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VCE Mathematical Methods ¾
AOS 1 Revision [0.8]
Workshop

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Section A: Cheat Sheets

[1.1.1] – Find the Maximal Domain and Range

- Inside of a log must be bigger than 0.
- Inside of a root must be bigger than or equal to 0.
- Denominator cannot be zero.
- The domain of sum or product of two functions is equal to the intersection of the two domains.

[1.1.2] – Find the Rule, Domain and Range of a Composite Function (Range Does Not Require Splitting to Find as the Function is Easy to Draw)

- $f(g(x)) = f \circ g\,(x)$.
- For composite function to exist, range (output) of inside $\subseteq$ domain (input) of outside
- The domain of composite is equal to the domain of inside (1^{st}) function.
- The range of composite is a subset of the range of the outside.

[1.1.3] – Find the Rule, Domain, and Range of Inverse Functions

- f needs to be $1:1$ for f^{-1} to exist.
- Domain of the inverse function equals to range of the original **and** vice versa.
- Symmetrical around $y = x$.
- For intersections of inverses, we can equate the function to $y = x$.

[1.1.4] – Find the Composite Function of the Inverse Function

- The composite function of inverses is always given by $f(f^{-1}(x)) = x$.

[1.2.1] – Find a New Domain to Fix Composite Functions

- The range of the inside function must be a subset of the domain of the outside function.
- We restrict the domain of the inside function so its range fits in the domain of the outside function.

[1.2.2] – Find the Range of Complex Composite Functions

- To find the range of a complicated function, we can break the function into a composition of two simpler functions.

[1.2.3] – Find the Gradient of Inverse Functions

- If the gradient of f at $(a, f(a)) = m$, then the gradient of f^{-1} at $(f(a), a) = \frac{1}{m}$.

[1.3.1] – Applying x' and y' Notation to Find Transformed Points, Find the Interpretation of Transformations and Altered Order of Transformations

- The transformed point is called the image and is denoted by (x', y').
- The dilation factor is multiplied to the original coordinate.
- Reflection makes the original coordinates the negative of their original values.
- Translation adds a unit to the original coordinate.
- Transformations should be interpreted when x' and y' are isolated.
- The order of transformation follows the BODMAS order.
- To change the order of transformations, we either factorise or expand.

Cheat Sheet

[1.3.2] – Find Transformed Functions

- To transform the function, replace its ___old variables___ with the new one.

[1.3.3] – Find Transformations From Transformed Functions (Reverse Engineering)

- To find the transformations, simply equate the ___LHS and RHS___ after separating the transformations of x and y.

[1.4.1] – Apply Quick Method to Find Transformations

- For applying transformations in the quick method: Apply everything for x in the ___opposite___ direction. Including the order!

- For interpreting transformations in the quick method: Read everything for x in the opposite direction. Including the ___order___!

[1.4.2] – Find Opposite Transformations

- Order is ___reversed___.

- All transformations are in the ___opposite___ direction.

[1.4.3] – Apply Transformations of Functions to Find Their Domain, Range, Transformed Points and Tangents

- Everything moves together as a function.

- **Steps:**

  1. Find the ___transformations___ between two functions.

  2. Apply the ___same___ transformations to domain, range, points and tangents.

[1.4.4] – Find Transformations of the Inverse Functions $f(x)$

- **Steps:**

  1. Find the transformations between the two original functions.

  2. Inverse the transformations found in **1**.

[1.4.5] – Find Multiple Transformations For the Same Functions

- Same transformations can be done ___differently___ by either putting it in or out of the $f()$.

- Commonly, look for basic algebra, index and ___log laws___

[1.4.6] – Apply Manipulation of the Functions to Find Appropriate Transformations

- **Steps:**

  1. Identify the region of ___x___.

  2. Identify the region of ___y___.

  3. ___Manipulate___ the function so that all the changes are within the region of x or y.

[1.5.1] – Find the Midpoint and Distance (Horizontal & Vertical) Between Two Points or Functions

- Midpoint is simply the ___average___ of 2 points.

- Distance formula is derived from ___Pythagoras theorem___

- Horizontal distance is the distance between ___x___ values.

- Vertical distance is the distance between ___y___ values.

[1.5.2] – Find Parallel and Perpendicular Lines

- Parallel lines have the ___same___ gradient.

- Perpendicular lines have ___negative reciprocal___ gradient.

Cheat Sheet

[1.5.3] – Find the Angle Between a Line and x-axis or Two Lines

- To find the angle between a line and the x-axis we can use the equation $m = \boxed{\tan(\theta)}$.
- To find the angle between two lines we can use

$$\theta = \boxed{|\tan^{-1}(m_1) - \tan^{-1}(m_2)|}$$

or

$$\tan\theta = \boxed{\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|}$$

[1.5.4] – Find The Unknown Value for Systems of Linear Equations

- Two linear equations have unique solutions if they have **different** gradients.
- Two linear equations have infinitely many solutions when they have **the same** gradient and **the same** constant.
- Two linear equations have no solution when they have **the same** gradient and **different** constant.

[1.5.5] – Sketching the Sum of Two Function's Graph by Using the Addition of Ordinates

- Addition of ordinates is used to sketch the **sum** of two functions.
- We always add their y values.
- When we have an x intercept for one graph, sum graph **intersects** the other graph.
- When we have an intersection between two graphs, the sum graph equals to **double** their y value.
- When we have an equidistance from the x-axis, sum graph has an x- intercept.

[1.6.1] – Apply Midpoint to Find a Reflected Point

- The line between a point and its reflection is **perpendicular** to the line it is reflected in.
- The **midpoint** of a line and its reflection lies on the line it is reflected in.
- **Steps** for finding the reflection of a point in a line:
  1. Find the **perpendicular** line passing through the point.
  2. Find the **intersection** between the original line and the perpendicular line.
  3. Find the reflected point (x, y) by treating the intersection from 2. as the **midpoint** between the original and reflected point.

[1.6.2] – Apply Parallel and Perpendicular Lines to Geometric Problems

- When solving geometric problems always draw a **diagram** of the situation.

[1.7.1] – Apply the Factor Theorem and Remainder Theorem to Identify the Roots, and Remainders and Find the Unknown of a Function

- The degree of a polynomial is the polynomial's **highest** power.
- The roots of a polynomial are its **x-intercepts**.
- For polynomial long division:

$$\frac{\textit{Dividend}}{\textit{Divisor}} = \textit{Quotient} + \boxed{\frac{\textit{Remainder}}{\textit{Divisor}}}$$

- When $P(x)$ is divided by $(x - a)$, the remainder is $P(a)$.
- If $P(a) = 0$, then $(x - a)$ is a **factor** of $P(x)$.

[1.7.2] - Find Factored Form of Polynomials

- Steps to factor a cubic polynomial are:

  1. Find a single root by trial and error.

     (Factor Theorem: Substitute into the function and see if we get zero).

  2. Use long division to find the quadratic factor.

  3. Factorise the quadratic.

- Rational Root Theorem **narrows down** the possible roots. If the roots are rational numbers, it must be that any:

$$\textit{Potential root} = \frac{\textit{Factors of constant term } a_0}{\textit{Factors of leading coefficient } a_n}$$

- Sum and difference of cubes:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

[1.7.3] – Graph Factored and Unfactored Polynomials

- Graphs of $a(x-h)^n + k$, where n is an odd positive integer that is not equal to 1:

  - The point (h, k) gives us the stationary point of inflection.

- Graphs of $a(x-h)^n + k$, where n is an even positive integer:

  - The point (h, k) gives us the turning point.

  - These graphs look like a quadratic.

- Steps to graphing factorised polynomials:

  1. Plot x-intercepts.

  2. Determine whether the polynomial is positive or negative.

  3. Use the repeated factors to deduce the shape:

     - Non-Repeated: Only x-intercept.

     - Even Repeated: x-intercept and a turning point.

     - Odd Repeated: x-intercept and a stationary point of inflection.

Cheat Sheet

[1.7.4] – Identify Odd, and Even Functions and Correct Power Functions

- **Odd Functions:**

$$f(-x) = -f(x)$$

- Property: Reflecting on the y-axis is the same as reflecting around the x-axis.

- **Even Functions:**

$$f(x) = f(x)$$

- Property: It is symmetrical about the y-axis.

- **Power Functions:**

$$y = x^{\frac{n}{m}}$$

- m: Dictates the number of **tails**.
  - **Odd m = Two tails.**
  - **Even m = One tail.**
- n: Dictates the **range**.
  - Odd n: Range could be all real.
  - Even n: Range must be non-negative.
- Power > 1: Looks like a polynomial function.
- Power < 1: Looks like a root function.

Cheat Sheet

[1.8.1] – Apply Transformations to Restrict the Number of Positive/Negative x-intercept(s)

- To solve these questions, figure out how to translate the relevant intercept to the origin.

[1.8.2] – Apply Discriminant to Solve Number of Solutions Questions

- There are no real solutions for a quadratic when Δ $<$ 0.
- There is one real solution for a quadratic when Δ $=$ 0.
- There are two unique real solutions for a quadratic when Δ $>$ 0.

[1.8.3] – Apply Shape/Graph to Solve Number of Solutions Questions

To find the number of solutions for $f(x) = k$, draw a horizontal line at y $= k$ and count the intersections.

[1.8.4] – Apply Odd and Even Functions (MHS Investigation 2023)

- For an odd function, $f(x) =$ $-f(-x)$.
- For an even function, $f(x) =$ $f(-x)$.

[1.8.5] – Identify Possible Rule(s) From a Graph

- A turning point x-intercept has $a(n)$ even power on its factor.
- A stationary point of inflection x intercept has $a(n)$ odd power on its factor.
- If the x-intercept passes straight through, the power of the factor is 1.

Section B: Questions (61 Marks)

Sub-Section: Exam 1

INSTRUCTION: 31 Marks. 5 Minutes Reading. 35 Minutes Writing.

Question 1 (4 marks)

Consider the points $A(-3, 5)$ and $B(4, -2)$.

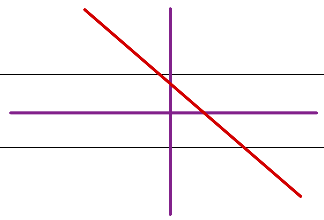
a. Find the equation of the line joining A and B and hence, find the angle that the line segment AB makes with the positive x-axis. (2 marks) **[1.5.2] [1.5.3]**

$m = \frac{5-(-2)}{-3-4} = \frac{7}{-7} = -1$

$y - 5 = -1(x-(-3))$

$y = -x - 3 + 5$

$\boxed{y = -x + 2}$



$\tan(\theta) = -1$

$\theta = -\frac{\pi}{4}$

$\boxed{\theta = \frac{3\pi}{4}}$

b. Find the equation of the line that the point A could be reflected in to map it onto the point B. (2 marks) **[1.5.2] [1.6.1]**

$m = \frac{-1}{-1} = 1$

midpoint $= \left(\frac{-3+4}{2}, \frac{5+-2}{2}\right)$

$= \left(\frac{1}{2}, \frac{3}{2}\right)$

$\boxed{y = x + 1}$!

Question 2 (3 marks) **[1.5.4]**

Consider the system of linear equations:

$$(a-4)x + 3y = 2 \qquad y = \frac{-(a-4)}{3}x + \frac{2}{3}$$

$$4x + (a+7)y = a+3 \qquad y = \frac{-4}{a+7}x + \frac{a+3}{a+7}$$

where $a \in \mathbb{R}$. Find the value of a such that the system of equations has infinitely many solutions.

$\frac{-4}{a+7} = \frac{-(a-4)}{3}$ AND $\frac{2}{3} = \frac{a+3}{a+7}$

$12 = (a-4)(a+7)$ $\qquad 2(a+7) = 3(a+3)$

$12 = a^2 + 3a - 28$ $\qquad 2a + 14 = 3a + 9$

$a^2 + 3a - 40 = 0$ $\qquad \boxed{a = 5}$

$(a+8)(a-5) = 0$

$\boxed{a = 5, -8}$

$\therefore \boxed{a = 5}$

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Question 3 (10 marks)

Consider the functions f and g, defined over their maximal domains where:

$$f(x) = 2\sqrt{x+2} - 2$$

$$g(x) = \log_2(3-x)$$

a. Find the maximal domain of $f(x) + \frac{1}{\sqrt{g(x)}}$. (2 marks) **[1.1.1]**

$x+2 \geq 0$ $\quad$ $\log_2(3-x) > 0$

$x \geq -2$ $\quad$ $3-x > 1$

$\cap$ $\quad$ $x < 2$

$\therefore$ $x \in [-2, 2)$

b. Show that $f(g(x))$ is not defined. (1 mark) **[1.1.2]**

ran $g(x) \in \mathbb{R}$ $\quad$ Dom $f(x) \in [-2, \infty)$

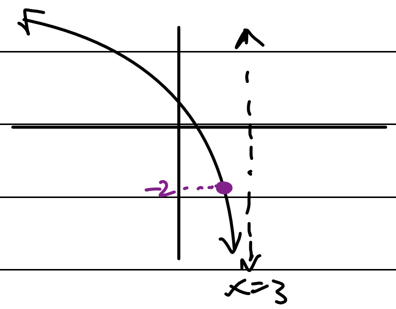
$\mathbb{R} \not\subseteq [-2, \infty)$

$\therefore$ $f(g(x))$ is not defined

c. The domain of g is restricted to $x \in (-\infty, a]$. Find the largest value of a such that $f(g(x))$ exists and write down its rule. (3 marks) **[1.1.2] [1.2.1]**

range $g \subseteq$ Dom f

range $g \subseteq [-2, \infty)$



$\log_2(3-x) = -2$

$3-x = 2^{-2}$

$x = 3 - \frac{1}{4} = \frac{11}{4}$

$\boxed{a = \frac{11}{4}}$

$\boxed{f(g(x)) = 2\sqrt{\log_2(3-x)+2} - 2}$

d. Define f^{-1}, the inverse function of f. (2 marks) **[1.1.3]**

let $y = 2\sqrt{x+2} - 2$

swap x & y.

$x = 2\sqrt{y+2} - 2$

$\sqrt{y+2} = \frac{x+2}{2}$

$y+2 = \frac{1}{4}(x+2)^2$

$y = \frac{1}{4}(x+2)^2 - 2$

Dom f^{-1} = Ran $f \in [-2, \infty)$

$\therefore f^{-1}: [-2, \infty) \to \mathbb{R}, f^{-1}(x) = \frac{1}{4}(x+2)^2 - 2$

e. Find all points of intersection between f and f^{-1}. (2 marks) **[1.1.3]**

$f(x) = x$

$2\sqrt{x+2} - 2 = x$

$\sqrt{x+2} = \frac{1}{2}(x+2)$

$x+2 = \frac{1}{4}(x+2)^2$

$4x+8 = x^2+4x+4$

$x^2 = 4, \quad x = \pm 2$

$y = \pm 2$

$\boxed{(2, 2) \quad (-2, -2)}$

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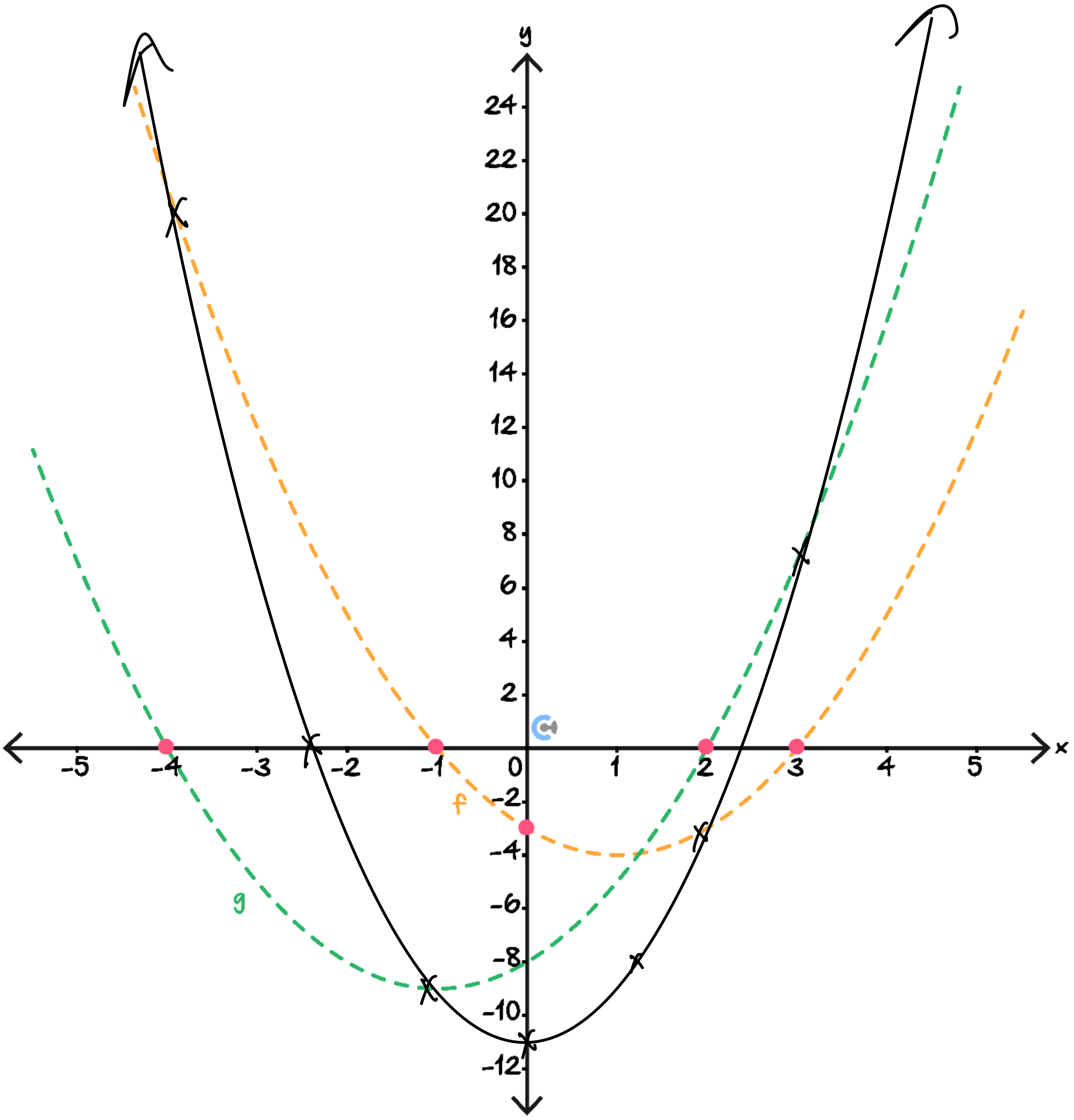
Question 4 (3 marks) **[1.5.5]**

The graphs of quadratic functions f and g are sketched on the axes below. Sketch the graph of $f + g$ on the same axes.



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Question 5 (4 marks)

old new

Consider the functions $f(x) = 2\log_2(x)$ and $g(x) = -4\log_2(3x-6)$.

a. Using dilations, reflections, and horizontal translations only, describe a sequence of transformations that map $f(x)$ to $g(x)$. (2 marks) **[1.4.5]**

$y = 2\log_2(x)$ $\quad$ $y' = -4\log_2(3x'-6)$

$\frac{y}{2} = \log_2(x)$ $\quad$ $\frac{y'}{-4} = \log_2(3x'-6)$

$\frac{y}{2} = \frac{y'}{-4}$ $\quad$ $3x'-6 = x$

$y' = -2y$ $\quad$ $x' = \frac{1}{3}(x+6)$

1. 6 right
2. Dil $\frac{1}{3}$ from y.
3. Dil 2 from x
4. Reflect in x.

b. Without using any dilations from the y-axis, describe a sequence of transformations that map $g(x)$ to $f(x)$. (2 marks) **[1.4.6]**

$y = -4\log_2(3x-6)$ $\quad$ $y' = 2\log_2(x')$

$= -4(\log_2(3) + \log_2(x-2))$ $\quad$ $\frac{y'}{2} = \log_2(x')$

$y = -4\log_2(3) - 4\log_2(x-2)$ $\quad$ $x' = x-2$

$\frac{y + 4\log_2(3)}{-4} = \log_2(x-2)$ $\quad$ $\frac{y'}{2} = \frac{y + 4\log_2(3)}{-4}$

$y' = -\frac{1}{2}(y + 4\log_2(3))$

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Question 6 (7 marks)

a. Let $f(x) = x^3 - 2x^2 - 11x + 12$. Solve the equation $f(x) = 0$. (2 marks) **[1.7.2]**

$f(1) = 1 - 2 - 11 + 12$
$= 0$

$(x-1)$ is factor

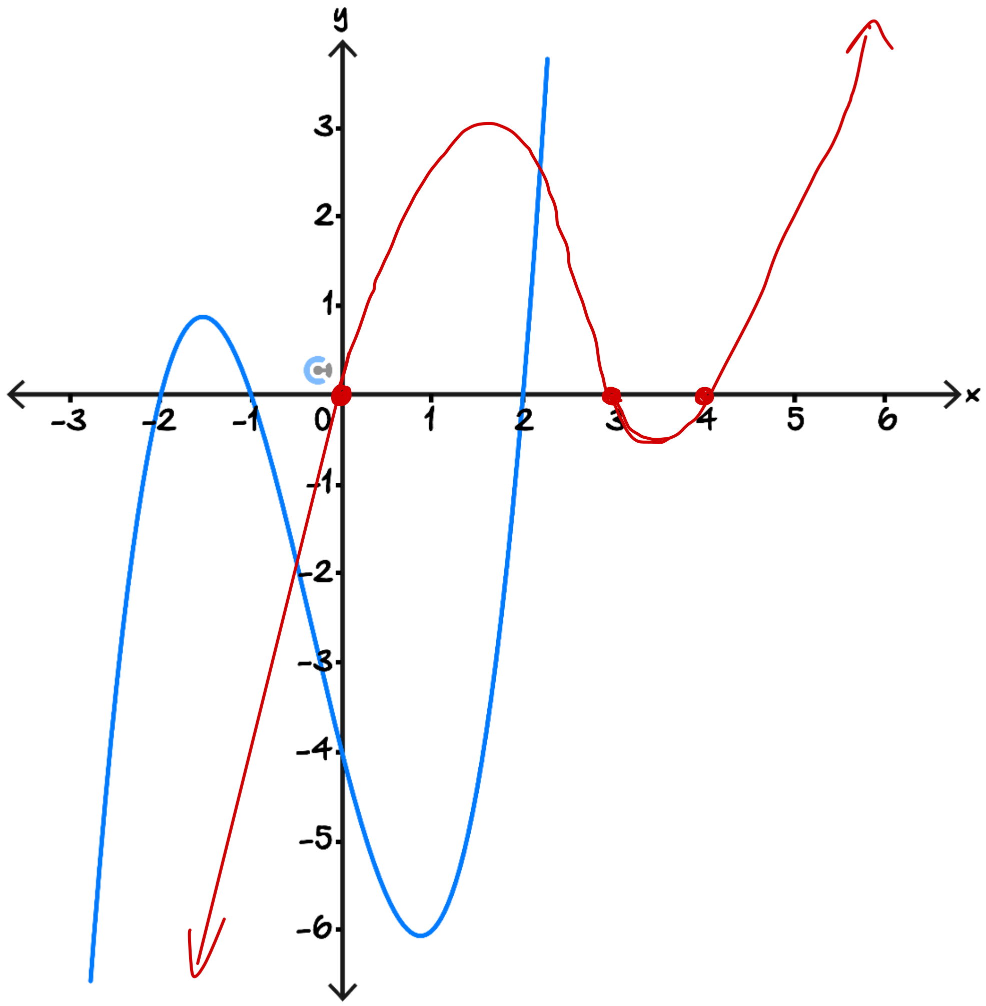
$$\begin{array}{r} x^2 - x - 12 \\ x-1 \overline{) x^3 - 2x^2 - 11x + 12} \\ \underline{x^3 - x^2} \\ -x^2 - 11x + 12 \\ \underline{-x^2 + x} \\ -12x + 12 \end{array}$$

$\therefore (x-1)(x^2 - x - 12) = 0$

$(x-1)(x+3)(x-4) = 0$

$x = 1, -3, 4$

Let $g(x) = (x-2)(x+1)(x+2)$. The graph of $y = g(x)$ is shown on the axes below:



b. Describe the transformations that map $g(x)$ to $h(x) = -\frac{1}{2}g(2-x)$. (2 marks) **[1.3.1]**

$y = g(x)$

$y' = -\frac{1}{2}g(2-x')$

$-2y' = g(2-x')$

$x = 2 - x'$

$y = -2y'$

$x' = -x+2$

$y' = -\frac{1}{2}y$

1. reflect in y
2. 2 right
3. reflect in x
4. Dil ½ from x.

c. Find the factored form of $h(x)$ and sketch the graph of $h(x)$ on the same axes as $g(x)$. Label all axes intercepts. (3 marks) **[1.3.2]**

$h(x) = -\frac{1}{2}\left[(2-x+2)(2-x+1)(2-x-2)\right]$

$= -\frac{1}{2}(4-x)(3-x)-x$

$= \frac{1}{2}x(x-4)(x-3)$

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Sub-Section: Exam 2

INSTRUCTION: 30 Marks. 5 Minutes Reading. 35 Minutes Writing.

Question 7 (1 mark) **[1.1.4]**

Consider the function $f: [-3, \infty) \to \mathbb{R}, f(x) = (x+3)^2 - 5$. Which of the following is the rule and domain of $f\left(f^{-1}(x)\right)$?

Dom f^{-1} = Ran f

A. $f\left(f^{-1}(x)\right) = x, x \in [-3, \infty)$

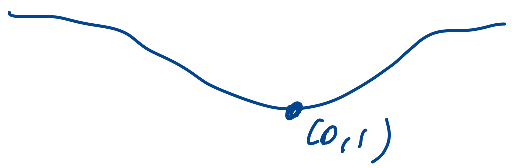
B. $f\left(f^{-1}(x)\right) = x, x \in [-5, \infty)$

C. $f\left(f^{-1}(x)\right) = -x, x \in (-\infty, -5]$

D. $f\left(f^{-1}(x)\right) = x, x \in (-\infty, -3]$

Question 8 (1 mark) **[1.2.2]**

The range of the function $f(x) = \log_2\left(\sqrt{x^2+4}\right)$ is:



A. $[2, \infty)$

B. $(2, \infty)$

C. $[1, \infty)$

D. $(1, \infty)$

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Question 9 (1 mark) **[1.2.3]**

The function f has an inverse function f^{-1}. It is known that $f(1) = 2, f(2) = 3$ and $f'(2) = 3, f'(3) = 5$. Find the gradient of f^{-1} when $x = 3$.

$\frac{1}{f'(2)}$

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. 2

D. $\frac{1}{5}$

Question 10 (1 mark) **[1.4.1]**

The function $T: \mathbb{R}^2 \to \mathbb{R}^2$ maps the graph of $y = (x-1)^2$ onto the graph $y = 2(x-3)^2 + 6$. The rule for T could be:

A. $T(x, y) = (x - 2, 2y - 6)$

B. $T(x, y) = (x + 2, 2y + 3)$

C. $T(x, y) = (x + 2, 2y - 3)$

D. $T(x, y) = (x + 2, 2y + 6)$

Question 11 (1 mark) **[1.7.1]**

The polynomial $x^3 + ax^2 + bx + 5$ is perfectly divisible by $x + 3$ and has a remainder of 1 when divided by $x - 2$.The values (a, b) are:

A. $(4, 12)$

B. $\left(\frac{4}{15}, -\frac{98}{15}\right)$

C. $\left(\frac{16}{3}, -\frac{26}{3}\right)$

D. $\left(-\frac{14}{3}, \frac{10}{3}\right)$

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Question 12 (1 mark) **[1.8.2]**

The function $f(x) = x^3 - x^2 + (k-6)x + 2k$, where $k \in \mathbb{R}$, has exactly one root for:

A. $k < \frac{9}{4}$

$k = -20$ (3 sol^ns) x-ints

B. $k > \frac{9}{4}$

C. $-\frac{9}{4} < k < \frac{9}{4}$

D. $k = \frac{9}{4}$

Question 13 (1 mark) **[1.4.2]**

$y = -\frac{1}{3} f(4x+8)$ $\quad y' = f(x')$ $\quad x' = 4x+8$

$-3y = f(4x+8)$ $\quad y' = -3y$

Let $f(x)$ and $g(x) = -\frac{1}{3} f(4x + 8)$ be functions. A sequence of transformations that maps $g(x)$ to $f(x)$ is:

A. A dilation by a factor $\frac{1}{3}$ from the x-axis, a dilation by a factor $\frac{1}{4}$ from the y-axis, a reflection in the x-axis, and a translation 2 units to the left.

B. A dilation by a factor 3 from the x-axis, a dilation by a factor 4 from the y-axis, a reflection in the x-axis. And a translation 2 units to the left.

C. A translation 2 units to the right, a reflection in the x-axis, a dilation by a factor 4 from the y-axis, and a dilation by a factor 3 from the x-axis.

D. A translation 2 units to the left, a reflection in the x-axis, a dilation by a factor 3 from the y-axis, and dilation by a factor 4 from the x-axis.

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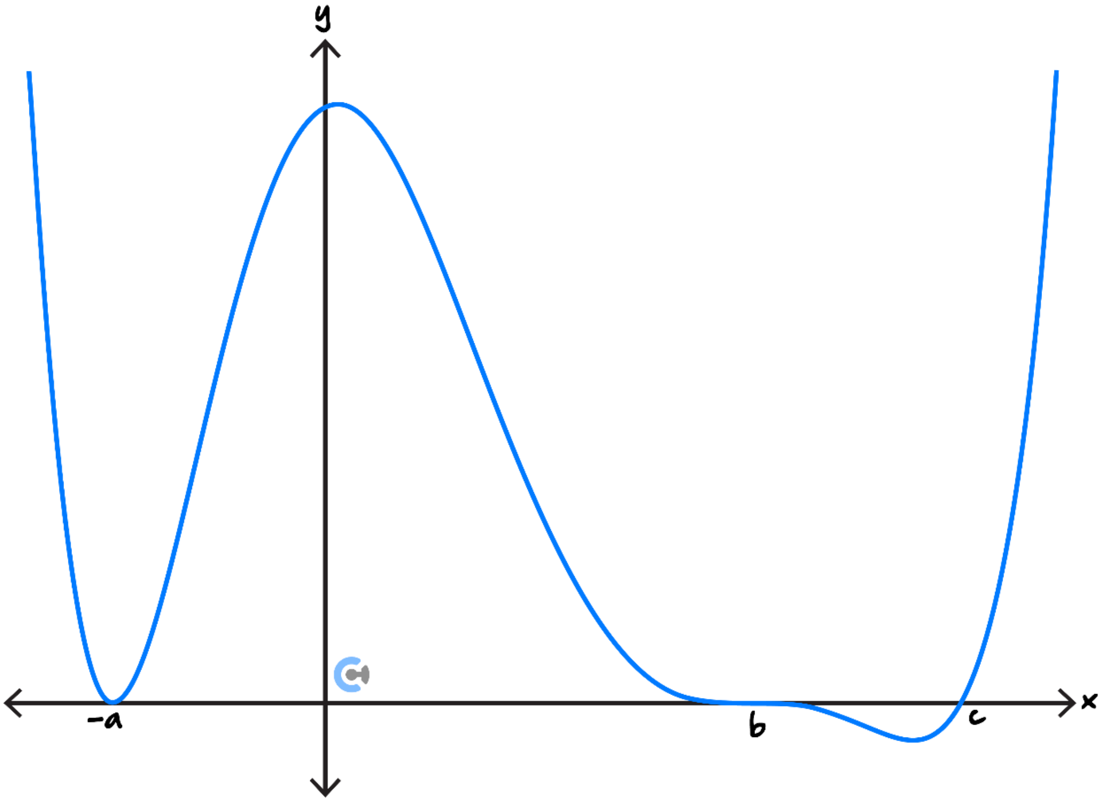
Question 14 (1 mark)

Consider the graph of a function f shown below, where $a, b, c > 0$. A possible rule for $f(x)$ is:



A. $f(x) = -(x+a)^2(x-b)^3(x-c)$

B. $f(x) = (x+a)^2(x-b)^3(x-c)$

C. $f(x) = (x-a)^2(x-b)^3(x-c)$

D. $f(x) = (x+a)^2(x+b)^3(x-c)$

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Question 15 (1 mark) **[1.7.4] [1.8.4]**

Consider the function $f(x) = x^5 + 3x^3 + (k^2 - 3k - 4)x^2 + 2kx + 2k^2 + k - 1$. The value(s) of k for which $f(x)$ is an odd function are:

A. $k = 1$

Define $f(x) =$

B. $k = 1$ or $k = -1$

C. $k = -1$

solve$(f(-x) = -f(x), k)$ $\quad$ $k = -1$

D. $k = 4$

Question 16 (12 marks)

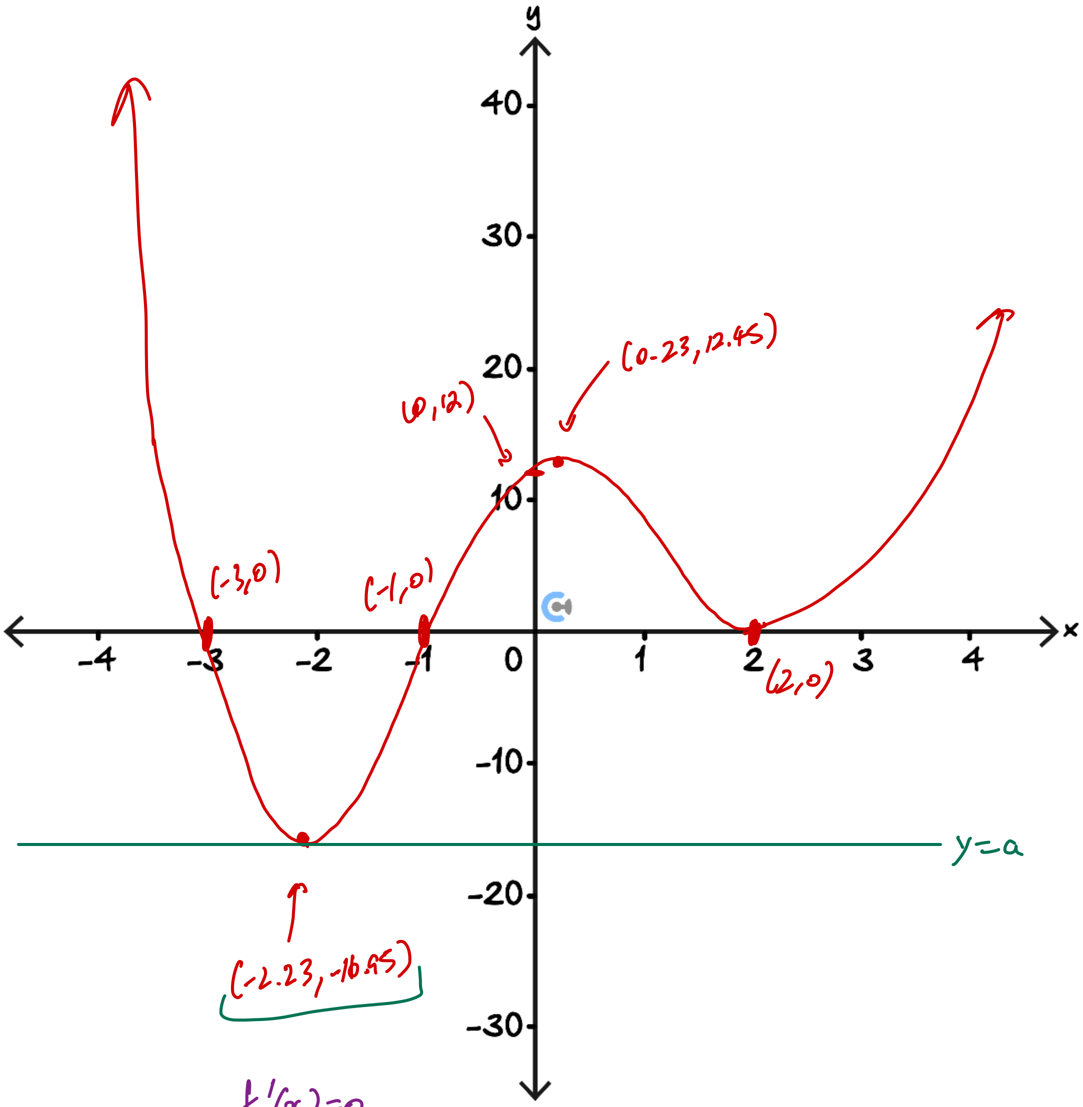
Consider the quadratic function $f(x) = x^4 - 9x^2 + 4x + 12$.

a. Fully factorise f and hence, find all its roots. (2 marks) **[1.7.2]**

$f(x) = (x-2)^2(x+1)(x+3)$

$f(x)=0$, $\boxed{x = 2, -1, -3}$

b. Sketch the graph of $y = f(x)$ on the axes below. Label all axes, intercepts, and turning points correct to two decimal places where appropriate. (3 marks) **[1.7.3]**



$f'(x)=0$

$x = \ldots$

$f(-\frac{3}{2}) = -6\sqrt{6} - \frac{9}{4}$

c.

k right

i. Find all values of $k \in \mathbb{R}$ such that $f(x-k) = 0$ has two positive solutions. (2 marks) **[1.8.1]**

$1 < k \leq 3$

ii. The equation $f(x) = a, a \in \mathbb{R}$ has no solutions. Find all possible values of a, exactly. (1 mark) **[1.8.3]**

$a < -6\sqrt{6} - \frac{9}{4}$

iii. Find the shortest horizontal distance between two points on the graph of $y = f(x)$ when $y = 10$. Give your answer correct to two decimal places. (2 marks) **[1.5.1]**

$f(x) = 10$, $x = 0.78, -0.30$

1.08

d. The graph of $y = f(x)$ and the graph of $y = -f(x) + k$ has exactly one point of intersection. Find the exact value of k. (2 marks) **[1.3.3]**

$2\left(\frac{9}{4}+6\sqrt{6}\right)$ (Distance to move down)

$\therefore k = -\frac{9}{2} - 12\sqrt{6}$

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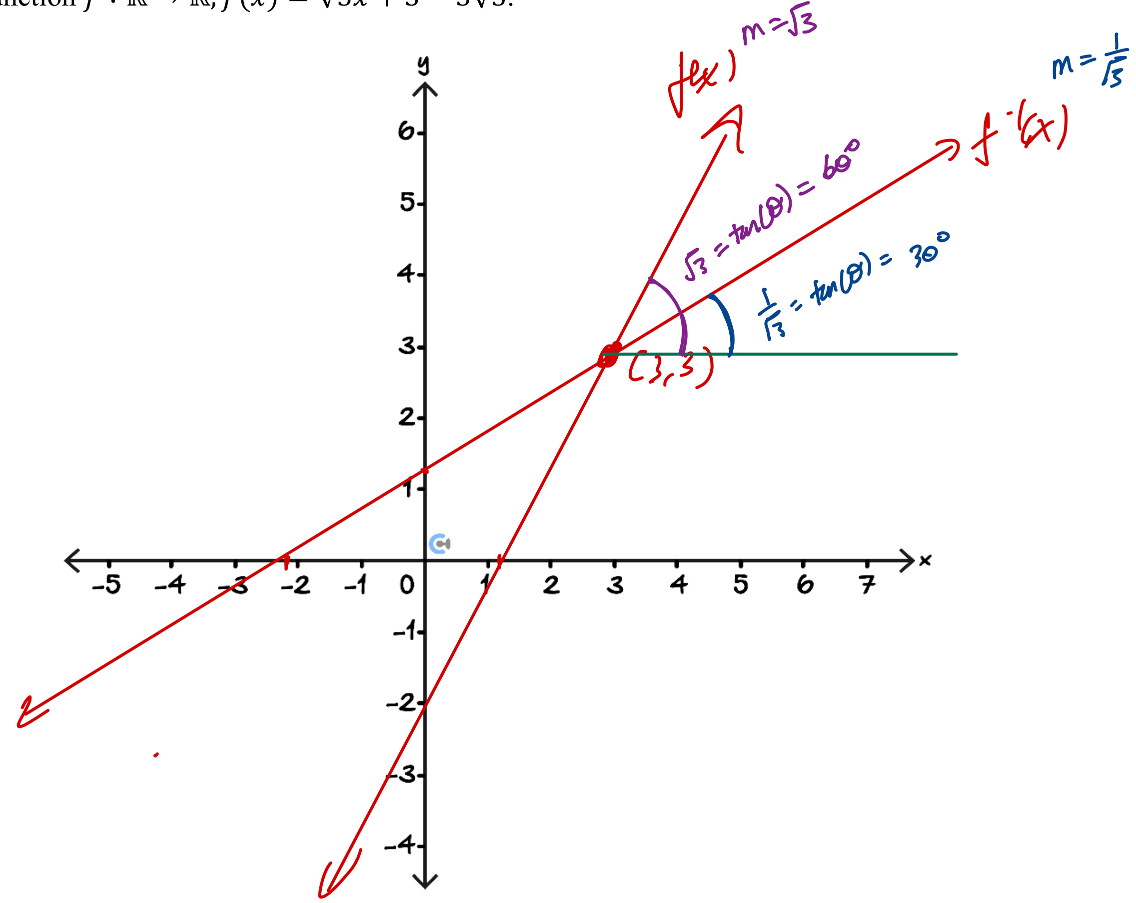
Question 17 (9 marks)

Consider the function $f : \mathbb{R} \to \mathbb{R}, f(x) = \sqrt{3}x + 3 - 3\sqrt{3}$.



a.

i. Sketch the graph of f and f^{-1} on the axes above. Label the point of intersection with coordinates. (2 marks) **[1.1.3]**

ii. Find the distance between the origin and the intersection. (1 mark) **[1.5.1]**

distance $= \sqrt{3^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$

iii. Find the exact size of the acute angle between f and f^{-1} at their intersection point. (1 mark) **[1.5.3]**

$\tan^{-1}(\sqrt{3}) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$

Consider the functions $g(x) = 2x - 5$ and $h : [-\sqrt{k}, \sqrt{k}] \to \mathbb{R}, h(x) = \frac{1}{\sqrt{k}}x - k$, where $k \in \mathbb{R}^+$.

b. Find the coordinates for any point of intersection between g and h in terms of k. (1 mark)

$2x - 5 = \frac{1}{\sqrt{k}}x - k$

P.O.I $\left(\frac{-\sqrt{k}(k-5)}{2\sqrt{k}-1}, \frac{19}{4(2\sqrt{k}-1)} - k - \frac{\sqrt{k}}{2} - \frac{1}{4}\right)$

Point of intersection exists

c. Find the values of k for which $g(x) = h(x)$ has a unique solution. (2 marks)

solve$\left(-\sqrt{k} \leq \frac{-\sqrt{k}(k-5)}{2\sqrt{k}-1} \leq \sqrt{k}, k\right)$

$k \in [8-2\sqrt{7}, 2\sqrt{5}+6]$

d. Find the shortest distance from any intersection of g and h to the origin. Give your answer correct to two decimal places. (2 marks) **[1.5.1]**

$d(k) = \sqrt{\left(\frac{-\sqrt{k}(k-5)}{2\sqrt{k}-1} - 0\right)^2 + \left(\ldots - 0\right)^2}$

$d(8-2\sqrt{7}) \approx 2.37$

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