Website: contoureducation.com.au | Phone: 1800 888 300
Email: hello@contoureducation.com.au

VCE Mathematical Methods ¾
AOS 1 Revision [1.0]
Contour Check (Part 1)

Contour Checklist

Section A: [1.1] - Functions and Relationships (Checkpoints)

Sub-Section [1.1.1]: Find the Maximal Domain and Range of Functions

Question 1

Find the maximal domain of the following functions:

a. $f(x) = \sqrt{x^2 + 1}$

b. $f(x) = \log_e(x + 4)$

c. $f(x) = \frac{1}{x+2} - 3$

Question 2

Find the maximal domain of the following functions:

a. $f(x) = \sqrt{(x + 1)^2 - 4}$

b. $f(x) = \log_e(4 - x^2)$

c. $f(x) = \frac{3+x^2}{x^2+5x+6}$

Question 3

Find the maximal domain of the following functions:

a. $f(x) = \cos(x)\log_e(2x) + \frac{1}{x^2-5}$

b. $f(x) = \sqrt{\frac{x-3}{x+1}}$

c. $f(x) = \frac{1}{2-x} \times \sqrt{x^2-4} \log_e(x^2-1)$

Question 4

Find the maximal domain and range of $f(x) = \frac{e^{2x}-1}{e^{2x}+1}$.

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Sub-Section [1.1.2]: Existence, Rule, Domain, and Range of Composite Functions

Question 5

The following functions are defined over their maximal domain:

$$f(x) = x^2 \text{ and } g(x) = 3 - x$$

a. Determine whether $f(g(x))$ and $g(f(x))$ exist.

b. Find the rule of any composition that exists.

c. State the domain of any composition that exists.

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Question 6

The following functions are defined over their maximal domain:

$$f(x) = e^{2x} \text{ and } g(x) = \log_e(2x)$$

a. Determine whether $f(g(x))$ and $g(f(x))$ exist.

b. Find the rule of any composition that exists.

c. State the domain of any composition that exists.

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Question 7

For the following functions:

$$f(x) = x^2 + 1 \text{ and } g(x) = \frac{1}{x^2-4}$$

a. Determine whether $f(g(x))$ and $g(f(x))$ exist.

b. Find the rule of any composition that exists.

c. State the domain of any composition that exists.

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Question 8

Functions are defined over their maximal domain unless specified otherwise.

For the functions f and g, determine whether $f(g(x))$ and $g(f(x))$ exist. State the rule and the domain of the composite function that do exist.

$$f(x) = e^x - e^{-x}$$

$$g(x) = \frac{1}{x(x-2)}$$

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Sub-Section [1.1.3]: Finding the Rule, Domain, and Range of Inverse Functions

Question 9

For the function:

$$f : (0, \infty) \to \mathbb{R}, f(x) = \log_e(3x)$$

a. Fully define the inverse function.

b. Find the range of the inverse function.

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Question 10

For the function:

$$f : (b, -\infty) \to \mathbb{R}, f(x) = \frac{1}{(x+2)^2} - 2$$

a. Find the largest value of b such that the inverse function exists.

b. Fully define the inverse function.

c. Find the range of the inverse function.

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Question 11

For the following functions:

$$f : (-\infty, k] \to \mathbb{R}, f(x) = 2x^2 - 8x + 4$$

a. Find the largest value of k such that the inverse function exists.

b. Fully define the inverse function.

c. Find the range of the inverse function.

d. Find the point of intersection between f and f^{-1}.

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Question 12

Find the inverse function of:

$$f(x) = e^{2x} + 4e^{x} + 1$$

And determine whether f and f^{-1} have any points of intersection.

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Sub-Section [1.1.4]: Finding the Composition of Inverse Functions

Question 13

Let $f: (3, \infty) \to \mathbb{R}, f(x) = x^2 - 4x + 7$.

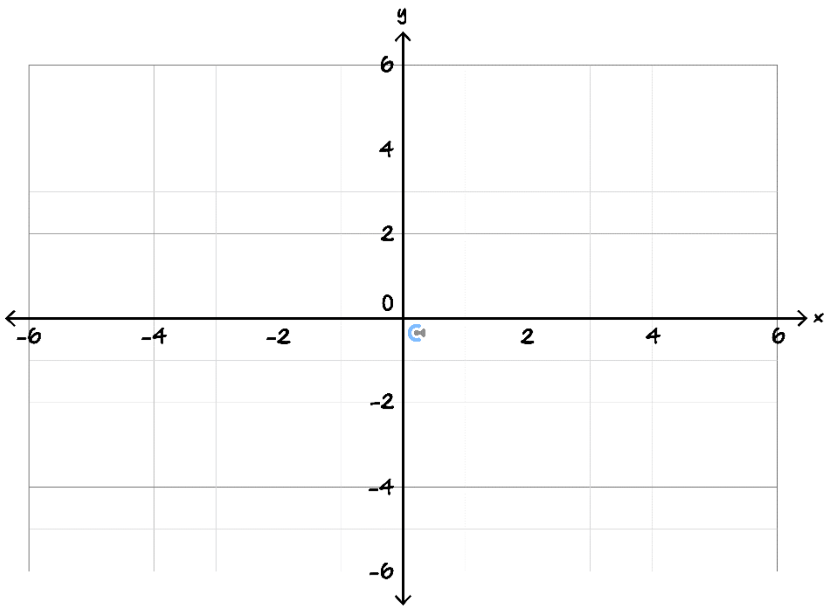
Find the rule and domain for $f^{-1}(f(x))$.

Question 14

Let $f: \mathbb{R} \setminus \{1\} \to \mathbb{R}, f(x) = \frac{5}{x-1} + 3$.

a. Find the rule and domain for $f^{-1}(f(x))$.

b. Sketch the graph of $f^{-1}(f(x))$ on the axis below.



Question 15

Let $f(x) = x^2 - 2kx + 9$, where $x \geq 0$ and $k \geq 0$.

The function $f^{-1} \circ f$ is defined on its maximal domain.

Find the rule and domain for $f^{-1}(f(x))$.

Question 16

Let $f^{-1}: \left[\frac{\pi}{2}, \pi\right] \to \mathbb{R}, f^{-1}(x) = \sin(x)$.

Define the function f and find the rule and domain for $f^{-1}(f(x))$.

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Section B: [1.2] - Functions and Relationships Exam Skills (Checkpoints)

Sub-Section: [1.2.1] - Finding a New Domain to Fix Composite Functions

Question 17

Consider the functions the following functions defined over their maximal domains:

$$f(x) = \log_e(x) \text{ and } g(x) = e^x - 1$$

a. Show that $f(g(x))$ does not exist.

b. Find the maximal domain of g such that $f(g(x))$ exists.

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Question 18

Consider the following functions defined over their maximal domains:

$$f(x) = (x^2 - 2)^2 \text{ and } g(x) = \sqrt{x-1}$$

Find the maximal domain of f such that $g(f(x))$ exists.

Space for Personal Notes

Question 19

Consider the following functions defined over their maximal domains:

$$f(x) = \frac{1}{1+x} \text{ and } g(x) = \sqrt{16 - (x-1)^2}$$

Find the maximal domain of f such that $g(f(x))$ exists.

Space for Personal Notes

Question 20

Consider the following functions:

$$f: [0, 2) \to \mathbb{R}, f(x) = \log_2(4 - x^2) \text{ and } g: (-\infty, 2) \to \mathbb{R}, g(x) = 3(x - 1)^2 - 1$$

Find the largest interval of x-values for which $f(g(x))$ and $g(f(x))$ both exist.

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Sub-Section: [1.2.2] - Finding the Range of Complex Composite Functions

Question 21

Find the range of $f(x) = e^{x^2+1}$.

Question 22

Find the range of $f: [0, \infty) \to \mathbb{R}, f(x) = \log_3(3^x + 8)$.

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Question 23

Find the range of $f(x) = \sqrt{\frac{x}{x+1}}$ where f is defined on its maximal domain.

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Question 24

Consider the following functions defined on all real numbers:

$$f(x) = \sin(x) \text{ and } g(x) = \log_3(4x^2 - 4x + 2)$$

Find the range of $g(f(x))$.

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Sub-Section: [1.2.3] - Finding the Gradient of Inverse Functions

Question 25

Consider the function $f: [0, \infty) \rightarrow \mathbb{R}, f(x) = x^2$.

The gradient of f at $x = a$ is $2a$.

Let g be the inverse function of f. Find the gradient of g when $x = 2$.

Question 26

Consider the one-to-one function f with the following properties:

$$f(2) = 5, f(5) = 7, f'(2) = 3 \text{ and } f'(5) = 1$$

Let g be the inverse function of f. Find the gradient of g when $x = 5$.

Question 27

Consider the function $f(x)$, the gradient of f at $x = a$ is $2f(a) + 2a$, and $f(0) = 1$.

From this information, we can tell that the gradient of f^{-1} at $x = b$ is c. Find b and c.

Question 28

Consider the differentiable, one-to-one, function $f: (0, 1) \to \mathbb{R}$. It is known that:

1. $f'(x) = -\left[f(x)\right]^2$, for all $x \in (0, 1)$.
2. $\text{ran } f = (1, \infty)$.

If g is the inverse function of f, find the domain and range of $g'(x)$.

Hint: $g'(a)$ denotes the gradient of g at $x = a$.

Sub-Section: Exam 1 Questions

Question 29

Let $f: [0, \infty) \to \mathbb{R}, f(x) = \sqrt{x+4}$.

a. State the range of f.

b. Let $g: (-\infty, c] \to \mathbb{R}, g(x) = x^2 + 6x + 7$, where $c < 0$.

Find the largest possible value of c such that the range of g is a subset of the domain of f.

c. For the value of c found in **part b.**, state the range of $f(g(x))$.

d. Let $h: \mathbb{R} \to \mathbb{R}, h(x) = x^2 + 5$.

State the range of $f(h(x))$.

Question 30

Let $f: (-2, \infty) \to \mathbb{R}, f(x) = 3 - \frac{4}{(x+2)^2}$.

State the rule and domain of f^{-1}.

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Question 31

a. Let $f: \mathbb{R}\backslash\{3\} \to \mathbb{R}$, $f(x) = \frac{1}{x-3}$. Find the rule for f^{-1}.

b. State the domain of f^{-1}.

c. Let $g(x) = f(x-c) + d$ for $c, d \in \mathbb{R}$.

Find the values of c and d, given that $g = f^{-1}$.

d. Given that $f'(1) = -\frac{1}{4}$ and $f'(4) = -1$, find the value of $g'(1)$.

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Question 32

Find the maximal domain of f, where $f(x) = \frac{1}{\sqrt{x^2-6x+5}}$.

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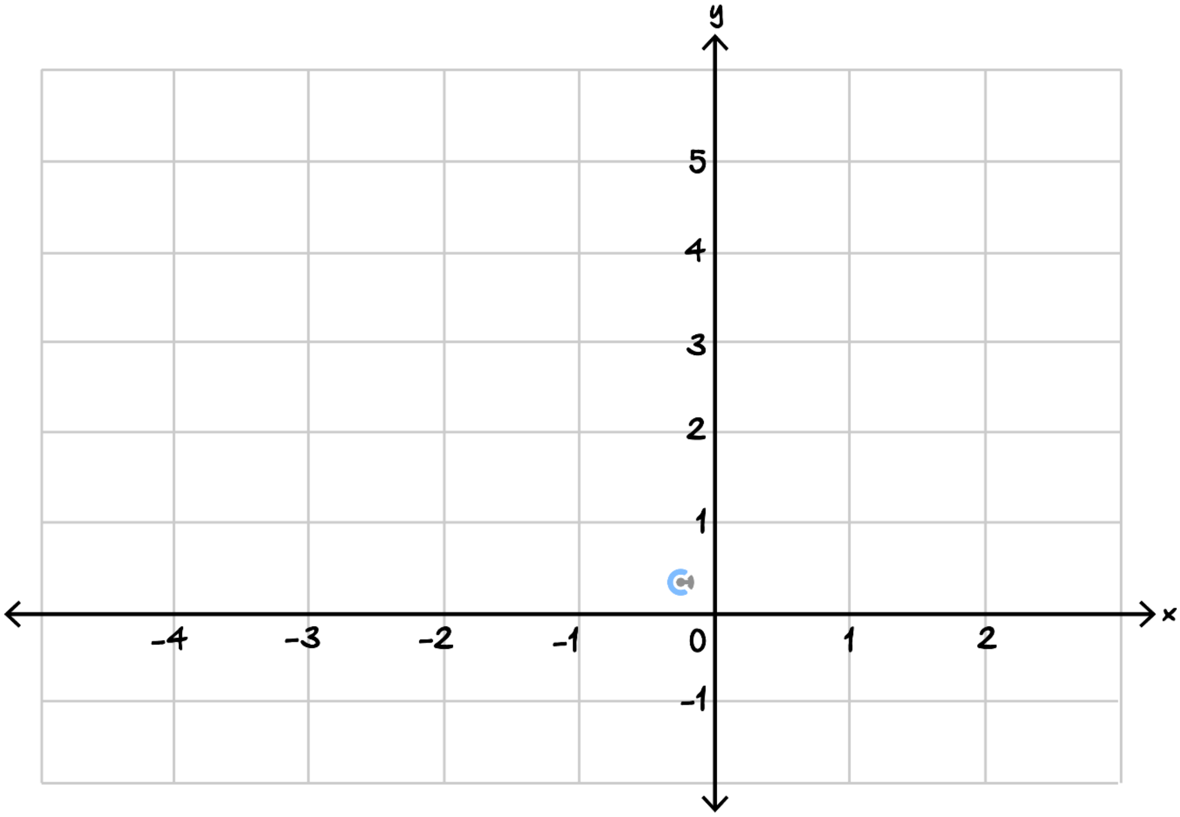
Question 33

a. Sketch the graph of $f(x) = 3 + \frac{1}{x+1}$ on the axes below, labelling all asymptotes with their equations and axial intercepts with their coordinates.



b. Find the values of x for which $f(x) \in (2, 4)$.

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Sub-Section: Exam 2 Questions

Question 34

Which one of the following is the inverse function of $g: (-\infty, 2] \to \mathbb{R}, g(x) = 4(x-2)^2 + 3$?

A. $f: [3, \infty) \to \mathbb{R}, f(x) = 2 + \frac{\sqrt{x-3}}{2}$

B. $f: [3, \infty) \to \mathbb{R}, f(x) = 2 - \frac{\sqrt{x-3}}{2}$

C. $f: [3, \infty) \to \mathbb{R}, f(x) = 4 + \frac{\sqrt{x-3}}{4}$

D. $f: [3, \infty) \to \mathbb{R}, f(x) = 4 - \frac{\sqrt{x-3}}{4}$

Question 35

The maximal domain of the function f is $(-\infty, 1 - \sqrt{5}\,] \cup [1 + \sqrt{5}, \infty)$.

A possible rule of f is:

A. $f(x) = \sqrt{5 - (x-1)^2}$

B. $f(x) = \log_e(5 - (x-1)^2)$

C. $f(x) = \frac{1}{\sqrt{5} - (x-1)^2}$

D. $f(x) = \frac{1}{\log_e(5 - (x-1)^2)}$

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Question 36

Let f be a one-to-one differentiable function and the following values are known:

$$f(-1) = 3, f(3) = 7, f'(-1) = 5 \text{ and } f'(3) = 2$$

Let $g(x) = f^{-1}(x)$, the value of $g'(3)$ is:

A. 5

B. 2

C. $\frac{1}{5}$

D. $\frac{1}{2}$

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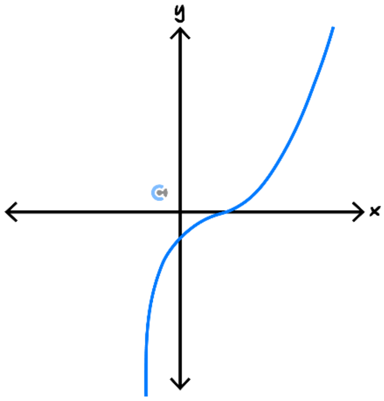
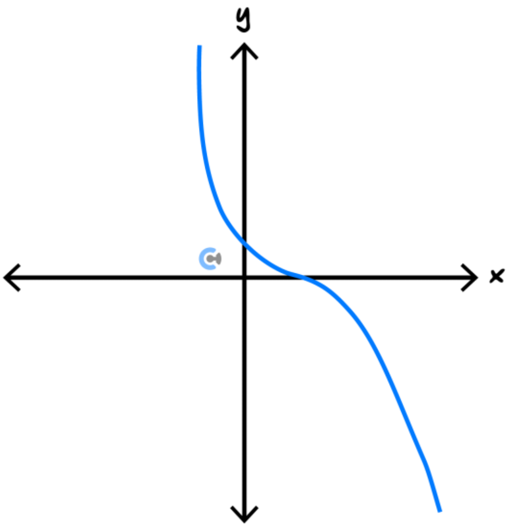
Question 37

Part of the graph of the function f is shown below. The same scale has been used on both axes.

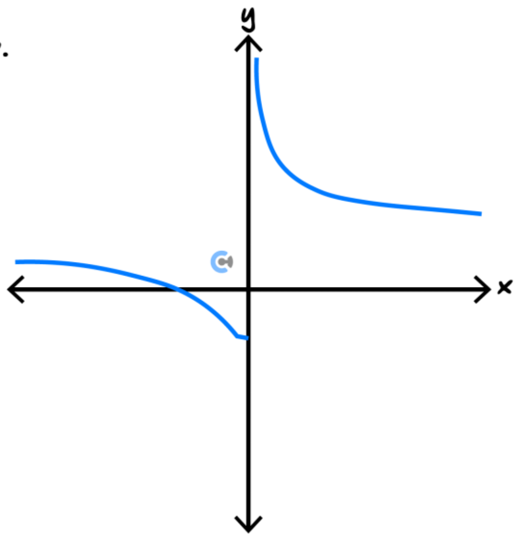


The corresponding part of the graph of the inverse function f^{-1} is best represented by:

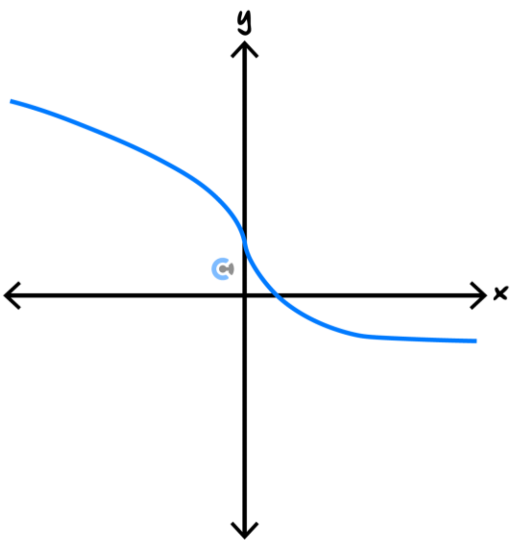
A.



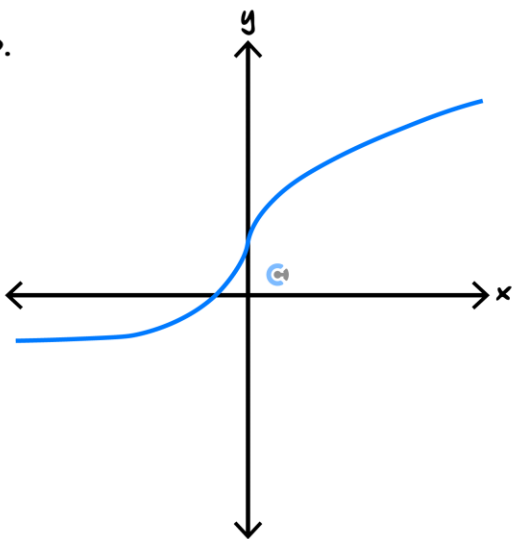
B.



C.



D.



Question 38

Consider the following functions:

$$f : \left(-\frac{\sqrt{3}}{2}, \infty\right) \to \mathbb{R}, f(x) = \log_e\left(x + \frac{\sqrt{3}}{2}\right)$$

$$g : (-\infty, 3) \to \mathbb{R}, g(x) = \cos(x)$$

The largest interval of x-values for which $f(g(x))$ and $g(f(x))$ both exist is:

A. $\left[-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right]$

B. $\left(-\frac{\sqrt{3}}{2}, \frac{5\pi}{6}\right)$

C. $\left(-\frac{5\pi}{6}, \frac{5\pi}{6}\right)$

D. $\left(-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right)$

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Question 39

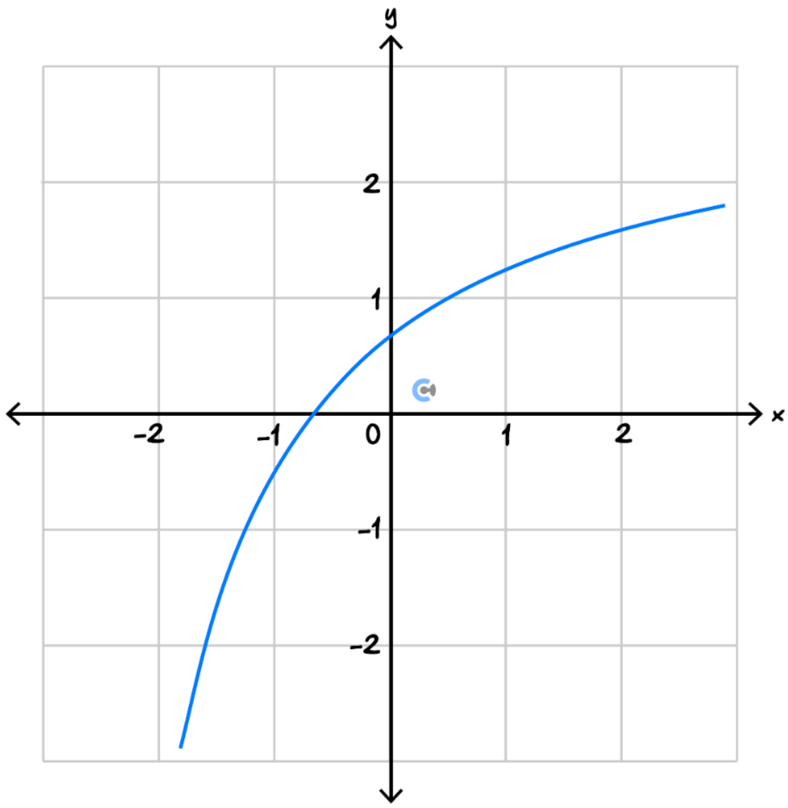
a. Express $\frac{3x+2}{x+3}$ in the form of $a + \frac{b}{x+2}$, where a and b are non-zero integers.

b. Let $f : \mathbb{R}\backslash\{-3\} \rightarrow \mathbb{R}, f(x) = \frac{3x+2}{x+3}$.

i. Find the rule and domain of f^{-1} and the inverse function of f.

ii. Part of the graph of f is shown in the diagram below.

Sketch the graph of $y = f^{-1}$, labelling all points of intersection with their coordinates.



c. Let $g(x) = -\sqrt{16 - x^2}$.

i. Show that both $f(g(x))$ and $g(f(x))$ do not exist.

ii. Find the largest interval on which both $f(g(x))$ and $g(f(x))$ are defined on.

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Question 40

Let $f(x) = 2^{-x}$ and $g(x) = 4x^2 - 4x + 3$.

a.

i. State the rule of $f(g(x))$.

ii. State the range of $f(g(x))$.

b. Let $h: [a, \infty) \to \mathbb{R}, h(x) = g(f(x))$. Find the smallest value of a such that h is a one-to-one function.

c. For the value of a found in **part b.**, state the rule and domain for h^{-1}.

d. How many solutions does the equation $f(g(x)) + g(f(x)) = 0$ have?

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Section C: [1.3] - Transformations (Checkpoints)

Sub-Section [1.3.1]: Applying Transformations to Points

Question 41

Consider the following transformations of the plane:

- S, a dilation by a factor of 2 from the y-axis, followed by a translation of 3 units up.
- T, a translation of 2 units left and 1 unit up.
- W, a reflection in the line $y = x$.

a. Find $S(x, y)$.

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b. Find $T(x, y) = (x', y')$. Express x and y in terms of x' and y'.

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c. Find $W(x, y)$.

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Question 42

Consider the following transformation, $T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (-2x + 4, 5(y + 3))$.

T can be described using the following sequence of transformations:

- A dilation by a factor of a from the x-axis, followed by,
- A dilation by a factor of b from the y-axis, followed by,
- A reflection in the y-axis, followed by,
- A translation c units in the positive direction of the x-axis, followed by,
- A translation of d units in the positive direction of the y-axis.

a. Find a, b, c, and d.

b. Describe T as a sequence of two translations, followed by two dilations, and a reflection.

c. The image of $(p, -5)$ under T is $(2, q)$. Find p and q.

Question 43

Consider the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ described by the following sequence of transformations:

- A dilation by a factor of $\frac{1}{5}$ from the x-axis, followed by,
- A translation of 2 units in the positive direction of the x-axis, followed by,
- A reflection in the y-axis, followed by,
- A translation of 3 units in the positive direction of the x-axis, followed by,
- A translation of 5 units in the negative direction of the y-axis, followed by,
- A dilation by a factor of 5 from the x-axis, followed by,
- A reflection in the x-axis, followed by,
- A dilation by a factor of 3 from the y-axis.

a. Find (x', y'), the image of (x, y) under T.

b. Express x in terms of x' and y in terms of y'.

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c. A transformation $S : \mathbb{R}^2 \to \mathbb{R}^2$ maps $T(x, y) = (x', y')$ to (x, y).

Describe S as a sequence of 2 translations followed by 2 reflections followed by a dilation.

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Question 44

a. Describe a reflection in the line $y = x + b$ using elementary transformations.

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A reflection in the line $y = ax$ can be described via the following transformation:

$$T(x, y) = \left(\frac{x(1-a^2)+2ay}{1+a^2}, \frac{y(a^2-1)+2ax}{1+a^2}\right).$$

b. Describe a reflection in the line $y = ax + b$ using elementary transformations and T.

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c. Find the image of the point $(2, 4)$ when it is reflected in the line $y = 3x + 5$.

d. Show using coordinate geometry that T describes a reflection in the line $y = ax$.

Hint: Find the line going through a point (x_0, y_0) with a gradient $-\frac{1}{a}$.

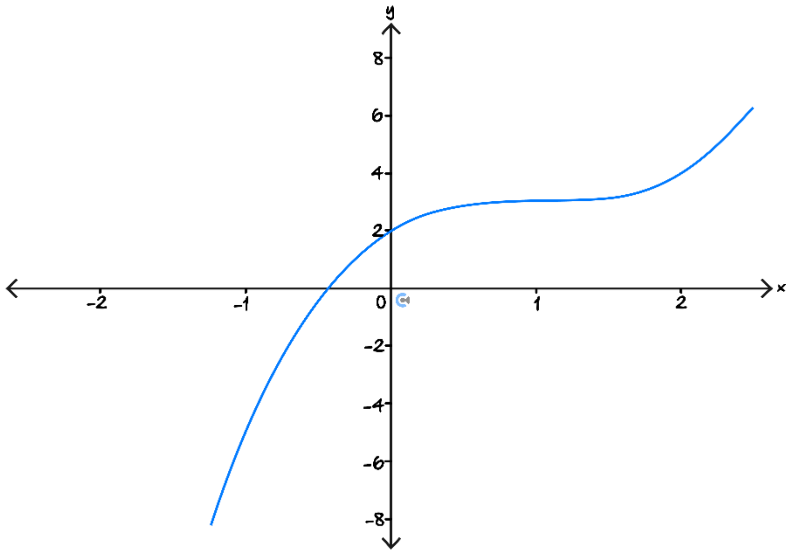
Then, equate that line to $y = ax$ to get a point (x_1, y_1).

Then, (x_1, y_1) is the midpoint of (x_0, y_0) and $(x_0', y_0') = T(x_0, y_0)$.

Sub-Section [1.3.2]: Transforming Graphs of Functions

Question 45

a. The graph of $f(x)$ is shown below.



On the same axes, sketch the graph of $g(x) = f(-2x)$.

b. Let $f(x) = e^x$. The transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, $T(x, y) = (x - 1, y + 2)$ maps the graph of $f(x)$ onto the graph of $g(x)$. Find the rule for $g(x)$.

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c. Find the rule for the image of the graph of $y = \cos(x)$ under the transformation,

$$S = \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = \left(-3x, \frac{1}{2}y\right).$$

Question 46

a. Let $f(x) = 5\sqrt{x} - 3$. The transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$, $T(x, y) = (4x, 3 - y)$ maps the graph of $f(x)$ onto the graph of $g(x)$. Find the rule for $g(x)$.

b. Find the rule for the image of the graph of $y = e^{x+2} - \log_e(-2x)$ under the transformation,

$$S : \mathbb{R}^2 \rightarrow \mathbb{R}^2, T(x, y) = (-2x - 1, y + 3).$$

c. Let $f(x) = (x - 1)(x + 2)(x - 3)$, and $g(x) = 4f(2 - x) + 5$.

Solve $g(x) = 5$.

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Question 47

a. Consider the transformation, $T : \mathbb{R}^2 \to \mathbb{R}^2$ which can be described by the following sequence of transformations:

- A translation is 3 units up and 2 units left, followed by,
- A dilation by a factor of 3 from the x-axis and $\frac{1}{2}$ from the y-axis followed by,
- A reflection in the x-axis.

T maps the graph of $f(x)$ onto the graph of $g(x) = \log_e(x)$. Find the rule of $f(x)$.

b. Consider the transformation $S : \mathbb{R}^2 \to \mathbb{R}^2$, which the following sequence of transformations can describe:

- A dilation by a factor of 2 from the x-axis and 5 from the y-axis, followed by,
- A translation 1 unit down and 4 units right.

Find the rule for the image of the graph of $y = 25x^2 + 5x - 1$ under S.

c. A transformation $U : \mathbb{R}^2 \to \mathbb{R}^2$, $U(x, y) = (2x + 5, 3 - 2y)$ maps the graph of $y = af(x) + b$ onto the graph of $y = f(cx + d)$. Find the values of a, b, c, and d.

Space for Personal Notes

Question 48

Consider the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, which is described by the following sequence of transformations:

- A translation of 3 units upwards and 5 units left, followed by,
- A reflection in the line $y = x$, followed by,
- A dilation by a factor of $\frac{1}{2}$ from the x-axis and $\frac{1}{4}$ from the y-axis, followed by,
- A reflection in the x-axis.

T maps the graph of $f : (-\infty, 2], f(x) = 3x^2 + 12x + 5$ onto the graph of g.

Find the rule of g.

Sub-Section [1.3.3]: Find Transformations from Transformed Function

Question 49

a. Let $f(x) = x^2$ and $g(x) = 3x^2 - 2$.

Describe a transformation that maps the graph of f onto the graph of g.

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b. The transformation $T : \mathbb{R}^2 \to \mathbb{R}^2, (x, y) \mapsto (ax + b, cx + d)$ maps the graph of $y = \log_e(x)$ to the graph of $y = 5 - \log_e(2x + 3)$.

Find the values of a, b, c, and d.

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c. A transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ maps the graph of $f(x) = \sqrt{x}$ onto the graph of $g(x) = 3\sqrt{x-1} + 5$.

T can be described by the following sequence of transformations,

- A dilation by a factor of ________ from the x-axis, followed by
- A translation of ________ unit(s) in the positive direction of the x-axis, followed by
- A translation of ________ unit(s) in the positive direction of the y-axis.

Fill in the blanks.

Question 50

a. Let $f(x) = 4(x-5)^2$.

The transformations:

$$S : \mathbb{R}^2 \to \mathbb{R}^2, (x, y) \mapsto (x + b, ay),$$

and

$$T : \mathbb{R}^2 \to \mathbb{R}^2, (x, y) \mapsto (cx + d, y).$$

Both map the graph of $y = x^2$ onto the graph of f.

Find the values of a, b, c, and d.

b. Consider a function $f : [0, \infty) \to \mathbb{R}, f(x) = 100 - 4x$.

A different function g has the property, that g decreases at half the rate of f at any point in time and that $g(0) = f(0)$. State a single transformation that maps the graph of f onto the graph of g.

c. Let $g(x) = -\frac{f(4x+12)}{5} - 20$.

Fill in the blank lines to make the following sequences of transformations map the graph of $f(x)$ onto the graph of $g(x)$.

- A dilation by a factor of ________ from the x-axis, followed by,
- A translation of ________ units in the positive direction of the x-axis, followed by,
- A translation of ________ units in the positive direction of the y-axis, followed by,
- A dilation by a factor of ________ from the y-axis, followed by,
- A reflection in the x-axis.

Question 51

a. Describe a sequence of three transformations that map the graph of $f(x) = \sqrt{7 - 6x - x^2}$ onto the graph of $g(x) = \sqrt{4 - x^2}$.

b. Let $f : [2, \infty) \to \mathbb{R}$, $f(x) = \sqrt{4x^2 - 16x + 16}$.

A transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, $(x, y) \mapsto (ax + b, y)$ maps the graph of $f(x)$ onto the graph of $g : [0, \infty) \to \mathbb{R}, g(x) = x$.

Find the values of a and b.

c. A function f has its only stationary point at $(2, 3)$ and its only x-axis intercept at $(-5, 0)$.

A function g has its only stationary point at $(6, -2)$ and only x-axis intercept at $(-8, 0)$.

A transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, $T(x, y) = (ax + b, cy)$ maps the graph of f onto the graph of g.

Find a, b, and c.

Question 52 Tech-Active.

Let $f(x) = x^4 + x^3 + x^2 + x + 1$ and $g(x) = x^4 + 2x^3 + 4x^2 + 8x + 11$.

A transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, $T(x, y) = (ax + b, cx + d)$ maps the graph of f onto the graph of g.

Find a, b, c, d and show that they are unique.

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Section D: [1.4] – Transformations Exam Skills (Checkpoints)

Sub-Section [1.4.1]: Apply Quick Method to Find Transformations

Question 53

Find the image of the graph of $y = x^2$ under the transformation, $T : \mathbb{R}^2 \to \mathbb{R}^2$, $T(x, y) = (1 - 2x, y + 5)$.

Question 54

Describe a sequence of transformations that maps the graph of $y = x^3$ onto the graph of $y = 2(3x + 2)^3 - 3$.

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Question 55

Find the image of the graph of $y = 2\log_2(x) - 3$ under the following sequence of transformations:

- A dilation by a factor of 3 from the x-axis, followed by,
- A translation of 2 units left and 3 units up, followed by,
- A reflection in the y-axis, followed by,
- A dilation by a factor of 5 from the y-axis.

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Question 56

Consider four linear functions, $p_1(x)$, $p_2(x)$, $q_1(x)$, and $q_2(x)$.

A transformation,

$$T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (x', y')$$

maps the graph of $y = f(x)$ onto the graph of $y = (p_1 \circ p_2 \circ f \circ q_2 \circ q_1)(x)$. Express x' in terms of x and y' in terms of y.

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Sub-Section [1.4.2]: Apply Transformations of Functions to Find its Domain and Range

Question 57

The function $f : \mathbb{R} \to \mathbb{R}$ has a range of $[2, \infty)$.

The transformation, $T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (5 - 2x, 3 + y)$ maps the graph of f onto the graph of g. State the domain and range of g.

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Question 58

The function $f : (-\infty, -1] \to \mathbb{R}$ has a range of $[-2, \infty)$.

Describe a sequence of transformations that maps the graph of f onto a graph of a function with a domain of $[0, \infty)$ and a range of $(-\infty, 2]$.

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Question 59

Consider the function, $f : \mathbb{R}\backslash\{-2\} \to \mathbb{R}, f(x) = \frac{3}{(x+2)^2} - 5$.

The following sequence of transformations maps the graph of f onto the graph of g:

- A reflection in the x-axis, followed by,
- A dilation by a factor of 3 from the x-axis, followed by,
- A dilation by a factor of $\frac{1}{2}$ from the y-axis, followed by,
- A translation of 3 units up and 2 units left.

State the domain and range of g.

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Question 60

Let $f : (-2, 1] \to \mathbb{R}, f(x) = 2(x+1)^2 - 3$.

Consider the transformation, $T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (ax + b, cy + d)$ where a and c are both non-zero.

The transformation T maps the graph of f onto the graph of g.

a. Explain why the range of g will always be of the form $[p, q]$ for some real $p < q$.

b. Explain why the domain of g will always be of the form $(p, q]$ or $[p, q)$ for some real $p < q$.

c. For what values of a, is the domain of g of the form $(p, q]$?

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Sub-Section [1.4.3]: Apply Transformations of Functions to Find Transformed Points and Tangents

Question 61

The equation of the tangent to the graph of $f(x)$ at the point $(1, 3)$ is $y = 2x + 1$.

The transformation, $T(x, y) = \left(x, \frac{y}{3} + 1\right)$ maps the graph of f onto the graph of g.

Find the equation of the tangent to the graph of g at the point $(1, 2)$.

Question 62

The points $(2, 4)$ and $(4, 7)$ lie on the graph of $f(x)$.

Evaluate $g(2)$, where $g(x) = 3f(6 - x) + 5$.

Question 63

Consider the transformation, $T : \mathbb{R}^2 \to \mathbb{R}^2$ described by the following sequence of transformations:

- A dilation by a factor of 2 from the x-axis, followed by,
- A translation by a factor of 4 in the negative direction of the x-axis, followed by,
- A dilation by a factor of $\frac{1}{3}$ from the y-axis, followed by,
- A translation by a factor of 5 in the positive direction of the y-axis.

The image of $A(u, v)$ under T is $(3, 7)$. Find the values of u and v.

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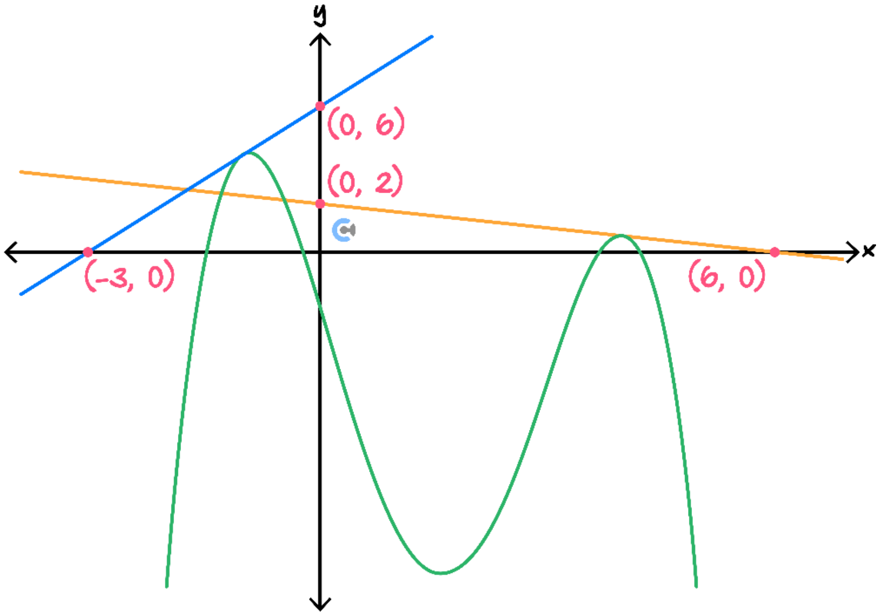
Question 64

The graph of $y = f(x)$ is drawn below along with two tangents at $x = 4$ and at $x = -1$.



Find the equation of the tangent to the graph of $g(x) = 1 - 3f(2 - 2x)$ when $x = -1$.

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Sub-Section [1.4.4]: Find Transformations with Constraints

Question 65

Consider the transformation, $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by the following sequence of transformations:

- A dilation by a factor of a from the x-axis.
- A translation by a factor of b in the positive direction of the y-axis.

T maps the graph of $f(x) = \sqrt{x}$ onto the graph of $g(x) = \sqrt{9x} + 6$.

Find the values of a and b.

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Question 66

The transformation, $T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (ax + b, y + c)$ maps the graph of $y = 2^x$ onto the graph of $y = 8 \times 2^{3x-1} - 5$.

Find the values of a, b, and c.

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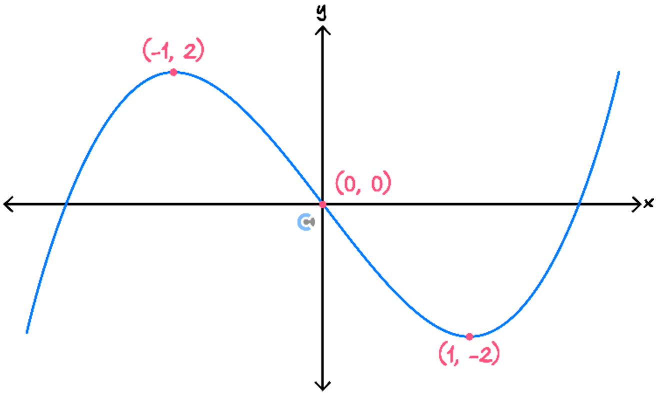
Question 67

The graph of $y = x^3 - 3x$ is drawn below.



The transformation,

$$T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (a - x, b - y)$$

Maps the graph of $y = x^3 - 3x$ onto the graph of $y = (x - 1)^3 - 3x + 5$.

Find the values of a and b.

Question 68

Consider the functions:

$$f : [-1, \infty) \to \mathbb{R}, f(x) = x^2 + 2x + 2$$

$$g : (-\infty, 1] \to \mathbb{R}, g(x) = 4(2x - 1)^2 + 3$$

Describe a sequence of a dilation followed by two translations and lastly a reflection that maps the graph of f onto the graph of g.

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Sub-Section [1.4.5]: Find Transformations of the Inverse Functions

Question 69

Consider the function, $f : \mathbb{R}\backslash\{1\} \to \mathbb{R}, f(x) = \frac{2}{x-1} + 4$.

The transformation, $T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (x + a, y + b)$ maps the graph of f onto the graph of its inverse function. Find the values of a and b.

Question 70

Consider the one-to-one functions, $f(x)$ and $g(x)$. The transformation $T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (3 - x, 2y + 7)$ maps the graph of f onto the graph of g.

Describe a sequence of transformations that maps the graph of f^{-1} onto the graph of g^{-1}.

Question 71

Let $f:(-\infty,2]\to\mathbb{R}, f(x)=3x^2-12x+11$ and $g:[-3,\infty)\to\mathbb{R}, g(x)=2\sqrt{x+3}+4$.

a. Describe a sequence of transformations that maps the graph of f onto the graph of g^{-1}.

b. Hence, or otherwise, describe a sequence of transformations that maps the graph of g onto the graph of f^{-1}.

Question 72

Consider the function f which has the property that $f(x-3)-3=f^{-1}(x)$.

The transformation, $T:\mathbb{R}^2 \to \mathbb{R}^2, T(x,y)=(4x+1, 2-y)$ maps the graph of f onto the graph of g.

Describe a sequence of basic transformations (translations, dilations, and reflections in the x- and y-axis only) that maps the graph of g onto the graph of g^{-1}.

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Sub-Section [1.4.6]: Find Opposite Transformations

Question 73

Describe a sequence of transformations that maps the graph of $y = 3e^{2x+1} - 4$ onto the graph of $y = e^x$.

Question 74

The transformation, $T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = \left(2x + 3, \frac{1}{3}y - 4\right)$ maps the graph of $y = f(x)$ onto the graph of $y = x^3$.

Find the rule of f.

Question 75

The following sequence of transformations maps the graph of f onto the graph of $y = \sqrt{x}$, for $x \in (2, \infty)$:

- A dilation by a factor of 3 from the x-axis, followed by,
- A translation of 2 units left and 4 units up, followed by,
- A reflection in both the x-axis and the y-axis.

State the rule and domain of f.

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Question 76

Describe a transformation different from $(x, y) \mapsto (x, y)$, that maps the graph of $y = a(x-k)^5 + b(x-k)^3 + h$ onto itself.

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Sub-Section: Exam 1 Questions

Question 77

The following sequence of transformations maps the graph of $y = f(x)$ onto the graph of $y = \frac{1}{2}\cos\left(\frac{\pi}{3} - 2x\right)$:

- A translation of $\frac{\pi}{6}$ units in the positive direction of the x-axis, followed by,
- A dilation by a factor of $\frac{1}{2}$ in from the y-axis, followed by,
- A dilation by a factor of 2 from the x-axis.

Find the rule of f.

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Question 78

Let $f : \mathbb{R} \to \mathbb{R}, f(x) = 2 - \frac{1}{2}x^3$, and let $g : \mathbb{R} \to \mathbb{R}, g(x) = 6 - 2x$.

a.

i. Find $(g \circ f)(x)$.

ii. Find $(f \circ g)(x)$ and express it in the form $k + m(x - h)^3$, where m, k and h are integers.

b. The transformation, $T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (x + b, ay + c)$, where a, b and c are integers, maps the graph of $y = (f \circ g)(x)$ onto the graph of $y = (g \circ f)(x)$.

Find the values of a, b, and c.

Question 79

Let $f : [1, \infty) \to \mathbb{R}, f(x) = 4(x - 1)^2 - 3$ and let $g : [2, \infty) \to \mathbb{R}, g(x) = 1 - \sqrt{x - 2}$.

a. Let g^{-1} be the inverse function of g.

i. State the domain and range of g^{-1}.

ii. Find the rule of g^{-1}.

b. The transformation, $T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (ax + b, y + c)$ maps the graph of f onto the graph of g^{-1}.

Find the values of a, b, and c.

Question 80

Let $f : \mathbb{R}\backslash\{a\} \to \mathbb{R}, f(x) = \frac{1}{x-a} + b$.

a. Find the rule and domain for the graph of f^{-1} in terms of a and b.

b. The following sequence of transformations maps the graph of f to the graph of f^{-1}:

- A translation of 4 units in the positive direction of the x-axis, followed by,
- A translation of 4 units in the negative direction of the y-axis.

Find the value of a in terms of b.

c. Let $g(x) = \frac{1}{x-c} + d$. A transformation,

$$T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (x + h, y + k)$$

maps the graph of g onto the graph of g^{-1}.

What restrictions are there on the values of h and k?

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Sub-Section: Exam 2 Questions

Question 81

The graph of the function f passes through the point $(2, -3)$.

If $h(x) = 3f(x-2)$, then the graph of the function h must pass through the point:

A. $(0, 1)$

B. $(4, -9)$

C. $(0, -9)$

D. $(4, -1)$

Question 82

The graph of the function $f : \mathbb{R} \to \mathbb{R}, f(x) = 2^x - 1$, is reflected in the y-axis and then translated 2 units to the left and then 3 units up.

Which one of the following is the rule of the transformed graph?

A. $y = 2^{2-x} + 2$

B. $y = 2^{2+x} + 2$

C. $y = \left(\frac{1}{2}\right)^{-2-x} + 2$

D. $y = \frac{1}{4}\left(\frac{1}{2}\right)^{x} + 2$

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Question 83

The transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, which maps the graph of $y = 4 - \log_e\left(\frac{x-1}{2}\right)$ onto the graph of $y = \log_e(x)$, has the rule:

A. $T(x, y) = \left(\frac{x-1}{2}, 4 - y\right)$

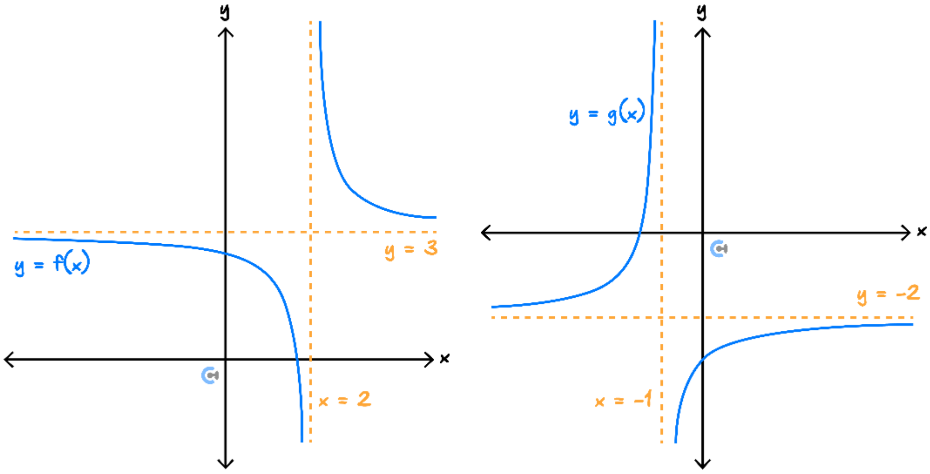
B. $T(x, y) = (2x + 1, -y - 4)$

C. $T(x, y) = (2x + 1, 4 - y)$

D. $T(x, y) = \left(\frac{x-1}{2}, -y - 4\right)$

Question 84

Consider the graph of f and g below, which have the same scale:



If T transforms the graph of f onto the graph of g, then:

A. $T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (1 - x, y - 5)$

B. $T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (x - 3, y - 5)$

C. $T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (x - 3, 5 - y)$

D. $T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (1 - x, 2 - y)$

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Question 85

The graph of the function g is obtained from the graph of the function $f: [-2, 3] \to \mathbb{R}, f(x) = 2x^2 - 4x + 5$, by a dilation of factor 2 from the y-axis, followed by a dilation of factor $\frac{1}{3}$, from the x-axis, followed by a reflection in the y-axis, and finally, followed by a translation of 1 unit in the negative direction of the y-axis.

The domain and range of g are respectively:

A. $[-6, 4]$ and $\left[\frac{8}{3}, 6\right]$

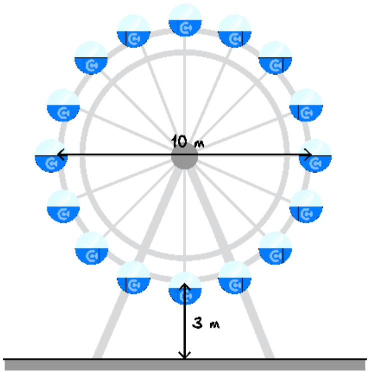
B. $\left[-1, \frac{2}{3}\right]$ and $[21, 41]$

C. $[-6, 4]$ and $\left[\frac{2}{3}, \frac{17}{3}\right]$

D. $[-6, 4]$ and $[0, 6]$

Question 86

The Contour Ferris Wheel pictured below takes 30 minutes to complete a trip.



Thus, the height of the bottom of a carriage t minutes after the start of a trip is given by,

$$h(t) = 8 - 5\cos\left(\frac{\pi t}{15}\right)$$

a. Describe a sequence of transformations that maps the graph of $\sin(t)$ onto the graph of h.

b. The horizontal displacement, d from the bottom of the carriage to the centre of the roller coaster t minutes after the start of a trip is,

$$d(t) = 5\sin\left(\frac{\pi t}{15}\right)$$

The transformation, $T(t, y) = (t + a, y + b)$ maps the graph of h onto the graph of d.

i. Find b.

---

ii. Find the possible value of a.

---

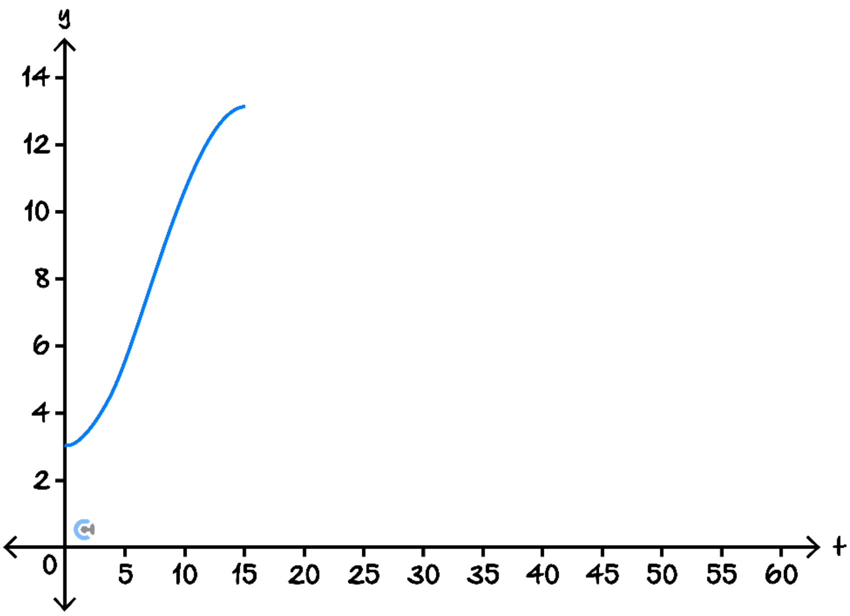
c. 15 minutes into a trip on the Ferris Wheel, Caitlin crashes her car into the Ferris Wheel. This causes the Ferris Wheel to stop for 5 minutes before starting again at half speed.

The height of the Ferris wheel in this trip, $h_1 : [0, r] \to \mathbb{R}$ is given by the following function:

$$h_1(t) = \begin{cases} h(t) & 0 \le t < 15 \\ k & 15 \le t < 20 \\ h(pt + q) & 20 \le t \le r \end{cases}$$

Find a set of possible values of p, q, k, and r.

d. Part of the graph of h_1 is drawn on the axis below. Draw the rest of the graph of h_1 labelling endpoints with their coordinates.



Question 87

Consider the function, $f : (-1, 1) \to \mathbb{R}, f(x) = (2x - 1)^2 (x + 1)$.

a. State the range of f.

b. The following sequence of transformations, T, maps the graph of f onto the graph of g:

- A dilation by a factor of 3 from the x-axis, followed by,
- A translation of 2 units down and 5 units left, followed by,
- A reflection in the y-axis.

i. State the rule of g.

ii. State the domain of g.

iii. State the range of g.

c. The tangent to the graph of f at the point $A\left(-\frac{1}{4}, \frac{27}{16}\right)$ is given by the equation:

$$y = \frac{9}{8} - \frac{9x}{4}$$

i. Find B, the image of A under T.

ii. Find the equation of the tangent to the graph of g at point B.

d. A transformation, $S: \mathbb{R}^2 \rightarrow \mathbb{R}^2, S(x, y) = (-x, a - y)$ maps the graph of f onto itself.

i. State the value of a.

ii. Hence, or otherwise, describe a sequence of transformations in terms of S and T as required, that maps the graph of g to itself, but does not map A to itself.

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