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VCE Mathematical Methods ¾
Differentiation [0.9]
Workshop

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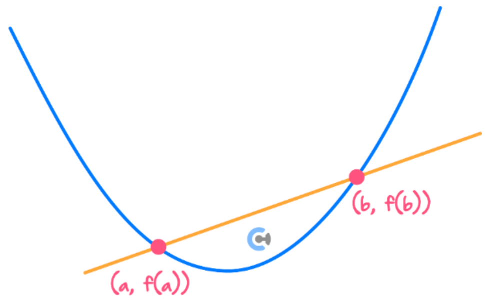
Section A: Recap

Average Rate of Change

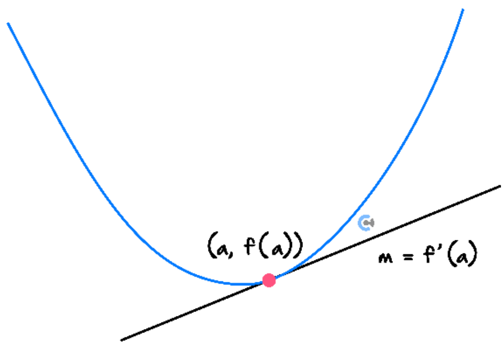


- The average rate of change of a function $f(x)$ over $x \in [a, b]$ is given by:

$$\textit{Average rate of change} = \frac{f(b) - f(a)}{b - a}$$

- It is the gradient of the line joining the two points.

Instantaneous Rate of Change



- **Instantaneous rate of change** is a gradient of a graph at a single point/moment.

$$\textit{Instantaneous rate of change} = f'(x)$$

- **Differentiation** is the process of finding the derivative of a function.
- **First principles** derivative definition:

$$f'(x) = \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} \right)$$

Definition

Alternative Notation for Derivative

$$f'(x) = \frac{dy}{dx}$$

Definition

Derivatives of Functions

- The derivatives of many of the standard functions are in the summary table below:

$f(x)$	$f'(x)$
x^n	$n \times x^{n-1}$
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$\sec^2(x)$
e^x	e^x
$\log_e(x)$	$\frac{1}{x}$

Definition

The Product Rule

- The derivative of $h(x) = f(x) \times g(x)$ is given by:

$$h'(x) = f'(x) \cdot g(x) + g'(x) \cdot f(x)$$

- Or, in another form:

$$\frac{d}{dx}(u \cdot v) = u'v + v'u$$

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The Quotient Rule

- The derivative of a $h(x) = \frac{f(x)}{g(x)}$ is given by:

$$h'(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g(x)^2}$$

- Or, written in another form:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - v'u}{v^2}$$

  - Always differentiate the top function first.

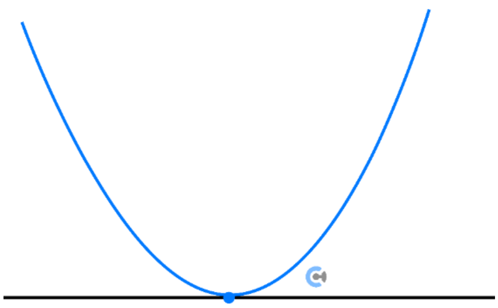
The Chain Rule

$$y = f\big(g(x)\big)$$

$$\frac{dy}{dx} = f'\big(g(x)\big)g'(x)$$

- The process for finding derivatives of **composite functions.**

Stationary Points



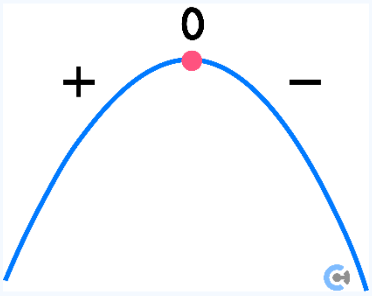
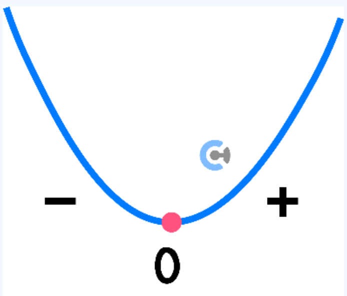
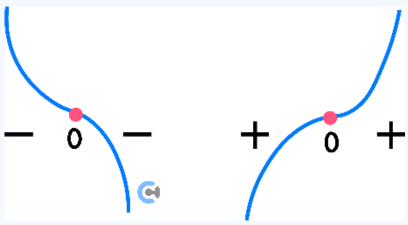
- The point where the gradient of the function is zero.

$$f'(x) = 0, \frac{dy}{dx} = 0$$

Calculator Commands: Finding Derivatives

Mathematica	TI	Casio
`f'[x]`	Shift Minus $\frac{d}{dx}(f(x))$	Math 2 $\frac{d}{dx}(f(x))$

Types of Stationary Points

Local Maximum	Local Minimum	Stationary Point of Inflection
		

- Sign test

- We can identify the nature of a stationary point by using the sign table.

x	Less than a	a	Bigger than a
$f'(x)$	Negative	0	Positive
Shape	∩ - Decreasing curve	Stationary Point	∪ - Increasing curve

- Find the gradient of the neighbouring points.

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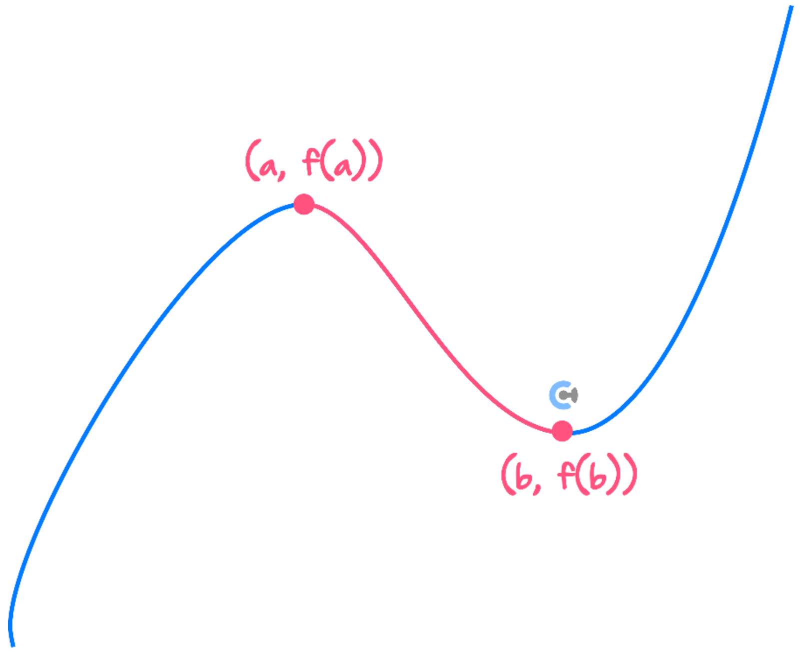
Strictly Increasing and Strictly Decreasing Functions



Strictly Increasing: $x \in (-\infty, a] \cup [b, \infty)$

Strictly Decreasing: $x \in [a, b]$

- Steps:

  1. Find the turning points.
  2. Consider the sign of the derivative between/outside the turning points.

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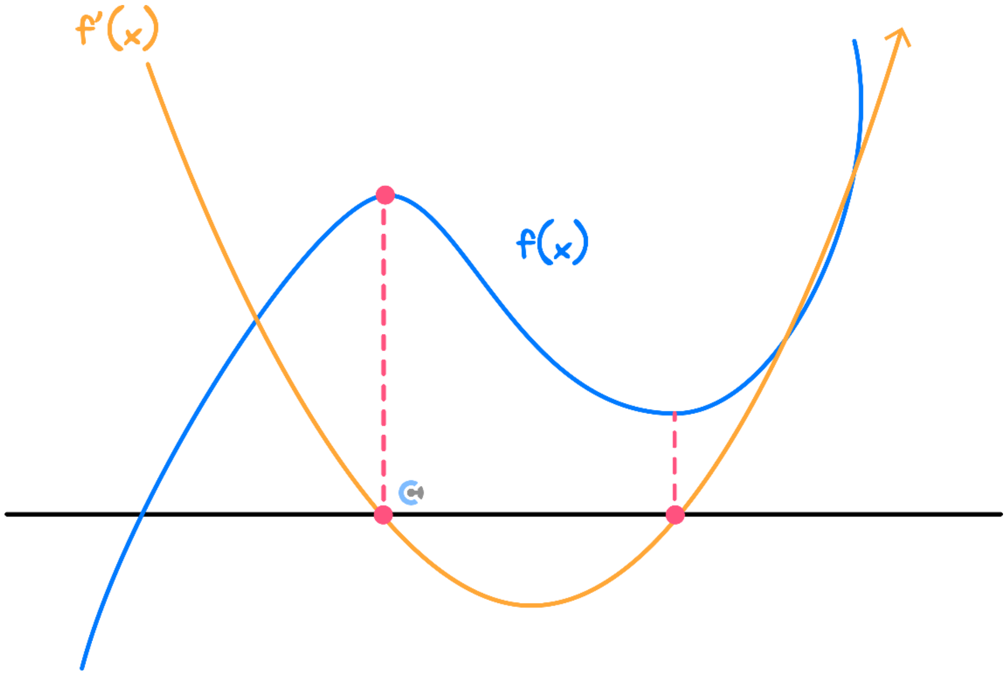
Graphs of the Derivative Function



$f(x)$	$f'(x)$
Stationary Point	x-intercepts
Increasing	Positive
Decreasing	Negative

$$y \text{ value of } f'(x) = \text{Gradient of } f(x)$$

- **Steps**

1. Plot x-intercept at the same x value as the stationary point of the original.
2. Consider the trend of the original function and sketch the derivative.
   - Original is increasing → Derivative is above the x-axis.
   - Original is decreasing → Derivative is below the x-axis.

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Section B: Warmup

Question 1

Let $f: \mathbb{R} \to \mathbb{R}, f(x) = x^3 + 3x^2 - 9x + 3$.

a. Find $f(1)$.

$$\boxed{f(1) = -2}$$

b. Find the average rate of change from $x = 0$ to $x = 2$.

$$\boxed{\frac{f(2) - f(0)}{2} = \frac{5-3}{2} = 1}$$

c.

i. Find $f'(x)$.

$$\boxed{f'(x) = 3x^2 + 6x - 9}$$

ii. Find $f'(-1)$.

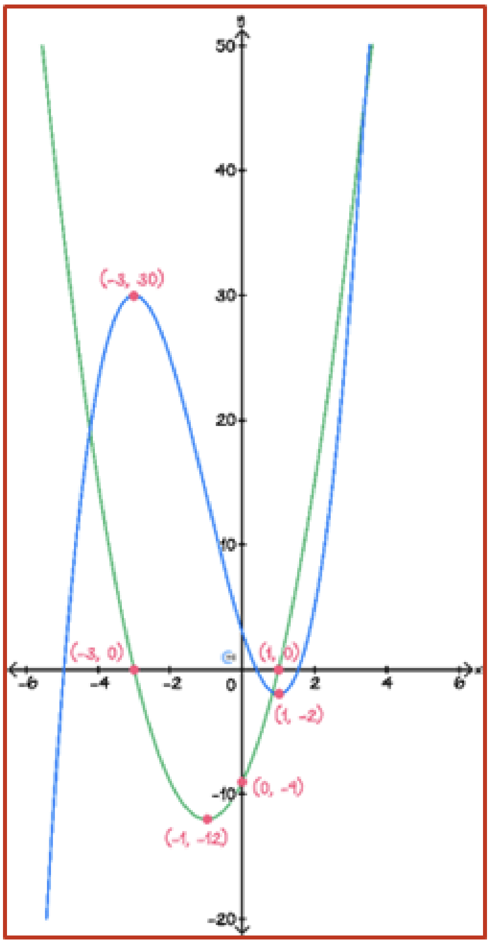
$$\boxed{f'(-1) = -12}$$

d. Determine the coordinates and nature of any stationary points.

$f'(x) = 0 \implies 3x^2 + 6x - 9 = 0 \implies x^2 + 2x - 3 = 0 \implies (x+3)(x-1) = 0$.
$f(-3) = 30$ and $f(1) = -2$. Stationary points $(-3, 30)$ and $(1, -2)$. Function is positive cubic so

$$(-3, 30) \text{ is a local maximum}$$
$$(1, -2) \text{ is a local minimum}$$

e. The graph of $y = f(x)$ is sketched on the axes below. Sketch the graph of $y = f'(x)$ on the same axes. Label all axial intercepts with coordinates.



f. Hence, state the values of x for which $f(x)$ is strictly decreasing.

$\boxed{-3 \leq x \leq 1}$

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Section C: Exam 1 Questions (17 Marks)

INSTRUCTION: 17 Marks. 25 Minutes Writing. (17 min Writing Extension)

Question 2 (3 marks)

a. Let $y = \frac{\tan(2x)}{x^3}$. Find $\frac{dy}{dx}$. (1 mark)

$$\frac{dy}{dx} = \frac{\sec^2(2x)\cdot 2\cdot x^3 - \tan(2x)\cdot 3x^2}{x^6}$$

$$= \frac{2x^3\sec^2(2x) - 3x^2\tan(2x)}{x^6} = \frac{2x\sec^2(2x) - 3\tan(2x)}{x^4}$$

b. Let $f(x) = x^3\tan(e^x)$. Evaluate $f'\left(\log_e\left(\frac{\pi}{6}\right)\right)$. (2 marks)

$$f'(x) = 3x^2\tan(e^x) + x^3\cdot\sec^2(e^x)e^x$$

$f\circ f^{-1}(x) = x$

$e^{\log_e\left(\frac{\pi}{6}\right)} = \frac{\pi}{6}$

$$f'\left(\log_e\left(\frac{\pi}{6}\right)\right) = 3\left(\log_e\left(\frac{\pi}{6}\right)\right)^2\tan\left(e^{\log_e\left(\frac{\pi}{6}\right)}\right)$$

$$+ \left(\log_e\left(\frac{\pi}{6}\right)\right)^3\cdot\sec^2\left(e^{\log_e\left(\frac{\pi}{6}\right)}\right)\cdot e^{\log_e\left(\frac{\pi}{6}\right)}$$

$\frac{1}{\cos^2\left(\frac{\pi}{6}\right)} = \frac{1}{3/4} = \frac{4}{3}$

$$= 3\left(\log_e\left(\frac{\pi}{6}\right)\right)^2\underbrace{\tan\left(\frac{\pi}{6}\right)}_{\frac{1}{\sqrt{3}}} + \left(\log_e\left(\frac{\pi}{6}\right)\right)^3\cdot\underbrace{\sec^2\left(\frac{\pi}{6}\right)}\cdot\frac{\pi}{6}$$

$$= \sqrt{3}\left(\log_e\left(\frac{\pi}{6}\right)\right)^2 + \frac{2\pi}{9}\left(\log_e\left(\frac{\pi}{6}\right)\right)^3$$

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$\left(\log_e\left(\frac{\pi}{6}\right)\right)^2 \neq 2\log_e\left(\frac{\pi}{6}\right)$

$\log_e\left(\left(\frac{\pi}{6}\right)^2\right) = 2\log_e\left(\frac{\pi}{6}\right)$

Question 3 (3 marks)

Let $f: \mathbb{R} \to \mathbb{R}, f(x) = xe^{kx^2}$, where $k \in \mathbb{R}$.

a. Show that $f'(x) = (2kx^2 + 1)e^{kx^2}$. (1 mark)

$f'(x) = 1 \cdot e^{kx^2} + x \cdot e^{kx^2} \cdot 2kx$

$= (2kx^2+1)e^{kx^2}$

"None then 1 soln"

b. Find the value(s) of k for which the graph of $y = f(x)$ and $y = f'(x)$ have exactly one point of intersection. (2 marks)

$(2kx^2+1)e^{kx^2} = xe^{kx^2}$

$2kx^2+1 = x$

$2kx^2 - x + 1 = 0$

$\Delta = (-1)^2 - 4 \cdot 2k \cdot 1 = 0$

$1 = 8k$

$k = \frac{1}{8}$

$k = 0$

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Question 4 (6 marks)

Consider the functions $f: \mathbb{R} \to \mathbb{R}, f(x) = -x^2 + 5x - 4$ and $g: \mathbb{R} \to \mathbb{R}, g(x) = e^x$.

a. State the rule of $g(f(x))$. (1 mark)

$e^{-x^2+5x-4} = g(f(x))$

b. Find the values of x for which $g(f(x))$ is strictly decreasing. (2 marks)

let $y = g(f(x))$

$\frac{dy}{dx} = e^{-x^2+5x-4} \times (-2x+5) < 0$

$-2x+5 < 0$

$5 < 2x$

$\frac{5}{2} \leq x$

c. Find the coordinates of the stationary point of the graph of $f(g(x))$ and state its nature. (3 marks)

let $y = \underbrace{f(g(x))}$

$f'(g(x)) \cdot g'(x)$

$\frac{dy}{dx} = (-2g(x)+5) \times e^x$

$= \underbrace{(-2e^x+5) \cdot e^x} = 0$

$-2e^x+5 = 0$

$e^x = 5/2$

$x = \ln(5/2)$

x	0	$\ln(5/2)$	10
$\frac{dy}{dx}$	$+$	0	$-$
shape	/	—	\

local max

$-\left(\frac{5}{2}\right)^2 + 5 \cdot \frac{5}{2} - 4$

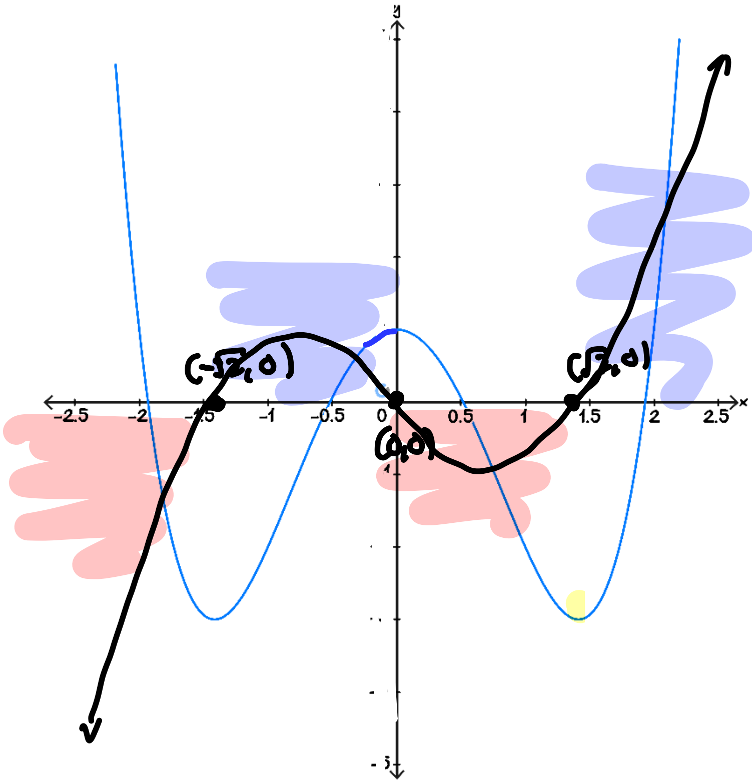
$-\frac{25}{4} + \frac{25}{2} - 4$

$= \frac{25}{4} - 4 = \frac{9}{4}$

$\left(\log_e\left(\frac{5}{2}\right), \frac{9}{4}\right)$

Question 5 (5 marks)

The graph of $f(x) = x^4 - 4x^2 + 1$ is shown below.



a. Sketch the graph of $y = f'(x)$ on the axes above. Label all axes intercepts with coordinates. (3 marks)

$f'(x) = 4x^3 - 8x$

$f'(x) = 4x^3 - 8x = 0$

$4x(x^2 - 2) = 0$

$x = 0, \pm\sqrt{2}$

b. For both $f(x)$ and $f'(x)$ state whether they are an odd function, even function or neither. (1 mark)

$f(x) =$ even

f' : odd

All pelm are evn   All pelm even −1

c. Let g be an even polynomial function of degree $n \geq 2$. Is it always true that g' is an odd function? (1 mark)

= odd

Odd ?

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Section D: Tech Active Exam Skills

Calculator Commands: Stationary Point

- ALWAYS sketch the graph first to get an idea of the nature of the stationary point.
- The turning points for a function $f(x)$ can be found by solving $f'(x) = 0$ and subbing the result into f.
- **Example:** Find the turning point for $f(x) = e^{-x^2+2x}$.
- **TI:**

Define $f(x)=e^{-x^2+2\cdot x}$	Done
$\text{solve}\left(\frac{d}{dx}(f(x))=0,x\right)$	$x=1$
$f(1)$	e

- **Casio:**

define $f(x) = e^{-x^2+2x}$	
	done
$\text{solve}(\frac{d}{dx}(f(x))=0, x)$	
	$\{x=1\}$
$f(1)$	
	e

- **Mathematica:**

```
In[4]:= f[x_] := Exp[-x^2 + 2 x]
In[5]:= Solve[f'[x] == 0 && y == f[x], Reals]
Out[5]= {{x → 1, y → e}}
```

Section E: Exam 2 Questions (20 Marks)

INSTRUCTION: 20 Marks. 25 Minutes Writing.

Question 6 (1 mark)

The average rate of change for the function with the rule $f(x) = y = -4e^{-\frac{2x}{5}}$ from $y = -3$ to $y = -1$ is closest to:

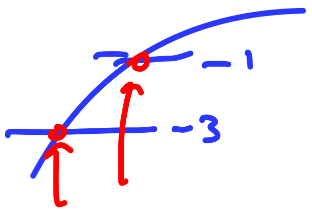
A. 1.02

B. -1.02

C. 1.37

D. 0.73

$f(x) = -3,$ $f(x) = -1,$ Real



$$\frac{(-1)-(-3)}{x_2 - x_1}$$

Question 7 (1 mark)

Let $f(x)$ and $g(x)$ be differentiable functions, with the following values given:

$$f(2) = 3,\ f'(4) = 5,\ g(2) = 4,\ g'(2) = 6.$$

Find the gradient of $f\big(g(x)\big)$ at $x = 2$.

A. 20

B. 15

C. 30

D. 24

$\frac{d}{dx}(f(g(x)))$.

$\Rightarrow f'(g(x))g'(x)$

$f'(g(2)) \cdot g'(2)$

$f'(4) \times 6$

5×6

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Question 8 (1 mark)

Given that $f(1) = 2, f'(1) = 3, g(1) = 4, g'(1) = 5$, find the gradient of $f(x)g(x)$ at $x = 1$.

$f'(x)g(x) + f(x) \cdot g'(x)$

A. 13

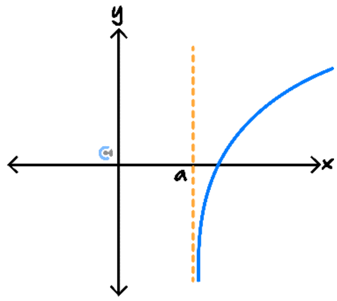
B. 22

C. 18

D. 20

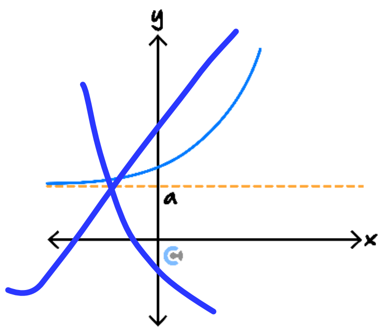
Question 9 (1 mark)

The graph of the function f is shown below:

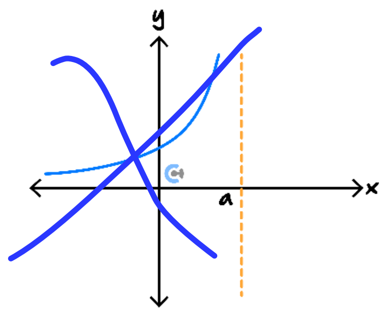


The graph corresponding to f' is:

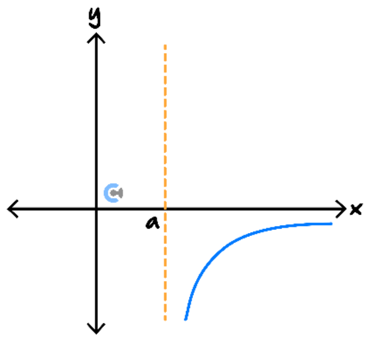
A.



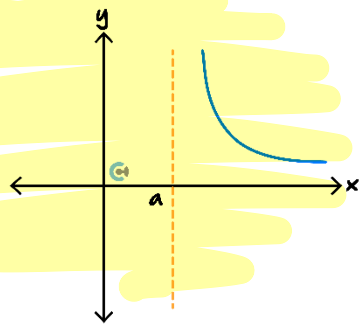
B.



C.



D.



Question 10 (1 mark)

Suppose $f(x)$ and $g(x)$ are differentiable, and the following values are given:

$$f(3) = 5,\ f'(3) = 4,\ g(3) = 2,\ g'(3) = 1.$$

Find the gradient of $\frac{f(x)}{g(x)}$ at $x = 3$.

A. $\frac{2}{5}$

B. $\frac{3}{4}$

C. $\frac{1}{2}$

D. $\frac{2}{3}$

$$\frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

Question 11 (1 mark)

Consider the graph of g with the rule $g(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$, has a y-intercept at $(0,10)$ and turning points when $x = -3$ and $x = 2$ and passes through $(3,64)$.

The rule of $g(x)$ is:

define $g(x) = a \cdot x^3 + b \cdot x^2 + c \cdot x + d$

A. $g(x) = -4x^3 - 6x^2 + 72x + 10$

B. $g(x) = -3x^3 - 9x^2 + 27x + 10$

TI: define $dg(x) = \frac{d}{dx}(g(x))$.

C. $g(x) = 3x^3 + 8x^2 + 10x + 10$

CAS: define $f(x) = \frac{d}{dx}(g(x))$

D. $g(x) = 2x^3 + 12x^2 + 6x + 10$

Space for Personal Notes

$g(0) = 10$

$g'(-3) = 0$

$g'(2) = 0$

$g(3) = 64$

M2

Sketch

& Trace (5.1)

Question 12 (1 mark)

The graph of $f(x) = ax^5 + bx^4 + x^3 - 3$, where a and b are real constants, will have three stationary points when:

A. $a > -\frac{4b^2}{15}$

B. $a \leq -\frac{4b^2}{15}$

C. $a < \frac{4b^2}{15}$

D. $a > \frac{4b^2}{15}$

$f'(x) = 5ax^4 + 4bx^3 + 3x^2$

$= x^2 [5ax^2 + 4bx + 3]$

$x^2 = 0$

$\underbrace{5ax^2 + 4bx + 3}_{\Delta > 0}$

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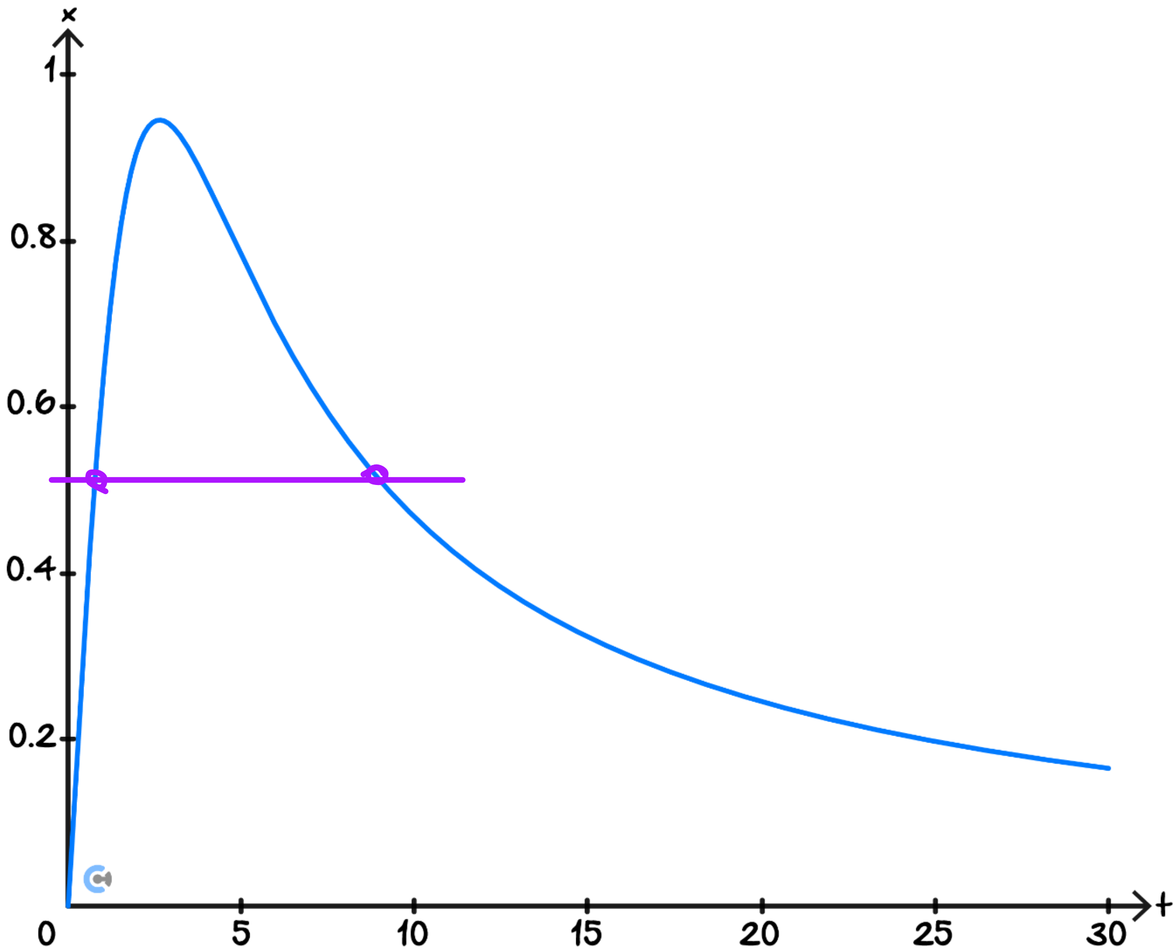
Question 13 (13 marks)

A tranquiliser is injected into a muscle from which it enters the bloodstream. The concentration, $x, mg/L$, of the tranquiliser in the bloodstream, may be modelled by the function:

$$x(t) = \frac{5t}{7+t^2}, t \geq 0$$

Where t is the number of hours after the injection is given. The graph of the equation is shown:



a. The tranquiliser is effective when the concentration is at least 0.5 mg/L. Find, correct to two decimal places, the length of time in hours for which the tranquiliser is effective according to this model. (2 marks)

$x(t)=0.5$

$t = 0.7573, 9.24264$

$9.24264 - 0.7573$

$\approx 8.49hr$

b.

i. Find the coordinates for the stationary point of $x(t)$. (2 marks)

$x'(t) = 0$

$t = \sqrt{7}$

$\left(\sqrt{7}, \frac{5}{2\sqrt{7}}\right)$

ii. State the nature of the stationary point from **part b.i.** (1 mark)

local max.

iii. Hence, find the maximum concentration of tranquiliser in the bloodstream. Give your answer correct to three decimal places. (1 mark)

$\frac{5}{2\sqrt{7}} = 0.945$ mg/L SACs

c. For what times, is the concentration of transquiliser in the bloodstream strictly decreasing? (1 mark)

$t \in [\sqrt{7}, \infty)$

d. According to this model, the derivative of x with respect to t gives the measure of the rate of absorption of the tranquiliser in the bloodstream.

How many hours after the injection is the rate of absorption into the bloodstream 0.3 $mg/L/h$?

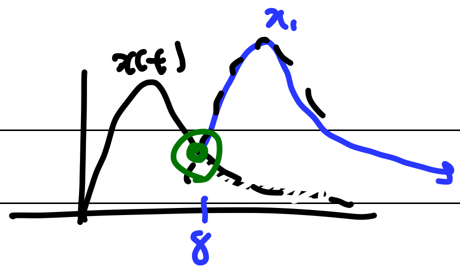
Give your answer correct to two decimal places. (1 mark)

$x'(t) = 0.3$

$t = 1.44$ hrs.

So that the tranquiliser concentration does not get too low, the exact same tranquiliser is re-applied when $t = 8$. From this point onwards the concentration of tranquiliser in the bloodstream is measured by a new function $x_1(t)$.

e. State the domain for $x_1(t)$. (1 mark)



$t \in [8, \infty)$

f. $x_1(t)$ has the rule $x_1(t) = x(t) + \underbrace{x(t-a)}$. State the value of a. (1 mark)

a right = 8 right

$\boxed{a = 8}$

g. Find the time it takes for the concentration of tranquiliser to double from its value at $t = 8$. Give your answer in hours correct to two decimal places. (2 marks)

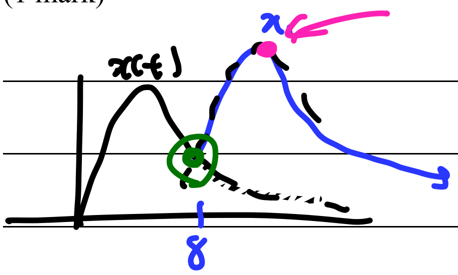
$x_1(t) = 2 \cdot x(8)$

$t = 8.977$

$8.977 - 8$

≈ 0.98 hrs

h. Determine the times, $t \geq 8$, when the concentration of tranquiliser in the bloodstream is strictly decreasing. (2dp) (1 mark)



$x_1'(t) = 0$

$t = 10.40$

$t \in [10.40, \infty)$

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Section F: Extension Exam 1 (13 Marks)

INSTRUCTION: 13 Marks. 16 Minutes Writing.

Question 14 (3 marks)

For the function $f(x) = 3x^3\tan(2x)$, $f'(x) = \frac{ax^2}{\cos(2x)}(b\sin(2x) + cx\sec(2x))$. Find the values of a, b and c.

$f'(x) = 9x^2\tan(2x)$
$+ 3x^3\sec^2(2x)\cdot 2$

$= 9x^2\tan(2x) + 6x^3\sec^2(2x)$

$= \frac{9x^2\sin(2x)}{\cos(2x)} + \frac{6x^3\sec(2x)}{\cos(2x)}$

$= \frac{3x^2}{\cos(2x)}\cdot(3\sin(2x) + 2x\sec(2x))$

$a = 3$

$b = 3$

$c = 2$

Space for Personal Notes

Question 15 (7 marks)

$\sqrt{x} \rightarrow \frac{1}{2\sqrt{x}}$ "Double & flip"

Let $f : [0, \infty) \rightarrow \mathbb{R},\ f(x) = 5\sqrt{x} - x - 4.$

a. Find the coordinates of any stationary point of f and determine its nature. (3 marks)

Sign #.

$f'(x) = 5 \cdot \frac{1}{2\sqrt{x}} - 1 = 0$

$\frac{5}{2\sqrt{x}} = 1$

$\frac{5}{2} = \sqrt{x}$

$x = \frac{25}{4}$

$f\left(\frac{25}{4}\right) = 5\sqrt{\frac{25}{4}} - \frac{25}{4} - 4$

$= 5 \cdot \frac{5}{2} - \frac{25}{4} - 4$

$= \frac{25}{2} - \frac{25}{4} - 4 = \frac{25}{4} - \frac{16}{4}$

$= \frac{9}{4}$

x	1	$\frac{25}{4}$	9
f'	$+$	0	$-$
Shape	/	—	\

$f'(1) = \frac{5}{2} \times \frac{1}{\sqrt{1}} - 1 = \frac{5}{2} - 1 = 1.5$

$f'(9) = \frac{5}{2} \cdot \frac{1}{\sqrt{9}} - 1 = \frac{5}{6} - 1 = -\frac{1}{6}$

$\boxed{\left(\frac{25}{4}, \frac{9}{4}\right)}$ $\boxed{\text{LOCAL MAX}}$

Let A and B be the coordinates of the x-intercepts of the graph $y = f(x)$. Let C be any point on the graph of $y = f(x)$ that lies between the points A and B.

b. Determine the coordinates of A and B. (2 marks)

"Something & something^2"

$0 = 5\sqrt{x} - x - 4$

Let $\sqrt{x} = A$

$0 = 5A - A^2 - 4$

$A^2 - 5A + 4 = 0$

$(A - 4)(A - 1) = 0$

$A = 4, 1$

$\sqrt{x} = 4, 1$

$x = 16, 1$

$(16, 0)$ & $(1, 0)$

c. Hence, determine the maximum possible area of the triangle ABC. (2 marks)

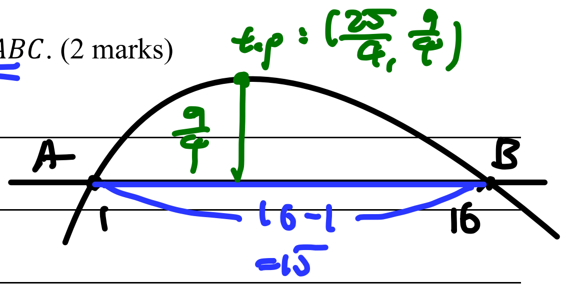
$\frac{1}{2} \times \text{Base} \times \text{Height}$

$= \frac{1}{2} \times 15 \times \frac{9}{4}$

$= \frac{135}{8}$

15×9

$15 \times (16-1)$



Question 16 (3 marks)

TIP: If some transformations are fixed (x), apply the transformation

Let $f: [0, 12\pi] \to \mathbb{R}, f(x) = 2\sin\left(\frac{x}{3}\right) - \frac{\pi}{2}$.

The rule for f' can be obtained from the rule of f under a transformation T given by:

$$T: \mathbb{R}^2 \to \mathbb{R}^2, (x, y) \to \left(x + \frac{3\pi}{2}, ay + b\right)$$

Find the values of a and b.

1) Find f'

$f'(x) = \frac{2}{3}\cos\left(\frac{x}{3}\right)$

2) Apply the "fixed transform"

$x' = x + \frac{3\pi}{2}$

$x = x' - \frac{3\pi}{2}$

$y = 2\sin\left(\frac{x}{3}\right) - \frac{\pi}{2}$

$y = 2\sin\left(\frac{x}{3} - \frac{\pi}{2}\right) - \frac{\pi}{2}$

$= -2\cos\left(\frac{x}{3}\right) - \frac{\pi}{2}$

3) Find a & b.

$y = -2\cos\left(\frac{x}{3}\right) - \frac{\pi}{2}$

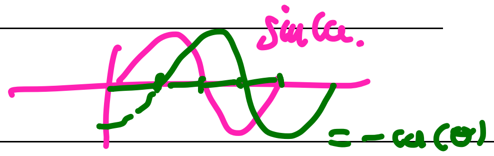
$\times\left(-\frac{1}{3}\right)$ a

$y = \frac{2}{3}\cos\left(\frac{x}{3}\right) + \frac{\pi}{6}$

$+\left(-\frac{\pi}{6}\right)$ b

$y = \frac{2}{3}\cos\left(\frac{x}{3}\right)$

$\boxed{a = -\frac{1}{3}, \quad b = -\frac{\pi}{6}}$



Complementary.

$\sin\left(\frac{x}{3} - \frac{\pi}{2}\right) = \sin\left(\theta - \frac{\pi}{2}\right)$

Section G: Extension Exam 2 (12 Marks)

INSTRUCTION: 12 Marks. 15 Minutes Writing.

Question 17 (1 mark)

Let f be a one-to-one differentiable function such that $f(3) = 7$, $f(7) = 8$, $f'(3) = 2$ and $f'(7) = 3$. The function g is differentiable and $g(x) = f^{-1}(x)$ for all x. $g'(7)$ is equal to:

A. $\frac{1}{2}$

B. 2

C. $\frac{1}{6}$

D. $\frac{1}{3}$

f: $(3, 7)$ $(7, 8)$

$m = 2$ $m = 3$

$\frac{dy}{dx} \to \frac{dx}{dy}$

$g = f^{-1}$: $(7, 3)$ $(8, 7)$

$m = \frac{1}{2}$

Question 18 (1 mark)

Consider the function $f(x) = xg(x)$.

It is known that $g(0) = -3$, $g(2) = -2$ and $g(3) = 3$.

Also $g'(0) = -2$, $g'(2) = 1$ and $g'(3) = 5$ and that f has only one stationary point.

Which of the following options lists the x-coordinate and nature of the stationary point of f?

A. ~~$x = 0$, local minimum.~~

B. $x = 2$, local maximum.

C. $x = 2$, local minimum.

D. ~~$x = 3$, local minimum.~~

$f'(x) = 1g(x) + x \cdot g'(x) = 0.$

$f'(0) = g(0) + 0 = -3$

$f'(2) = g(2) + 2 \cdot g'(2)$

$= -2 + 2 \times 1 = 0$

x	0	2	3
f'	-3	0	18
	(−)	(0)	(+)

Space for Personal Notes

Question 19 (1 mark)

$\frac{d}{dx}(\log_e(f(x)))$

Let f be a differentiable function. The derivative of $\log_e(f(x))$ with respect to x is:

A. $\frac{f'(x)}{f(x)}$

B. $\frac{f(x)}{(f(x))^2}$

C. $\frac{f'(x)}{(f(x))^2}$

D. $f'(x)\log_e(f(x))$

$\frac{1}{f(x)} \times f'(x)$

Question 20 (1 mark)

Consider the differentiable function f. It is known that $f'(1) = 2$, $f'(2) = 4$ and $f'(6) = 1$.

The gradient of $3f(2x+1)+4$ when $x = \frac{1}{2}$ is:

A. 6

B. 24

C. 3

D. 5

$f(x)$

$3f(2x+1)+4$

$2x+1 = x$

$2 \cdot \frac{1}{2} + 1 = x$

$2 = x$

$y = 3f(2x+1)+4$

$\frac{dy}{dx} = 3 \cdot f'(2x+1) \cdot 2$

$= 6f'(2x+1)$

$= 2$

$6 \times 4 = 24$

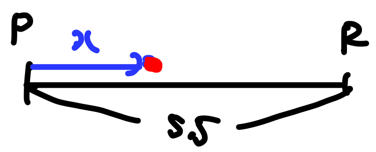
Space for Personal Notes

Question 21 (8 marks)

A train is moving along a railway. The train driver sees a large rock ahead and immediately applies the brakes when the front of the train is at a point P that is 5.5 km away from the rock at the point R.

The train's initial speed is w km/h and x km after the train passes the point P the train speed is given by:

$$v = \boxed{k} \log_e\left(\frac{x+1}{6}\right)$$



Assume that $w > 0$.

a. Find the value of k in terms of w. (1 mark)

$v = w$
$x = 0$

$w = k \cdot \ln\left(\frac{1}{6}\right)$

$k = \underbrace{\frac{w}{\ln\left(\frac{1}{6}\right)} = \frac{-w}{\ln(6)}}$

b. If $v = \frac{50\log_e(2)}{\log_e(6)}$ when $x = 2$, find the value of w. (2 marks)

$\frac{50\ln(2)}{\ln(6)} = k \cdot \ln\left(\frac{1}{2}\right)$

$\frac{50\ln(2)}{\ln(6) \cdot \ln\left(\frac{1}{2}\right)} = k$

$-\ln(2)$

$k = \frac{-50}{\ln(6)}$

$w = 50$

c. Show that the location where the train stops is independent of its initial speed w. (2 marks)

$0 = k \cdot \log_e\left(\frac{x+1}{6}\right)$

$0 = \frac{-w}{\ln(6)} \cdot \ln\left(\frac{x+1}{6}\right)$

$0 = \frac{-1}{\ln(6)} \cdot w \cdot \ln\left(\frac{x+1}{6}\right)$

$\neq 0$ as $w > 0$

$= 0$

$\ln\left(\frac{x+1}{6}\right) = 0$

$\frac{x+1}{6} = e^0$

$x = 6 - 1$

$x = 5$

d. Find a general formula for $\frac{d^n v}{dx^n}$, the n^{th} derivative of v, where $n \geq 1$. Leave your answer in terms of x, n and k. (3 marks)

$\frac{d}{dx}\frac{d}{dx}(v)$

$\frac{d^2(v)}{dx^2}$

$(-1)^{n+1} = (-1)^{n-1}$

$n=1: \quad \frac{dv}{dx} = \frac{k}{x+1}$ $\times 1$

$\frac{d^n v}{dx^n} = \boxed{\frac{-(-1)^n \cdot (n-1)!\, k}{(x+1)^n}}$

$n=2: \quad \frac{d^2v}{dx^2} = \frac{-k}{(x+1)^2}$ $\times 2$

$\to f(n)$

$n=3: \quad \frac{d^3v}{dx^3} = \frac{2k}{(x+1)^3}$ $\times 3$

$f(1)$

$n=4: \quad \frac{d^4v}{dx^4} = \frac{-6k}{(x+1)^4}$ $\times 4$

$f(2)$

$f(3)$

Space for Personal Notes

$n=5: \quad \frac{d^5v}{dx^4} = \frac{24k}{(x+1)^4}$

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