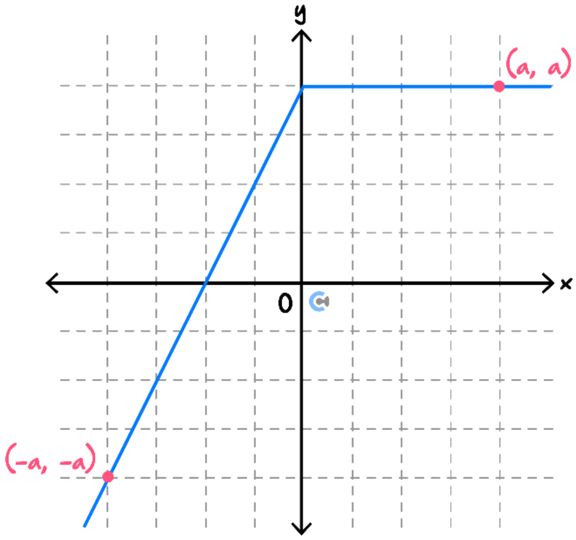
C. $-5 < k < 11$

D. $3 - 2\sqrt{2} \leq k \leq 3 + 2\sqrt{2}$

E. $k \in R^+$

Question 278 (1 mark)

Part of the graph of a function f is shown below.



Which one of the following is the average value of the function f over the interval $[-a, a]$?

A. 0

B. $\frac{3a}{4}$

C. $\frac{3a}{8}$

D. $\frac{a}{2}$

E. $\frac{a}{4}$

Question 279 (1 mark)

The simultaneous linear equations $ax - 3y = 5$ and $3x - ay = 8 - a$ have **no solution** for:

A. $a = 3$

B. $a = -3$

C. both $a = 3$ and $a = -3$

D. $a \in R \setminus \{3\}$

E. $a \in R \setminus [-3, 3]$

Question 280 (1 mark)

Let $p(x) = x^3 - 2ax^2 + x - 1$, where $a \in R$. When p is divided by $x + 2$, the remainder is 5.

The value of a is:

A. 2

B. $-\frac{7}{4}$

C. $\frac{1}{2}$

D. $-\frac{3}{2}$

E. -2

Question 281 (1 mark)

If $x + a$ is a factor of $8x^3 - 14x^2 - a^2x$, where $a \in R \setminus \{0\}$, then the value of a is:

A. 7

B. 4

C. 1

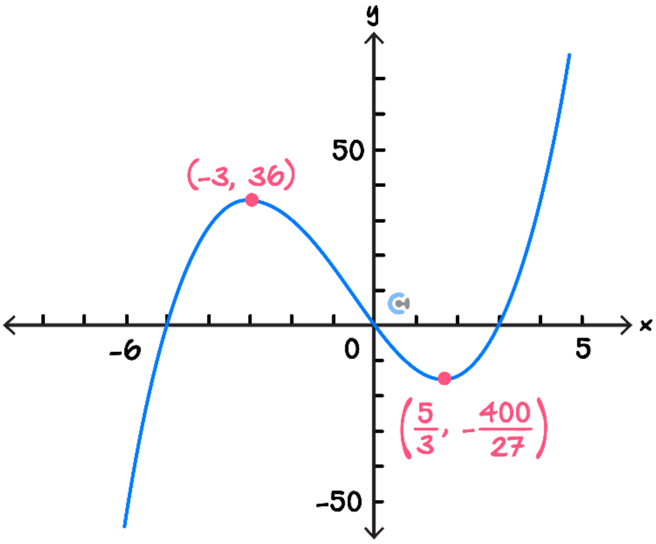
D. -2

E. -1

Space for Personal Notes

Question 282 (1 mark)

Part of the graph of a cubic polynomial function f and the coordinates of its stationary points are shown below.



$f'(x) < 0$ for the interval:

A. $(0, 3)$

B. $(-\infty, -5) \cup (0, 3)$

C. $(-\infty, -3) \cup \left(\frac{5}{3}, \infty\right)$

D. $\left(-3, \frac{5}{3}\right)$

E. $\left(\frac{-400}{27}, 36\right)$

Question 283 (1 mark)

The equation $(p-1)x^2 + 4x = 5 - p$ has no real roots when:

A. $p^2 - 6p + 6 < 0$

B. $p^2 - 6p + 1 > 0$

C. $p^2 - 6p - 6 < 0$

D. $p^2 - 6p + 1 < 0$

E. $p^2 - 6p + 6 > 0$

Question 284 (1 mark)

The simultaneous linear equations $(m-1)x+5y=7$ and $3x+(m-3)y=0.7m$ have infinitely many solutions for:

A. $m \in R \setminus \{0, -2\}$

B. $m \in R \setminus \{0\}$

C. $m \in R \setminus \{6\}$

D. $m = 6$

E. $m = -2$

Question 285 (1 mark)

The simultaneous linear equations,

$$kx - 3y = 0$$

$$5x - (k+2)y = 0$$

Where k is a real constant, have a unique solution provided.

A. $k \in \{-5, 3\}$

B. $k \in R \setminus \{-5, 3\}$

C. $k \in \{-3, 5\}$

D. $k \in R \setminus \{-3, 5\}$

E. $k \in R \setminus \{0\}$

Space for Personal Notes

Question 286 (1 mark)

The simultaneous linear equations,

$$ax + 3y = 0$$

$$2x + (a + 1)y = 0$$

Where a is a real constant, have infinitely many solutions for:

A. $a \in R$

B. $a \in \{-3, 2\}$

C. $a \in R \setminus \{-3, 2\}$

D. $a \in \{-2, 3\}$

E. $a \in R \setminus \{-2, 3\}$

Question 287 (1 mark)

The simultaneous linear equations,

$$mx + 12y = 24$$

$$3x + my = m$$

Have a unique solution only for:

A. $m = 6$ or $m = -6$

B. $m = 12$ or $m = 3$

C. $m \in R \setminus \{-6, 6\}$

D. $m = 2$ or $m = 1$

E. $m \in R \setminus \{-12, -3\}$

Space for Personal Notes

Question 288 (1 mark)

The graph of $y = kx - 3$ intersects the graph of $y = x^2 + 8x$ at two distinct points for:

A. $k = 11$

B. $k > 8 + 2\sqrt{3}$ or $k < 8 - 2\sqrt{3}$

C. $5 \leq k \leq 6$

D. $8 - 2\sqrt{3} \leq k \leq 8 + 2\sqrt{3}$

E. $k = 5$

Question 289 (1 mark)

The solution set of the equation $e^{4x} - 5e^{2x} + 4 = 0$ over R is:

A. $\{1, 4\}$

B. $\{-4, -1\}$

C. $\{-2, -1, 1, 2\}$

D. $\{-\log_e(2), 0, \log_e(2)\}$

E. $\{0, \log_e(2)\}$

Question 290 (1 mark)

The simultaneous linear equations $(m-2)x + 3y = 6$ and $2x + (m-3)y = m - 1$ have **no solution** for:

A. $m \in R \setminus \{0, 5\}$

B. $m \in R \setminus \{0\}$

C. $m \in R \setminus \{6\}$

D. $m = 5$

E. $m = 0$

Question 291 (1 mark)

The set of values of k for which $x^2 + 2x - k = 0$ has two real solutions is:

A. $\{-1, 1\}$

B. $(-1, \infty)$

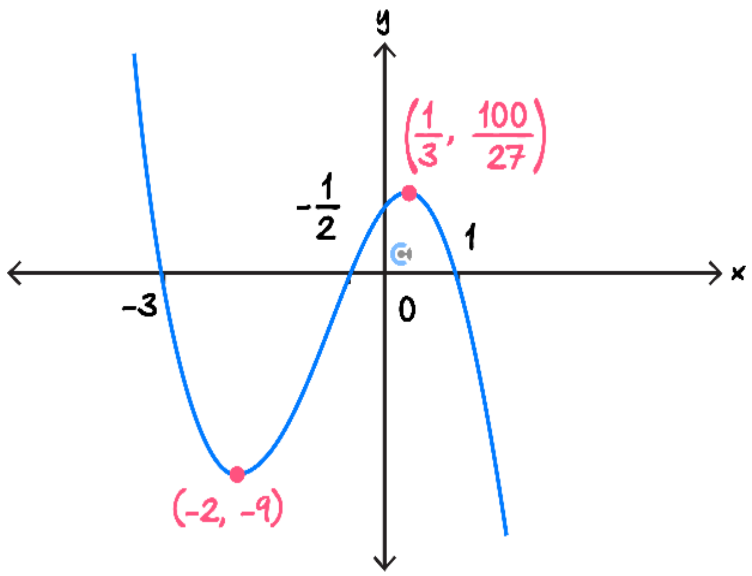
C. $(-\infty, -1)$

D. $\{-1\}$

E. $[-1, \infty)$

Question 292 (1 mark)

Part of the graph $y = f(x)$ of the polynomial function f is shown below.



$f'(x) < 0$ for

A. $x \in (-2,0) \cup \left(\frac{1}{3}, \infty\right)$

B. $x \in \left(-9, \frac{100}{27}\right)$

C. $x \in (-\infty, -2) \cup \left(\frac{1}{3}, \infty\right)$

D. $x \in \left(-2, \frac{1}{3}\right)$

E. $x \in (-\infty, -2] \cup (1, \infty)$

Question 293 (1 mark)

The line with equation $y = mx + 1$ and the curve with equation $y = 3x^2 + 2x + 4$ intersect at two distinct points. The values of m are:

A. $-4 < m < 8$

B. $m < -4$

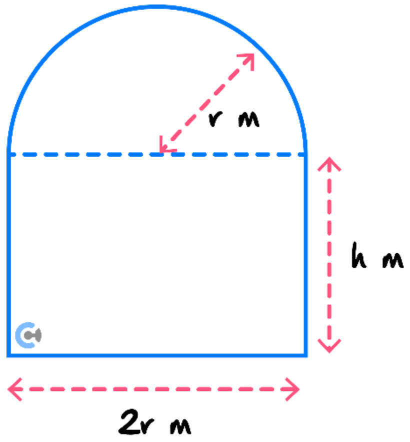
C. $m > 8$

D. $m < -4$ or $m > 8$

E. $m = -4$ or $m = 8$

Question 294 (1 mark)

The diagram below shows a glass window consisting of a rectangle of height h metres and width $2r$ metres, and a semicircle of radius r metres. The perimeter of the window is $8\ m$.



An expression for the area of the glass window, A, in terms of r is:

A. $A = 8r - 2r^2 - \frac{3\pi r^2}{2}$

B. $A = 8r - 2r^2 + \frac{\pi r^2}{2}$

C. $A = 8r - 4r^2 - \frac{3\pi r^2}{2}$

D. $A = 8r - 4r^2 - \frac{\pi r^2}{2}$

E. $A = 8r - 2r^2 - \frac{\pi r^2}{2}$

Question 295 (1 mark)

The simultaneous linear equations $2y + (m - 1)x = 2$ and $my + 3x = k$ have infinitely many solutions for:

A. $m = 3$ and $k = -2$

B. $m = 3$ and $k = 2$

C. $m = 3$ and $k = 4$

D. $m = -2$ and $k = -2$

E. $m = -2$ and $k = 3$

Question 296 (1 mark)

The simultaneous linear equations $mx + 7y = 12$ and $7x + my = m$ have a unique solution only for:

A. $m = 7$ and $m = -7$

B. $m = 12$ and $m = 3$

C. $m \in R \setminus \{-7, 7\}$

D. $m = 4$ and $m = 3$

E. $m \in R \setminus \{12, 1\}$

Space for Personal Notes

Question 297 (1 mark)

Let $f: [0, \infty) \to R, f(x) = x^2 + 1$.

The equation $f(f(x)) = \frac{185}{16}$ has real solution(s):

A. $x = \pm\frac{\sqrt{13}}{4}$

B. $x = \frac{\sqrt{13}}{4}$

C. $x = \pm\frac{\sqrt{13}}{2}$

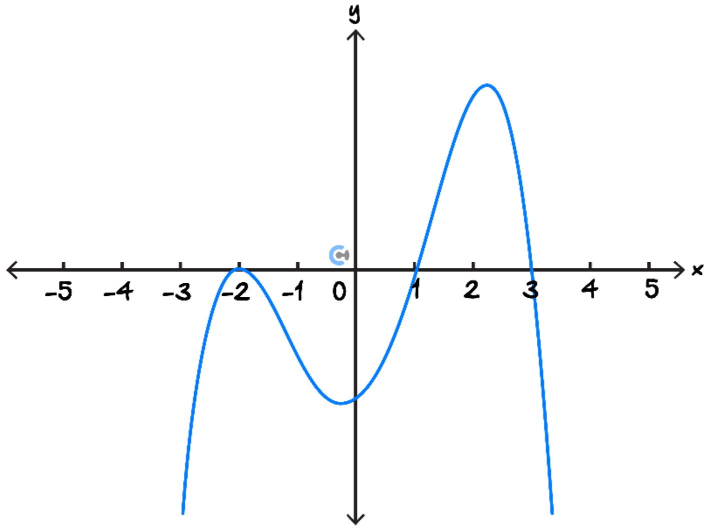
D. $x = \frac{3}{2}$

E. $x = \pm\frac{3}{2}$

Space for Personal Notes

Question 298 (1 mark)

The diagram below shows part of the graph of a polynomial function.



A possible rule for this function is:

A. $y = (x + 2)(x - 1)(x - 3)$

B. $y = (x + 2)^2(x - 1)(x - 3)$

C. $y = (x + 2)^2(x - 1)(3 - x)$

D. $y = -(x - 2)^2(x - 1)(3 - x)$

E. $y = -(x + 2)(x - 1)(x - 3)$

Space for Personal Notes

Question 299 (1 mark)

A set of three numbers that could be the solutions of $x^3 + ax^2 + 16x + 84 = 0$ is:

A. $\{3, 4, 7\}$

B. $\{-4, -3, 7\}$

C. $\{-2, -1, 21\}$

D. $\{-2, 6, 7\}$

E. $\{2, 6, 7\}$

Question 300 (1 mark)

The transformation $T: R^2 \to R^2$ maps the graph of $y = x^3 - x$ onto the graph of $y = 2(x-1)^3 - 2(x-1) + 4$. The transformation T could be given by:

A. $T(x, y) = (x + 1, 2y + 4)$

B. $T(x, y) = \left(x + 1, \frac{1}{2}y + 4\right)$

C. $T(x, y) = (2x + 1, y + 2)$

D. $T(x, y) = \left(\frac{1}{2}x + 1, y + 2\right)$

E. $T(x, y) = (x + 1, 2y + 2)$

Question 301 (1 mark)

The transformation $T: R^2 \to R^2$, which maps the graph of $y = -\sqrt{2x+1} - 3$ onto the graph of $y = \sqrt{x}$, has rules:

A. $T(x, y) = \left(\frac{1}{2}x - 1, -y - 3\right)$

B. $T(x, y) = \left(\frac{1}{2}x - 1, -y + 3\right)$

C. $T(x, y) = \left(\frac{1}{2}x + 1, -y - 3\right)$

D. $T(x, y) = (2x + 1, -y - 3)$

E. $T(x, y) = (2x - 1, -y + 3)$

Question 302 (1 mark)

The point (a, b) is transformed by:

$$T(x, y) = \left(\frac{1}{2}x - \frac{1}{2}, -2y - 2\right)$$

If the image of (a, b) is $(0, 0)$, then (a, b) is:

A. $(1, 1)$

B. $(-1, 1)$

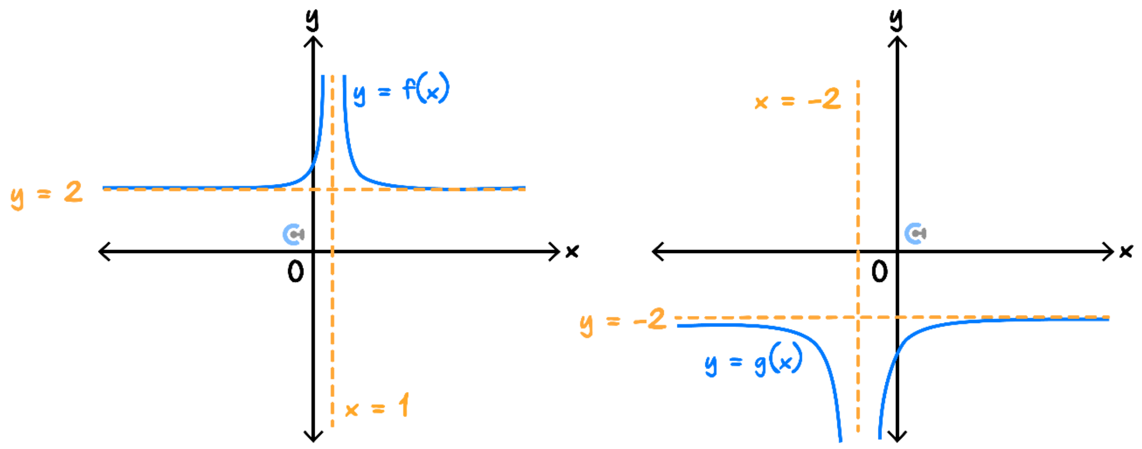
C. $(-1, 0)$

D. $(0, 1)$

E. $(1, -1)$

Question 303 (1 mark)

Consider the graphs of f and g below, which have the same scale.



If T transforms the graph of f onto the graph of g, then:

A. $T(x, y) = (x - 3, y - 4)$

B. $T(x, y) = (-x - 3, y - 4)$

C. $T(x, y) = (x - 3, -y)$

D. $T(x, y) = (-2x, -y)$

E. $T(x, y) = (-x, -2y)$

Question 304 (1 mark)

The graph of the function $f: [0, \infty) \to R$, where $f(x) = 4x^{\frac{1}{3}}$, is reflected in the x-axis and then translated five units to the right and six units vertically down.

Which one of the following is the rule of the transformed graph?

A. $y = 4(x-5)^{\frac{1}{3}} + 6$

B. $y = -4(x+5)^{\frac{1}{3}} - 6$

C. $y = -4(x+5)^{\frac{1}{3}} + 6$

D. $y = -4(x-5)^{\frac{1}{3}} - 6$

E. $y = 4(x-5)^{\frac{1}{3}} + 1$

Question 305 (1 mark)

The point $A(3, 2)$ lies on the graph of the function f. A transformation maps the graph of f to the graph of g where $g(x) = \frac{1}{2}f(x-1)$. The same transformation maps the point A to the point P.

The coordinates of the point P are:

A. $(2, 1)$

B. $(2, 4)$

C. $(4, 1)$

D. $(4, 2)$

E. $(4, 4)$

Space for Personal Notes

Question 306 (1 mark)

The graph of a function f is obtained from the graph of the function g with rule $g(x) = \sqrt{2x - 5}$ by a reflection in the x-axis followed by a dilation from the y-axis by a factor of $\frac{1}{2}$.

Which one of the following is the rule for the function f?

A. $f(x) = \sqrt{5 - 4x}$

B. $f(x) = -\sqrt{x - 5}$

C. $f(x) = \sqrt{x + 5}$

D. $f(x) = -\sqrt{4x - 5}$

E. $f(x) = -\sqrt{4x - 10}$

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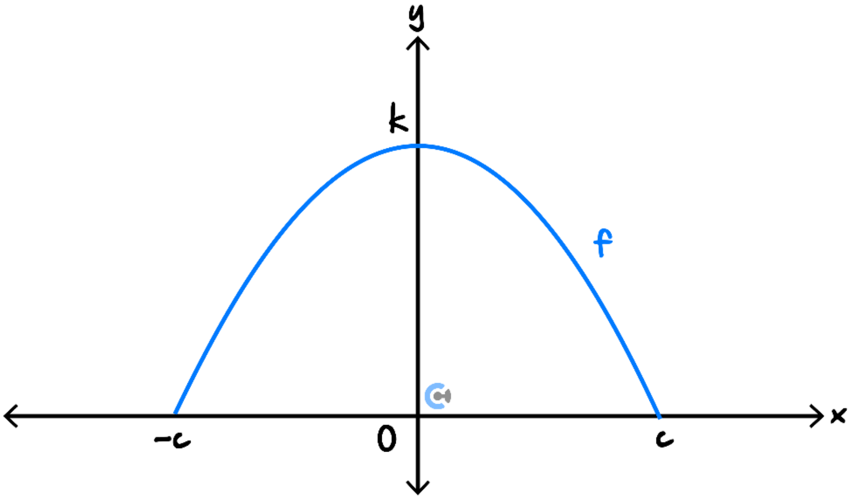
Question 307 (10 marks)

The parabolic arch of a tunnel is modelled by the function $f: [-c, c] \to R, f(x) = ax^2 + b$, where $a < 0$, $b \in R$ and $c > 0$.

Let x be the horizontal distance, in metres, from the origin and let y be the vertical distance, in metres, above the base of the arch.

The graph of f is shown below, where the coordinates of the y-intercept are $(0, k)$ and the coordinates of the x-intercepts are $(-c, 0)$ and $(c, 0)$.



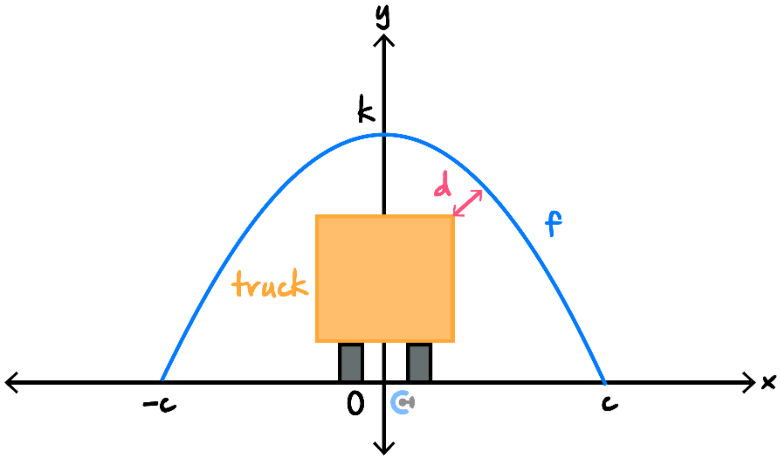
a. Express a and b in terms of c and k. (2 marks)

A particular tunnel has an arch modelled by f. It has a height of 6 m at the centre and a width of 8 m at the base.

b.

i. Find the rule for this arch. (1 mark)

ii. A truck that has a height of 3.7 m and a width of 2.7 m will fit through the arch with the function f found in **part b. i.**

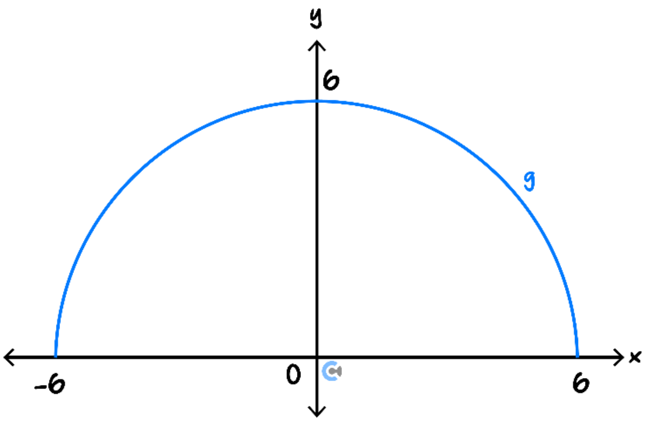


Assuming that the truck drives directly through the middle of the arch, let d be the minimum distance between the arch and the top corner of the truck.

Find d and the value of x for which this occurs, correct to three decimal places. (3 marks)

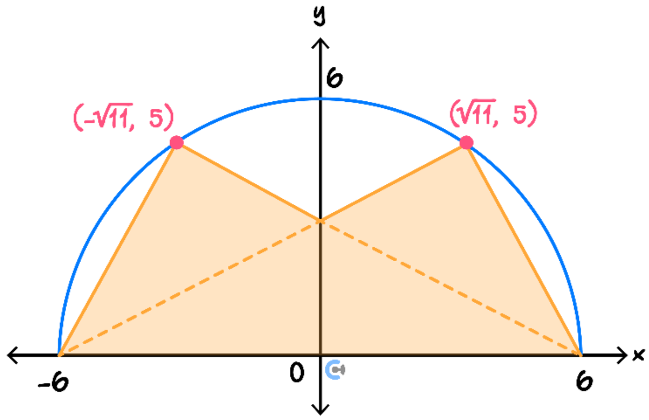
A different tunnel has a semicircular arch. This arch can be modelled by the function $g: [-6, 6] \to R$, $g(x) = \sqrt{r^2 - x^2}$, where $r > 0$.

The graph of g is shown below.



c. State the value of r. (1 mark)

d. Two lights have been placed on the arch to light the entrance of the tunnel. The positions of the lights are $(-\sqrt{11}, 5)$ and $(\sqrt{11}, 5)$. The area that is lit by these lights is shaded in the diagram below.



Determine the proportion of the cross-section of the tunnel entrance that is lit by the lights. Give your answer as a percentage, correct to the nearest integer. (3 marks)

Space for Personal Notes

Question 308 (3 marks)

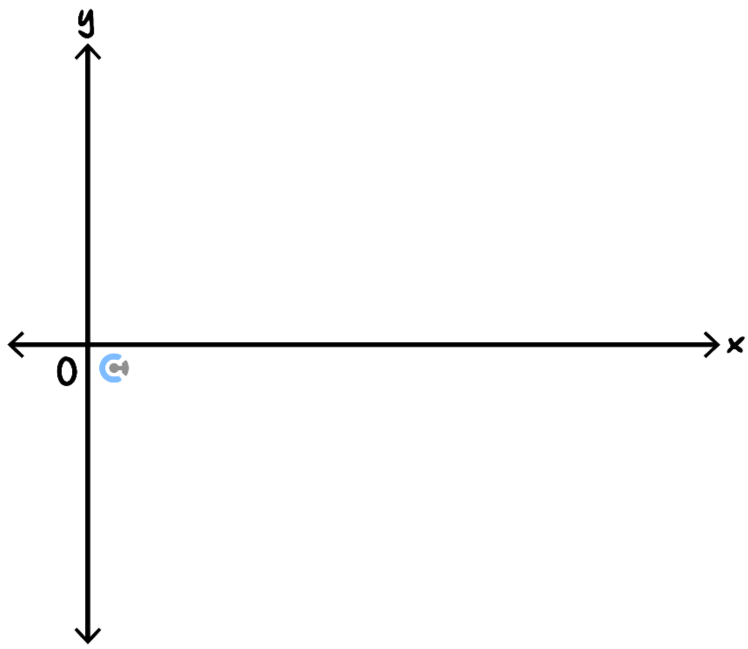
A well-designed computer screen display aims to make it quick and easy for a user to do tasks such as clicking on a screen button. Fitts' Law models the way in which the time taken to move to and click on a screen button depends on the distance the mouse is moved and the width of the screen button.

According to Fitts' Law, for a fixed distance travelled by the mouse, the time taken, in seconds, is given by $a - b\log_e(x)$, $0 \le x \le 5$, where $x\ cm$ is the button width and a and b are positive constants for a particular user.

a. Minnie discovers that, for her, $a = 1.1$ and $b = 0.5$.

i. Let $f(0,5] \to R, f(x) = 1.1 - 0.5\log_e(x)$.

Sketch the graph of $y = f(x)$ on the axes below. Label any asymptote with its equation and any end-point with its exact coordinates. (3 marks)

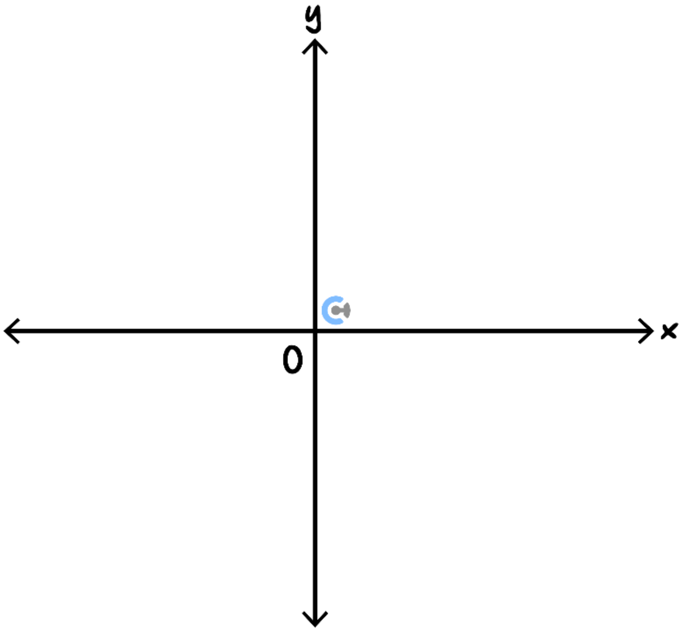


ii. Explain why f^{-1}, the inverse function of f, exists.

iii. Find $f^{-1}(x)$, the rule for f^{-1}.

iv. State the domain of f^{-1}.

v. Sketch the graph of $y = f^{-1}(x)$ on the axes below. Label any asymptote with its equation and any end-point with its exact coordinates.



b. Mickey decides to find the values of a and b for his use. He finds that when x is 1, his time is 0.5 seconds, and when x is 1.5, his time is 0.3 seconds.

Find the exact values of a and b for Mickey.

c. Show that, when the button width is halved, the time taken by Minnie (for whom $a = 1.1$ and $b = 0.5$) is increased by $\log_2 \sqrt{2}$ seconds.

Space for Personal Notes

Question 309 (12 marks)

Sally is using graph sketching software to design the landscape of the four hills shown in Figure 1 below.

She starts by using the square root functions h, h_1, and h_2 to model the shapes of three of the four hills, as shown in Figure 2 below.

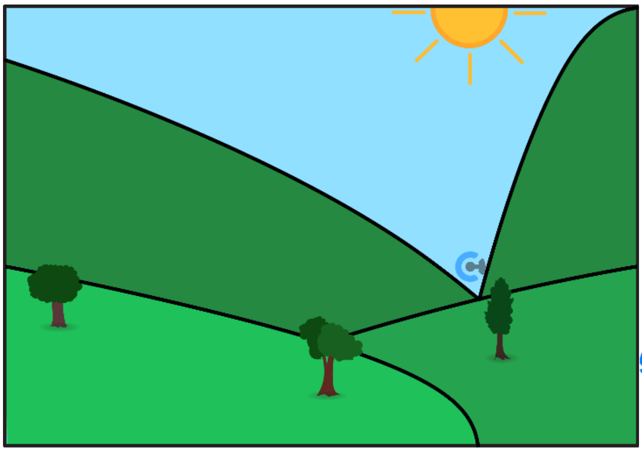


Figure 1

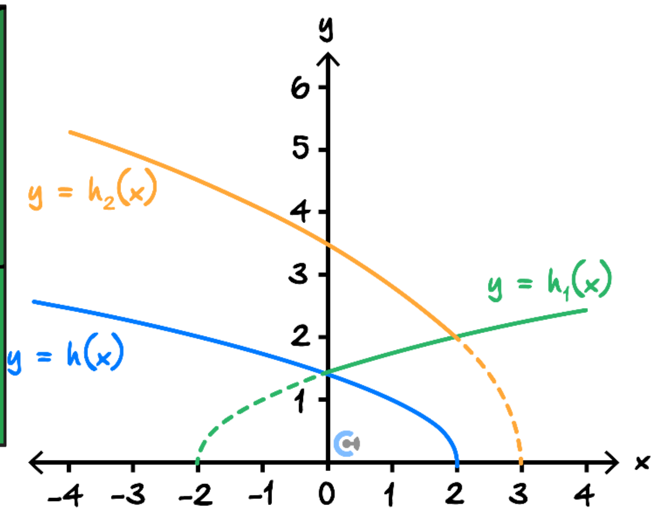


Figure 2

The rule for the function h is $h(x) = \sqrt{2-x}$.

a.

i. State the maximal domain for h. (1 mark)

ii. The rule for the function h_1 is obtained by reflecting the graph of h in the vertical axis.

State the rule for the function h_1. (1 mark)

b. The rule for the function h_2 is $h_2(x) = 2\sqrt{3-x}$.

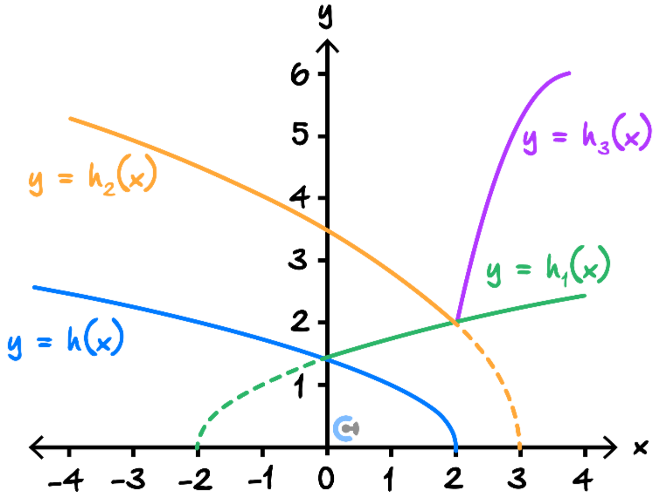
i. Write a sequence of two transformations that map the graph of h onto the graph of h_2. (1 mark)

ii. Let $T_1(x, y) = (ax + c, by + d)$ be a transformation that maps the graph of h onto the graph of h_2.

Find **one** set of possible values for a, b, c and d. (2 marks)

iii. Find the value of x for which the slope of the hill defined by the function h is equal to the slope of the hill defined by the function h_2. (1 mark)

Sally decides to use a quadratic function, h_3, to model the shape of the fourth hill in her landscape.



c. Find the rule for h_3, a quadratic function with a stationary point at $(4, 6)$ and which passes through $(2, 2)$. (2 marks)

Sally believes the function h_3 is closely related to the inverse of h.

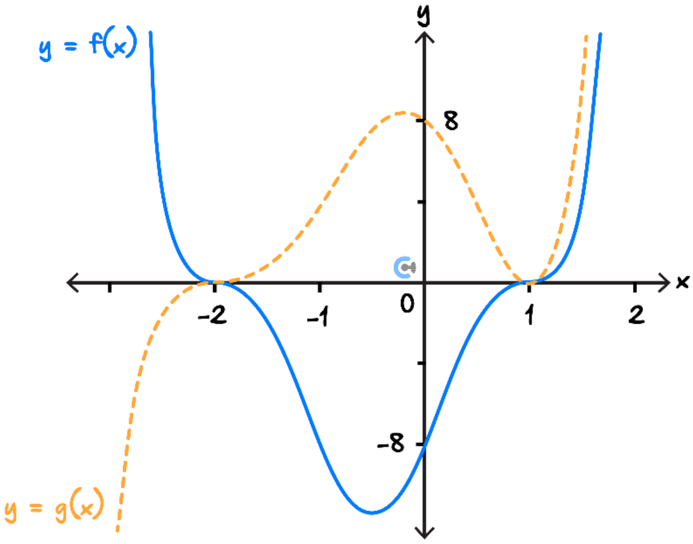
d. Find the domain and the rule for the function h^{-1}, the inverse of $h(x) = \sqrt{2 - x}$. (2 marks)

e. Consider the transformation $T_2(x, y) = (y + 4, x + 4)$.

Does the transformation above map the function h onto the function h_3? Give a reason to justify your answer. (2 marks)

Question 310 (9 marks)

Parts of the graphs of $f(x) = (x - 1)^3(x + 2)^3$ and $g(x) = (x - 1)^2(x + 2)^3$ are shown on the axes below.



The two graphs intersect at three points, $(-2, 0)$, $(1, 0)$ and (c, d). The point (c, d) is not shown in the diagram above.

a. Find the values of c and d. (2 marks)

b. Find the values of x such that $f(x) > g(x)$. (1 mark)

c. State the values of x for which:

i. $f'(x) > 0$. (1 mark)

ii. $g'(x) > 0$. (1 mark)

d. Show that $f(1+m) = f(-2-m)$ for all m. (1 mark)

e. Find the values of h such that $g(x+h) = 0$ has exactly one negative solution. (2 marks)

f. Find the values of k such that $f(x) + k = 0$ has no solutions. (1 mark)

Question 311 (8 marks)

Inspired from VCAA Mathematics Exam 2019
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2019/NHT/mm2nht_examrep19.pdf

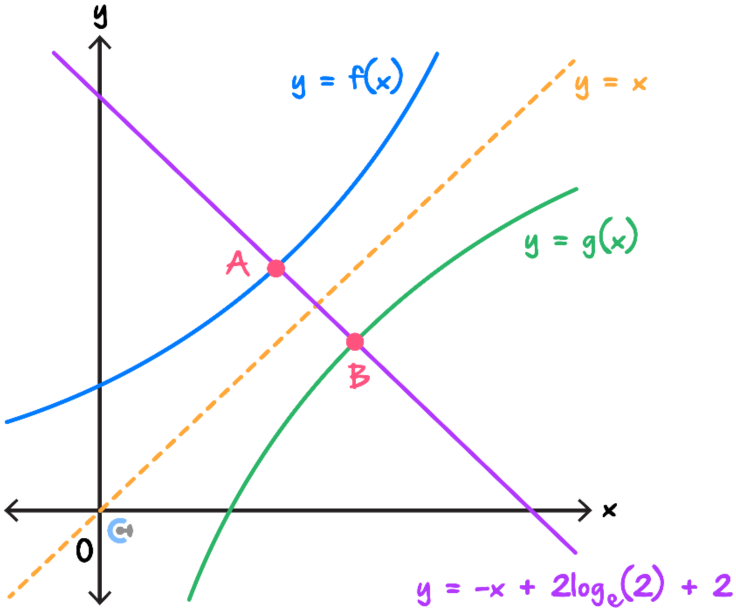
Let $f: R \to R, f(x) = e^{\left(\frac{x}{2}\right)}$ and $g: R^+ \to R, g(x) = 2\log_e(x)$.

a. Find $g^{-1}(x)$. (1 mark)

b. Find the coordinates of point A, where the tangent to the graph of f at A is parallel to the graph of $y = x$. (2 marks)

c. Show that the equation of the line that is perpendicular to the graph of $y = x$ and goes through point A is $y = -x + 2\log_e(2) + 2$. (1 mark)

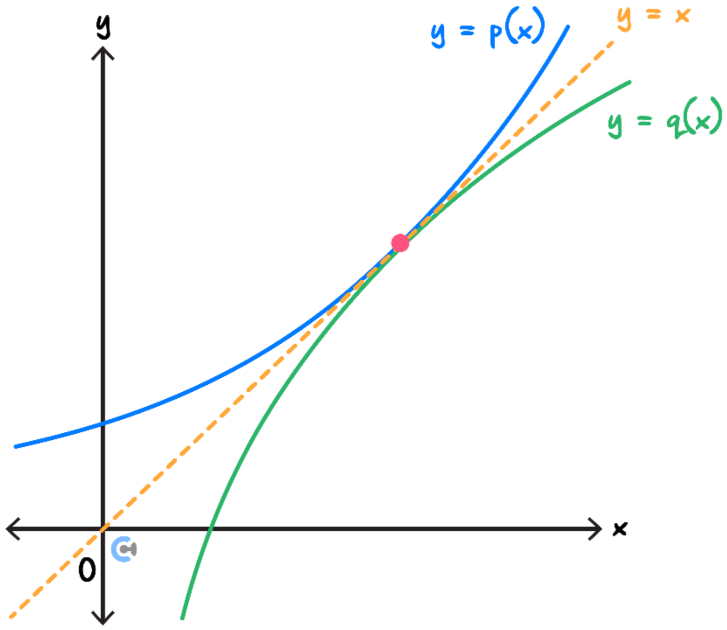
Let B be the point of intersection of the graphs of g and $y = -x + 2\log_e(2) + 2$, as shown in the diagram below.



d. Determine the coordinates of point B. (1 mark)

Let $p: R \to R, p(x) = e^{kx}$ and $q: R^+ \to R, q(x) = \frac{1}{k}\log_e(x)$.

e. The graphs of p, q and $y = x$ are shown in the diagram below. The graphs of p and q touch but do not cross.



Find the value of k. (2 marks)

f. Find the value of k, $k > 0$, for which the tangent to the graph of p at its y-intercept and the tangent to the graph of q at its x-intercept are parallel. (1 mark)

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Question 312 (16 marks)

Inspired from VCAA Mathematics Exam 2023
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2023/NHT/2023mathsmethods2-NHT-report.docx

Let $g: R \to R, g(x) = (x+2)^2 - 1$.

a. Express the rule for g in the form $g(x) = ax^2 + bx + c$, where $a, b, c \in R$. (1 mark)

b. The function g can also be written in the form $g(x) = (x-p)(x-q)$, where $p, q \in Z$. Give the values of p and q. (1 mark)

c. Find the value of k for which the graph of $y = g(x) + k$ passes through the origin. (2 marks)

d. Using algebra, find the value(s) of d such that the graph of $y = g(x - d)$ will pass through the origin. (2 marks)

e. Describe the transformation from the graph of $y = g(x)$ to the graph of $y = g(3x)$. (1 mark)

Let $h: R \to R, h(x) = mx + n$, where m and n are real numbers.

f. Find the value of m, such that the graph of the sum function $y = g(x) + h(x)$ has a turning point on the y-axis. (2 marks)

g. Find n in terms of m, such that the graph of the sum function $y = g(x) + h(x)$ has a turning point on the x-axis. (2 marks)

h. Find **two** pairs of values for m and n, such that the graph of the product function $y = g(x)h(x)$ has exactly two x-intercepts. (3 marks)

i. Find the coordinates of the turning point of the graph of $y = g(h(x))$, giving your answer in terms of m and n. (2 marks)

Question 313 (9 marks)

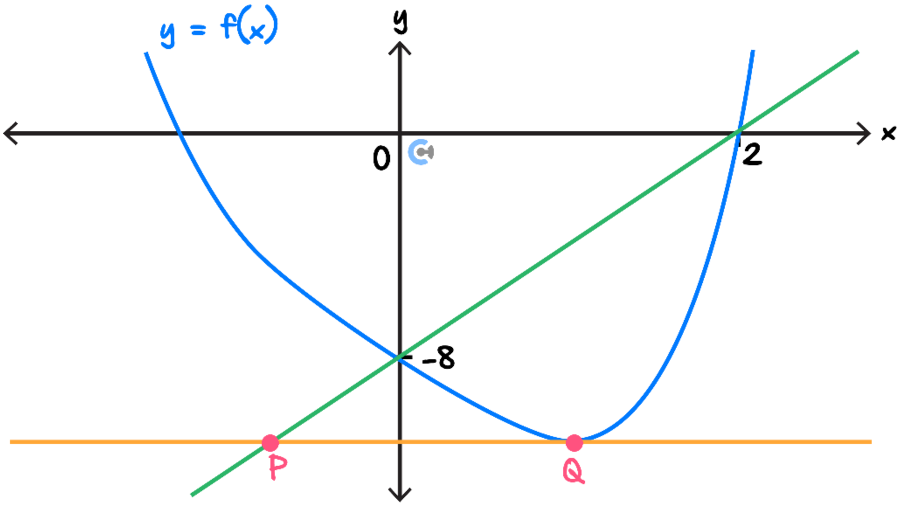
Inspired from VCAA Mathematics Exam 2018
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2018/nht/mathsmethods2nht_examrep18.pdf

Let $f: R \to R, f(x) = x^4 - 4x - 8$.

a. Given $f(x) = (x-2)(x^3 + ax^2 + bx + c)$, find a, b and c. (1 mark)

b. Find two consecutive integers m and n such that a solution to $f(x) = 0$ is in the interval (m, n), where $m < n < 0$. (2 marks)

The diagram below shows part of the graph of f and a straight line drawn through the points $(0, -8)$ and $(2, 0)$. A second straight line is drawn parallel to the horizontal axis and it touches the graph off at the point Q. The two straight lines intersect at the point P.



c.

i. Find the equation of the line through $(0, -8)$ and $(2, 0)$. (1 mark)

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ii. State the equation of the line through the points P and Q. (1 mark)

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iii. State the coordinates of the points P and Q. (2 marks)

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d. A transformation $T: R^2 \rightarrow R^2, T(x, y) = (x + d, y)$ is applied to the graph of f.

i. Find the value of d for which P is the image of Q. (1 mark)

ii. Let $(m', 0)$ and $(n', 0)$ be the images of $(m, 0)$ and $(1, 0)$ respectively, under the transformation T, where m and n are defined in **part b**.

Find the values of m' and n'. (1 mark)

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