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VCE Mathematical Methods ¾
AOS 1 Revision [0.8]
Workshop Solutions

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Section A: Cheat Sheets

Cheat Sheet

[1.1.1] - Find the Maximal Domain and Range

- Inside of a log must be bigger than 0.
- Inside of a root must be bigger than or equal to 0.
- Denominator cannot be zero.
- The domain of sum or product of two functions is equal to the intersection of the two domains.

[1.1.2] - Find the Rule, Domain and Range of a Composite Function (Range Does Not Require Splitting to Find as the Function is Easy to Draw)

- $f(g(x)) = f \circ g\,(x)$.
- For composite function to exist, range (output) of inside $\subseteq$ domain (input) of outside
- The domain of composite is equal to the domain of inside (1^{st}) function.
- The range of composite is a subset of the range of the outside.

[1.1.3] - Find the Rule, Domain, and Range of Inverse Functions

- f needs to be $1:1$ for f^{-1} to exist.
- Domain of the inverse function equals to range of the original and vice versa.
- Symmetrical around $y = x$.
- For intersections of inverses, we can equate the function to $y = x$.

[1.1.4] - Find the Composite Function of the Inverse Function

- The composite function of inverses is always given by $f(f^{-1}(x)) = x$.

[1.2.1] - Find a New Domain to Fix Composite Functions

- The range of the inside function must be a subset of the domain of the outside function.
- We restrict the domain of the inside function so its range fits in the domain of the outside function.

[1.2.2] - Find the Range of Complex Composite Functions

- To find the range of a complicated function, we can break the function into a composition of two simpler functions.

[1.2.3] - Find the Gradient of Inverse Functions

- If the gradient of f at $(a, f(a)) = m$, then the gradient of f^{-1} at $(f(a), a) = \frac{1}{m}$.

[1.3.1] - Applying x' and y' Notation to Find Transformed Points, Find the Interpretation of Transformations and Altered Order of Transformations

- The transformed point is called the image and is denoted by (x', y').
- The dilation factor is multiplied to the original coordinate.
- Reflection makes the original coordinates the negative of their original values.
- Translation adds a unit to the original coordinate.
- Transformations should be interpreted when x' and y' are isolated.
- The order of transformation follows the BODMAS order.
- To change the order of transformations, we either factorise or expand.

Cheat Sheet

[1.3.2] - Find Transformed Functions

- To transform the function, replace its old variables with the new one.

[1.3.3] - Find Transformations From Transformed Functions (Reverse Engineering)

- To find the transformations, simply equate the LHS and RHS after separating the transformations of x and y.

[1.4.1] - Apply Quick Method to Find Transformations

- For applying transformations in the quick method: Apply everything for x in the opposite direction. Including the order!
- For interpreting transformations in the quick method: Read everything for x in the opposite direction. Including the order!

[1.4.2] - Find Opposite Transformations

- Order is reversed.
- All transformations are in the opposite direction.

[1.4.3] - Apply Transformations of Functions to Find Their Domain, Range, Transformed Points and Tangents

- Everything moves together as a function.
- **Steps:**
  1. Find the transformations between two functions.
  2. Apply the same transformations to domain, range, points and tangents.

[1.4.4] - Find Transformations of the Inverse Functions $f(x)$

- **Steps:**
  1. Find the transformations between the two original functions.
  2. Inverse the transformations found in **1.**

[1.4.5] - Find Multiple Transformations For the Same Functions

- Same transformations can be done differently by either putting it in or out of the $f()$.
- Commonly, look for basic algebra, index and log laws

[1.4.6] - Apply Manipulation of the Functions to Find Appropriate Transformations

- **Steps:**
  1. Identify the region of x.
  2. Identify the region of y.
  3. Manipulate the function so that all the changes are within the region of x or y.

[1.5.1] - Find the Midpoint and Distance (Horizontal & Vertical) Between Two Points or Functions

- Midpoint is simply the average of 2 points.
- Distance formula is derived from Pythagoras theorem
- Horizontal distance is the distance between x values.
- Vertical distance is the distance between y values.

[1.5.2] - Find Parallel and Perpendicular Lines

- Parallel lines have the same gradient.
- Perpendicular lines have negative reciprocal gradient.

Cheat Sheet

[1.5.3] - Find the Angle Between a Line and x-axis or Two Lines

- To find the angle between a line and the x-axis we can use the equation m = $\tan(\theta)$.
- To find the angle between two lines we can use

$$\theta = \left| \tan^{-1}(m_1) - \tan^{-1}(m_2) \right|$$

or

$$\tan\theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

[1.5.4] - Find The Unknown Value for Systems of Linear Equations

- Two linear equations have unique solutions if they have different gradients.
- Two linear equations have infinitely many solutions when they have the same gradient and the same constant.
- Two linear equations have no solution when they have the same gradient and different constant.

[1.5.5] - Sketching the Sum of Two Function's Graph by Using the Addition of Ordinates

- Addition of ordinates is used to sketch the sum of two functions.
- We always add their y values.
- When we have an x intercept for one graph, sum graph intersects the other graph.
- When we have an intersection between two graphs, the sum graph equals to double their y value.
- When we have an equidistance from the x-axis, sum graph has an x-intercept.

[1.6.1] - Apply Midpoint to Find a Reflected Point

- The [line between the] point and its reflection is perpendicular to the line it is reflected in.
- The midpoint of a line and its reflection lies on the line it is reflected in.
- **Steps** for finding the reflection of a point in a line:
  1. Find the perpendicular line passing through the point.
  2. Find the intersection between the original line and the perpendicular line.
  3. Find the reflected point (x, y) by treating the intersection from **2.** as the midpoint between the original and reflected point.

[1.6.2] - Apply Parallel and Perpendicular Lines to Geometric Problems

- When solving geometric problems always draw a diagram of the situation.

[1.7.1] - Apply the Factor Theorem and Remainder Theorem to Identify the Roots, and Remainders and Find the Unknown of a Function

- The degree of a polynomial is the polynomial's highest power.
- The roots of a polynomial are its x-intercepts.
- For polynomial long division:

$$\frac{\textbf{Dividend}}{\textbf{Divisor}} = \textbf{Quotient} + \frac{\textbf{Remainder}}{\textbf{Divisor}}$$

- When $P(x)$ is divided by $(x - \alpha)$, the remainder is $P(\alpha)$.
- If $P(\alpha) = 0$, then $(x - \alpha)$ is a factor of $P(x)$.

[1.7.2] - Find Factored Form of Polynomials

- Steps to factor a cubic polynomial are:
  1. Find a single root by trial and error.

     (Factor Theorem: Substitute into the function and see if we get zero).
  2. Use long division to find the quadratic factor.
  3. Factorise the quadratic.
- Rational Root Theorem **narrows down** the possible roots. If the roots are rational numbers, it must be that any:

$$\textit{Potential root} = \frac{\textit{Factors of constant term } a_0}{\textit{Factors of leading coefficient } a_n}$$

- Sum and difference of cubes:

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

[1.7.3] - Graph Factored and Unfactored Polynomials

- Graphs of $a(x-h)^n + k$, where n is an odd positive integer that is not equal to 1:
  - The point (h, k) gives us the stationary point of inflection.
- Graphs of $a(x-h)^n + k$, where n is an even positive integer:
  - The point (h, k) gives us the turning point.
  - These graphs look like a quadratic.

- Steps to graphing factorised polynomials:
  1. Plot x-intercepts.
  2. Determine whether the polynomial is positive or negative.
  3. Use the repeated factors to deduce the shape:
     - Non-Repeated: Only x-intercept.
     - Even Repeated: x-intercept and a turning point.
     - Odd Repeated: x-intercept and a stationary point of inflection.

Cheat Sheet

[1.7.4] - Identify Odd, and Even Functions and Correct Power Functions

- **Odd Functions:**

$$f(-x) = -f(x)$$

- Property: Reflecting on the __y-axis__ is the same as reflecting around the __x-axis__.

- **Even Functions:**

$$f(-x) = f(x)$$

- Property: It is symmetrical about the __y-axis__.

- **Power Functions:**

$$y = x^{\frac{n}{m}}$$

- m: Dictates the number of **tails**.
  - Odd m = __Two__ tails.
  - Even m = __One__ tail.
- n: Dictates the **range**.
  - Odd n: Range could be __all real__.
  - Even n: Range must be __non-negative__
- Power > 1: Looks like a __polynomial__ function.
- Power < 1: Looks like a __root__ function.

Cheat Sheet

[1.8.1] – Apply Transformations to Restrict the Number of Positive/Negative x-intercept(s)

- To solve these questions, figure out how to translate the relevant intercept to the __ origin __.

[1.8.2] – Apply Discriminant to Solve Number of Solutions Questions

- There are no real solutions for a quadratic when Δ __ $<$ __ 0.
- There is one real solution for a quadratic when Δ __ $=$ __ 0.
- There are two unique real solutions for a quadratic when Δ __ $>$ __ 0.

[1.8.3] – Apply Shape/Graph to Solve Number of Solutions Questions

To find the number of solutions for $f(x) = k$, draw a __ horizontal __ line at __ y __ $= k$ and count the intersections.

[1.8.4] – Apply Odd and Even Functions (MHS Investigation 2023)

- For an odd function, $f(x) =$ __ $-f(-x)$ __.
- For an even function, $f(x) =$ __ $f(-x)$ __.

[1.8.5] – Identify Possible Rule(s) From a Graph

- A turning point x-intercept has $a(n)$ __ even __ power on its factor.
- A stationary point of inflection x intercept has $a(n)$ __ odd __ power on its factor.
- If the x-intercept passes straight through, the power of the factor is __ 1 __.

Section B: Questions (61 Marks)

Sub-Section: Exam 1

INSTRUCTION: 31 Marks. 5 Minutes Reading. 35 Minutes Writing.

Question 1 (4 marks)

Consider the points $A(-3, 5)$ and $B(4, -2)$.

a. Find the equation of the line joining A and B and hence, find the angle that the line segment AB makes with the positive x-axis. (2 marks) **[1.5.2] [1.5.3]**

Solution: Gradient -1 and through the point $(-3, 5)$. Therefore

$$y = 2 - x$$

Makes an angle of $135°$ with the positive x-axis.

b. Find the equation of the line that the point A could be reflected in to map it onto the point B. (2 marks) **[1.5.2] [1.6.1]**

The desired line is the perpendicular bisector of AB.

$$y = x + 1$$

Question 2 (3 marks) **[1.5.4]**

Consider the system of linear equations:

$$(a-4)x + 3y = 2$$

$$4x + (a+7)y = a+3$$

where $a \in \mathbb{R}$. Find the value of a such that the system of equations has infinitely many solutions.

Solution: Gradients must be equal so

$$\frac{a-4}{3} = \frac{4}{a+7}$$
$$\implies a = -8, 5$$

y-intercepts must also be equal $\frac{2}{3} = \frac{a+3}{a+7} \implies a = 5$

Infinitely many solutions when $a = 5$.

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Question 3 (10 marks)

Consider the functions f and g, defined over their maximal domains where:

$$f(x) = 2\sqrt{x+2} - 2$$

$$g(x) = \log_2(3 - x)$$

a. Find the maximal domain of $f(x) + \frac{1}{\sqrt{g(x)}}$. (2 marks) **[1.1.1]**

Solution: $f(x)$ has domain $x \geq -2$ and $g(x)$ has domain $x < 3$. But for $\frac{1}{\sqrt{g(x)}}$ to exist we require $g(x) > 0 \implies x < 2$.
So the maximal domain of $f(x) + \frac{1}{\sqrt{g(x)}}$ is $x \in [-2, 2]$.

b. Show that $f\big(g(x)\big)$ is not defined. (1 mark) **[1.1.2]**

Solution: dom $f = [-2, \infty)$ and ran $f = [-2, \infty)$.
dom $g = (-\infty, 3)$ and ran $g = \mathbb{R}$
$f(g(x))$ is not defined since ran $g \not\subseteq$ dom f.

c. The domain of g is restricted to $x \in (-\infty, a]$. Find the largest value of a such that $f\big(g(x)\big)$ exists and write down its rule. (3 marks) **[1.1.2] [1.2.1]**

Solution: We must restrict the range of g to $y \geq -2$. Solve

$$\log_2(3 - x) = -2 \implies 3 - x = \frac{1}{4} \implies x = \frac{11}{4}$$

Therefore $a = \frac{11}{4}$ and

$$f(g(x)) = 2\sqrt{\log_2(3 - x) + 2} - 2$$

d. Define f^{-1}, the inverse function of f. (2 marks) **[1.1.3]**

$$f^{-1} : [-2, \infty) \to \mathbb{R},\ f^{-1}(x) = \frac{1}{4}(x+2)^2 - 2.$$

e. Find all points of intersection between f and f^{-1}. (2 marks) **[1.1.3]**

$$(-2, -2) \text{ and } (2, 2)$$

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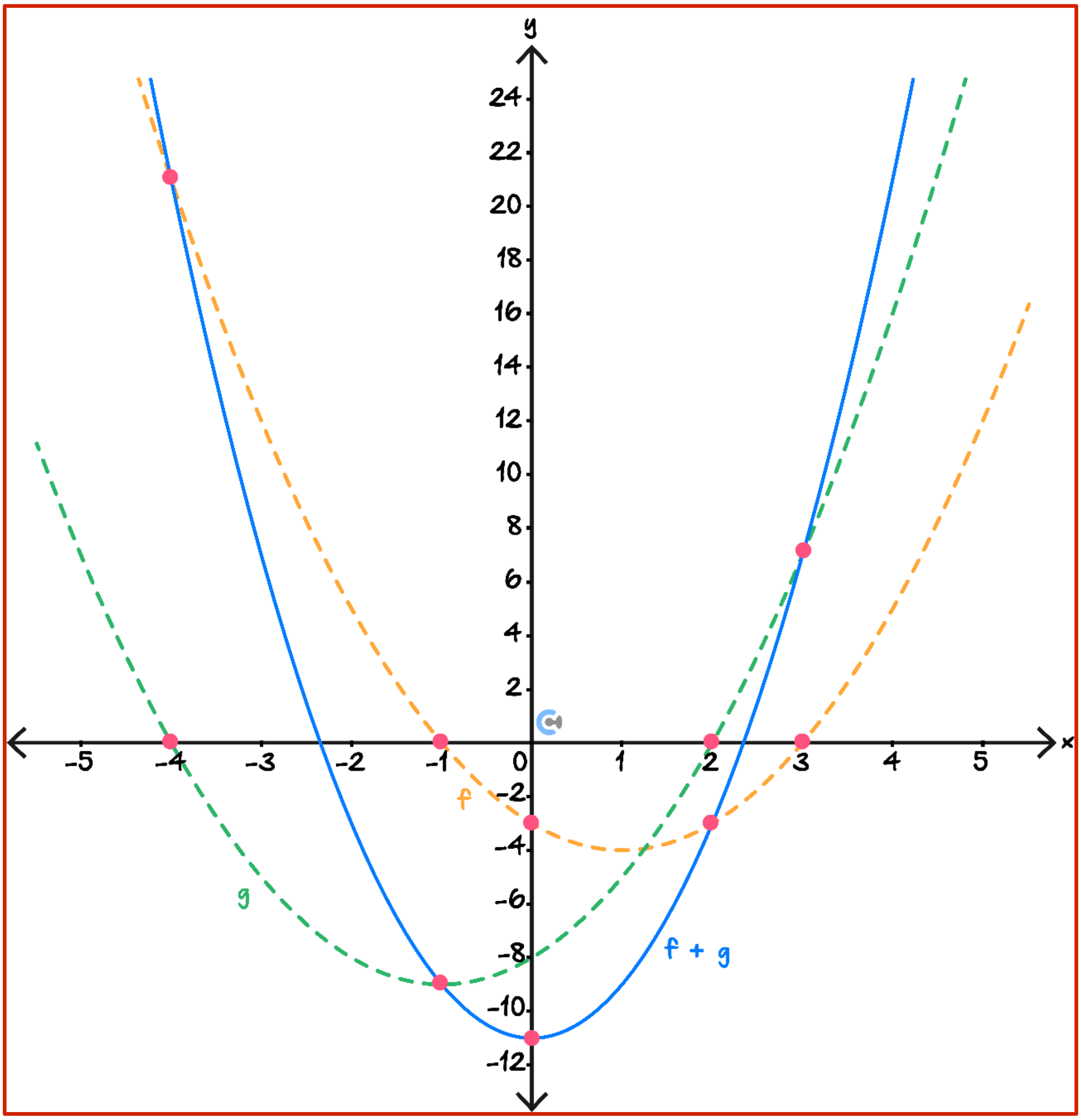
Question 4 (3 marks) **[1.5.5]**

The graphs of quadratic functions f and g are sketched on the axes below. Sketch the graph of $f + g$ on the same axes.



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Question 5 (4 marks)

Consider the functions $f(x) = 2\log_2(x)$ and $g(x) = -4\log_2(3x - 6)$.

a. Using dilations, reflections, and horizontal translations only, describe a sequence of transformations that map $f(x)$ to $g(x)$. (2 marks) **[1.4.5]**

- A dilation by factor 2 from the x-axis
- A reflection in the x-axis
- A dilation by factor $\frac{1}{3}$ from the y-axis
- A translation 2 units to the right.

b. Without using any dilations from the y-axis, describe a sequence of transformations that map $g(x)$ to $f(x)$. (2 marks) **[1.4.6]**

- A translation 2 units to the left
- A reflection in the x-axis
- A dilation by factor $\frac{1}{2}$ from the x-axis.
- A translation $2\log_2(3)$ units down.

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Question 6 (7 marks)

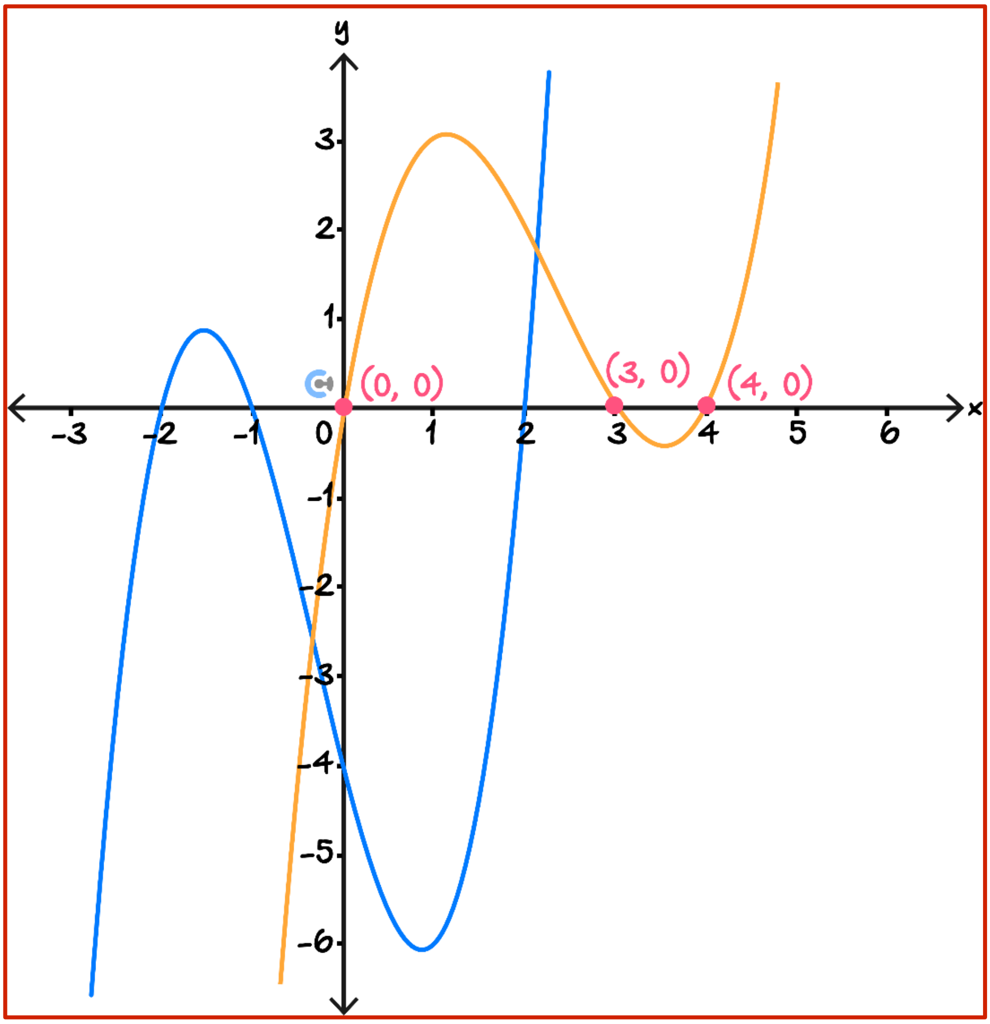
a. Let $f(x) = x^3 - 2x^2 - 11x + 12$. Solve the equation $f(x) = 0$. (2 marks) **[1.7.2]**

Solution: Note that $f(1) = 0$ so $x - 1$ is a factor. Then

$$f(x) = (x-1)(x^2 - x - 12) = (x-1)(x-4)(x+3).$$

So solutions to $f(x) = 0$ are $x = -3, 1, 4$

Let $g(x) = (x-2)(x+1)(x+2)$. The graph of $y = g(x)$ is shown on the axes below:



b. Describe the transformations that map $g(x)$ to $h(x) = -\frac{1}{2}g(2-x)$. (2 marks) **[1.3.1]**

- A dilation by factor $\frac{1}{2}$ from the y-axis
- A reflection in the x-axis
- A reflection in the y-axis
- A translation 2 units to the right.

c. Find the factored form of $h(x)$ and sketch the graph of $h(x)$ on the same axes as $g(x)$. Label all axes intercepts. (3 marks) **[1.3.2]**

$$h(x) = \frac{1}{2}x(x-3)(x-4)$$

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Sub-Section: Exam 2

INSTRUCTION: 30 Marks. 5 Minutes Reading. 35 Minutes Writing.

Question 7 (1 mark) **[1.1.4]**

Consider the function $f: [-3, \infty) \to \mathbb{R}, f(x) = (x+3)^2 - 5$. Which of the following is the rule and domain of $f\left(f^{-1}(x)\right)$?

A. $f\left(f^{-1}(x)\right) = x, x \in [-3, \infty)$

B. $f\left(f^{-1}(x)\right) = x, x \in [-5, \infty)$

C. $f\left(f^{-1}(x)\right) = -x, x \in (-\infty, -5]$

D. $f\left(f^{-1}(x)\right) = x, x \in (-\infty, -3]$

Question 8 (1 mark) **[1.2.2]**

The range of the function $f(x) = \log_2\left(\sqrt{x^2+4}\right)$ is:

A. $[2, \infty)$

B. $(2, \infty)$

C. $[1, \infty)$

D. $(1, \infty)$

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Question 9 (1 mark) **[1.2.3]**

The function f has an inverse function f^{-1}. It is known that $f(1) = 2, f(2) = 3$ and $f'(2) = 3, f'(3) = 5$. Find the gradient of f^{-1} when $x = 3$.

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. 2

D. $\frac{1}{5}$

Question 10 (1 mark) **[1.4.1]**

The function $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ maps the graph of $y = (x-1)^2$ onto the graph $y = 2(x-3)^2 + 6$. The rule for T could be:

A. $T(x, y) = (x - 2, 2y - 6)$

B. $T(x, y) = (x + 2, 2y + 3)$

C. $T(x, y) = (x + 2, 2y - 3)$

D. $T(x, y) = (x + 2, 2y + 6)$

Question 11 (1 mark) **[1.7.1]**

The polynomial $x^3 + ax^2 + bx + 5$ is perfectly divisible by $x + 3$ and has a remainder of 1 when divided by $x - 2$.The values (a, b) are:

A. $(4, 12)$

B. $\left(\frac{4}{15}, -\frac{98}{15}\right)$

C. $\left(\frac{16}{3}, -\frac{26}{3}\right)$

D. $\left(-\frac{14}{3}, \frac{10}{3}\right)$

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Question 12 (1 mark) **[1.8.2]**

The function $f(x) = x^3 - x^2 + (k - 6)x + 2k$, where $k \in \mathbb{R}$, has exactly one root for:

A. $k < \frac{9}{4}$

B. $k > \frac{9}{4}$

C. $-\frac{9}{4} < k < \frac{9}{4}$

D. $k = \frac{9}{4}$

Question 13 (1 mark) **[1.4.2]**

Let $f(x)$ and $g(x) = -\frac{1}{3}f(4x + 8)$ be functions. A sequence of transformations that maps $g(x)$ to $f(x)$ is:

A. A dilation by a factor $\frac{1}{3}$ from the x-axis, a dilation by a factor $\frac{1}{4}$ from the y-axis, a reflection in the x-axis, and a translation 2 units to the left.

B. A dilation by a factor 3 from the x-axis, a dilation by a factor 4 from the y-axis, a reflection in the x-axis. And a translation 2 units to the left.

C. A translation 2 units to the right, a reflection in the x-axis, a dilation by a factor 4 from the y-axis, and a dilation by a factor 3 from the x-axis.

D. A translation 2 units to the left, a reflection in the x-axis, a dilation by a factor 3 from the y-axis, and dilation by a factor 4 from the x-axis.

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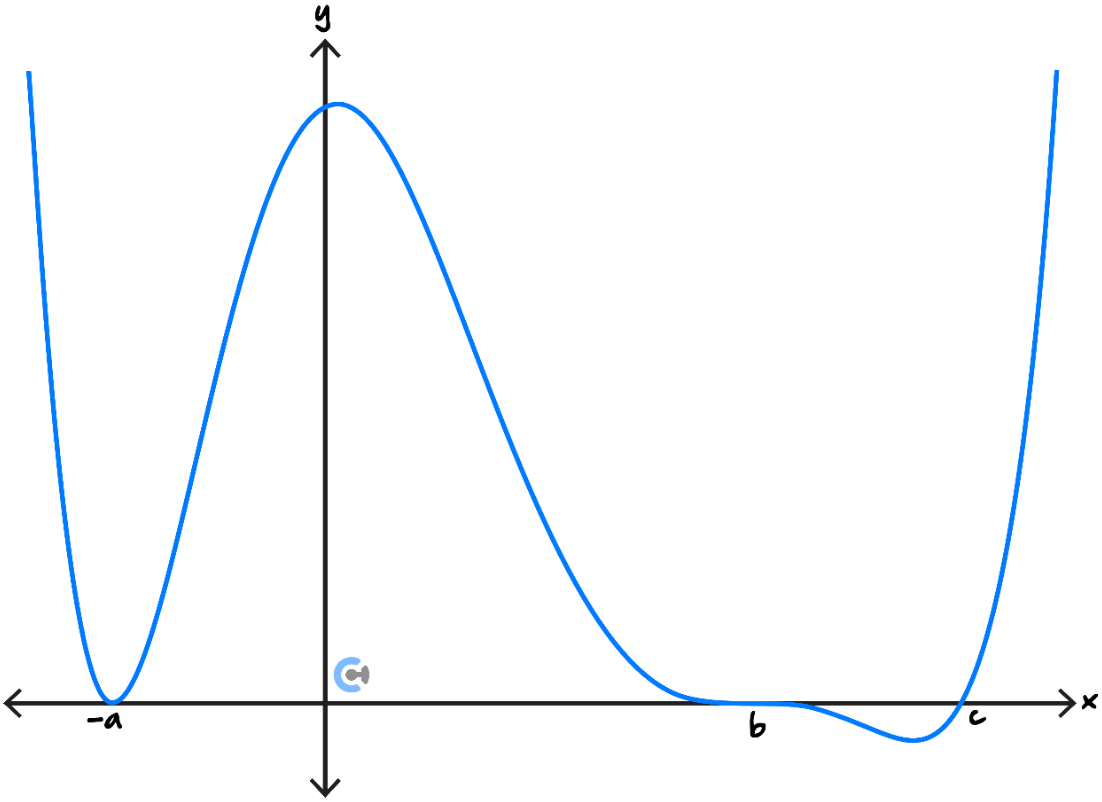
Question 14 (1 mark)

Consider the graph of a function f shown below, where $a, b, c > 0$. A possible rule for $f(x)$ is:



A. $f(x) = -(x+a)^2(x-b)^3(x-c)$

B. $f(x) = (x+a)^2(x-b)^3(x-c)$

C. $f(x) = (x-a)^2(x-b)^3(x-c)$

D. $f(x) = (x+a)^2(x+b)^3(x-c)$

Question 15 (1 mark) **[1.7.4] [1.8.4]**

Consider the function $f(x) = x^5 + 3x^3 + (k^2 - 3k - 4)x^2 + 2kx + 2k^2 + k - 1$. The value(s) of k for which $f(x)$ is an odd function are:

A. $k = 1$

B. $k = 1$ or $k = -1$

C. $k = -1$

D. $k = 4$

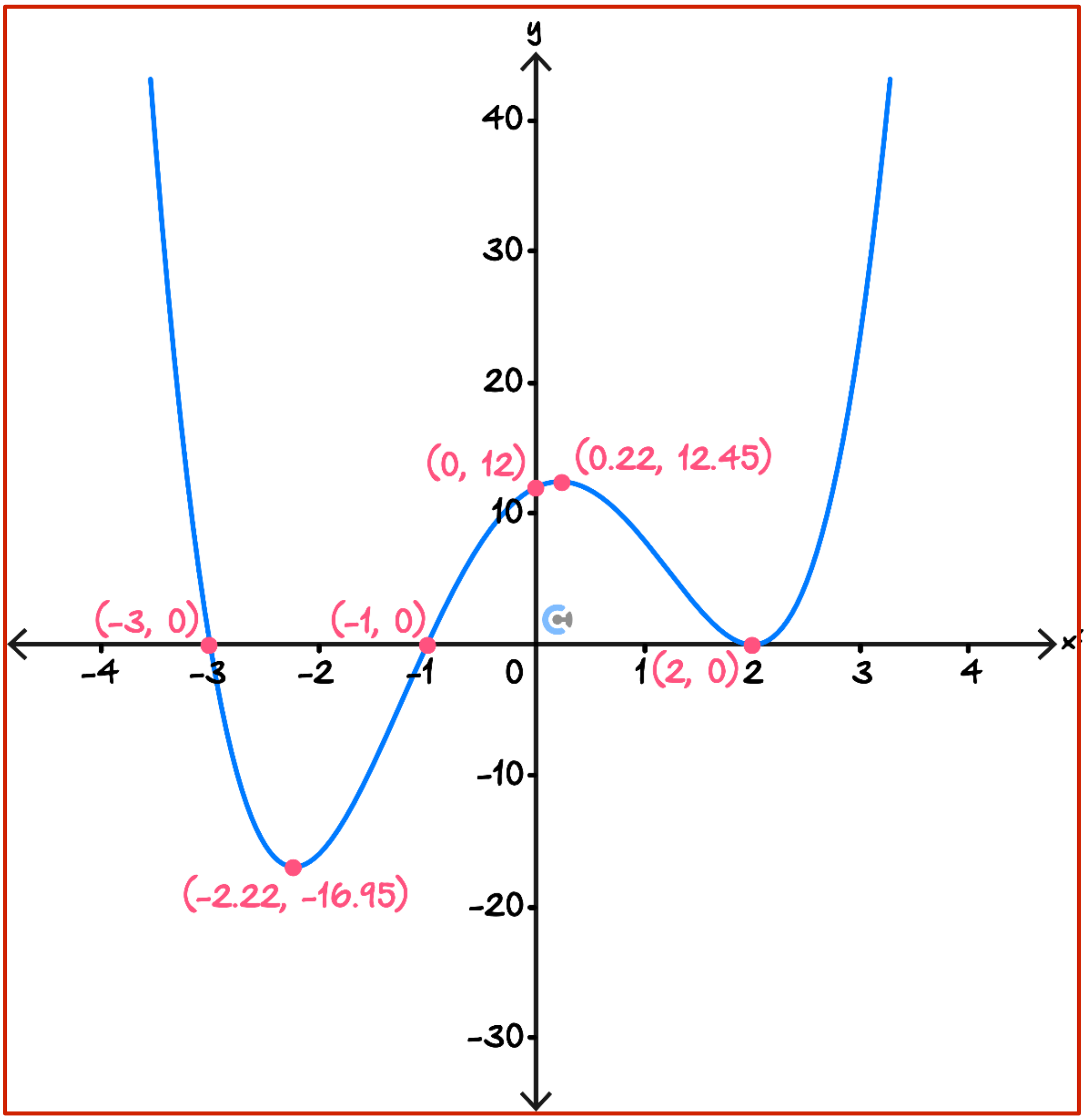
Question 16 (12 marks)

Consider the quadratic function $f(x) = x^4 - 9x^2 + 4x + 12$.

a. Fully factorise f and hence, find all its roots. (2 marks) **[1.7.2]**

Solution: $f(x) = (x-2)^2(x+1)(x+3)$.
Roots are $x = -3, -1, 2$

b. Sketch the graph of $y = f(x)$ on the axes below. Label all axes, intercepts, and turning points correct to two decimal places where appropriate. (3 marks) **[1.7.3]**



c.

i. Find all values of $k \in \mathbb{R}$ such that $f(x-k) = 0$ has two positive solutions. (2 marks) **[1.8.1]**

Solution: Must translate more than 1 unit to the right but less than or equal to 3 units right.
$1 < k \leq 3$

ii. The equation $f(x) = a, a \in \mathbb{R}$ has no solutions. Find all possible values of a, exactly. (1 mark) **[1.8.3]**

$$a < -\frac{9}{4} - 6\sqrt{6}$$

iii. Find the shortest horizontal distance between two points on the graph of $y = f(x)$ when $y = 10$. Give your answer correct to two decimal places. (2 marks) **[1.5.1]**

Solution: Solve $f(x) = 10 \implies x = -3.1721$ or $x = -0.299795$ or $x = 0.781785$ or $x = 2.69011$
Shortest distance is 1.08.

d. The graph of $y = f(x)$ and the graph of $y = -f(x) + k$ has exactly one point of intersection. Find the exact value of k. (2 marks) **[1.3.3]**

Solution: One intersection if $y = -f(x)$ is translated down $2\left(\frac{9}{4} + 6\sqrt{6}\right) = \frac{9}{2} + 12\sqrt{6}$ units.
Therefore $k = -\frac{9}{2} - 12\sqrt{6}$.

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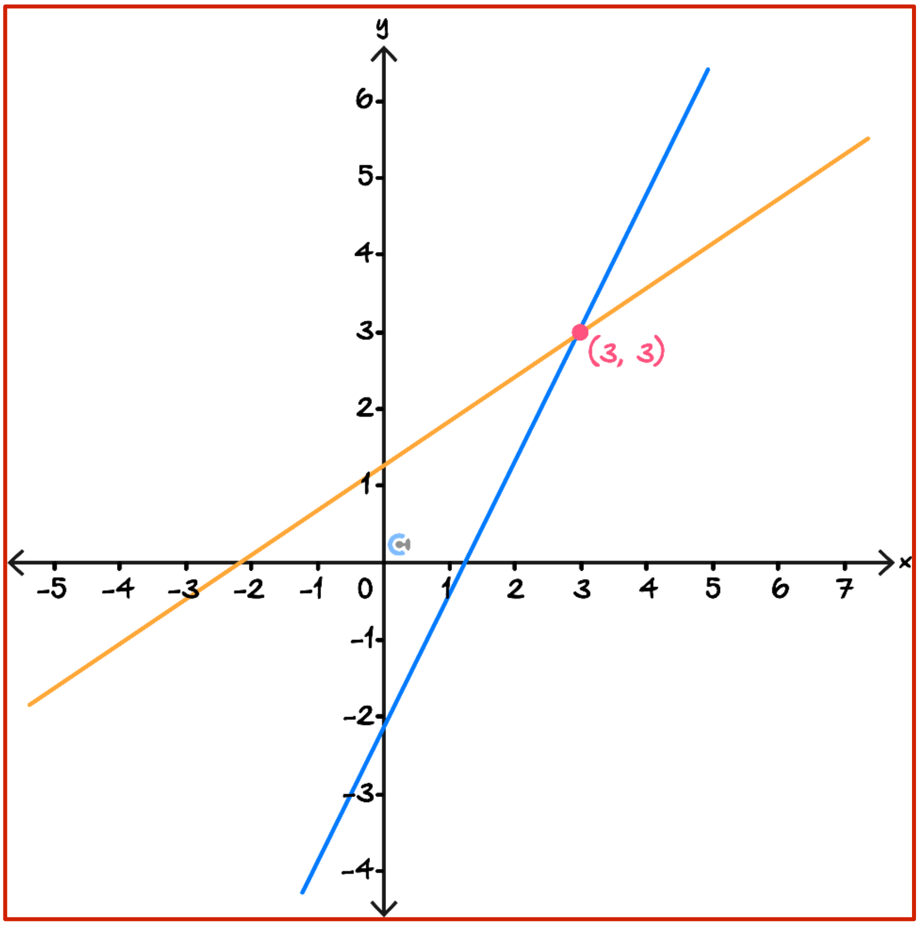
Question 17 (9 marks)

Consider the function $f : \mathbb{R} \to \mathbb{R}, f(x) = \sqrt{3}x + 3 - 3\sqrt{3}$.



a.

i. Sketch the graph of f and f^{-1} on the axes above. Label the point of intersection with coordinates. (2 marks) **[1.1.3]**

ii. Find the distance between the origin and the intersection. (1 mark) **[1.5.1]**

$3\sqrt{2}$

iii. Find the exact size of the acute angle between f and f^{-1} at their intersection point. (1 mark) **[1.5.3]**

$30°$

Consider the functions $g(x) = 2x - 5$ and $h : [-\sqrt{k}, \sqrt{k}] \to \mathbb{R}, h(x) = \frac{1}{\sqrt{k}}x - k$, where $k \in \mathbb{R}^+$.

b. Find the coordinates for any point of intersection between g and h in terms of k. (1 mark)

$$\left(\frac{(5-k)\sqrt{k}}{2\sqrt{k}-1}, \frac{2(5-k)\sqrt{k}}{2\sqrt{k}-1} - 5\right)$$

c. Find the values of k for which $g(x) = h(x)$ has a unique solution. (2 marks)

Solution: The intersection must be within $[-\sqrt{k}, \sqrt{k}]$. Solve

$$-\sqrt{k} \le \frac{(5-k)\sqrt{k}}{2\sqrt{k}-1} \le \sqrt{k}$$

We get $k \in [8 - 2\sqrt{7}, 6 + 2\sqrt{5}]$

d. Find the shortest distance from any intersection of g and h to the origin. Give your answer correct to two decimal places. (2 marks) **[1.5.1]**

Solution: Shortest distance between $g(x)$ and origin occurs along line $y = -\frac{1}{2}x$ at point $(2, -1)$. However this happens when $k < 8 - 2\sqrt{7}$, so not valid.
The shortest distance therefore occurs when $k = 8 - 2\sqrt{7}$ and we use the distance function

$$d(k) = \sqrt{\left(\frac{(5-k)\sqrt{k}}{2\sqrt{k}-1}\right)^2 + \left(\frac{2(5-k)\sqrt{k}}{2\sqrt{k}-1} - 5\right)^2}$$

to get a shortest distance of approximately 2.37.

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