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VCE Mathematical Methods ¾
AOS 1 Revision [0.8]
Workshop

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Section A: Cheat Sheets

Cheat Sheet

[1.1.1] - Find the Maximal Domain and Range

- Inside of a log must be ____________________.
- Inside of a root must be ____________________________.
- Denominator __________________________.
- The domain of sum or product of two functions is equal to the ________________________ of the two domains.

[1.1.2] - Find the Rule, Domain and Range of a Composite Function (Range Does Not Require Splitting to Find as the Function is Easy to Draw)

- $f(g(x)) =$____ $\circ$____ (x).
- For composite function to exist, ______________________ $\subseteq$ ______________________.
- The domain of composite is equal to the domain of ___________________ function.
- The range of composite is a ___________ of the range of the outside.

[1.1.3] - Find the Rule, Domain, and Range of Inverse Functions

- f needs to be ________ for f^{-1} to exist.
- Domain of the inverse function equals to _________________________________ and vice versa.
- Symmetrical around _________.
- For intersections of inverses, we can equate the function to _______.

[1.1.4] - Find the Composite Function of the Inverse Function

- The composite function of inverses is always given by $f(f^{-1}(x)) =$_________.

[1.2.1] - Find a New Domain to Fix Composite Functions

- The range of the ________function must be a subset of the __________of the outside function.
- We restrict the ____________ of the inside function so its __________ fits in the domain of the outside function.

[1.2.2] - Find the Range of Complex Composite Functions

- To find the range of a complicated function, we can break the function into a _____________ of two simpler functions.

[1.2.3] - Find the Gradient of Inverse Functions

- If the gradient of f at $(a, f(a)) = m$, then the gradient of f^{-1} at $(f(a), a) =$ ___________.

[1.3.1] - Applying x' and y' Notation to Find Transformed Points, Find the Interpretation of Transformations and Altered Order of Transformations

- The transformed point is called the ____________ and is denoted by ______________.
- The dilation factor is ________________ to the original coordinate.
- Reflection makes the original coordinates the _______________ of their original values.
- Translation ___________ a unit to the original coordinate.
- Transformations should be interpreted when _______________ are isolated.
- The order of transformation follows the ______________ order.
- To change the order of transformations, we either ___________________________.

Cheat Sheet

[1.3.2] - Find Transformed Functions

- To transform the function, replace its ____________________ with the new one.

[1.3.3] - Find Transformations From Transformed Functions (Reverse Engineering)

- To find the transformations, simply equate the __________________ after separating the transformations of x and y.

[1.4.1] - Apply Quick Method to Find Transformations

- For applying transformations in the quick method: Apply everything for x in the ______________ direction. Including the order!

- For interpreting transformations in the quick method: Read everything for x in the opposite direction. Including the _______________!

[1.4.2] - Find Opposite Transformations

- Order is ______________.

- All transformations are in the ______________ direction.

[1.4.3] - Apply Transformations of Functions to Find Their Domain, Range, Transformed Points and Tangents

- Everything moves together as a function.

- **Steps:**

  1. Find the ______________________ between two functions.

  2. Apply the _____________ transformations to domain, range, points and tangents.

[1.4.4] - Find Transformations of the Inverse Functions $f(x)$

- **Steps:**

  1. Find the transformations between the two original functions.

  2. Inverse the transformations found in **1.**

[1.4.5] - Find Multiple Transformations For the Same Functions

- Same transformations can be done ________________ by either putting it in or out of the $f()$.

- Commonly, look for basic algebra, index and ___________.

[1.4.6] - Apply Manipulation of the Functions to Find Appropriate Transformations

- **Steps:**

  1. Identify the region of _______.

  2. Identify the region of _______.

  3. ________________ the function so that all the changes are within the region of x or y.

[1.5.1] - Find the Midpoint and Distance (Horizontal & Vertical) Between Two Points or Functions

- Midpoint is simply the ___________ of 2 points.

- Distance formula is derived from ___________________.

- Horizontal distance is the distance between ____ values.

- Vertical distance is the distance between _____values.

[1.5.2] - Find Parallel and Perpendicular Lines

- Parallel lines have the _______ gradient.

- Perpendicular lines have _____________________ gradient.

Cheat Sheet

[1.5.3] - Find the Angle Between a Line and x-axis or Two Lines

- To find the angle between a line and the x-axis we can use the equation $m\ =$ __________.
- To find the angle between two lines we can use

$\theta = |\qquad\qquad\qquad\qquad|$

or

$\tan\theta =$

[1.5.4] - Find The Unknown Value for Systems of Linear Equations

- Two linear equations have unique solutions if they have ______________ gradients.
- Two linear equations have infinitely many solutions when they have _________________ gradient and _________________ constant.
- Two linear equations have no solution when they have _________________ gradient and _________________ constant.

[1.5.5] - Sketching the Sum of Two Function's Graph by Using the Addition of Ordinates

- Addition of ordinates is used to sketch the ________ of two functions.
- We always add their ______ values.
- When we have an x intercept for one graph, sum graph ______________________ the other graph.
- When we have an intersection between two graphs, the sum graph equals to ______________ their ________ value.
- When we have an equidistance from the x-axis, sum graph has an _______intercept.

[1.6.1] - Apply Midpoint to Find a Reflected Point

- The line between a point and its reflection is ______________________ to the line it is reflected in.
- The ___________________ of a line and its reflection lies on the line it is reflected in.
- **Steps** for finding the reflection of a point in a line:
  1. Find the _________________________ line passing through the point.
  2. Find the _______________________ between the original line and the perpendicular line.
  3. Find the reflected point (x, y) by treating the intersection from **2.** as the __________________ between the original and reflected point.

[1.6.2] - Apply Parallel and Perpendicular Lines to Geometric Problems

- When solving geometric problems always draw a ________________ of the situation.

[1.7.1] - Apply the Factor Theorem and Remainder Theorem to Identify the Roots, and Remainders and Find the Unknown of a Function

- The degree of a polynomial is the polynomial's ________________________ power.
- The roots of a polynomial are its _____________________.
- For polynomial long division:

$$\frac{\textbf{Dividend}}{\textbf{Divisor}} = \textbf{Quotient} + \_\_\_\_\_\_\_\_\_\_\_\_$$

- When $P(x)$ is divided by $(x-\alpha)$, the remainder is ________.
- If $P(\alpha) = 0$, then $(x-\alpha)$ is a _________________ of $P(x)$.

[1.7.2] - Find Factored Form of Polynomials

- Steps to factor a cubic polynomial are:

  1. Find a single root by trial and error.

     (Factor Theorem: Substitute into the function and see if we get ________________).

  2. Use __________________ to find the quadratic factor.

  3. Factorise the quadratic.

- Rational Root Theorem **narrows down** the possible roots. If the roots are rational numbers, it must be that any:

$$\textit{Potential root} = \pm \frac{\textit{Factors of } \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \ a_0}{\textit{Factors of } \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \ a_n}$$

- Sum and difference of cubes:

$$a^3 + b^3 = (\_\_\_\_\_\_\_\_\_\_\_\_)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (\_\_\_\_\_\_\_\_\_\_\_\_\_\_)(a^2 + ab + b^2)$$

[1.7.3] - Graph Factored and Unfactored Polynomials

- Graphs of $a(x-h)^n + k$, where n is an odd positive integer that is not equal to 1:

  - The point (h, k) gives us the stationary point of ______________.

- Graphs of $a(x-h)^n + k$, where n is an even positive integer:

  - The point (h, k) gives us the ____________________.

  - These graphs look like a __________________.

- Steps to graphing factorised polynomials:

  1. Plot x-intercepts.

  2. Determine whether the polynomial is positive or negative.

  3. Use the repeated factors to deduce the shape:

     - Non-Repeated: Only ___________________.

     - Even Repeated: x-intercept and a ________________________.

     - Odd Repeated: x-intercept and a _____________________________________.

Cheat Sheet

[1.7.4] – Identify Odd, and Even Functions and Correct Power Functions

- **Odd Functions:**

$$f(-x) = -f(x)$$

  - Property: Reflecting on the ______________ is the same as reflecting around the ______________.

- **Even Functions:**

$$f(-x) = f(x)$$

  - Property: It is symmetrical about the ______________.

- **Power Functions:**

$$y = x^{\frac{n}{m}}$$

  - m: Dictates the number of **tails**.
    - Odd m = ____________ tails.
    - Even m = ____________tail.
  - n: Dictates the **range**.
    - Odd n: Range could be _______________.
    - Even n: Range must be ________________.
  - Power > 1: Looks like a ____________________ function.
  - Power < 1: Looks like a ______________ function.

Cheat Sheet

[1.8.1] – Apply Transformations to Restrict the Number of Positive/Negative x-intercept(s)

- To solve these questions, figure out how to translate the relevant intercept to the ____________.

[1.8.2] – Apply Discriminant to Solve Number of Solutions Questions

- There are no real solutions for a quadratic when Δ _________ 0.
- There is one real solution for a quadratic when Δ_____________ 0.
- There are two unique real solutions for a quadratic when Δ__________________ 0.

[1.8.3] – Apply Shape/Graph to Solve Number of Solutions Questions

To find the number of solutions for $f(x) = k$, draw a ________________________ line at _______________ $= k$ and count the intersections.

[1.8.4] – Apply Odd and Even Functions (MHS Investigation 2023)

- For an odd function, $f(x) =$ ____________________.
- For an even function, $f(x) =$___________________.

[1.8.5] – Identify Possible Rule(s) From a Graph

- A turning point x-intercept has $a(n)$ ______________________ power on its factor.
- A stationary point of inflection x intercept has $a(n)$ __________________ power on its factor.
- If the x-intercept passes straight through, the power of the factor is ___________________.

Section B: Questions (61 Marks)

Sub-Section: Exam 1

INSTRUCTION: 31 Marks. 5 Minutes Reading. 35 Minutes Writing.

Question 1 (4 marks)

Consider the points $A(-3, 5)$ and $B(4, -2)$.

a. Find the equation of the line joining A and B and hence, find the angle that the line segment AB makes with the positive x-axis. (2 marks) **[1.5.2] [1.5.3]**

b. Find the equation of the line that the point A could be reflected in to map it onto the point B. (2 marks) **[1.5.2] [1.6.1]**

Question 2 (3 marks) **[1.5.4]**

Consider the system of linear equations:

$$(a-4)x+3y=2$$

$$4x+(a+7)y=a+3$$

where $a \in \mathbb{R}$. Find the value of a such that the system of equations has infinitely many solutions.

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Question 3 (10 marks)

Consider the functions f and g, defined over their maximal domains where:

$$f(x) = 2\sqrt{x+2} - 2$$

$$g(x) = \log_2(3 - x)$$

a. Find the maximal domain of $f(x) + \frac{1}{\sqrt{g(x)}}$. (2 marks) **[1.1.1]**

b. Show that $f\big(g(x)\big)$ is not defined. (1 mark) **[1.1.2]**

c. The domain of g is restricted to $x \in (-\infty, a]$. Find the largest value of a such that $f\big(g(x)\big)$ exists and write down its rule. (3 marks) **[1.1.2] [1.2.1]**

d. Define f^{-1}, the inverse function of f. (2 marks) **[1.1.3]**

e. Find all points of intersection between f and f^{-1}. (2 marks) **[1.1.3]**

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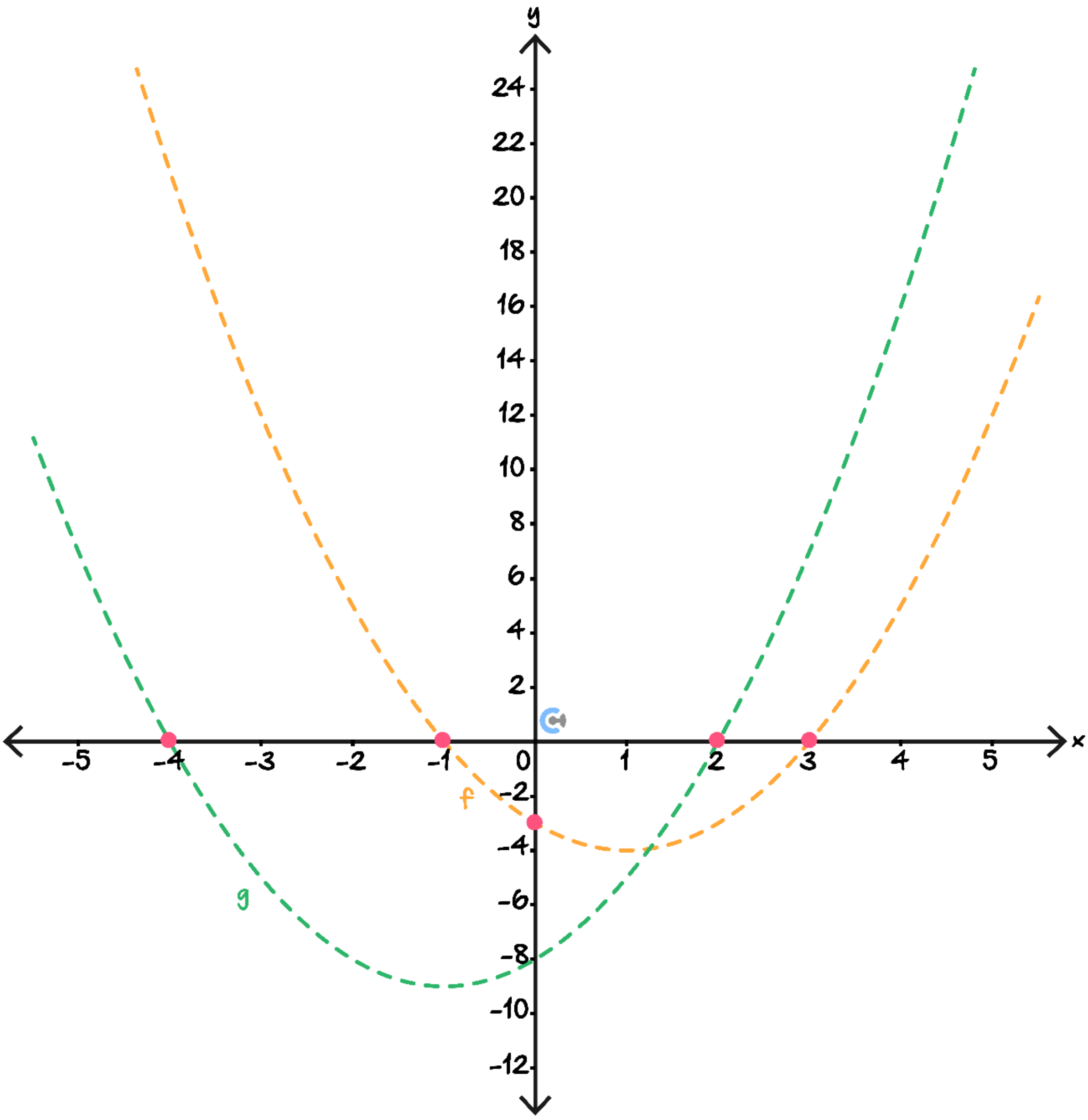
Question 4 (3 marks) **[1.5.5]**

The graphs of quadratic functions f and g are sketched on the axes below. Sketch the graph of $f + g$ on the same axes.



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Question 5 (4 marks)

Consider the functions $f(x) = 2\log_2(x)$ and $g(x) = -4\log_2(3x - 6)$.

a. Using dilations, reflections, and horizontal translations only, describe a sequence of transformations that map $f(x)$ to $g(x)$. (2 marks) **[1.4.5]**

b. Without using any dilations from the y-axis, describe a sequence of transformations that map $g(x)$ to $f(x)$. (2 marks) **[1.4.6]**

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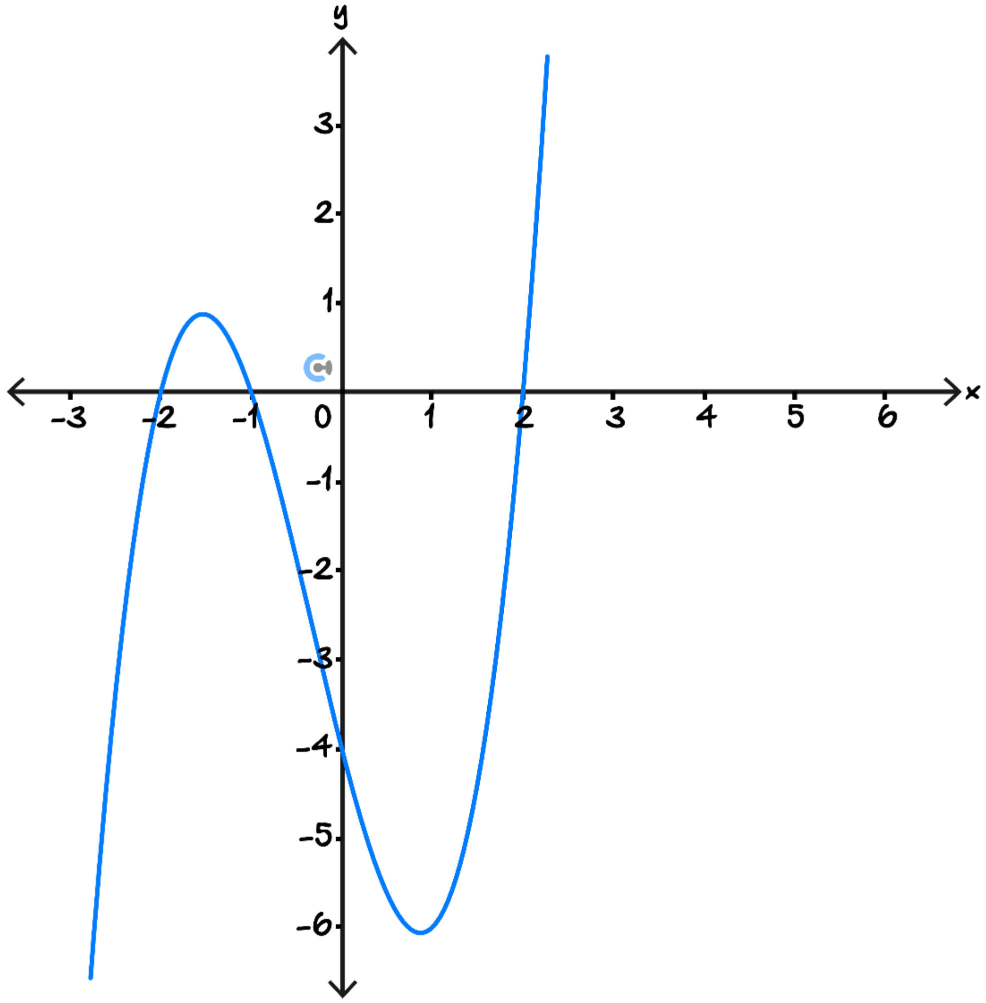
Question 6 (7 marks)

a. Let $f(x) = x^3 - 2x^2 - 11x + 12$. Solve the equation $f(x) = 0$. (2 marks) **[1.7.2]**

Let $g(x) = (x - 2)(x + 1)(x + 2)$. The graph of $y = g(x)$ is shown on the axes below:



b. Describe the transformations that map $g(x)$ to $h(x) = -\frac{1}{2}g(2-x)$. (2 marks) **[1.3.1]**

c. Find the factored form of $h(x)$ and sketch the graph of $h(x)$ on the same axes as $g(x)$. Label all axes intercepts. (3 marks) **[1.3.2]**

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Sub-Section: Exam 2

INSTRUCTION: 30 Marks. 5 Minutes Reading. 35 Minutes Writing.

Question 7 (1 mark) **[1.1.4]**

Consider the function $f: [-3, \infty) \to \mathbb{R}, f(x) = (x+3)^2 - 5$. Which of the following is the rule and domain of $f(f^{-1}(x))$?

A. $f(f^{-1}(x)) = x, x \in [-3, \infty)$

B. $f(f^{-1}(x)) = x, x \in [-5, \infty)$

C. $f(f^{-1}(x)) = -x, x \in (-\infty, -5]$

D. $f(f^{-1}(x)) = x, x \in (-\infty, -3]$

Question 8 (1 mark) **[1.2.2]**

The range of the function $f(x) = \log_2\left(\sqrt{x^2+4}\right)$ is:

A. $[2, \infty)$

B. $(2, \infty)$

C. $[1, \infty)$

D. $(1, \infty)$

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Question 9 (1 mark) **[1.2.3]**

The function f has an inverse function f^{-1}. It is known that $f(1) = 2, f(2) = 3$ and $f'(2) = 3, f'(3) = 5$. Find the gradient of f^{-1} when $x = 3$.

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. 2

D. $\frac{1}{5}$

Question 10 (1 mark) **[1.4.1]**

The function $T: \mathbb{R}^2 \to \mathbb{R}^2$ maps the graph of $y = (x-1)^2$ onto the graph $y = 2(x-3)^2 + 6$. The rule for T could be:

A. $T(x, y) = (x - 2, 2y - 6)$

B. $T(x, y) = (x + 2, 2y + 3)$

C. $T(x, y) = (x + 2, 2y - 3)$

D. $T(x, y) = (x + 2, 2y + 6)$

Question 11 (1 mark) **[1.7.1]**

The polynomial $x^3 + ax^2 + bx + 5$ is perfectly divisible by $x + 3$ and has a remainder of 1 when divided by $x - 2$.The values (a, b) are:

A. $(4, 12)$

B. $\left(\frac{4}{15}, -\frac{98}{15}\right)$

C. $\left(\frac{16}{3}, -\frac{26}{3}\right)$

D. $\left(-\frac{14}{3}, \frac{10}{3}\right)$

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Question 12 (1 mark) **[1.8.2]**

The function $f(x) = x^3 - x^2 + (k - 6)x + 2k$, where $k \in \mathbb{R}$, has exactly one root for:

A. $k < \frac{9}{4}$

B. $k > \frac{9}{4}$

C. $-\frac{9}{4} < k < \frac{9}{4}$

D. $k = \frac{9}{4}$

Question 13 (1 mark) **[1.4.2]**

Let $f(x)$ and $g(x) = -\frac{1}{3}f(4x + 8)$ be functions. A sequence of transformations that maps $g(x)$ to $f(x)$ is:

A. A dilation by a factor $\frac{1}{3}$ from the x-axis, a dilation by a factor $\frac{1}{4}$ from the y-axis, a reflection in the x-axis, and a translation 2 units to the left.

B. A dilation by a factor 3 from the x-axis, a dilation by a factor 4 from the y-axis, a reflection in the x-axis. And a translation 2 units to the left.

C. A translation 2 units to the right, a reflection in the x-axis, a dilation by a factor 4 from the y-axis, and a dilation by a factor 3 from the x-axis.

D. A translation 2 units to the left, a reflection in the x-axis, a dilation by a factor 3 from the y-axis, and dilation by a factor 4 from the x-axis.

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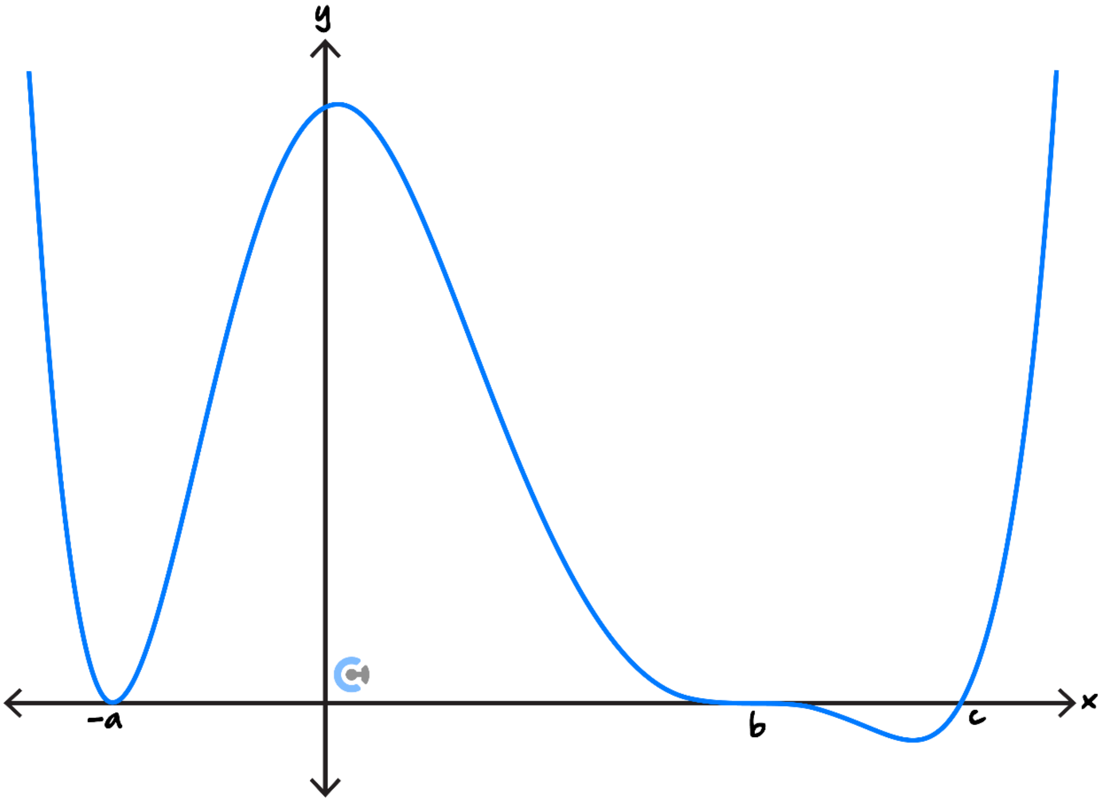
Question 14 (1 mark)

Consider the graph of a function f shown below, where $a, b, c > 0$. A possible rule for $f(x)$ is:



A. $f(x) = -(x+a)^2(x-b)^3(x-c)$

B. $f(x) = (x+a)^2(x-b)^3(x-c)$

C. $f(x) = (x-a)^2(x-b)^3(x-c)$

D. $f(x) = (x+a)^2(x+b)^3(x-c)$

Question 15 (1 mark) **[1.7.4] [1.8.4]**

Consider the function $f(x) = x^5 + 3x^3 + (k^2 - 3k - 4)x^2 + 2kx + 2k^2 + k - 1$. The value(s) of k for which $f(x)$ is an odd function are:

A. $k = 1$

B. $k = 1$ or $k = -1$

C. $k = -1$

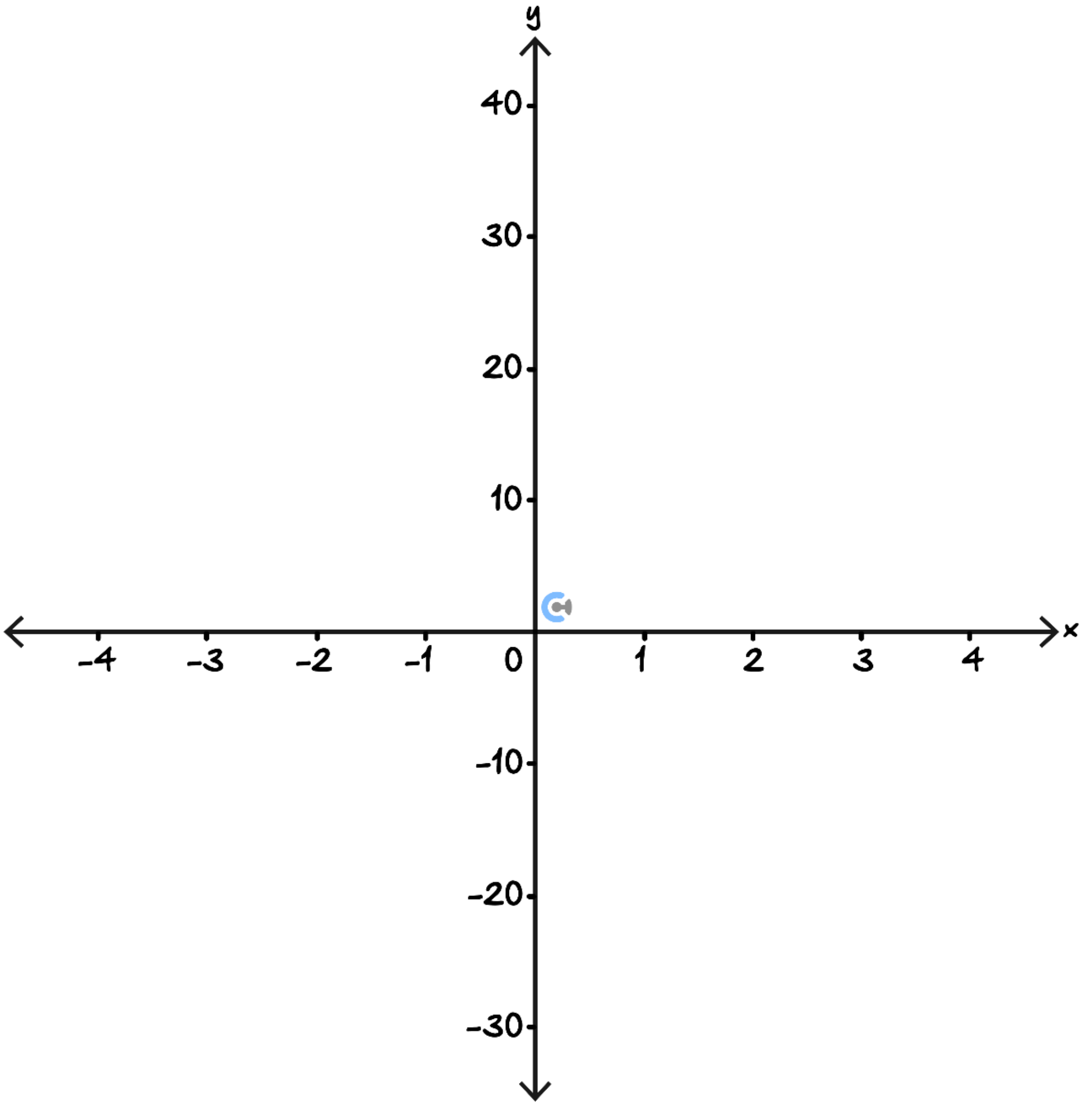
D. $k = 4$

Question 16 (12 marks)

Consider the quadratic function $f(x) = x^4 - 9x^2 + 4x + 12$.

a. Fully factorise f and hence, find all its roots. (2 marks) **[1.7.2]**

b. Sketch the graph of $y = f(x)$ on the axes below. Label all axes, intercepts, and turning points correct to two decimal places where appropriate. (3 marks) **[1.7.3]**



c.

i. Find all values of $k \in \mathbb{R}$ such that $f(x-k) = 0$ has two positive solutions. (2 marks) **[1.8.1]**

ii. The equation $f(x) = a, a \in \mathbb{R}$ has no solutions. Find all possible values of a, exactly. (1 mark) **[1.8.3]**

iii. Find the shortest horizontal distance between two points on the graph of $y = f(x)$ when $y = 10$. Give your answer correct to two decimal places. (2 marks) **[1.5.1]**

d. The graph of $y = f(x)$ and the graph of $y = -f(x) + k$ has exactly one point of intersection. Find the exact value of k. (2 marks) **[1.3.3]**

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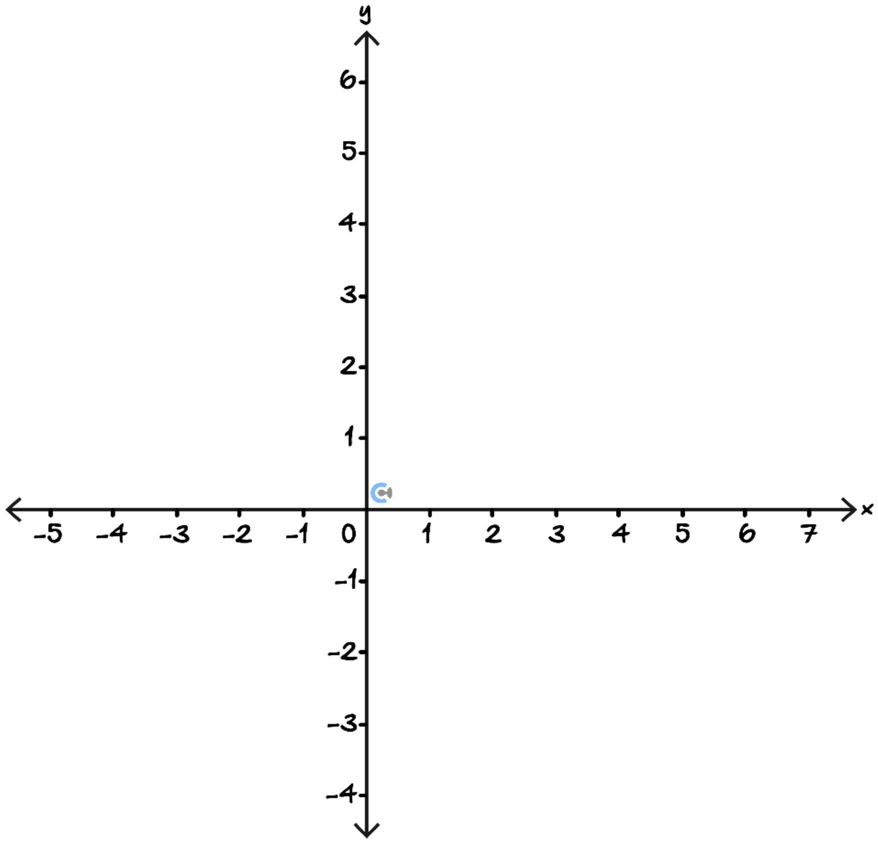
Question 17 (9 marks)

Consider the function $f : \mathbb{R} \to \mathbb{R}, f(x) = \sqrt{3}x + 3 - 3\sqrt{3}$.



a.

i. Sketch the graph of f and f^{-1} on the axes above. Label the point of intersection with coordinates. (2 marks) **[1.1.3]**

ii. Find the distance between the origin and the intersection. (1 mark) **[1.5.1]**

iii. Find the exact size of the acute angle between f and f^{-1} at their intersection point. (1 mark) **[1.5.3]**

Consider the functions $g(x) = 2x - 5$ and $h : [-\sqrt{k}, \sqrt{k}] \to \mathbb{R}, h(x) = \frac{1}{\sqrt{k}}x - k$, where $k \in \mathbb{R}^+$.

b. Find the coordinates for any point of intersection between g and h in terms of k. (1 mark)

c. Find the values of k for which $g(x) = h(x)$ has a unique solution. (2 marks)

d. Find the shortest distance from any intersection of g and h to the origin. Give your answer correct to two decimal places. (2 marks) **[1.5.1]**

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