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VCE Mathematical Methods ¾
AOS 1 Revision [0.8]
Workshop

Error Logbook:

Mistake/Misconception #1		Mistake/Misconception #2	
Question #:	Page #:	Question #:	Page #:
Notes:		Notes:	
Mistake/Misconception #3		**Mistake/Misconception #4**	
Question #:	Page #:	Question #:	Page #:
Notes:		Notes:	

Section A: Cheat Sheets

Cheat Sheet

[1.1.1] - Find the Maximal Domain and Range

- Inside of a $\log$ must be positive.
- Inside of a root must be non-negative.
- Denominator cannot = 0 . .
- The domain of sum or product of two functions is equal to intersection of the two domains.

[1.1.2] - Find the Rule, Domain and Range of a Composite Function (Range Does Not Require Splitting to Find as the Function is Easy to Draw)

- $f(g(x)) =$ $f \circ g$ (x).
- For composite function to exist, Range Ins $\subseteq$ Dom Out.
- The domain of composite is equal to the domain of inside function.
- Range of composite is a subset of the range of the outside.

[1.1.3] - Find the Rule, Domain, and Range of Inverse Functions

- f needs to be 1:1 for f^{-1} to exist.
- Domain of the inverse function equals to range of the original and vice versa.
- Symmetrical around $y = x$.
- For intersections of inverses, we can equate the function to $y = x$.

[1.1.4] - Find the Composite Function of the Inverse Function

- The composite function of inverses is always equal to x.

Cheat Sheet

[1.2.1] - Find a new domain to fix composite functions

- The range of the __outer__ function must be a subset of the __dom__ of the outside function.
- We restrict the __domain__ of the inside function so its __range__ fits in the domain of the outside function.

[1.2.2] - Find the range of complex composite functions

- To find the range of a complicated function, we can break the function into a __series__ of two simpler functions.

[1.2.3] - Find the gradient of inverse functions

- If the gradient of f at $(a, f(a)) = m$, then the gradient of f^{-1} at $(f(a), a) = \frac{1}{m}$.

Cheat Sheet

[1.3.1] - Applying x' and y' notation to find transformed points, find the interpretation of transformations and altered order of transformations.

- The transformed point is called the image and is denoted by x', y'.
- The dilation factor is multiplied. to the original coordinate.
- Reflection makes the original coordinates the negative of their original values.
- Translation adds a unit to the original coordinate.
- Transformations should be interpreted when x' & y' are isolated.
- The order of transformation follows the bodmas order.
- To change the order of transformations, we either factorise or expand.

[1.3.2] - Find transformed functions.

- To transform the function, replace its old variables with the new one.

[1.3.3] - Find transformations from transformed functions (Reverse Engineering).

- To find the transformations, simply equate the LHS & RHS after separating the transformations of x and y.

Cheat Sheet

[1.4.1] - Apply quick method to find transformations

[1.4.2] - Find opposite transformations

[1.4.3] - Apply transformations of functions to find their domain, range, transformed points and tangents.

[1.4.4] - Find transformations of the inverse functions $f(x)$

[1.4.5] - Find multiple transformations for the same functions

[1.4.6] - Apply manipulation of the functions to find appropriate transformations.

Cheat Sheet

[1.5.1] - Find the Midpoint and Distance (Horizontal & Vertical) Between Two Points Or Functions

- Midpoint is simply the aveage of 2 points.
- Distance formula is derived from pythag.
- Horizontal distance is the distance between x values.
- Vertical distance is the distance between y values.

[1.5.2] - Find Parallel and Perpendicular Lines

- Parallel lines have the same gradient.
- Perpendicular lines have negative reciprocal gradient.

[1.5.3] - Find the Angle Between a Line and x-axis or Two Lines

- To find the angle between a line and the x-axis we can use the equation $m = \tan(\theta)$.
- To find the angle between two lines we can use $\theta = |\tan^{-1}(m_1) - \tan^{-1}(m_2)|$ or $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

[1.5.4] - Find The Unknown Value for Systems of Linear Equations

- Two linear equations have unique solutions if they have different gradients.
- Two linear equations have infinitely many solutions when they have same gradient and same constant.
- Two linear equations have no solution when they have same gradient and different constant.

[1.5.5] - Sketching the sum of two function's graph by using the addition of ordinates

- Addition of ordinates is used to sketch the sum of two functions.
- We always add their y values.
- When we have an x intercept for one graph, sum graph equals to the other graph.
- When we have an intersection between two graphs, the sum graph equals to double their y value.
- When we have an equidistance from the x-axis, sum graph has an x intercept.

Cheat Sheet

[1.6.1] - Apply Midpoint to Find a Reflected Point.

- The line between a point and its reflection is ____perpendicular____ to the line it is reflected in.
- The ____________________ of a line and its reflection lies on the line it is reflected in.
- ~~Steps for finding the reflection of a point in a line:~~
  1. ~~Find the __________________________ line passing through the point.~~
  2. ~~Find the ________________________ between the original line and the perpendicular line.~~
  3. ~~Find the reflected point (x, y) by treating the intersection from 2. as the ___________________ between the original and reflected point.~~

[1.6.2] - Apply parallel and perpendicular lines to geometric problems

Cheat Sheet

[1.7.1] - Apply the Factor Theorem and Remainder Theorem to identify the roots, and remainders and find the unknown of a function.

- The degree of a polynomial is the polynomial's ____highest____ power.
- The roots of a polynomial are its ____x intercept.____.
- For polynomial long division:

$$\frac{\textit{Dividend}}{\textit{Divisor}} = \textit{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$$

- When $P(x)$ is divided by $(x-\alpha)$, the remainder is ____$p(\alpha)$____.
- If $P(\alpha) = 0$, then $(x-\alpha)$ is a ____factor.____ of $P(x)$.

[1.7.2] - Find factored form of polynomials.

- Steps to factor a cubic polynomial are:

1. Find a single root by trial and error.

   (Factor Theorem: Substitute into the function and see if we get ____0____).
2. Use ____long division____ to find the quadratic factor.
3. Factorise the quadratic.

- Rational Root Theorem **narrows down** the possible roots. If the roots are rational numbers, it must be that any:

$$\textit{Potential root} = \pm\frac{\textit{Factors of } \underline{\text{constant term}}\ a_0}{\textit{Factors of } \underline{\text{leading co.ef}}\ a_n}$$

- Sum and difference of cubes:

$$a^3 + b^3 = (\underline{\ a+b\ })(a^2 - ab + b^2)$$

$$a^3 - b^3 = (\underline{\ a-b\ })(a^2 + ab + b^2)$$

[1.7.3] - Graph factored and unfactored polynomials

- Graphs of $a(x-h)^n + k$, where n is an odd positive integer that is not equal to 1:
  - The point (h,k) gives us the stationary point of ____inflection____.
- Graphs of $a(x-h)^n + k$, where n is an even positive integer:
  - The point (h,k) gives us the ____t.p____.
  - These graphs look like a ____quadratic.____.
- Steps to graphing factorised polynomials:

1. Plot x-intercepts.
2. Determine whether the polynomial is positive or negative.
3. Use the repeated factors to deduce the shape:
   - Non-Repeated: Only ____x int____.
   - Even Repeated: x-intercept and a ____t.p____.
   - Odd Repeated: x-intercept and a ____stationary point of inf. (SPI)____.

[1.7.4] – Identify odd, and even functions and correct power functions.

- **Odd functions:**

$$f(-x) = -f(x)$$

- Property: Reflecting on the __y axis__ is the same as reflecting around the __x axis.__.

- **Even functions:**

$$f(-x) = f(x)$$

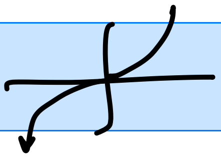
- Property: It is symmetrical about the __y axis__.

- **Power Functions:**

$$y = x^{\frac{n}{m}}$$

- m: Dictates the number of **tails**.
  - Odd m = __2__ tails.
  - Even m = __1__ tail.
- n: Dictates the **range**.
  - Odd n: Range could be __$\ominus$__.
  - Even n: Range must be __non-negative.__.
- Power > 1: Looks like a __polynomial__ function.
- Power < 1: Looks like a __root__ function.

Cheat Sheet



[1.8.1] – Apply Transformations to Restrict the Number of Positive/Negative x-intercept(s)

- To solve these questions, figure out how to translate the relevant intercept to the condition.

[1.8.2] – Apply Discriminant to Solve Number of Solutions Questions

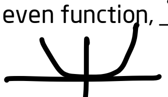
- There are no real solutions for a quadratic when Δ $<$ 0.
- There is one real solution for a quadratic when Δ $=$ 0.
- There are two unique real solutions for a quadratic when Δ $\geq$ 0.

[1.8.3] – Apply Shape/Graph to Solve Number of Solutions Questions

- To find the number of solutions for $f(x) = k$, draw a horizontal line at y $= k$ and count the intersections.

[1.8.4] – Apply Odd and Even Functions (MHS Investigation 2023)

- For an odd function, $f'(a)=f'(-a)$.
- For an even function, $-f'(a) = f'(-a)$.



[1.8.5] – Identify Possible Rule(s) From a Graph

- A turning point x-intercept has $a(n)$ even power on its factor.
- A stationary point of inflection x intercept has $a(n)$ odd power on its factor.
- If the x-intercept passes straight through, the power of the factor is 1.

Section B: Questions (66 Marks)

Sub-Section: Exam 1

INSTRUCTION: 30 Marks. 5 Minutes Reading. 35 Minutes Writing.

Question 1 (4 marks)

Consider the points $A(-3,5)$ and $B(4,-2)$.

a. Find the equation of the line joining A and B and hence find the angle that the line segment AB makes with the positive x-axis. (2 marks) **[1.5.2] [1.5.3]**

$m = \frac{5-(-2)}{-3-4} = -1$

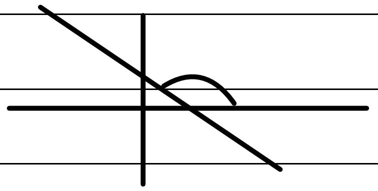
$\therefore \ y = -x + c$

sub $(-3,5)$

$5 = 3 + c$

$c = 2$

$\therefore \ y = -x + 2.$



$\theta = 135^\circ$ or 45°

CRa

b. Find the equation of the line that the point A could be reflected in to map it onto the point B. (2 marks) **[1.5.2] [1.6.1]**

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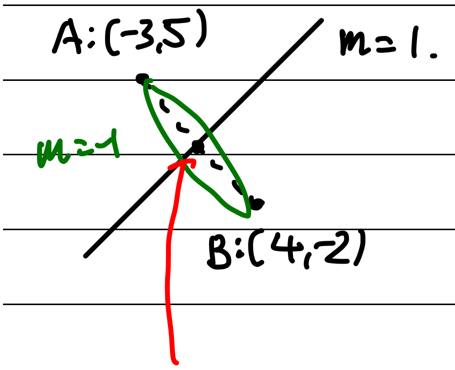


$\therefore \ y = \boxed{x} + c$

sub $(\frac{1}{2}, \frac{3}{2})$

$\frac{3}{2} = \frac{1}{2} + c$

$c = 1$

$\therefore y = x + 1$

Question 2 (3 marks) **[1.5.4]**

Consider the system of linear equations:

$$\begin{aligned}(a-4)x + 3y &= 2\\ 4x + (a+7)y &= a+3\end{aligned}$$

where $a \in \mathbb{R}$. Find the value of a such that the system of equations has infinitely many solutions.

$m_1 = m_2$

$$\frac{a-4}{3} = \frac{4}{a+7}$$

$(a-4)(a+7) = 12$

$a^2 + 3a - 28 = 12$

$a^2 + 3a - 40 = 0$

$(a+8)(a-5) = 0$

$a = 5, -8$

$c_1 = c_2$

$$\frac{3}{2} = \frac{a+7}{a+3}$$

$3a + 9 = 2a + 14$

$a = 5$

$$\boxed{a = 5}$$

Space for Personal Notes

Question 3 (9 marks)

Consider the functions f and g, defined over their maximal domains where:

$$f(x) = 2\sqrt{x+2} - 2$$

$$g(x) = \log_2(3-x)$$

a. Find the maximal domain of $f(x) + \frac{1}{\sqrt{g(x)}}$. (2 marks) **[1.1.1]**

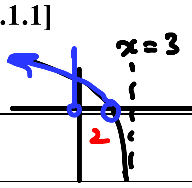
Dom f: $[-2, \infty)$

Dom $\frac{1}{\sqrt{g(x)}}$: $g(x) > 0$

$\log_2(3-x) > 0$

$y > 0$

$x \in (-\infty, 2)$



$0 = \log_2(3-x)$

$2^0 = 3-x$

$1 = 3-x$

$x = 2$

Dom $f + \frac{1}{\sqrt{g}}$: $[-2, 2)$

b. Show that $f(g(x))$ is not defined. (1 mark) **[1.1.2]**

Range $g = \mathbb{R}$

Dom $f = [-2, \infty)$

$\therefore$ Range $g \not\subseteq$ Dom f. $\therefore$ Not defined

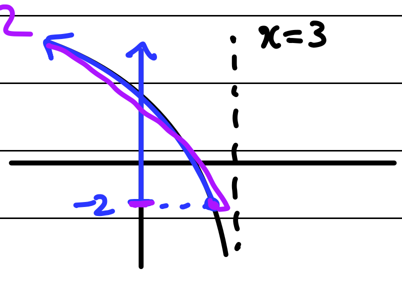
c. The domain of g is restricted to $x \in (-\infty, a]$. Find the largest value of a such that $f(g(x))$ exists and write down its rule. (2 marks) **[1.1.2] [1.2.1]**

Range $g \subseteq$ Dom f

$\log_2(3-x) \subseteq [-2, \infty)$

y value ≥ −2

fun



$\log_2(3-x) = -2$

$3-x = 2^{-2}$

$3-x = \frac{1}{4}$

$3 - \frac{1}{4} = x$

$\frac{11}{4} = x$

$\therefore x \in (-\infty, \frac{11}{4}]$

$\boxed{a = \frac{11}{4}}$

$f(g(x)) =$

$2\sqrt{\log_2(3-x)+2} - 2$

d. Define f^{-1}, the inverse function of f. (2 marks) **[1.1.3]**

let $y = f^{-1}(x)$

$\therefore x = 2\sqrt{y+2} - 2$

$\frac{x+2}{2} = \sqrt{y+2}$

$\frac{(x+2)^2}{4} = y+2$

$f^{-1}(x) = \frac{(x+2)^2}{4} - 2$

Dom f^{-1} = Range f = $(-2, \infty)$

f

e. Find all points of intersection between f and f^{-1}. (2 marks) **[1.1.3]**

$\underbrace{2\sqrt{x+2} - 2}_{\text{Increasing func.}} = x$

$2\sqrt{x+2} = x + 2$

$4(x+2) = x^2 + 4x + 4$

$4x + 8 = x^2 + 4x + 4$

$4 = x^2$

$\boxed{x = \pm 2}$

$\boxed{(-2, -2) \quad \& \quad (2, 2)}$

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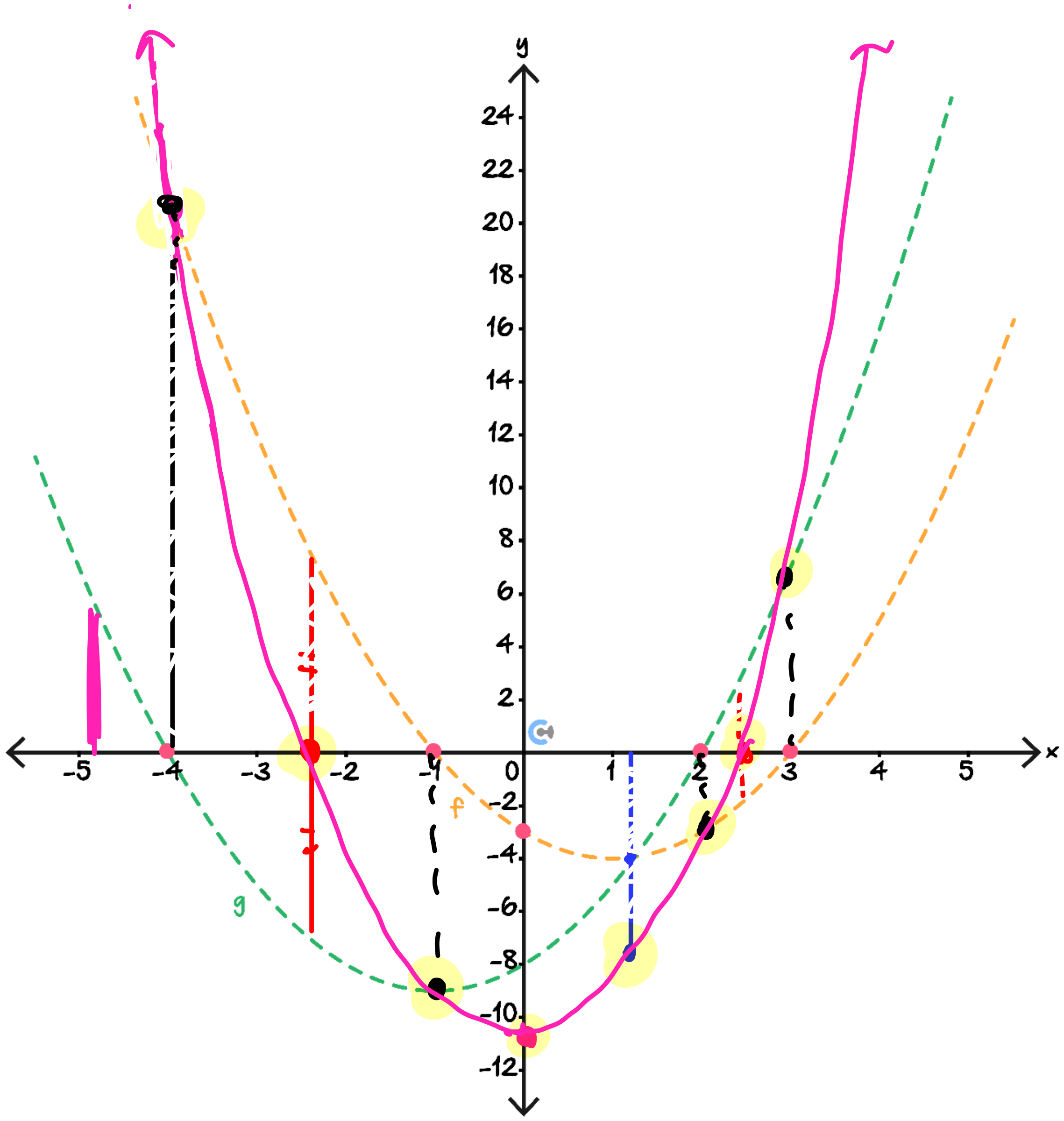
Question 4 (3 marks) **[1.5.5]**

The graphs of quadratic functions f and g are sketched on the axes below. Sketch the graph of $f + g$ on the same axes.



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Question 5 (4 marks)

Consider the functions $f(x) = 2\log_2(x)$ and $g(x) = -4\log_2(3x-6)$.

a. Using dilations, reflections and horizontal translations only, describe a sequence of transformations that map $f(x)$ to $g(x)$. (2 marks) **[1.4.5]**

$y = 2\log_2(x)$

$y = -4\log_2(3x'-6)$

$3x'-6 = x$

$3x' = x+6$

$x' = \frac{1}{3}x + 2$

Dil 2 from x axis
Reflection in x axis } y

Dil $\frac{1}{3}$ from y axis
Translate 2 units right } x

b. Without using any dilations from the y-axis, describe a sequence of transformations that map $g(x)$ to $f(x)$. (2 marks) **[1.4.6]**

$y = -4\log_2(3x-6)$

$= -4\log_2(x-2) \boxed{-4\log_2(3)}$

$y = 2\log_2(x')$

$x' = x-2$

Translate 2 units left.
Translate $4\log_2(3)$ units up
Dil $\frac{1}{2}$ from x axis
Reflect in x axis

OR

Translate 2 units left.
Dil $\frac{1}{2}$ from x axis
Reflect in x axis
Translate $2\log_2(3)$ units down

Space for Personal Notes

Question 6 (7 marks)

a. Let $f(x) = x^3 - 2x^2 - 11x + 12$. Solve the equation $f(x) = 0$. (2 marks) **[1.7.2]**

$f(1) = 1-2-11+12 = 0$

$\therefore x-1$ is a factor

$$\begin{array}{r|l} & x^2 - x - 12 \\ x-1 & x^3 - 2x^2 - 11x + 12 \\ & -(x^3 - x^2) \\ & -x^2 - 11x \\ & -(-x^2 + x) \\ & -12x + 12 \\ & -(-12x + 12) \\ & 0 \end{array}$$

$(x-1)(x-4)(x+3) = 0$

$\boxed{\therefore x = 1, 4, -3.}$

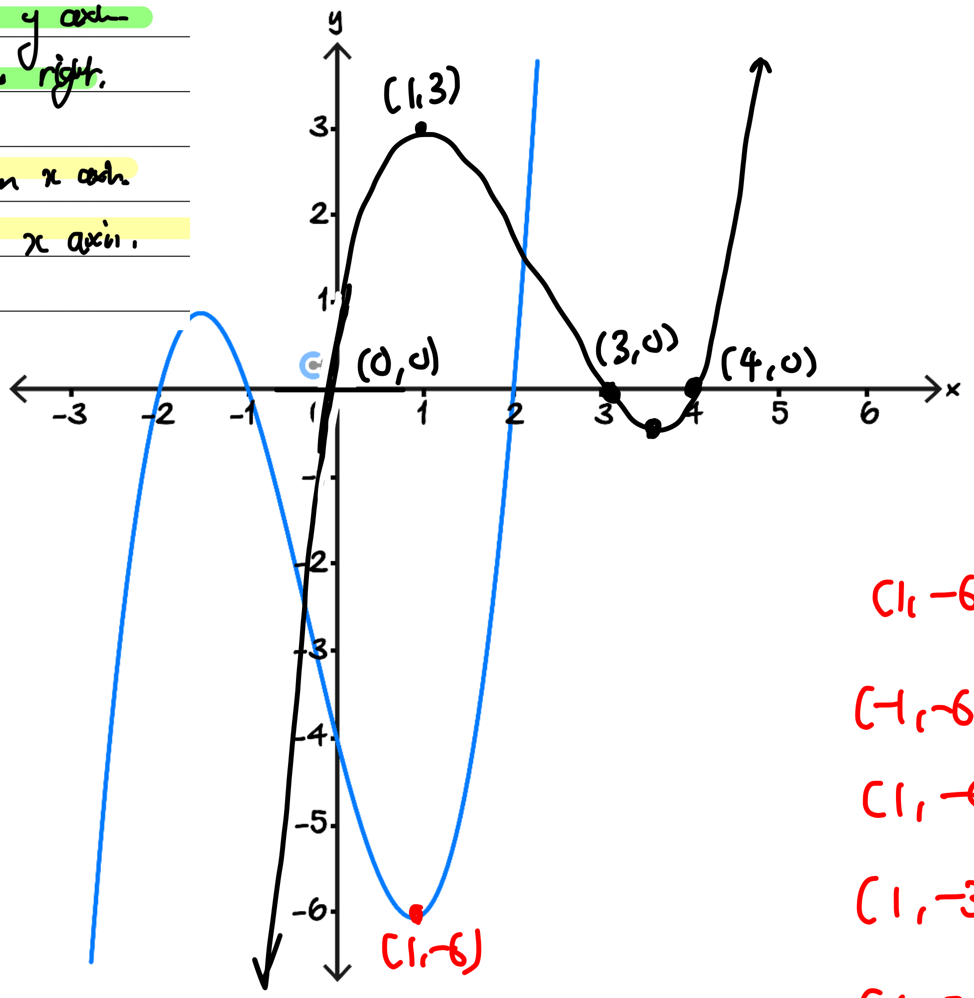
Let $g(x) = (x - 2)(x + 1)(x + 2)$. The graph of $y = g(x)$ is shown on the axes below:

Reflection in y axis

Translate 2 units right.

Dilate $\frac{1}{2}$ from x axis

Reflect in x axis.



$(1, -6)$ ↓

$(-1, -6)$ ↓

$(1, -6)$

$(1, -3)$ ↓

$(1, 3)$

b. Describe the transformations that map $g(x)$ to $h(x) = -\frac{1}{2}g(2-x)$. (2 marks) **[1.3.1]**

Reflection in y axis

Translate 2 units right.

Dilate $\frac{1}{2}$ from x axis

Reflect in x axis.

$y = g(x)$

$y = -\frac{1}{2}g(2-x')$

$2-x' = x$

$2-x = x'$

c. Find the factored form of $h(x)$ and sketch the graph of $h(x)$ on the same axes as $g(x)$. Label all axes intercepts. (3 marks) **[1.3.2]**

$g(x)=(x-2)(x+1)(x+2)$

$-\frac{1}{2}g(2-x) = -\frac{1}{2}(-x)(3-x)(4-x)$

$= \frac{1}{2}x(3-x)(4-x)$

↳ Use key points.

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Sub-Section: Exam 2

INSTRUCTION: 30 Marks. 5 Minutes Reading. 35 Minutes Writing. (No Page 25)

Question 7 (1 mark) **[1.1.4]**

Consider the function $f : [-3, \infty) \to \mathbb{R}, f(x) = (x+3)^2 - 5$. Which of the following is the rule and domain of $f(f^{-1}(x))$?

A. $f(f^{-1}(x)) = x, x \in [-3, \infty)$

B. $f(f^{-1}(x)) = x, x \in [-5, \infty)$

C. $f(f^{-1}(x)) = -x,$ ~~$x \in (-\infty, -5]$~~

D. $f(f^{-1}(x)) = x,$ ~~$x \in (-\infty,$~~ $-3]$

Dom = Dom f^{-1} = Range f

= $[-5, \infty)$

Question 8 (1 mark) **[1.2.2]**

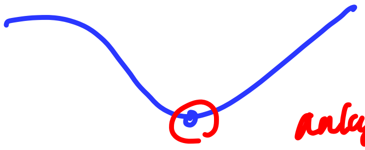
The range of the function $f(x) = \log_2\left(\sqrt{x^2+4}\right)$ is:

A. $[2, \infty)$

B. $(2, \infty)$

C. $[1, \infty)$

D. $(1, \infty)$



Analyse min. Graph (63)

Absolute fn

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Question 9 (1 mark) **[1.2.3]**

The function f has an inverse function f^{-1}. It is known that $\underbrace{f(1) = 2, f(2) = 3 \text{ and}}$ $f'(2) = 3, f'(3) = 5$ Find the gradient of f^{-1} when $x = 3$.

A. $\frac{1}{2}$

[✓ **B.** $\frac{1}{3}$]

C. 2

D. $\frac{1}{5}$

$\frac{dy}{dx} \rightarrow \frac{dx}{dy}$

$f:$ $(1,2)$ & $(2,3)$ $(3,2)$

$m=3$. $m=5$

$f^{-1}:$ $(2,1)$ & $(3,2)$

$m=\frac{1}{3}$

Question 10 (1 mark) **[1.4.1]**

Check if you get

The function $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ maps the graph of $y = (x-1)^2$ onto the graph $y = 2(x-3)^2 + 6$. The rule for T could be:

A. $T(x, y) = (x - 2, 2y - 6)$

B. $T(x, y) = (x + 2, 2y + 3)$

C. $T(x, y) = (x + 2, 2y - 3)$

[✓ **D.** $T(x, y) = (x + 2, 2y + 6)$]

1) define $f(x)=(x-1)^2$

2) A: $2f(x+2)-6$: enter

D: $2f(x-2)+6$ enter : =

Question 11 (1 mark) **[1.7.1]**

The polynomial $x^3 + ax^2 + bx + 5$ is perfectly divisible by $x + 3$ and has a remainder of 1 when divided by $x - 2$.The values (a, b) are:

A. $(4, 12)$

[✓ **B.** $\left(\frac{4}{15}, -\frac{98}{15}\right)$]

C. $\left(\frac{16}{3}, -\frac{26}{3}\right)$

D. $\left(-\frac{14}{3}, \frac{10}{3}\right)$

$$\begin{cases} f(-3) = 0 \\ f(2) = 1. \end{cases}$$

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Question 12 (1 mark) **[1.8.2]**

The function $f(x) = x^3 - x^2 + (k-6)x + 2k$, where $k \in \mathbb{R}$, has exactly one root for:

-100

A. ~~$k < \frac{9}{4}$~~

$\text{solve}(x^3 - x^2 + (k-6)x + 2k = 0, x) \mid k = \bigcirc$ Try value

B. $k > \frac{9}{4}$

Ctrl Enter

C. ~~$-\frac{9}{4} < k < \frac{9}{4}$~~

D. ~~$k = \frac{9}{4}$~~

NSolve[{ eg, k = 5, {x, k}

Question 13 (1 mark) **[1.4.2]**

$4x + 8 = x'$

$4(x+2)$

Let $f(x)$ and $g(x) = -\frac{1}{3}f(4x+8)$ be functions. A sequence of transformations that maps $g(x)$ to $f(x)$ is:

A. A dilation by a factor $\frac{1}{3}$ from the x-axis, a dilation by factor $\frac{1}{4}$ from the y-axis, a reflection in the x-axis and a translation 2 units to the left.

B. A dilation by a factor 3 from the x-axis, a dilation by factor 4 from the y-axis, a reflection in the x-axis and a translation 2 units to the left.

C. A translation 2 units to the right, a reflection in the x-axis, a dilation by factor 4 from the y-axis and a dilation by a factor 3 from the x-axis.

D. A translation 2 units to the left, a reflection in the x-axis, a dilation by factor 3 from the y-axis and dilation by a factor 4 from the x-axis.

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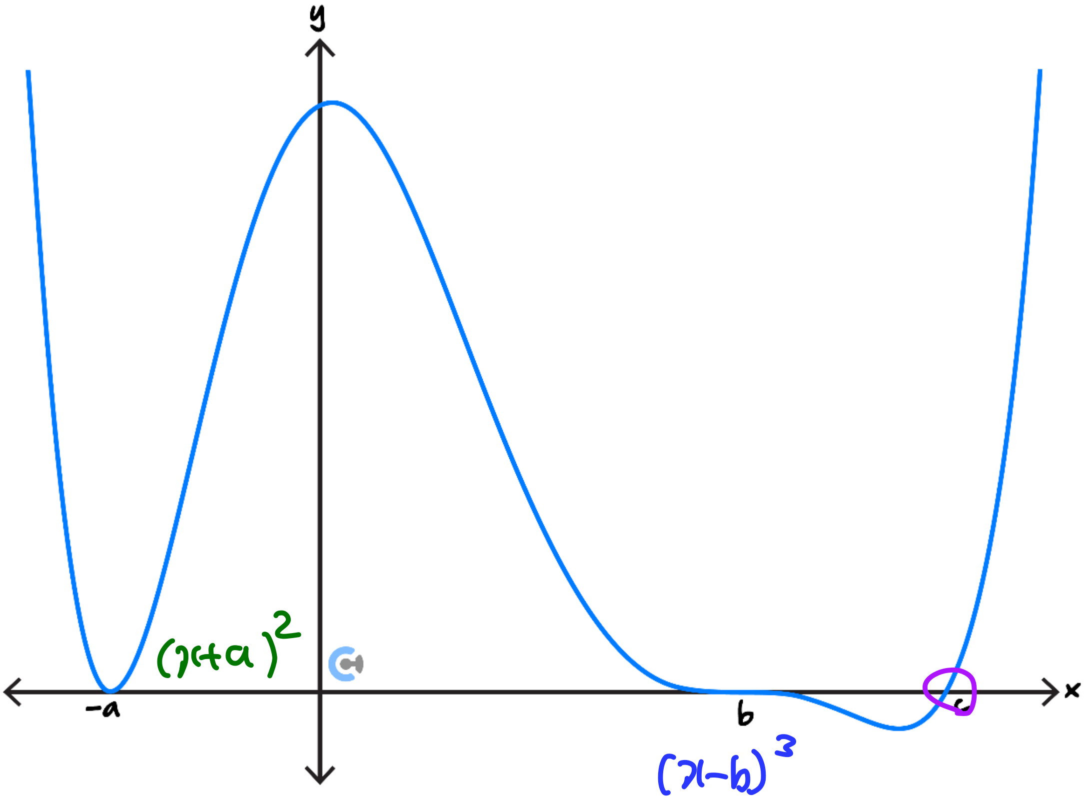
Question 14 (1 mark)

Consider the graph of a function f shown below, where $a, b, c > 0$. A possible rule for $f(x)$ is:



A. $f(x) = -(x+a)^2(x-b)^3(x-c)$

B. $f(x) = (x+a)^2(x-b)^3(x-c)$

~~**C.** $f(x) = (x-a)^2(x-b)^3(x-c)$~~

~~**D.** $f(x) = (x+a)^2(x+b)^3(x-c)$~~

Question 15 (1 mark) **[1.7.4] [1.8.4]**

Consider the function $f(x) = x^5 + 3x^3 + (k^2 - 3k - 4)x^2 + 2kx + 2k^2 + k - 1$. The value(s) of k for which $f(x)$ is an odd function are:

A. $k = 1$

B. $k = 1$ or $k = -1$

C. $k = -1$

D. $k = 4$

solve(f(-x) = -f(x), k).

$k = -1.$

$\text{SolveAlways}[\underbrace{f[-x] == -f[x]}, k]$

Question 16 (12 marks)

Consider the quartic function $f(x) = x^4 - 9x^2 + 4x + 12$.

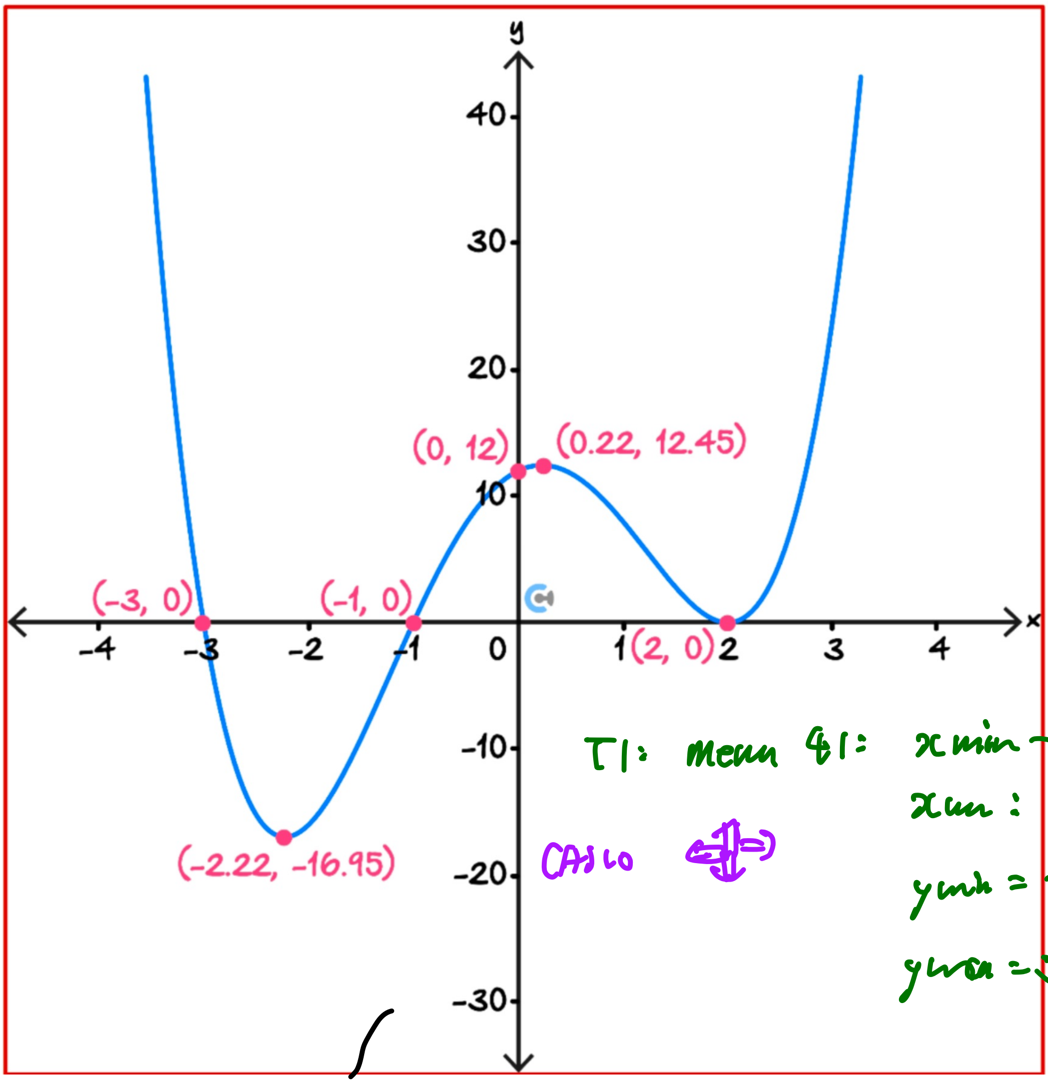
a. Fully factorise f and hence find all its roots. (2 marks) **[1.7.2]**

menu 32 factor

$(x-2)^2(x+1)(x+3)$ CASIO: Action → Transform

$x = -3, -1, 2$

b. Sketch the graph of $y = f(x)$ on the axes below. Label all axes intercepts and turning points correct to two decimal places where appropriate. (3 marks) **[1.7.3]**



TI: menu 41: xmin -5

xmax : 5

ymin = -40

ymax = 50

CASIO

c.

k right

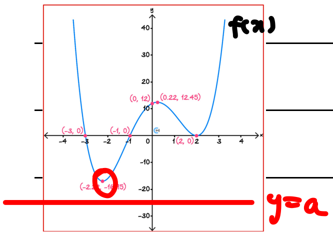
i. Find all values of $k \in \mathbb{R}$ such that $f(x - k) = 0$ has two positive solutions. (2 marks) **[1.8.1]**

1) $x = -3 + k, \ -1 + k, \ 2 + k.$

$-3 + k \leq 0 \qquad 0 < -1 + k$

$k \in (1, 3]$

ii. The equation $f(x) = a, a \in \mathbb{R}$ has no solutions. Find all possible values of a, exactly. (1 mark) **[1.8.3]**

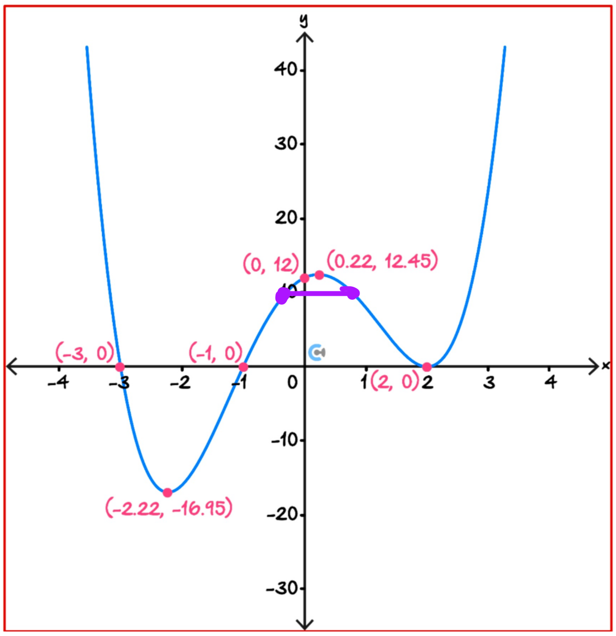


Below tp's y val

$a < \underbrace{-\frac{9}{4} - 6\sqrt{6}}_{\text{y value of lowest tp}}$

y=10

iii. Find the shortest horizontal distance between two points on the graph of $y = f(x)$ when $x = 10$. Give your answer correct to two decimal places. (2 marks) **[1.5.1]**



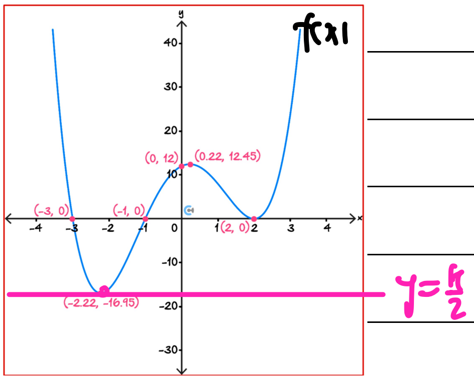
1) $f(x) = 10.$

$x = -3.1721, \ -0.299795$

2) $(-0.299795) - (-3.1721)$

$= 1.08$

d. The graph of $y = f(x)$ and the graph of $y = -f(x) + k$ has exactly one point of intersection. Find the exact value of k. (2 marks) **[1.3.3]**



$f(x) = -f(x) + k$

$2f(x) = k$

$f(x) = \frac{k}{2}$

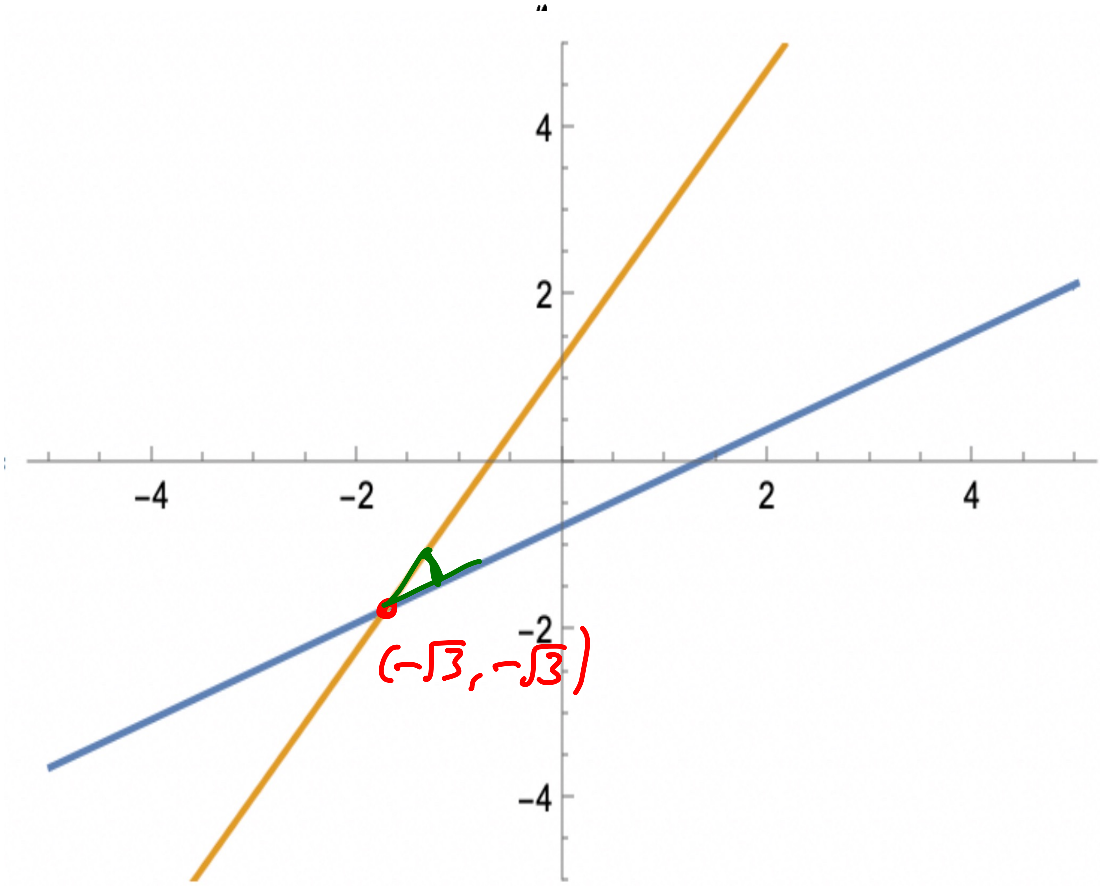
$\frac{k}{2} = -\frac{9}{4} - 6\sqrt{6}$

$\boxed{k = -\frac{9}{2} - 12\sqrt{6}}$

Space for Personal Notes

Question 17 (9 marks)

Consider the function $f : \mathbb{R} \to \mathbb{R}, f(x) = \sqrt{3}x + 3 - \sqrt{3}$. The graph of f is shown below:



a.

i. Sketch the graph of f^{-1}, the inverse function of f, on the same axes as f above. Label the point of intersection with coordinates. (2 marks) **[1.1.3]**

ii. Find the distance between the origin and the intersection. (1 mark) **[1.5.1]**

$\sqrt{6}$

iii. Find the exact size of the acute angle between f and f^{-1} at their intersection point. (1 mark) **[1.5.3]**

$\tan^{-1}(\sqrt{3}) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$= 30^\circ. \quad \left(\frac{\pi}{6}^c\right)$

Consider the functions $g(x) = 2x - 5$ and $h: [-\sqrt{k}, \sqrt{k}] \to \mathbb{R}, h(x) = \frac{1}{\sqrt{k}}x - k$, where $k \in \mathbb{R}^+$.

b. Find the coordinates for any point of intersection between g and h in terms of k. (1 mark)

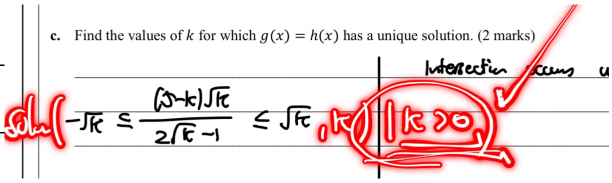
$$\left(\frac{(5-k)\sqrt{k}}{2\sqrt{k}-1}, \frac{2(5-k)\sqrt{k}}{2\sqrt{k}-1} - 5\right)$$

c. Find the values of k for which $g(x) = h(x)$ has a unique solution. (2 marks)

Intersection occurs within the dom

$$-\sqrt{k} \leq \frac{(5-k)\sqrt{k}}{2\sqrt{k}-1} \leq \sqrt{k}$$

$k \in [8-2\sqrt{7}, 6+2\sqrt{5}]$



c. Find the values of k for which $g(x) = h(x)$ has a unique solution. (2 marks)

Intersection occurs u

$\text{solve}\left(-\sqrt{k} \leq \frac{(5-k)\sqrt{k}}{2\sqrt{k}-1} \leq \sqrt{k}, k\right) \mid k>0$

d. Find the shortest distance from any intersection of g and h to the origin. Give your answer correct to two decimal places. (2 marks) **[1.5.1]**

1)

$d(k) = \sqrt{(\sim - 0)^2 + (\_ - 0)^2}$

2)

$\boxed{2.37}$

1) Find distance b/w I.P & (0,0)

2) fmin or Minimize

TI: fmin(d(k), k, $8-2\sqrt{7}$, $6+2\sqrt{5}$)

k=

d(~)

Space for Personal Notes

CASIO: fMin(d(k), k, $8-2\sqrt{7}$, $6+2\sqrt{5}$]

M: NMinimize[{ d[k], $8-2\sqrt{7} \leq k \leq 6+2\sqrt{5}$}, k]

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