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VCE Mathematical Methods ¾

AOS 1 Revision [1.0]

Contour Check (Part 2)

Contour Checklist

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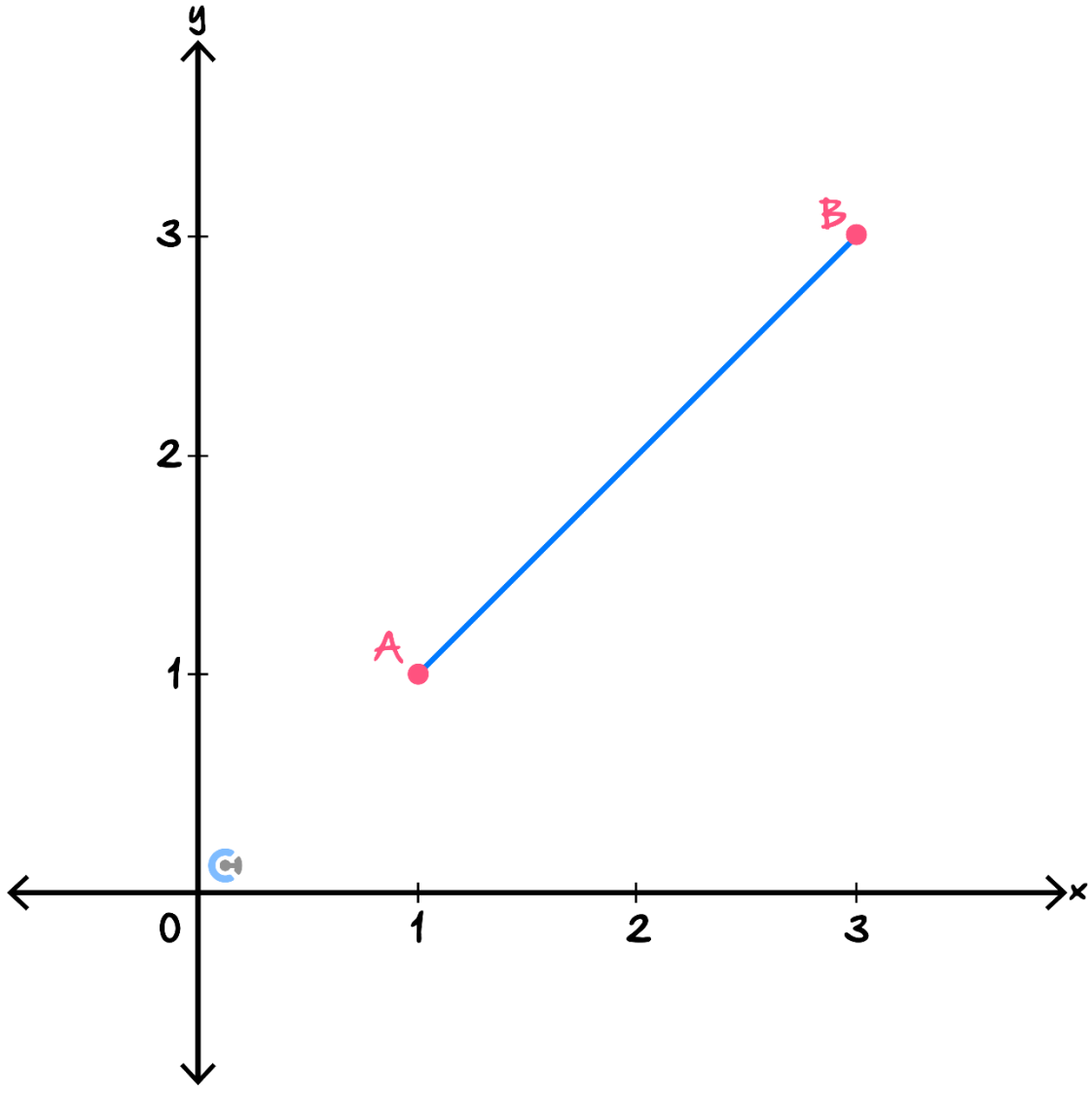
Section A: [1.5] – Linear and Coordinate Geometry (Checkpoints)

Sub-Section [1.5.1]: Finding the Midpoint and Distance Between Points and Functions

Question 88

The line segment AB is shown on the axis below. Draw the midpoint, M of AB.



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Question 89

Find the midpoints of the following points.

a. $A(3, 7)$ and $B(5, 9)$.

b. $C(-2, -3)$ and $D(6, 4)$.

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Question 90

The midpoint of points A and B is $M(2, 2)$.

a. If the coordinates of A are $(6, -4)$, find the coordinates of B.

Consider the points $C(c, 5)$ and $D(-3, d)$. The midpoint of the line CD is the origin.

b. Find the values of c and d.

c. Find the midpoint of $E(x_1, y_1)$ and $F(x_2, y_2)$ in terms of x_1, x_2, y_1, and y_2.

d. The graph of $y = x^2 + k$ and the line $y = 1$ has a minimum vertical distance of 4. Find the value of k.

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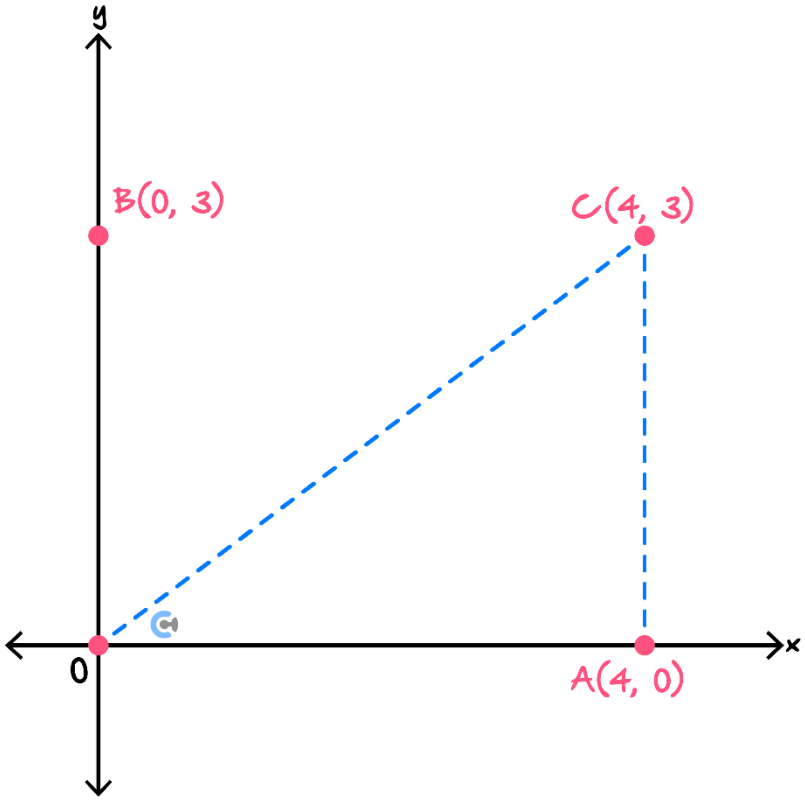
Sub-Section [1.5.2]: Finding Distances Between Points

Question 91

Consider the points, A, B, C as well as the origin drawn below.



a. Find the distance between the origin and point A.

b. Find the distance between the origin and point B.

c. Use Pythagoras' theorem to find the distance between the origin and point C.

Question 92

Find the distance between the following pairs of points.

a. $A(2, 5)$ and $B(-2, 2)$.

b. $C(-1, -7)$ and $D(4, 5)$.

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Question 93

A point $P(u, v)$ lies on the line $y = 3 - x$.

a. Express the distance between P and the origin in terms of u only.

Consider the points $A(-1, -1), B(5, 7)$ and $C(x, y)$.

The length of AC is equal to the length of BC which is equal to halve the length of AB.

b. Find the coordinates of C.

c. **Tech-Active.** The distance between the point $P(u, v)$ is 3 units away from the origin and 4 units away from the point $Q(1, 4)$. Find the coordinates of P.

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Sub-Section [1.5.3]: Finding Parallel and Perpendicular Lines

Question 94

State whether the following lines are parallel or perpendicular to each other.

a. $y = 2x + 1$ and $y = 2x + 5$.

b. $y = 3x + 2$ and $y = -\frac{1}{3}x - 2$.

c. $2x + 3y = 5$ and $4x + 6y = 12$.

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Question 95

A line l_1 goes through the points $(2, 3)$ and $(3, 5)$.

a. Find the gradient of l_1.

b. Find the equation of l_1.

The line l_2 is perpendicular to l_1 and goes through the point $(2, 3)$.

c. Find the gradient of l_2.

d. Find the equation of l_2.

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Question 96

The line l_1 is parallel to the line $l_2 = \{(x, y) \in \mathbb{R}^2 : 2y + 3x = 5\}$ and goes through the origin.

a. Find the equation of l_1.

b. Find the equation of the line that is perpendicular to the line with the equation $y = -5x + 7$ and passes through the point $(2, -5)$.

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Question 97

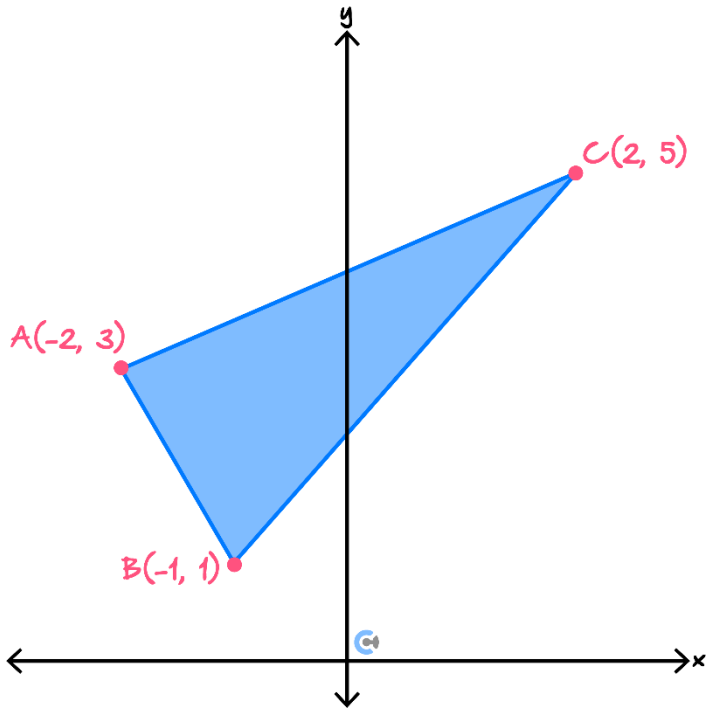
a. Find the perpendicular bisector of the points $A(2, 3)$ and $B(4, 9)$.

b. A point $P(u, v)$ lies on the line $y = 2x$.

Find the value of u and v for which the distance between P and the point $Q(0, 1)$ is minimum.

Hint: The line PQ is perpendicular to the line $y = 2x$.

c. Consider the triangle ABC drawn below.



i. Show that the line AB is perpendicular to the line AC.

ii. Hence, find the area of the triangle ABC.

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Sub-Section [1.5.4]: Angles Between Lines

Question 98

a. Find the angle of the line $y = x + 1$ makes with the positive direction of the x-axis.

b. Find the equation of the line that passes through the origin and makes an angle of 30 degrees with the positive direction of the x-axis.

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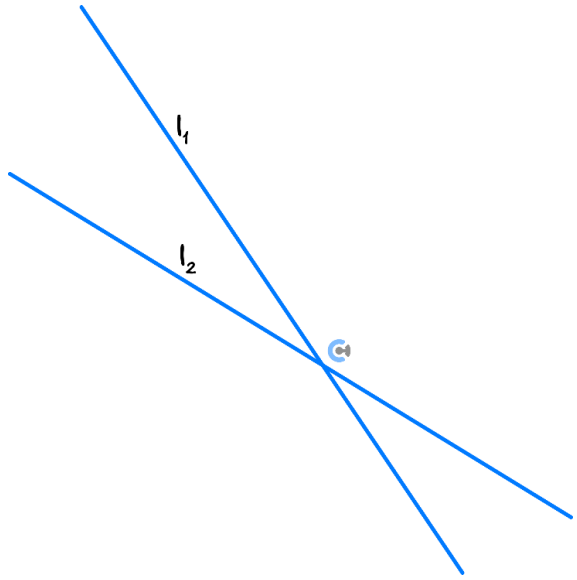
Question 99

a. Find the acute angle between the lines $y = \frac{1}{\sqrt{3}}x + 2$ and $y = \frac{-1}{\sqrt{3}}x$.

b. **Tech-Active.** Consider the line l_1, with the equation $2y + 3x = 5$.

The line l_2 intersects l_1 at an acute angle 25°. Both l_1 and l_2 are drawn below.



Find the slope of l_2 correct to 2 decimal places.

c. **Tech-Active.** Find the acute angle of intersection between the lines $y = 3x + 5$ and $-2x + 3y = 7$.

Give your answer in degrees correct to the nearest degree.

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Question 100

The line l intersects the positive y-axis at $30°$

a. Find the gradient, m of l if $m < 0$.

b. **Tech-Active.** Find the acute angle of intersection between the lines $y = 2x + 3$ and $3x + 5y = -4$.

Give your answer in degrees correct to the nearest degree.

c. Find the equation of all lines that intersect the line $y = x + 3$ at the point $(1, 4)$ at an acute angle of $15°$.

Sub-Section [1.5.5]: Simultaneous Equations

Question 101

Solve the following equations simultaneously.

a. $3x + 4y = 7$ and $5x - 2y = 3$.

b. $y = 5x + 3$ and $3y + 4x = 8$.

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Question 102

a. Find the point of intersection between the lines $y = 3x + 7$ and $2x + 5y = 1$.

b. Explain why the equations $2x + 4y = 6$ and $3x + 6y = 5$ have no solutions.

c. **Tech-Active.** For each pair of simultaneous equations, state whether they have, no solution, a unique solution or infinitely many solutions.

i. $2x + 5y = 7$ and $3x + 2y = 8$.

ii. $y = -3x + 6$ and $2y + 6x = 6$.

iii. $6x + y = 2$ and $y = -6x + 2$.

Question 103

a. Consider the following pair of simultaneous equations,

$$kx - y = 6$$
$$7x + (k - 8)y = 4$$

For what value(s) of k do they have:

i. A unique solution?

ii. No solution?

b. Consider the following pair of simultaneous equations,

$$ax + 3y = 6$$
$$x + (4 - a)y = 2$$

For what value(s) of a do they have:

i. No solution?

ii. Infinitely many solutions?

iii. A unique solution?

c. **Tech-Active.** Consider the following pair of simultaneous equations,

$$3x + (1 - a)y = 2$$
$$ax - 2y = b$$

Find all pairs (a, b) such that the equations have infinitely many solutions.

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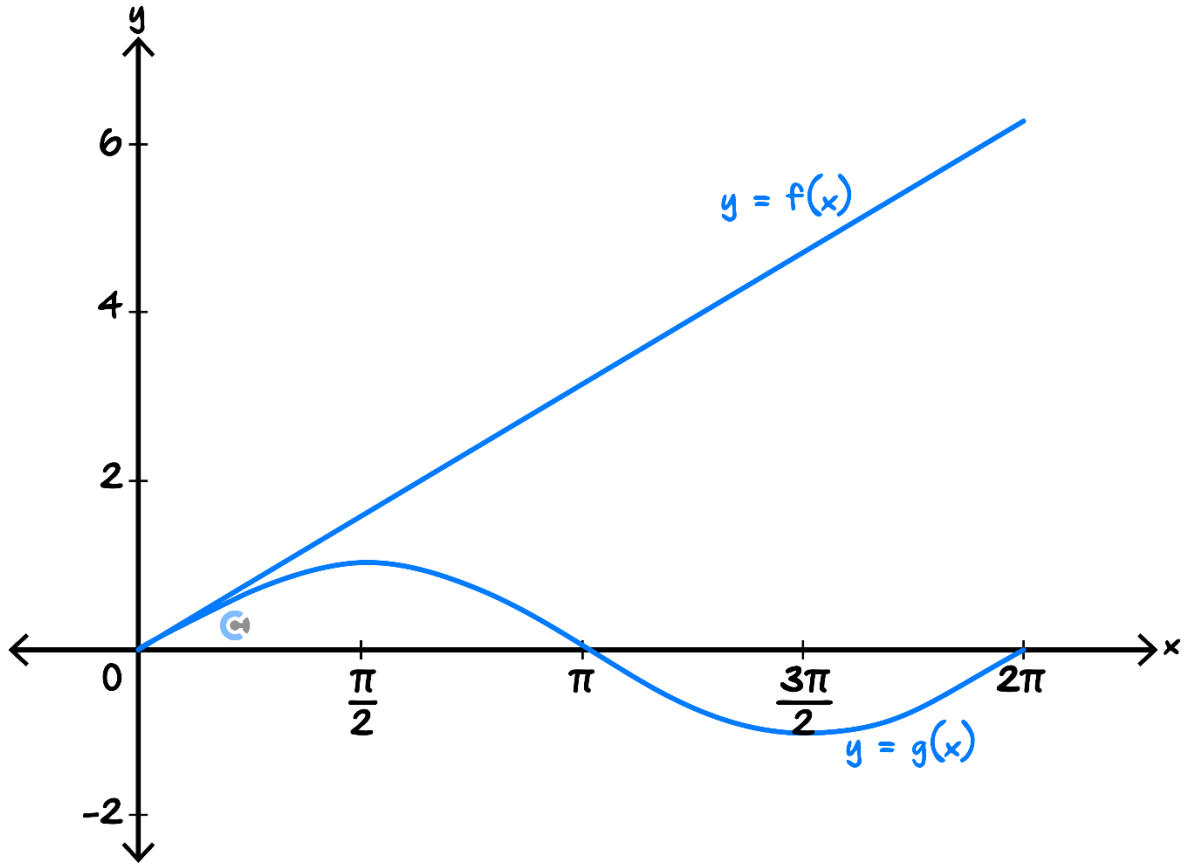
Sub-Section [1.5.6]: Addition of Ordinates

Question 104

The graphs of $f : [0, 2\pi] \longrightarrow \mathbb{R}, f(x) = x$, and $g : [0, 2\pi] \longrightarrow \mathbb{R}, g(x) = \sin(x)$ are drawn below.

Sketch the graph of $h(x) = f(x) + g(x)$ on the axis below, labelling all points of intersection between f and h with their co-ordinates.



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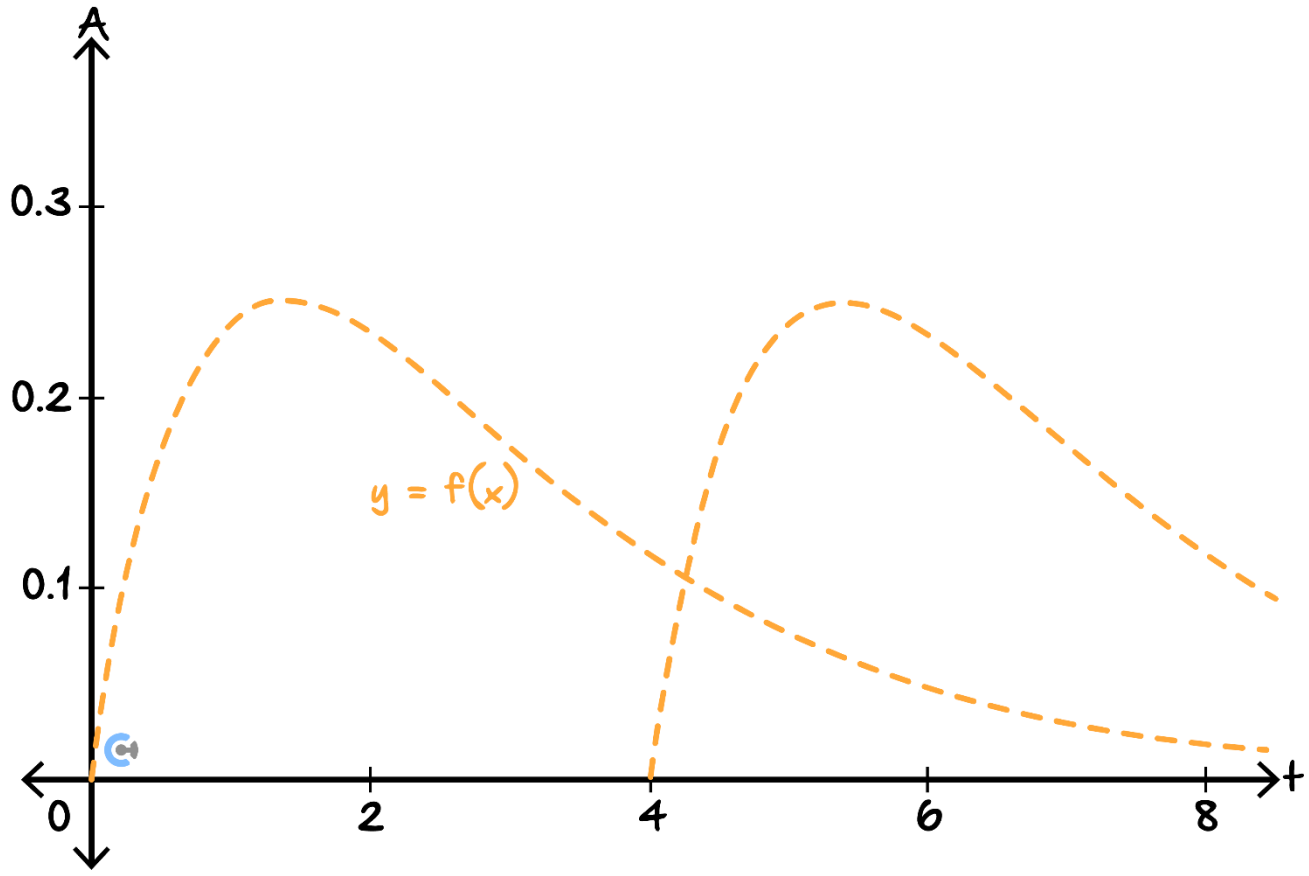
Question 105

t hours after taking a mystery pill, the concentration of dopamine in a patient's bloodstream is $A = f(t)$ milligrams per litre. The graph of f is shown below.

4 hours after taking one mystery pill, the patient takes another mystery pill.

On the axis below, sketch the concentration of dopamine in the patient's bloodstream during the first 8 hours after they take the first mystery pill.



Question 106 Tech-Active.

Let $f(x) = e^x - e^{-2x}$ and $g(x) = e^{x-x^2}$.

How many solutions does the equation $f(x) + g(x) = 0$ have?

_____

_____

Sub-Section [1.5.7]: Boss Question

Question 107

Consider the points $A(1, 0)$ and $B(4, 3)$.

a. Find the equation of the line segment AB.

There is another point C, such that A is the midpoint of the line segment CB.

b. Find the coordinates of C.

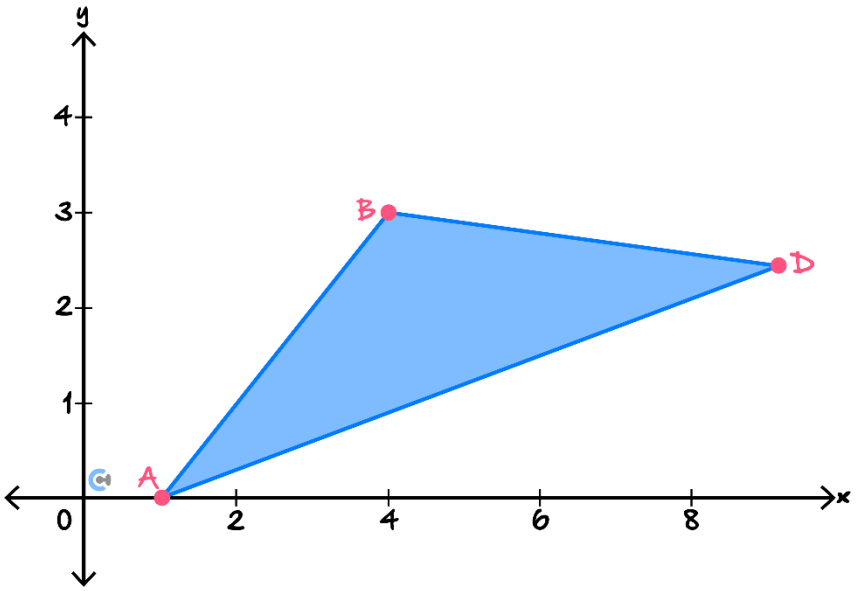
c. Hence or otherwise, find the length of BC.

d. Another point $D(u, v)$ has the following properties,

- The length of AD is equal to twice the length of AB.
- The angle between AD and AB is 30°.
- The gradient of AB is larger than the gradient of AD.
- Both u and v are positive.

Find the values of u and v correct to 3 decimal places.

e. The triangle ABD is drawn below.



i. Find the equation of the line, l perpendicular to AD that goes through B.

ii. Hence or otherwise, find the area of ABD correct to the nearest integer.

Section B: [1.6] – Linear and Coordinate Geometry Exam Skills (Checkpoints)

Sub-Section [1.6.1]: Apply Midpoint to Find a Reflected Point

Question 108

The point $(-1, 5)$ is reflected in the line $y = 2$. Find the coordinates of the reflected point.

Question 109

The point $(2, -3)$ is reflected in a line to become the point $(-10, -3)$. State the equation of the line.

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Question 110

Find the perpendicular bisector of the line segment joining the points $(4, -2)$ and $(-1, 0)$.

Question 111

The point $(1, -6)$ is reflected in a line to become the point $(5, -4)$. Find the equation of the line.

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Sub-Section [1.6.2]: Apply Parallel and Perpendicular Lines to Geometric Problems

Question 112

Find the equation of the line that passes through the point $(-2, 3)$ and is perpendicular to $y = x + 7$.

Question 113

Find the area of the triangle formed by the lines $y = 2x - 8$, $y = 6x - 4$, and $y = 2$.

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Question 114

Find the distance between the point $(2, 7)$ and the line $y = 3x - 1$.

Space for Personal Notes

Question 115

Consider the points $A(2,1)$, $B(1,-2)$, $C(5,0)$ and $D(m,n)$, where $m, n \in \mathbb{R}^+$. It is known that $\angle ABC = 45°$. Find the values of m and n such that $\angle BCD = 135°$.

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Sub-Section [1.6.3]: Solve Coordinate Geometry Problems with Transformations

Question 116

The area bound by the lines $y = 2x - 4$, $y = -1 - x$, and $y = \frac{1}{2}x + 2$ is $\frac{27}{2}$ square units. Hence, find the area bound by:

a. The lines $y = 8x - 4$, $y = -1 - 4x$ and $y = 2x + 2$.

b. The lines $y = -2x + 4$, $y = 1 + x$ and $y = -\frac{1}{2}x - 2$.

c. The lines $y = 6x - 4$, $y = 5 - 3x$, $y = \frac{3}{2}x + 14$.

Question 117

a. The original function is $f(x) = \frac{2}{(x-5)^2} - 16$, and the tangent line to the graph of $y = f(x)$ at $x = 6$ is $y = -4x + 8$. The graph of $f(x)$ is reflected in the x-axis translated 2 units down, then dilated by a factor of $\frac{1}{2}$ from the x-axis. Find the equation of the tangent to the transformed graph when $x = 6$.

b. The graph of $f(x) = 2x^2 - 3x + 1$ has a tangent line at $x = -1$ with an equation of $y = -7x - 1$. $f(x)$ undergoes a translation 3 units right, followed by a dilation by a factor of 4 from the x-axis. Find the equation of the tangent to the transformed graph when $x = 2$.

c. Consider the graph $f(x) = x^2 - 6x + 4$. The line $y = 2x - 12$ is a tangent to $f(x)$ at $x = 4$. Find the equation of the tangent to $y = 4x^2 - 28x + 32$ at $x = 4$.

Question 118

a. Find the value of a such that the area bound by the graphs $y = x - 2$, $y = ax + a$ and the y-axis is 2 square units.

b. It is known that the triangle formed by the lines $y = 2x + 6$, $y = -x - a$, and the x-axis has an area of 5. Find the values of a.

c. Find the values of a where the area between the lines $y = ax$, $y = x - 4$ and the y-axis is 12.

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Question 119

a. The shape bound by the lines $y = -\frac{1}{2}x - 1$, $y = x + 5$ and $y = ax - 1$ has an area of 8 square units. Find the value of a if $a \in (-\infty, 1)$.

b. Hence or otherwise, find the values of m and c such that the area bound by the graphs $y = -2x + 2$, $y = 4x + 8$, and $y = mx + c$ is 2 square units. Assume $m, c \in (1, \infty)$.

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Sub-Section: Exam 1 Questions

Question 120

Consider the simultaneous linear equations:

$$2ax - (a+1)y = -1$$

$$\frac{x}{2a+1} + 3y = 4a + 5$$

Where a is a real constant.

a. Find the values of a for which there is a unique solution to the set of equations.

b. Find the value of a for which there are no unique solutions.

c. Find the value of a for which there are infinitely many solutions.

Question 121

Consider the points $A(8,-2)$ and $B(2,6)$.

a. Find the equation of the line that is parallel to the line segment AB, and also contains the point $C(6,9)$.

b. Find the equation of the perpendicular bisector of AB.

c. Find the coordinates of D, the point of intersection between the lines found in **part a.** and **b.**

d. Find the area of the quadrilateral $ABCD$.

e. Let $E\left(\frac{8}{3}, -4\right)$, $F\left(\frac{2}{3}, 12\right)$, $G(2, 18)$, and $H(3, 10)$. Find the area of $EFGH$.

Space for Personal Notes

Question 122

The point $P(4, 1)$ is reflected in the line $y = 2x - 2$ to become the point P'.

a. Find the coordinates of P'.

b. Find the point of intersection between the lines $y = 2x - 2$ and $y = 7x - 27$.

c. The line $y = 7x - 27$ is reflected in the line $2x - 2$. Find the equation of the new line.

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Question 123

At $x = -2$, the graph $y = f(x)$ has a tangent line with the equation $y = 3 - 2x$, and a normal line is given by $y = \frac{1}{2}x + 8$.

a. Find the area bounded by the tangent line, normal line, and the x-axis.

The graph of $f(x)$ is translated down 3 units, dilated by a factor of 2 from the x-axis, and dilated by a factor of 5 from the y-axis to become the graph $g(x)$.

b. Find the equation of the normal line to $y = g(x)$ at $x = -4$.

c. Find the area bounded by the x-axis, the tangent line and the normal line of the graph $y = g(x)$ at $x = -4$.

Sub-Section: Exam 2 Questions

Question 124

The set of simultaneous equations:

$$\frac{5}{3k-4}y-\frac{x}{2}=\frac{3}{8}k+\frac{3}{2}$$

$$(k-6)x+2ky=\frac{4}{3}-k$$

Has no solutions for:

A. $k=3$ or $k=-\frac{10}{3}$

B. $k=-\frac{10}{3}$

C. $k=3$

D. $k\neq-\frac{2}{3}$ or $k\neq-\frac{10}{3}$

Question 125

The area of the triangle formed by the points $(2,3)$, $(-4,7)$ and $(4,6)$ is:

A. 13 square units.

B. 25 square units.

C. 26 square units.

D. 19 square units.

Space for Personal Notes

Question 126

The graph $f(x) = x^2 - 4x + 3$ has a tangent line and a normal line constructed at $x = 1$. The area bound by the tangent line, the normal line, and the y-axis is $\frac{5}{4}$ square units. The area bound by the y-axis, tangent line, and normal line to the graph $y = -\frac{1}{2}x^2 + 4x - 3$ at $x = -2$ is:

A. $\frac{5}{8}$ square units.

B. $\frac{5}{4}$ square units.

C. 5 square units.

D. 8 square units.

Question 127

The acute angle formed between the lines $y = 3x - 1$ and $y = mx + 5$ is at least 45° when:

A. $m \in \left[\frac{1}{2}, \infty\right)$

B. $m \in \left[-2, \frac{1}{2}\right]$

C. $m \in (-\infty, -2] \cup \left[\frac{1}{2}, \infty\right)$

D. $m \in \left[-2, 0\right) \cup \left(0, \frac{1}{2}\right]$

Question 128

The equation of the tangent line to $f(x)$ at $x = 2$ is $y = 1 - 4x$. The equation of the normal line to $f(x)$ at $x = 2$ is:

A. $y = \frac{1}{4}x - \frac{15}{2}$

B. $y = -\frac{1}{4}x + 1$

C. $y = 4x - 2$

D. Cannot be determined.

Question 129

Consider the points $A(6, -2)$ and $B(3, 4)$.

a. Find the perpendicular bisector of AB.

b. Find the values of m such that the line $y = mx$ forms a 45° angle with the line segment AB.

c. Point $C(m, n)$ and point $D(p, q)$ are different points that lie on the perpendicular bisector of AB, where m, $n \in \mathbb{R}^+$. Find the coordinates of C and D such that the triangles ABC and ABD are both right-angle triangles.

d. The point C can be mapped onto the point D by a reflection in the line $y = a$ followed by a reflection in the line
$x = b$. State the values of a and b.

e. Find the area of $ACBD$.

f. Find the area of the square that has opposite corners at $(7, -4)$ and $(1, 8)$.

Space for Personal Notes

Question 130

The function $f(x) = 2(x+3)^2 - 5$ has a tangent line with the equation $y = 4x + 5$.

a. Show that $y = 4x + 5$ is a tangent to $f(x)$ at the point $(-2, -3)$.

b. Find the equation of the normal line to $f(x)$ at $x = -2$.

c. State the obtuse angle formed between the line $y = 4x + 5$ and the x-axis, correct to 2 decimal places.

d. Find the area enclosed by the tangent line, the normal line, and the x-axis.

The graph of $y = f(x)$ is translated 4 units right, dilated by a factor of 4 from the x-axis, and dilated by a factor of $\frac{2}{3}$ from the y-axis to become the graph $y = g(x)$.

e. Find the equation of the tangent line to $y = g(x)$ at $x = \frac{4}{3}$.

f. State the obtuse angle formed between the new tangent of $y = g(x)$ at $x = \frac{4}{3}$, correct to 2 decimal places.

g. Find the area of the triangle formed between the x-axis, the tangent, and the normal line to $y = g(x)$ at $x = \frac{4}{3}$.

Space for Personal Notes

Section C: [1.7] – Polynomials (Checkpoints)

Sub-Section [1.7.1]: Applying Factor and Remainder Theorems

Question 131

a. State the remainder when $x^2 + 5x - 3$ is divided by $x + 2$.

b. Is $x - 2$ a factor of $f(x) = x^4 - 16$?

c. Is $x + 4$ a factor of $g(x) = x^3 + 4x^2 + 2$?

Question 132

Let $f(x) = 2x^3 + ax^2 + ax + 3$. Find the value of a such that $f(x)$ has a factor of $2x + 3$.

Question 133

Let $f(x) = x^2 + ax + b$. Find the values of a and b such that f has a factor of -1, and when f is divided by $2x - 3$, it has a remainder of -5.

Question 134

A cubic polynomial, $g(x)$ has the following properties:

1. $g(x) - 3$ has a factor of $(x - 2)^2$.

2. $g(x)$ divided by $x^2 - 1$ leaves a remainder of 2.

Find the rule for $g(x)$.

Sub-Section [1.7.2]: Finding Factored Forms of Polynomials

Question 135

Factorise the following polynomials:

a. $x^3 - 8$

b. $x^3 - 7x^2 + 10x$

c. $x^3 + 3x^2 - 4x - 12$

Space for Personal Notes

Question 136

a. Factorise $f(x) = x^3 + x^2 - 17x + 15$.

b. Factorise $g(x) = x^3 - 4x^2 + x + 6$.

c. Find all of the real roots of $h(x) = x^3 - 3x^2 + 4$.

Space for Personal Notes

Question 137

a. Factorise $f(x) = x^3 - 5x^2 - 29x + 105$.

b. Factorise $g(x) = 18x^3 - 3x^2 - 28x - 12$.

c. Factorise $h(x) = 2x^3 + 14x^2 - 10x - 150$.

Space for Personal Notes

Question 138

Let $f(x) = ax^2 + bx + c$ with a, b, c being co-prime non-zero integers, and assume that $\frac{p}{q}$ is a root of f with p and q co-prime and both non-zero.

a. Show that p divides c.

b. Show that q divides a.

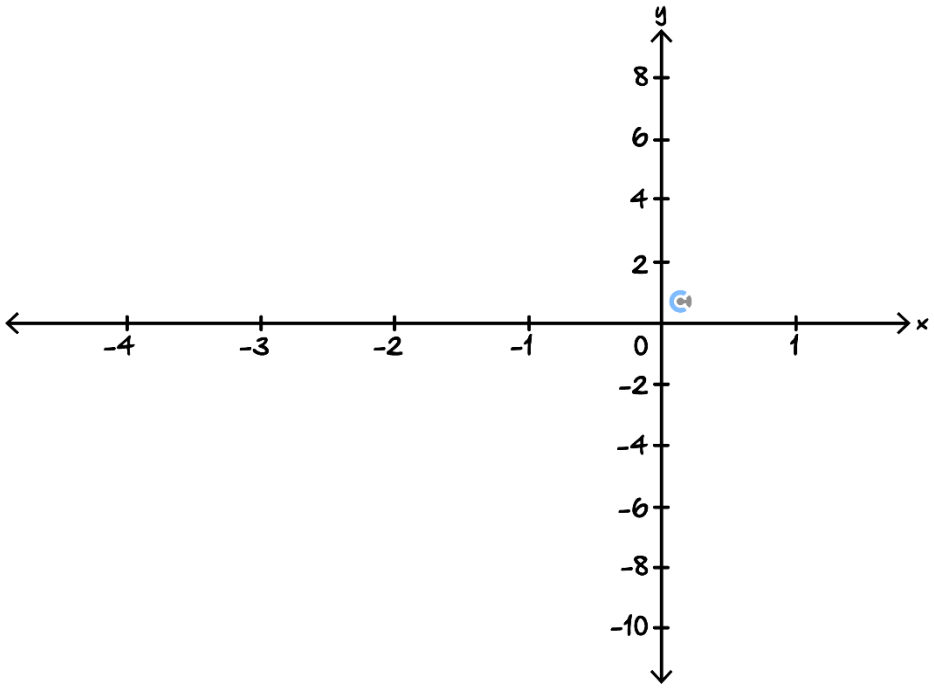
c. If a, b, c are not co-prime integers, where would your arguments for **parts a.** and **b.** breakdown?

Space for Personal Notes

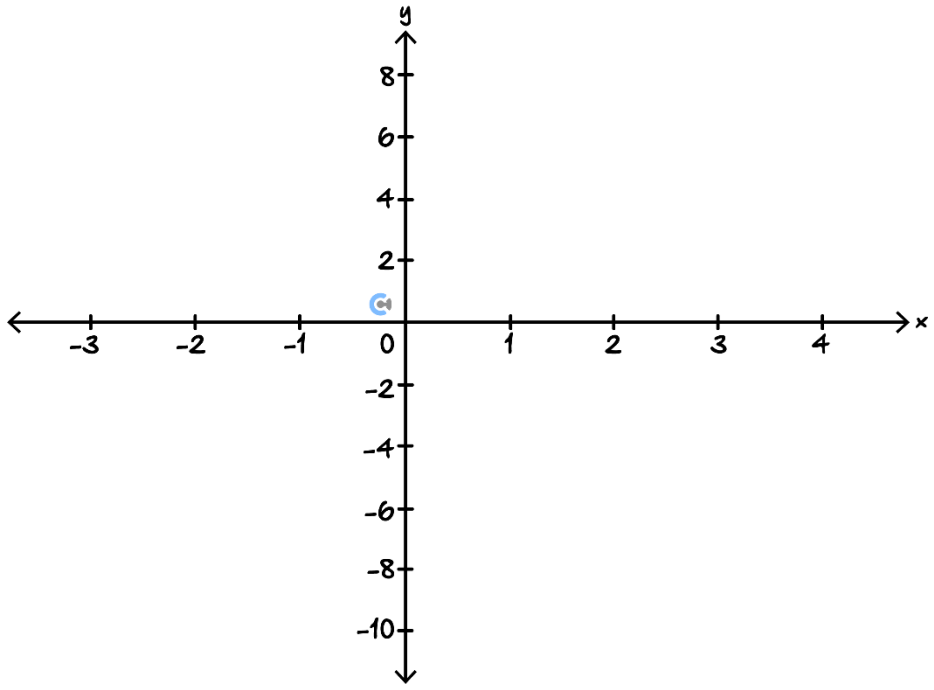
Sub-Section [1.7.3]: Graphing Factored and Unfactored Polynomials

Question 139

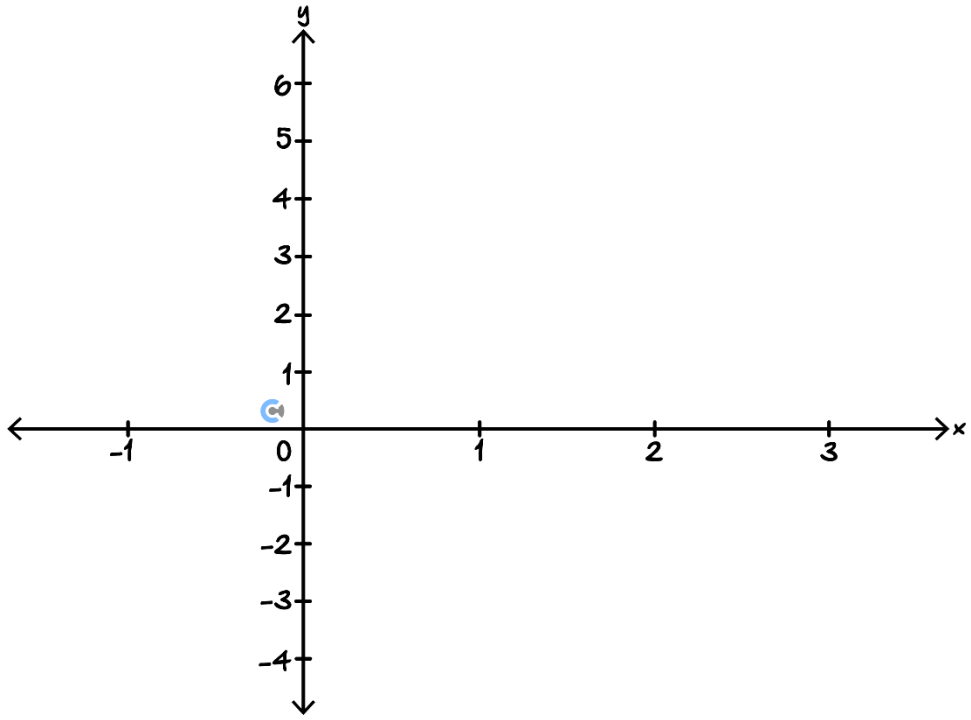
a. Sketch the graph of $y = (x + 2)^3 - 1$ on the axis below.



b. Sketch the graph of $y = x(x - 1)\ (x + 2)(x - 3)$ on the axis below.

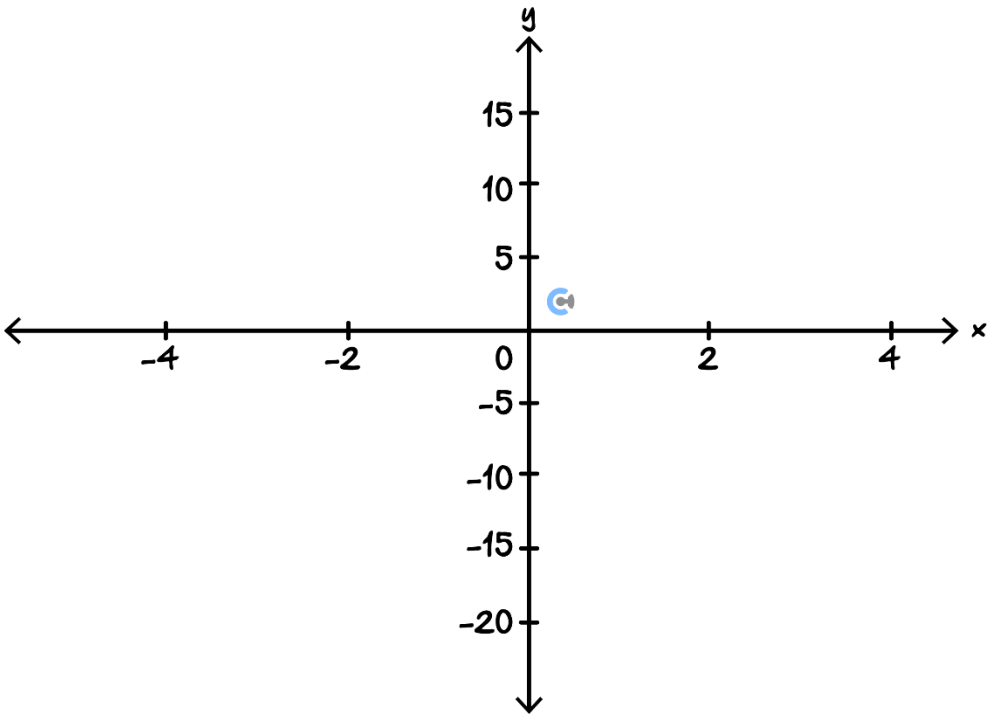


c. Sketch the graph of $y = 2(x-1)^3 + 2$ on the axis below.

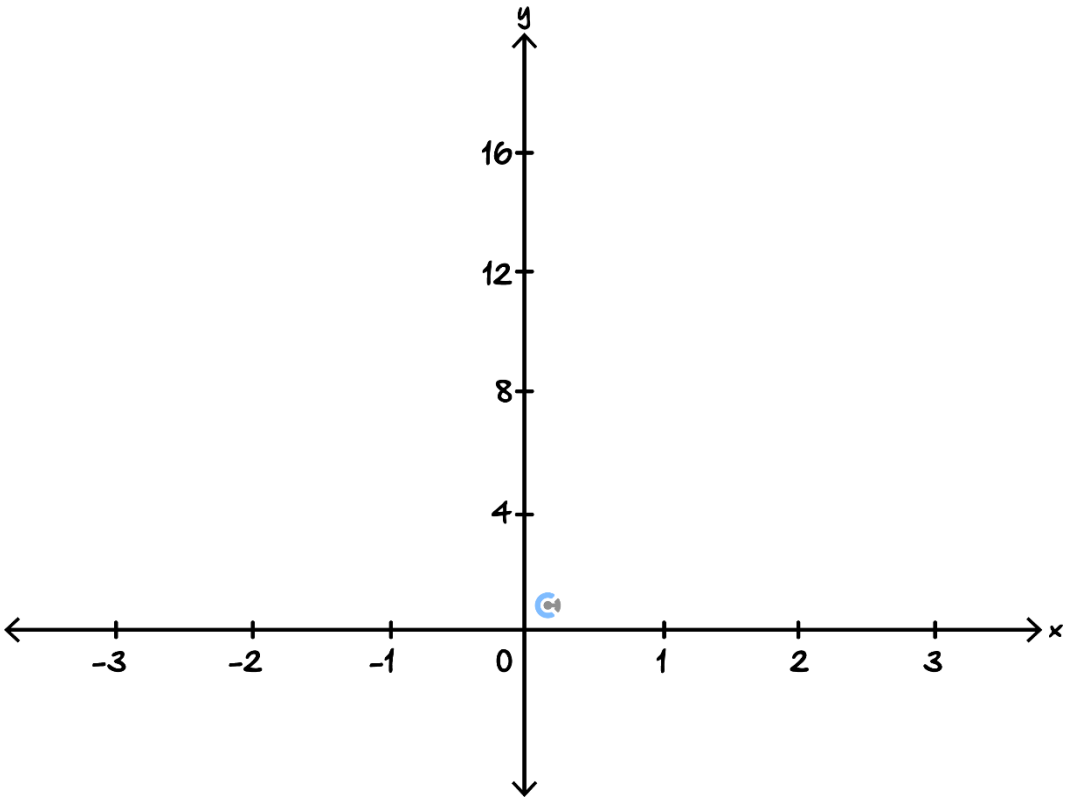


Question 140

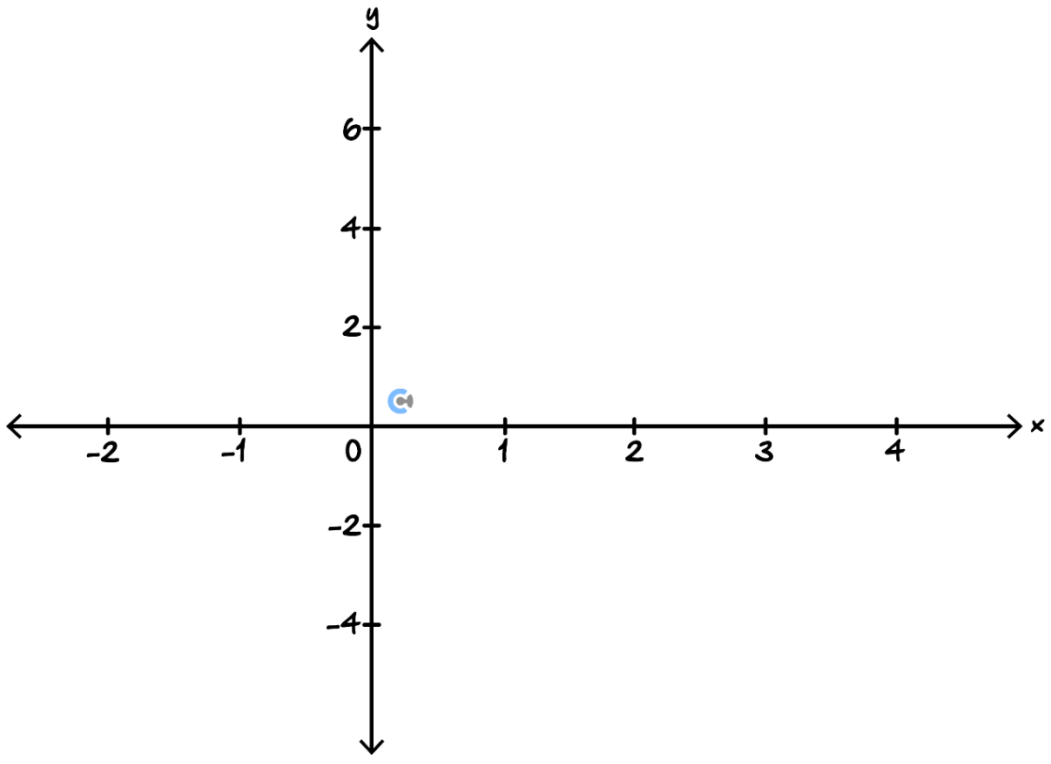
a. Sketch the graph of $y = x^3 + 2x^2 - 11x - 12$ on the axis below, the labelling axis intercepts with their coordinates.



b. Sketch the graph of $y = x^4 - 8x^2 + 16$ on the axis below, the labelling axis intercepts with their coordinates.

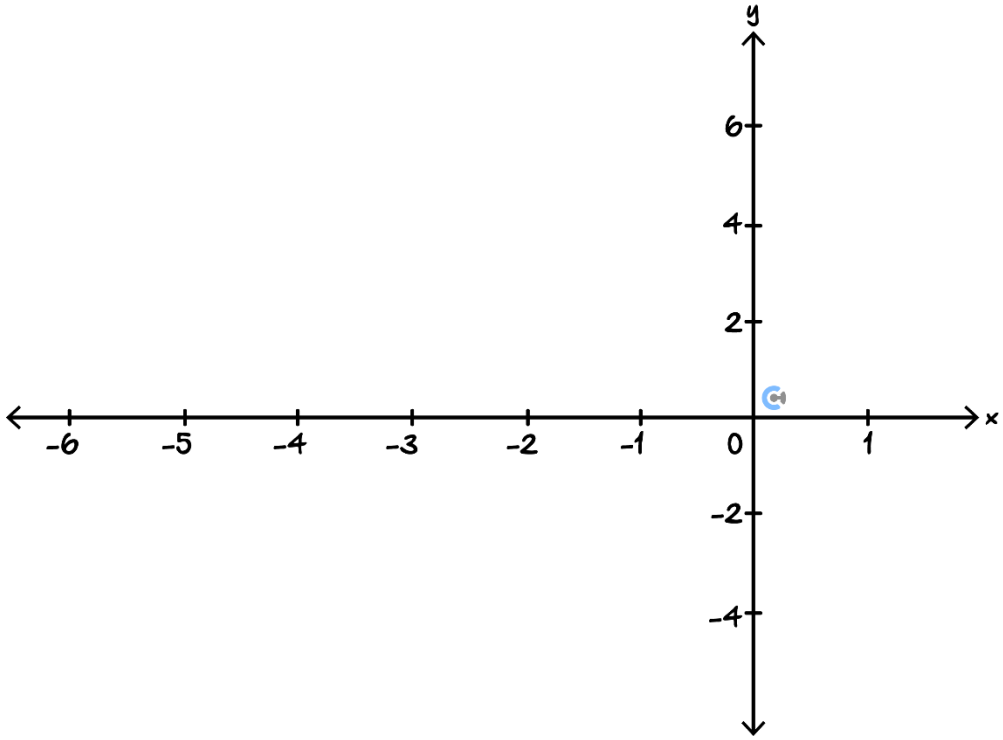


c. Sketch the graph of $y = x^3 - 4x^2 + x + 6$ on the axis below, the labelling axis intercepts with their coordinates.

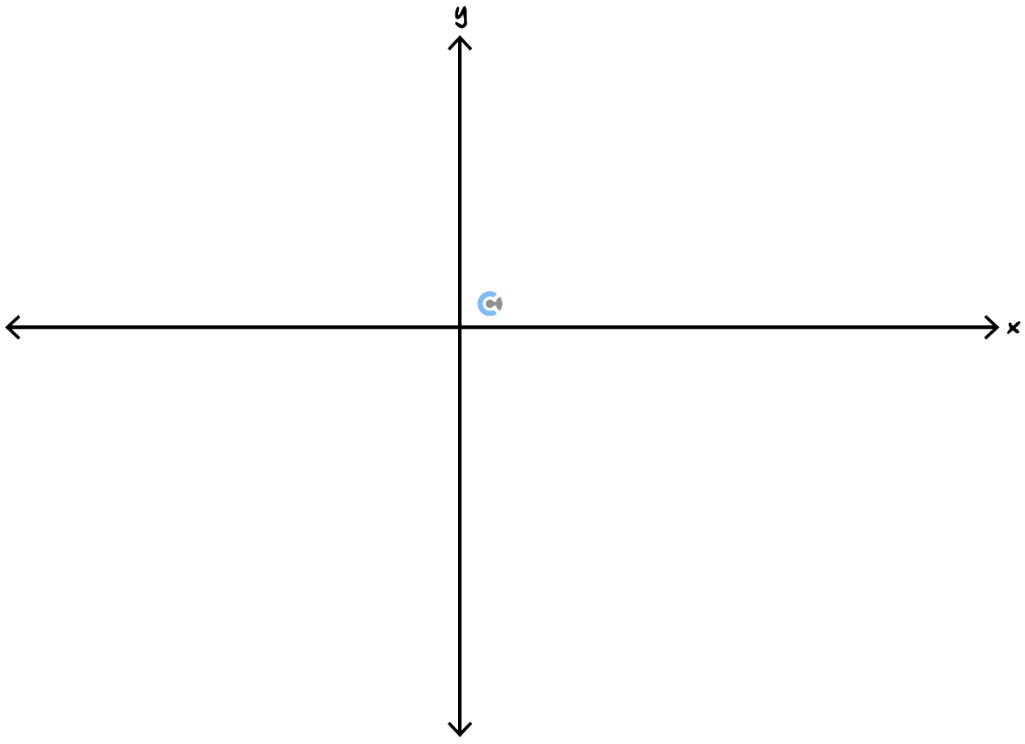


Question 141

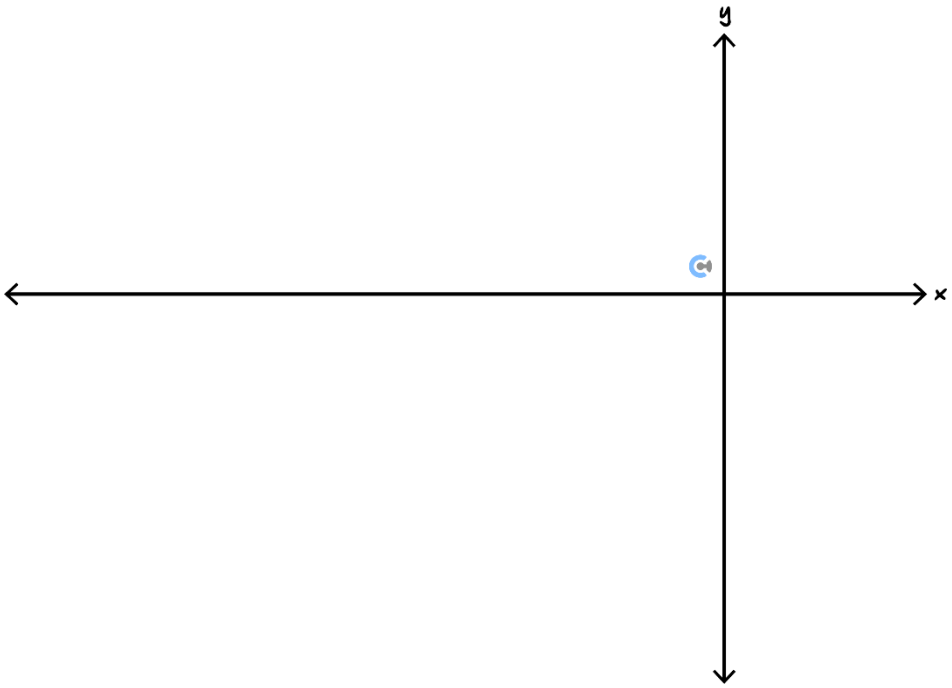
a. Sketch the graph of $y = x^3 + 8x^2 + 16x + 5$ on the axis below, the labelling axis intercepts with their coordinates.



b. Sketch the graph of $y = x^2(2x - 3)^3(x + 1)^2$ on the axis below, the labelling axis intercepts with their coordinates.

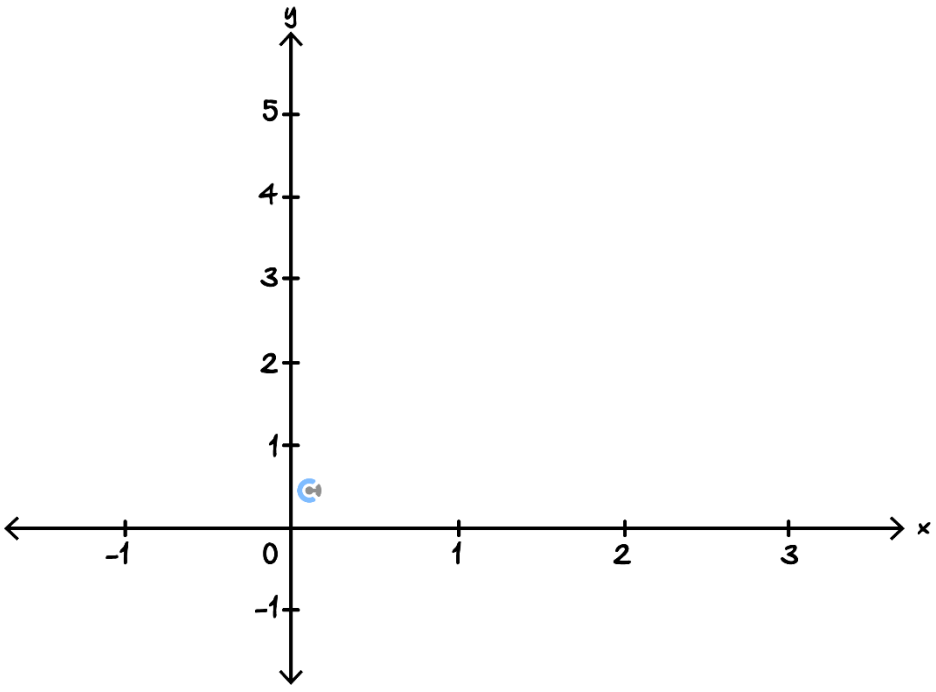


c. Sketch the graph of $y = x^4 + 5x^3 + 3x^2 - 7x - 2$ on the axis below, the labelling axis intercepts with their coordinates.



Question 142

Let $f_k(x) = x^4 - 4x^3 + 4x^2 + k$. By considering f_0 and f_{-1}, sketch the graph of f_2 on the axis below, the labelling axis intercepts and turning points with their coordinates.



Sub-Section [1.7.4]: Identify Odd and Even Functions

Question 143

a. Let $f(x)$ and $g(x)$ both be an odd function.

i. State whether $f(x) + g(x)$ is an even or an odd function.

_____

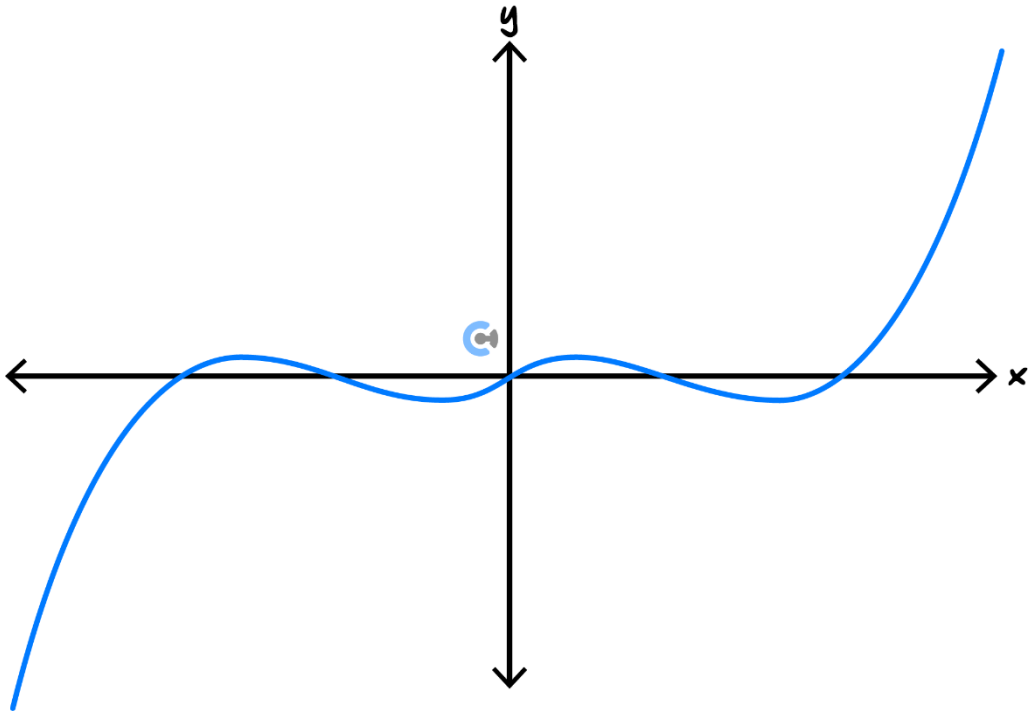
_____

ii. State whether $(f(x))^2 + 2f(x)g(x) + \big(g(x)\big)^2$ is an even or an odd function.

_____

_____

b. Part of the graph of $f(x)$ is drawn below. State whether f is an odd or an even function.



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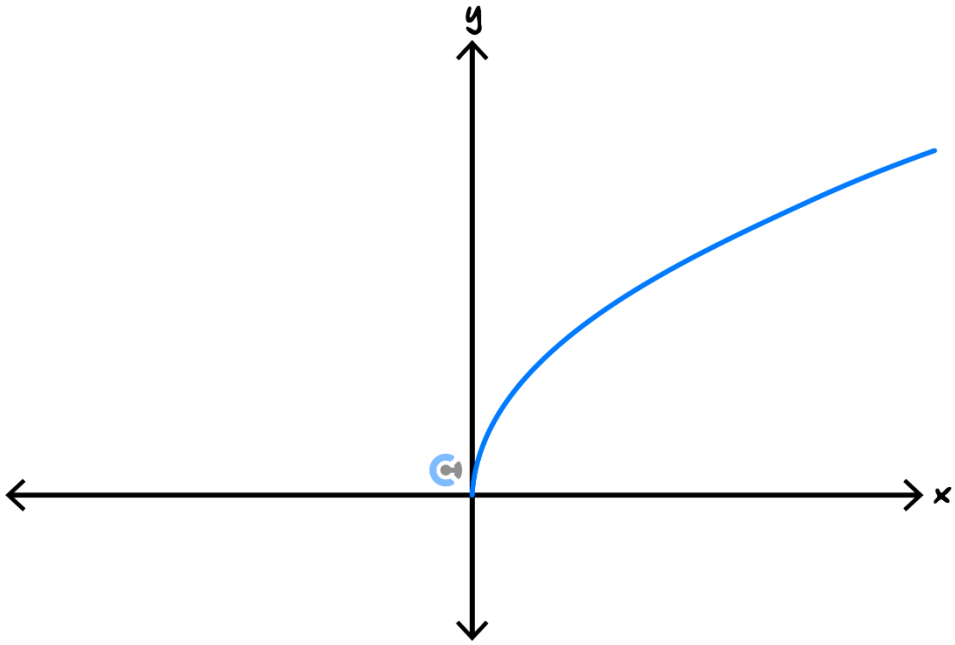
c. Part of the graph of $y = x^{\frac{m}{n}}$ is drawn below where m and n are co-prime.



State whether m and n are even or odd.

_____

_____

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Question 144

a. Show that $f(x) = x^4 - 2x^3$ is neither an even nor an odd function.

b. Describe a translation that maps the graph of $y = x^2 + 6x + 7$ onto the graph of an even function.

c. Consider the function $f(x)$. It is known that $f(2x + 3)$ is an odd function.

If $f(5) = 4$ and $f(-1) = -3$, find the value of $f(1)$.

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Question 145

a. Let $f(x)$ be a strictly increasing function with $f(0) = 0$.

If $\left(f(x)\right)^2$ is an even function, show that $f(x)$ is an odd function.

b. Let $f(x) = x^4 + 2x^3 + x^2$.

Describe a transformation that maps the graph of f onto the graph of an even function.

c. Let $f(x)$ be an even function.
The function,

$$g(x) = \begin{cases} f(x) + c & x \geq 0 \\ -f(x) + d & x < 0 \end{cases}$$

is an odd function.

Find the values of c and d.

Question 146

Let $f(x) = x^4 - 4x^3 + x^2 + 6x + k$, where k is a real number.
The function $g(x) = f(x - h)$ is an even function.
Find the value of h.

Section D: [1.8] – Polynomials (Checkpoints)

Sub-Section [1.8.1]: Apply Transformations to Restrict the Number of Positive/Negative x-Intercept(s)

Question 147

Let $f(x) = (x-1)(x+4)(x-2)^2$. Find the values of k such that $f(x+k)$ has no positive x-intercepts.

Question 148

Let $f(x) = x^3 - 2x^2 - 5x + 6$. Find the values of k such that $f(x+k)$ has exactly one negative x-intercept.

Question 149

Let $f(x) = 2x^2 - 15x + 14$ and $g(x) = x^2 - 10x + 8$. Find the values of k such that $f(x+k)$ and $g(x+k)$ have exactly two intersections with negative x-coordinates.

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Question 150

Let $f(x) = \frac{1}{2}x + 3$ and $g(x) = 2x^2 - 4x - 22$. Find the values of k such that $f\big(g(x+k)\big)$ has exactly one negative x-intercept.

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Sub-Section [1.8.2]: Apply Discriminant to Solve Number of Solutions Questions

Question 151

Find the values of k such that the equation $x^2 - 2^k x + 4$ has no solutions.

Question 152

Find the values of k such that the equation $x^2 - 2kx + 5k$ has exactly two solutions.

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Question 153

Find the values of k such that the equation $(x^2 - kx + 4)(x^2 - 2\sqrt{3}x + k) = 0$ has exactly three solutions.

Question 154

Let $f(x) = x^2 - 4x + 3$ and $g(x) = x^2 - 6x + k$. Find the values of k such that $f\big(g(x)\big)$ has exactly four solutions.

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Sub-Section [1.8.3]: Apply Shape/Graph to Solve Number of Solutions Questions

Question 155

Suppose $f(x) = x^2 - kx + 3$. Find the value of $k > 0$ so that $f(x) = k$ has exactly one solution.

Question 156

It is known that the quartic $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 8.5$ has turning points at $(1, -0.5), (2, 0.5)$ and $(3, -0.5)$. Find the values of k such that $f(x) = k$ has exactly two solutions.

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Question 157

It is known that the quartic $f(x) = x^4 - 4x^3 - 8x^2 + 48x + 3$ has turning points at $(-2, -77), (2, 51)$ and $(3, 48)$. Find the values of k such that $f(x) = k$ has exactly two solutions.

Question 158

Let $f(x) = x^4 - 16x^3 + 46x^2 - 48x + 20$ and $g(x) = -x^4 + 2x^2 + 3$. It is known that the quartic $h(x) = 2x^4 - 16x^3 + 44x^2 - 48x + 17$ has turning points at $(1, -1), (2,1)$ and $(3, -1)$. Hence or otherwise, find the value of k such that $f(x) = g(x) + k$ has exactly three solutions.

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Sub-Section [1.8.4]: Apply Odd and Even Functions

Question 159

Show that the function given by $f(x) = x^5 - 2x^2 + 1$ is neither even nor odd.

Question 160

Let $f(x) = x^4 - (k^2 - 5k + 6)x^3 + k^3x^2 + 10$. Find the value(s) of k so that $f(x)$ is an even function.

Question 161

The tangent to the graph of $f(x) = x^2 - 4$ at the point $x = 2$ is given by $h(x) = 4x - 8$. Denote the tangent to $f(x)$ at $x = -2$ by $k(x)$. Find the rule for $k(x)$ by applying a reflection to $h(x)$.

Question 162

The tangent to the graph of $f(x) = x^3 - 3x$ at the point $x = 2$ is given by $h(x) = 9x - 16$. Denote the tangent to $f(x)$ at $x = -2$ by $k(x)$. The rule for $k(x)$ can be obtained from the rule of $h(x)$ via the following sequence of transformations:

- A translation of a units in the positive direction of the x-axis.
- A translation of b units in the positive direction of the y-axis.

State the values of a and b and hence or otherwise, find the rule of $k(x)$.

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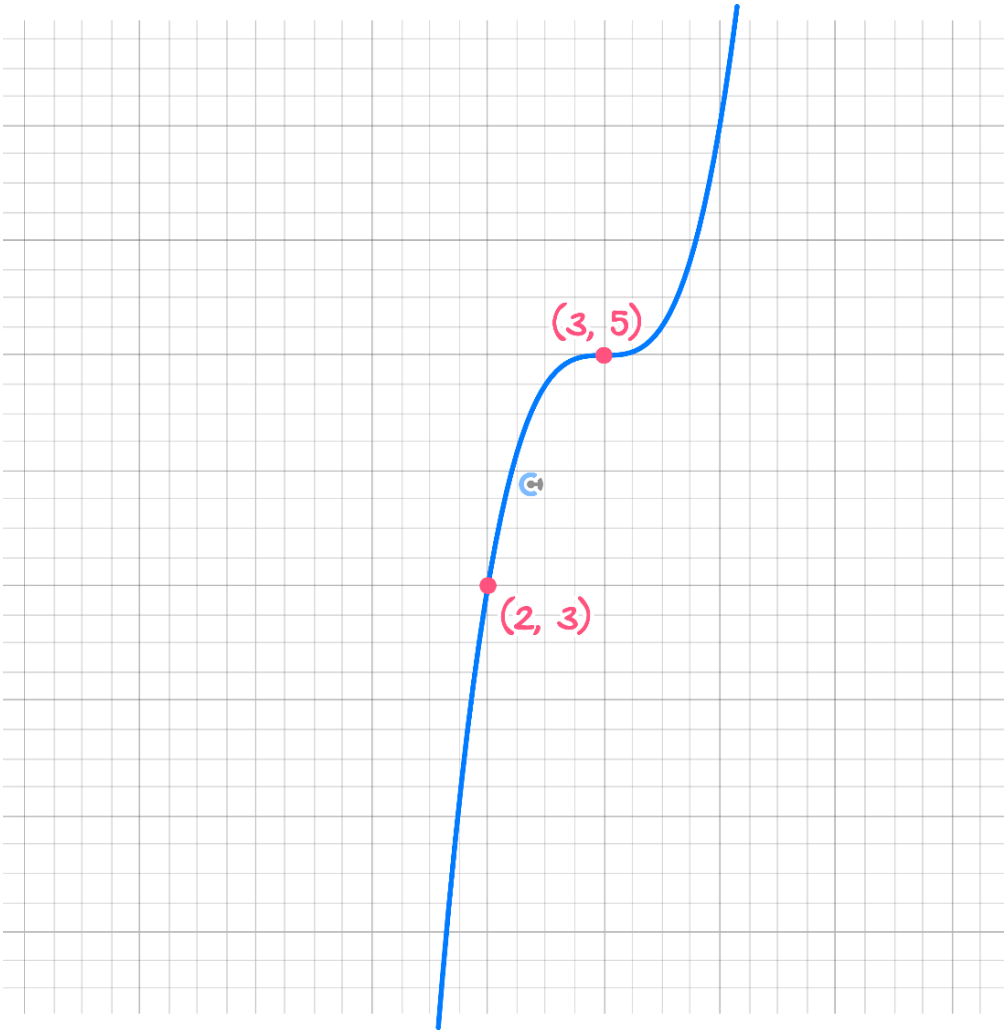
Sub-Section [1.8.5]: Identify Possible Rule(s) from a Graph

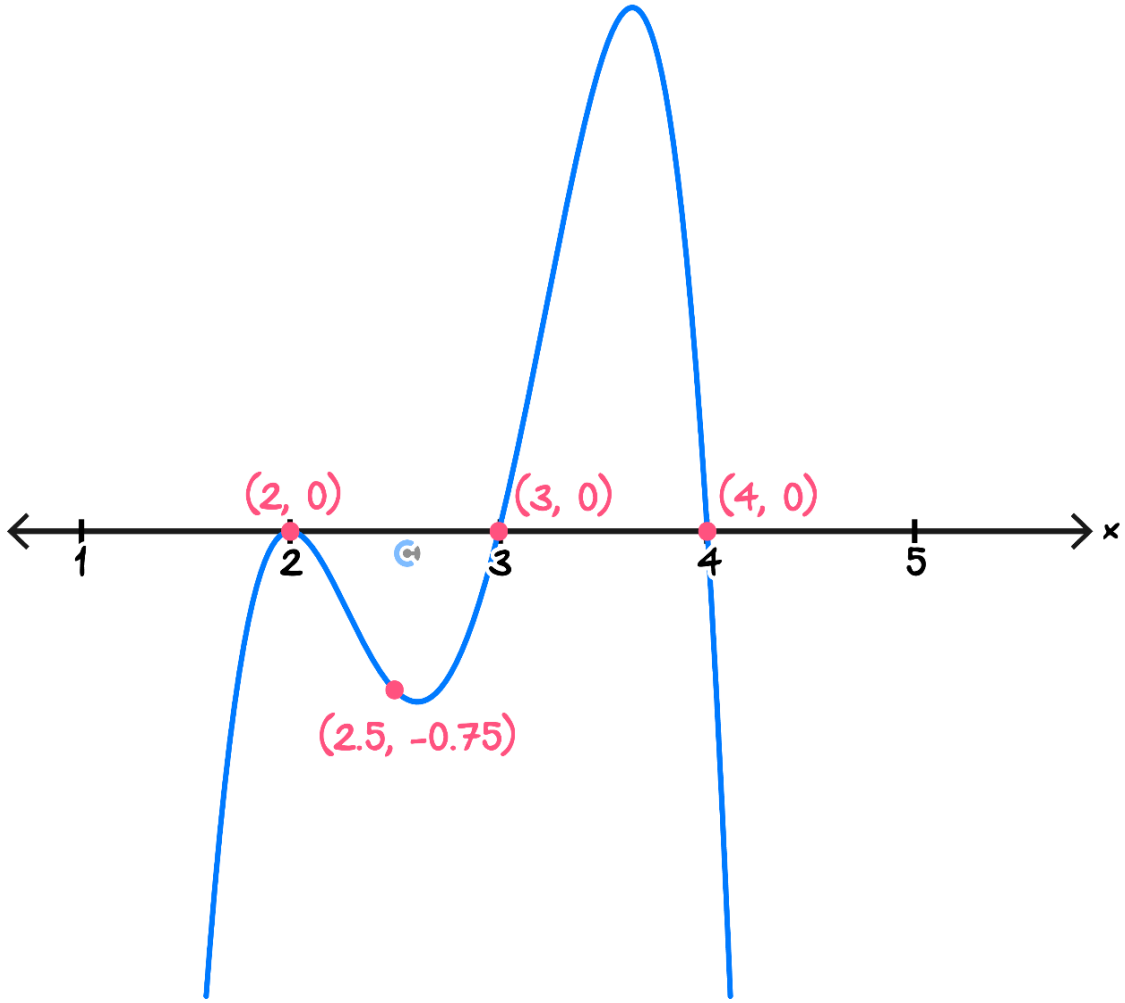
Question 163

Part of the graph of $f(x)$ is plotted below. The point $(3,5)$ is a stationary point of inflection. Find a possible rule for the function.



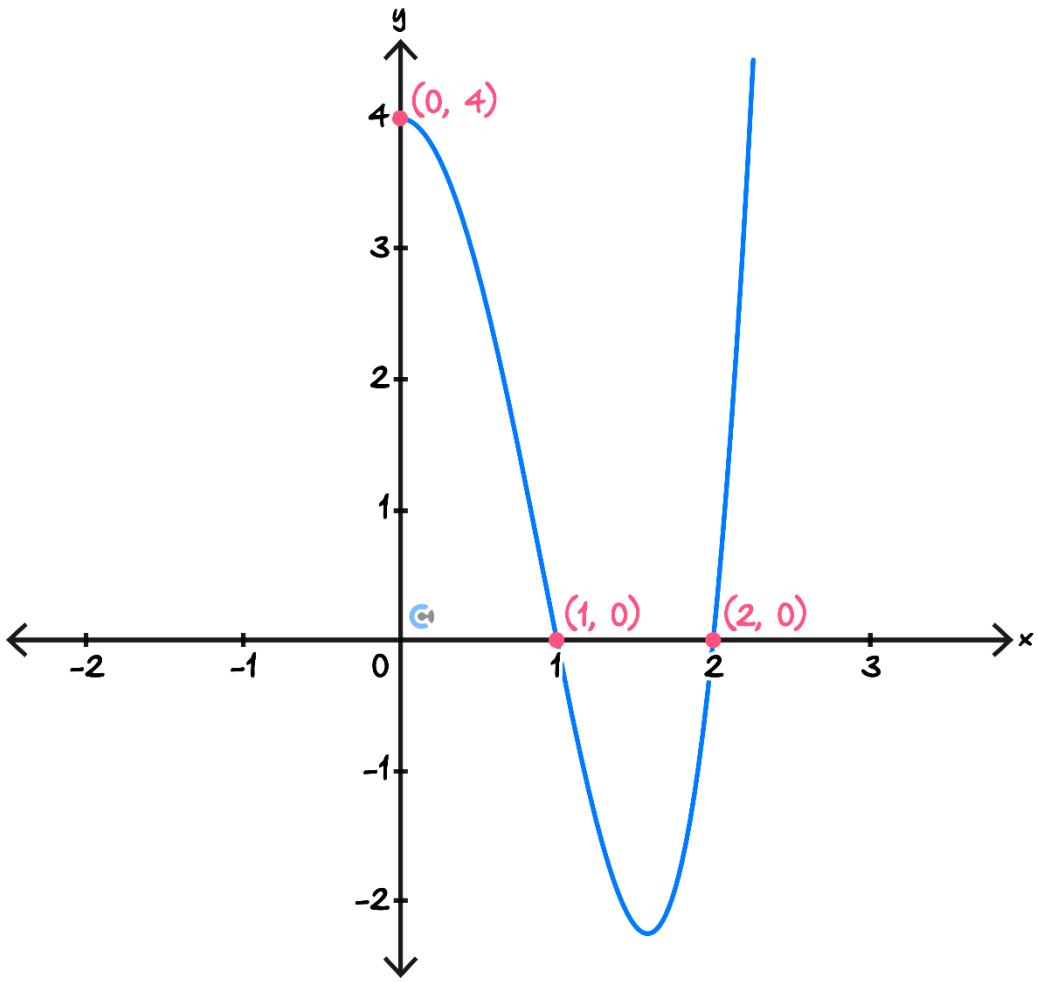
Question 164

Part of the graph of $f(x)$ is plotted below. Find a possible rule for the function.



Question 165

Part of the graph $f(x)$ is plotted below. Find a possible rule for the function if the function is known to be even.



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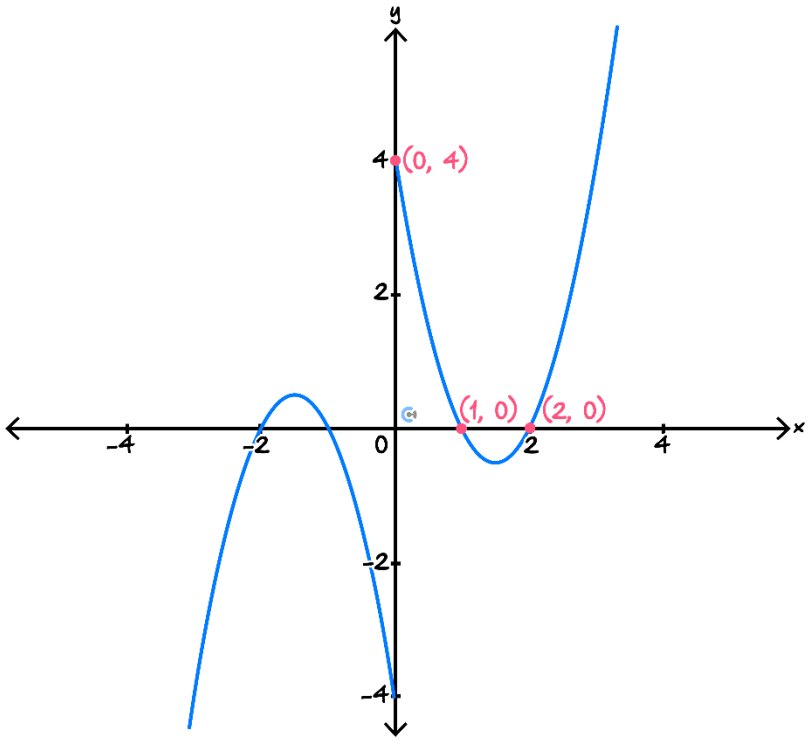
Question 166

Part of the graph $f(x)$ is plotted below.



Find a possible rule for the function if the function is known to be odd. Write your answer in the form.

$$f(x) = \begin{cases} f_1(x), & x < 0 \\ f_2(x), & x > 0 \end{cases}$$

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Sub-Section: Exam 1 Questions

Question 167

Find the value(s) of k so that the equation $(x^2 - kx + 16)(x^2 - 2\sqrt{7}x + k) = 0$ has:

a. Exactly one solution.

b. Exactly four solutions.

Question 168

Suppose that $f(x) = x^2 - 7x + 6$ and $g(x) = x^2 - kx + 1$. Find the values of k so that the equation $f\big(g(x)\big)$ has:

a. Exactly two solutions.

b. Exactly four solutions.

Question 169

Suppose that $f(x)$ is an odd function such that $f(x) = (x-2)^2$ for $x > 0$.

a. Write down a possible rule for $f(x)$ in the form:

$$f(x) = \begin{cases} f_1(x), & x < 0 \\ f_2(x), & x > 0 \end{cases}$$

b. It is known that the tangent to $f(x)$ at the point $x = 3$ is given by the rule $h(x) = 2x - 5$. By applying an appropriate sequence of transformations to $h(x)$, find the rule for the tangent at the point $x = -3$.

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Question 170

Consider a quartic of the form $f(x) = ax^4 + bx^3 + cx^2 + dx + e$. It is known that the quartic satisfies the following conditions:

- $f(1) = 0$
- $f(2) = 0$
- $f(0) = 4$
- Also, $f(x)$ is even.

a. Find the values of a, b, c, d and e.

b. Verify that $f(x)$ can be factorised to $(x - 1)(x + 1)(x - 2)(x + 2)$.

c. Find the values of k so that $f(x + k)$ has exactly two positive x-intercepts.

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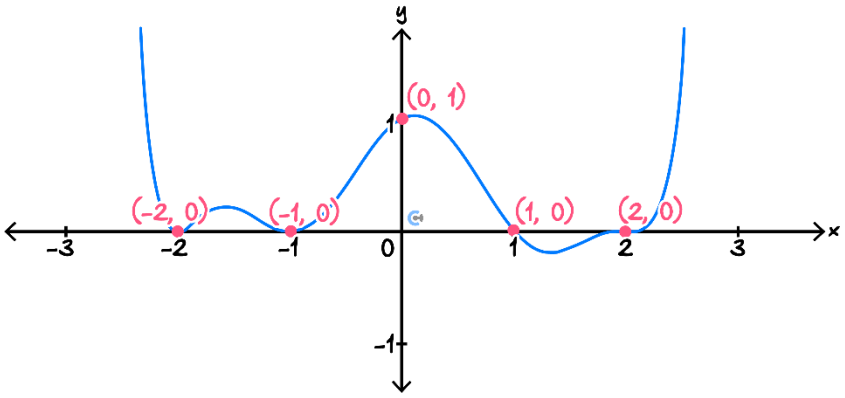
Sub-Section: Exam 2 Questions

Question 171

The minimum degree of the following polynomial is:



A. 2

B. 4

C. 6

D. 8

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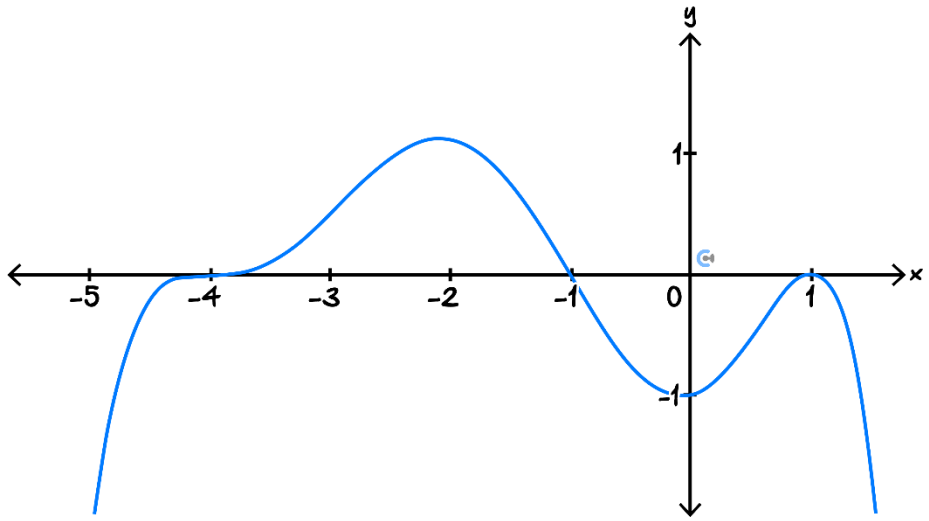
Question 172

A possible rule for the following function given below is:



A. $a(x-1)^3(x+4)^2(x+1)$ where $a<0$.

B. $a(x-1)^3(x+4)^2(x+1)^3$ where $a>0$.

C. $a(x-1)^2(x+4)^3(x+1)$ where $a<0$.

D. $a(x-1)\,(x+4)^3(x+1)$ where $a>0$.

Question 173

Let $f(x)=x^3-(k^2-5k+6)x^2-(k^3+5k)x$. If $f(x)$ is odd, then k must equal:

A. 1 or 3

B. 1 or 2

C. 2 or 3

D. 2 or 6

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Question 174

Let $g(x) = (x - 1)^2(x - 5)^2 - 4$. There will be exactly four solutions to the equation given by $g(x) = k$ whenever:

A. $-16 < k < 8$

B. $-4 < k < 12$

C. $-4 < k < 0$

D. $-4 < k < 16$

Question 175

Let $h(x) = x^4 - 10x^2 + 9$. The function $h(x + k)$ will have exactly three negative x-intercepts whenever:

A. $1 < k \leq 3$

B. $1 \leq k \leq 3$

C. $-3 < k \leq 1$

D. $-3 \leq k \leq 1$

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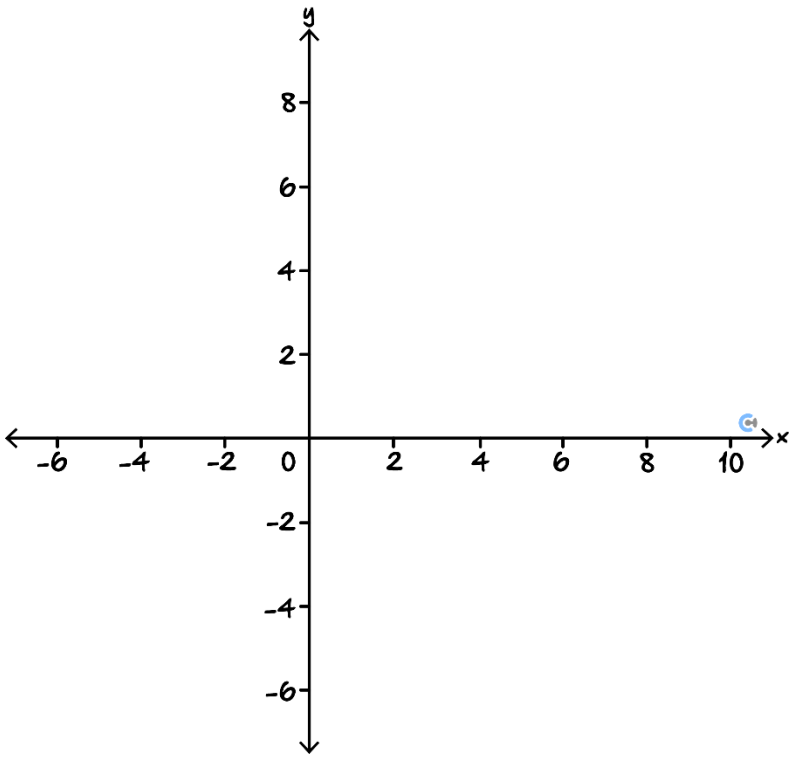
Question 176

Consider a cubic of the form $f(x) = ax^3 + bx^2 + cx + d$. Suppose that $f(x)$ satisfies the following conditions:

- $f(0) = 4$
- $f(1) = 0$
- $f(-2) = 0$
- $f(4) = 0$

a. Calculate the values of a, b, c and d.

b. Sketch the graph of the function $y = f(x)$, labelling all turning points and intercepts.



c. Find the value(s) of k such that $f(x) - k = 0$ has exactly:

i. 2 solutions.

ii. 3 solutions.

d. Let $g(x) = x^2 - kx + 5$. State the values of k such that $f\big(g(x)\big) = 0$ gives the maximum number of solutions possible.

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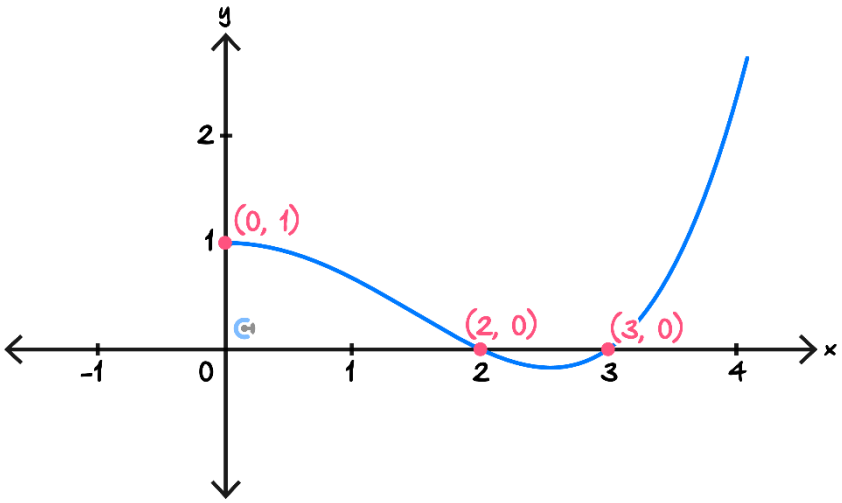
Question 177

The part of the graph of $f(x)$ is shown below. Furthermore, it is known that the function $f(x)$ is a quartic and also even.



a. State the rule for $f(x)$.

b. The tangent to the graph of $f(x)$ at $x = 3$ is given by $y = \frac{5}{6}x - \frac{5}{2}$.

i. Describe a sequence of transformation(s) that can be applied to $h(x)$ to obtain the tangent to the graph of $f(x)$ at $x = -3$.

ii. Hence, write down the rule for the tangent to the graph of $f(x)$ at $x = -3$.

c. State the values of k so that $f(x-k)$ has exactly:

i. 3 positive x-intercepts.

_____

_____

ii. 3 negative x-intercepts.

_____

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