Website: contoureducation.com.au | **Phone:** 1800 888 300
Email: hello@contoureducation.com.au

VCE Mathematical Methods ¾
Coordinate Geometry Exam Skills [0.6]
Workshop Solutions

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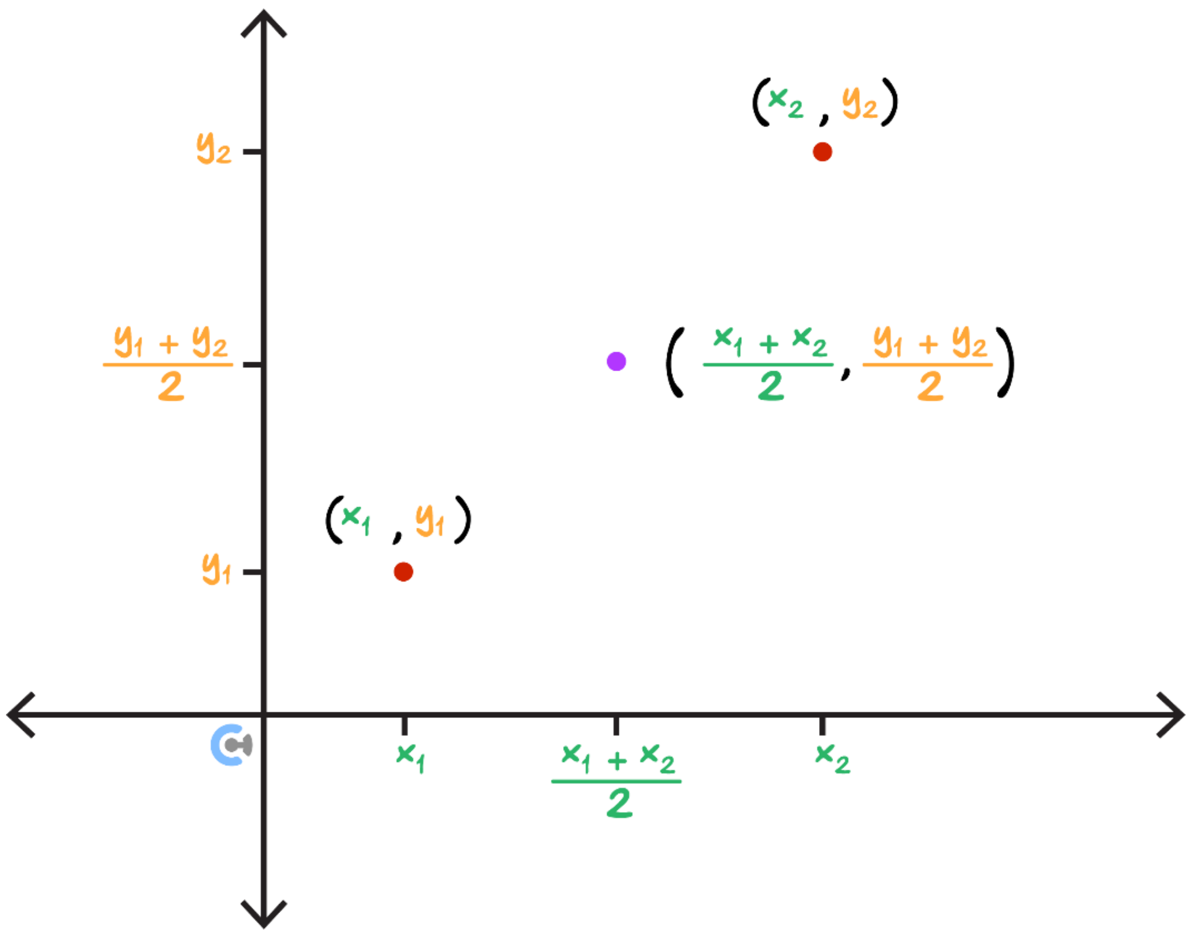
Section A: Recap

Midpoint



- The midpoint, M, of two points A and B is simply the point halfway between A and B.

$$M(x_m, y_m) = \left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}\right)$$

- The midpoint can be found by taking the average of the x-coordinate and y-coordinate of the two points.

Distance Between Two Points

- The distance between two points (x_1, x_2) and (y_1, y_2) can be found using Pythagoras' theorem:

$$Distance = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Horizontal Distance

$$Horizontal\ Distance = x_2 - x_1, where\ x_2 > x_1$$

- Find the difference between their x-values.

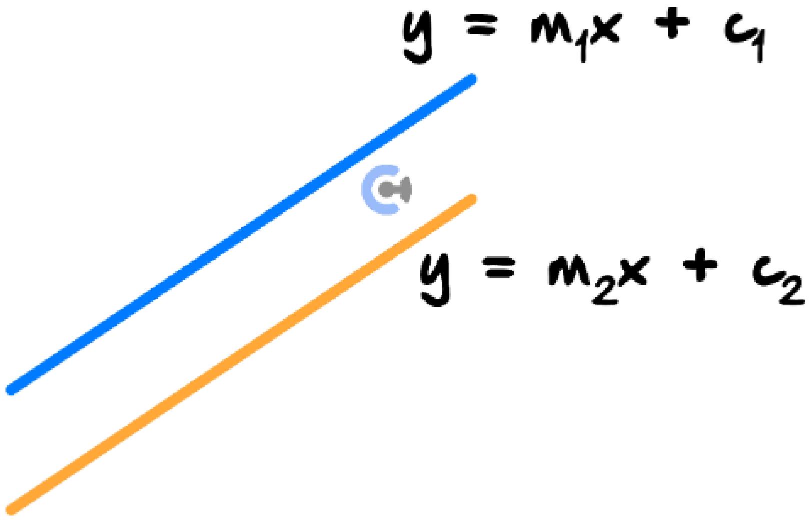
Vertical Distance

$$Vertical\ Distance = y_2 - y_1, where\ y_2 > y_1$$

- Find the difference between their y-values.

Definition

Parallel Lines

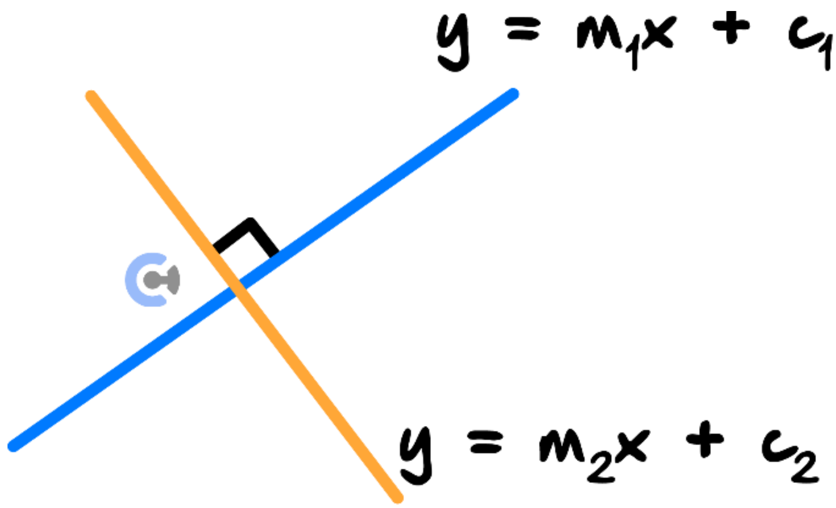


- Parallel lines have the same gradient.

$$m_1 = m_2$$

Definition

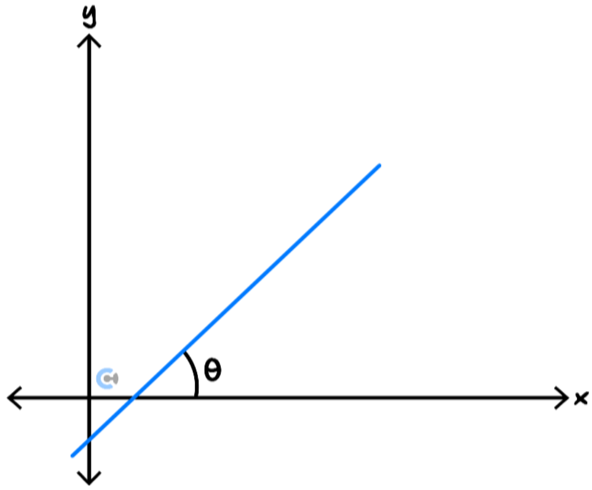
Perpendicular Lines



- A line that is perpendicular to another line has a gradient which is the negative reciprocal of the gradient of the first line.

$$m_{\perp} = -\frac{1}{m}$$

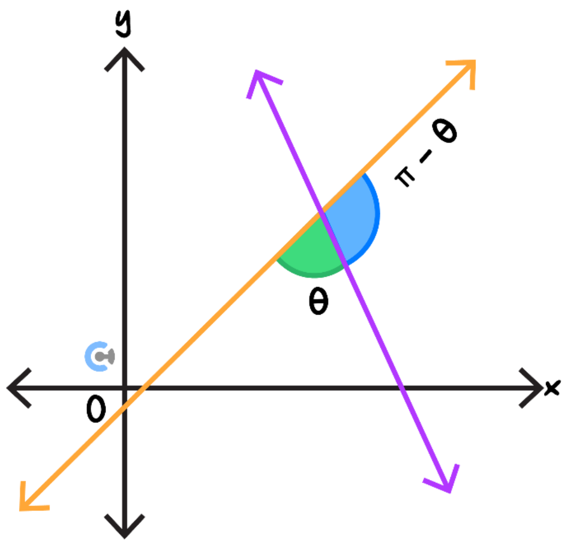
Angle Between a Line and the x-axis



- The angle between a line and the **positive direction of the x-axis** (anticlockwise) is given by:

$$\tan(\theta) = m$$

Acute Angle Between Two Lines



$$\theta = |\tan^{-1}(m_1) - \tan^{-1}(m_2)|$$

- **Alternatively:**

$$\tan(\theta) = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

Simultaneous Linear Equations

- Elimination Method.
- Substitution Method.

General Solutions of Simultaneous Linear Equations

- Two linear equations are either:
  - The same line, expressed in a different form. In this case, they have infinite solutions.
  - Unique lines which are **parallel**. In this case, they have NO solutions.
  - Unique lines which are not parallel. In this case, they have exactly one solution.

Solving Systems of Linear Equations with Parameters

- Occurs when solving for three variables with two equations. We simply,

$$\text{Let } x = k, \text{ or}$$

$$\text{Let } y = k, \text{ or}$$

$$\text{Let } z = k$$

- And solve simultaneously.

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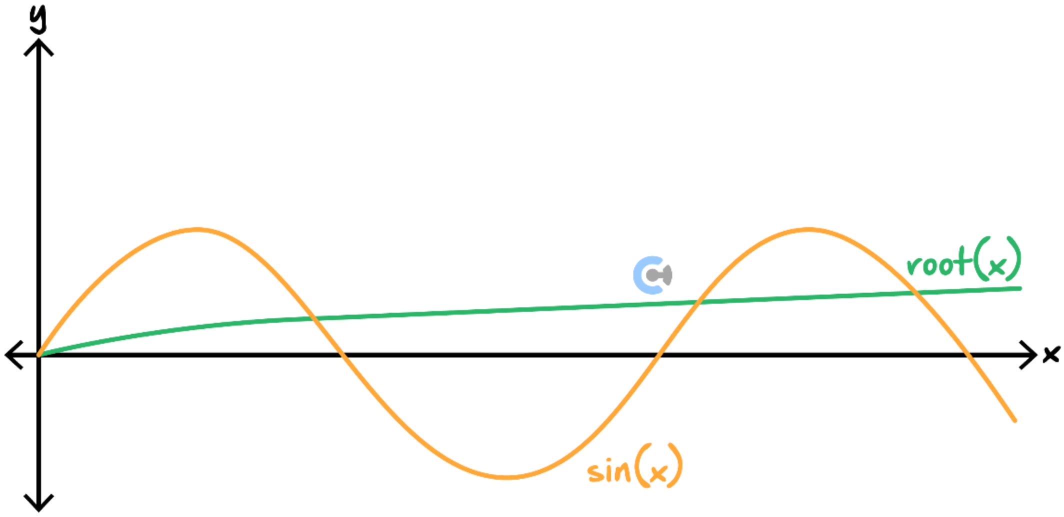
Addition of Ordinates

- **Definition:**

  - Technique used to graph the sum/difference of two functions.

$$\text{E.g., } y = \sin(x) + \sqrt{x}$$



- The addition of ordinates involves adding the y-values of two functions.

- **Steps to sketching $f(x) + g(x)$:**

1. Sketch $f(x)$ and $g(x)$ on the same axes.

2. Plot points for $f(x) + g(x)$ by adding the **y-values** of $f(x)$ and $g(x)$.

   - At x-intercepts, the sum equals to the other function.

   - At intersections, the sum equals to the y-value.

   - When functions are equidistant from x-axis, the sum equals to 0.

3. Join the plotted points.

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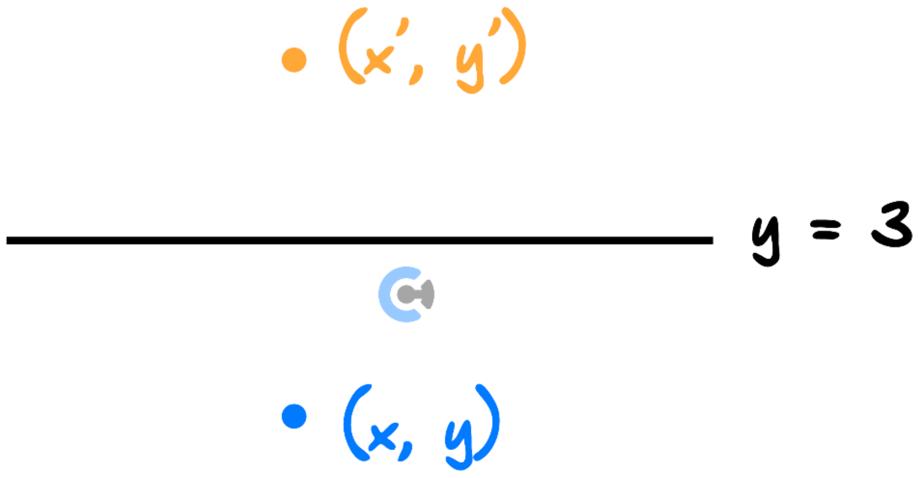
Definition

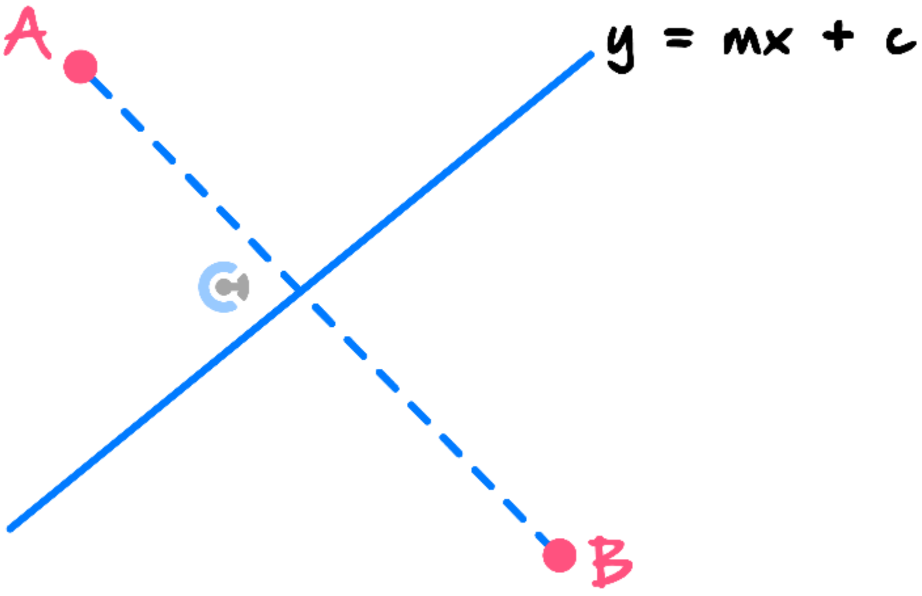
Reflection of a Point Around a Vertical/Horizontal Line



- Midpoint must be on the line of reflection.

Definition

Finding the Reflection of a Point In a Line



- **Steps:**

1. Find the perpendicular line passing through the point.
2. Find the intersection between the original line and the perpendicular line.
3. Find the reflected point (x, y) by treating the intersection from **step (2)** as the midpoint between the original and reflected point.

Section B: Warm-Up

INSTRUCTION: 5 Minutes Writing.

Question 1

Consider the line segment AB with coordinates $A(1, 2)$ and $B(5, 6)$.

a. Find the midpoint of AB.

$$(3, 4)$$

b. Find the equation of the line segment AB.

$m = 1$ and through $(1, 2)$. Therefore,

$$y = x + 1$$

c. Show that the equation of the perpendicular bisector of AB is $y = -x + 7$.

$m = -1$ and through the point $(3, 4)$

$$y - 4 = -(x - 3)$$

$$y = -x + 7$$

d. The point $A(2, 5)$ is reflected in the line $y = x + 1$ to become the point A'. Find the coordinates of A'.

Solution: $(2, 5)$ is on the line $y = -x + 7$.
$y = x + 1$ and $y = -x + 7$ intersect at $(3, 4)$.
Therefore $A'(4, 3)$

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Section C: Exam 1 Questions (28 Marks)

Question 2 (5 marks)

Consider the points $A(2,3)$ and $B(5,1)$.

a. Find the distance between points A and B. (1 mark)

$$d = \sqrt{3^2 + 2^2} = \sqrt{13}$$

b. The distance between point A and a point $C(5,k)$ is 4. Find the possible value(s) of k. (2 marks)

$$\sqrt{3^2 + (k-3)^2} = 4 \implies (k-3)^2 = 7 \implies k = 3 \pm \sqrt{7}$$

c. Find the coordinates of the point A' obtained by reflecting A in the line $x = -1$ and then in the line $y = 2$. (2 marks)

Solution: $(2,3) \mapsto (-4,3) \mapsto (-4,1)$
So $A'(-4,1)$

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Question 3 (5 marks)

Consider the linear equations:

$$(k+2)x + 4y = 6$$

$$3x + 2(k-3)y = k - 1$$

a. For what value(s) of k, will the system have a unique solution? (2 marks)

Solution: Unique solution if gradients are the same. Solve

$$\frac{k+2}{4} = \frac{3}{2(k-3)}$$
$$k = -3, 4$$

Therefore, $k \neq -3, 4$ for unique solution.

b. For what value of k, will the system have infinitely many solutions? (2 marks)

Solution: Need gradient and y-intercept to be the same. Same y intercept if

$$\frac{6}{4} = \frac{k-1}{2k-6}$$
$$k = 4$$

$k = 4$

c. For what value of k, will the system have no solutions? (1 mark)

$k = -3$

Question 4 (3 marks)

It is known that the acute angle made by the lines $y = 3x + 3$ and $y = mx - 4$ is 45°. Find the possible value(s) of m.

Solution: We solve the equation

$$\left|\frac{m-3}{1+3m}\right| = 1$$

$$\frac{m-3}{1+3m} = 1 \implies m = -2$$

OR

$$\frac{m-3}{1+3m} = -1 \implies m = \frac{1}{2}$$

so $m = -2$ or $m = \frac{1}{2}$

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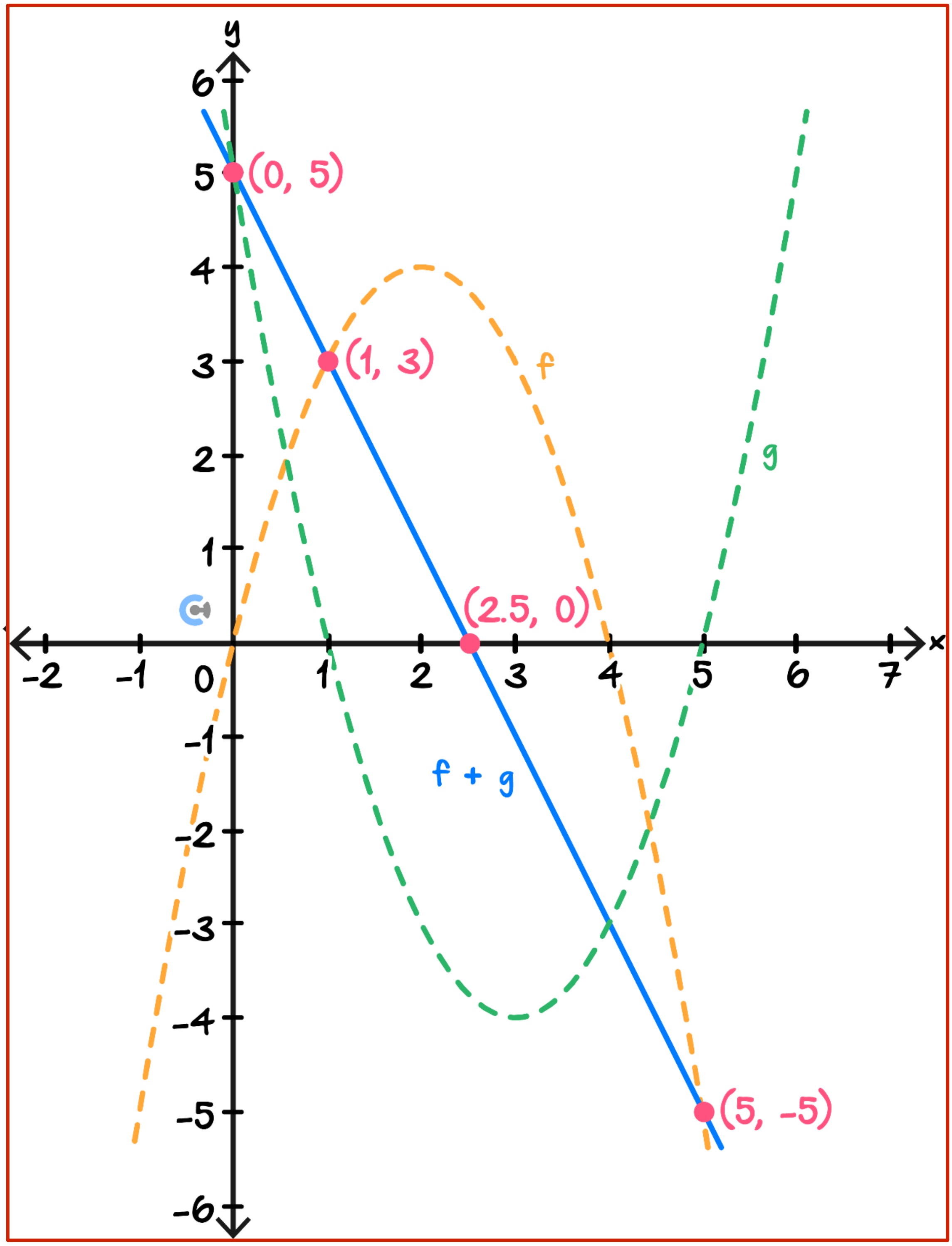
Question 5 (2 marks)

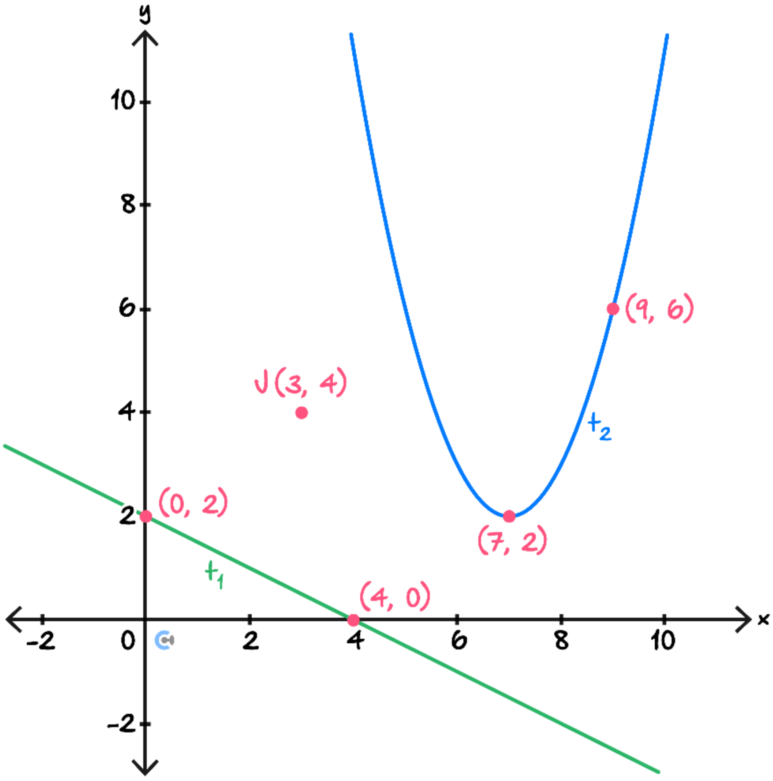
The graphs of the functions f and g are sketched on the axes below. Sketch the graph of $f + g$ on the same set of axes and label its axis intercepts with coordinates.



Question 6 (8 marks)

James is leaving the gym located at the point $J(3, 4)$. He decides to go for a walk to warm down.

James's location and two walking trails t_1 and t_2 are shown in the diagram below. The walking trail t_1 is modelled by a linear function and the trail t_2 is modelled by a quadratic function.



a. Show that t_1 can be modelled by the line $y = -\frac{1}{2}x + 2$. (1 mark)

Gradient $= \frac{-2}{4} = -\frac{1}{2}$ and y-intercept $(0, 2)$. Therefore

$$y = -\frac{1}{2}x + 2$$

b. Show that t_2 can be modelled by the quadratic $y = (x - 7)^2 + 2$. (1 mark)

Solution: Turning point at $(7, 2)$ so $y = a(x - 7)^2 + 2$ the point $(9, 6)$ is on the quadratic so

$$6 = a(4) + 2 \implies a = 1$$

Therefore $y = (x - 7)^2 + 2$.

c. Find the distance from J to t_2 when $x = 5$. (2 marks)

Calculate the distance between $(3, 4)$ and $(5, 6)$.

$$d = \sqrt{4 + 4} = 2\sqrt{2}$$

d. Find the shortest horizontal distance between t_1 and t_2 when $y = 2$. (1 mark)

7

e.

i. Find the equation of the line perpendicular to t_1 that passes through $J(3, 4)$. (1 mark)

$y = 2x - 2$

ii. Hence, find the shortest distance between J and t_1. (2 marks)

Solution: Intersection of $y = -\frac{1}{2}x + 2$ and $y = 2x - 2$ is $\left(\frac{8}{5}, \frac{6}{5}\right)$ distance between $(3, 4)$ and $\left(\frac{8}{5}, \frac{6}{5}\right)$

$$d = \sqrt{\left(\frac{7}{5}\right)^2 + \left(\frac{14}{5}\right)^2} = \frac{\sqrt{245}}{5} = \frac{7\sqrt{5}}{5}$$

Question 7 (5 marks)

A gardener is designing a symmetrical flower bed. One pair of flowers are located at the points $F_1(2,2)$ and $F_2(10,6)$ respectively.

a. Find the equation of the line that F_1 is reflected in to map to F_2. (2 marks)

Solution: The line we want is the perpendicular bisector of $(2,2)$ and $(10,6)$. This line is

$$y = 16 - 2x$$

b. The gardener plants another pair of flowers. He plants the first flower at the point $A(2,6)$. The location of the second flower is obtained by reflecting the point A in the line $y = 2x - 8$. Find the coordinates of where the second flower is planted. (3 marks)

Solution: Perp line: $y = -\frac{1}{2}x + 7$
Intersection of two lines $(6,4)$.
Therefore $A'(10,2)$

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Section D: Tech Active Exam Skills

Calculator Commands: Simultaneous Equations on CAS

System of Linear Equations

Example:

The simultaneous linear equations $ax - 3y = 5$ and $3x - ay = 8 - a$ have no solution for:

A. $a = 3$

B. $a = -3$

C. Both $a = 3$ and $a = -3$.

D. $a \in R\backslash\{3\}$

E. $a \in R\backslash[-3, 3]$

```
system_solve(a·x−3·y=5,3·x−a·y=8−a,a)
    ▸ Solving: [ a·x−3·y=5   ]
               [ 3·x−a·y=8−a ]
    ▸ Unique Solution: a≠−3 and a≠3
    ▸ No Solutions: a=−3
    ▸ Infinite Solutions: a=3
```

Overview:

This program takes two linear equations and a parameter and finds the parameter values for the system to obtain a unique solution, no solution, or infinite solutions.

Input:

$$system\_solve(<\ equation\,1\ >, <\ equation\,2\ >, <\ parameter\ >)$$

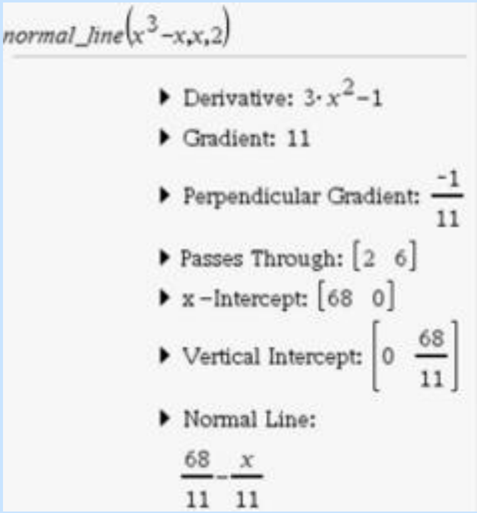
Other Notes:

The program can only handle one parameter.

- Or menu – 3 – 7.

- UDF line functions:

Normal Line



Overview:

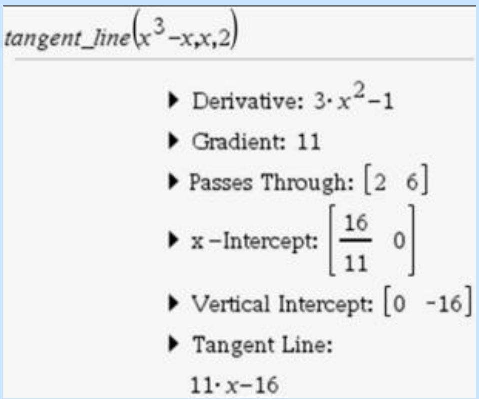
This program will find all the necessary information related to a normal line at a point on a function, which includes:

- The derivative.
- The gradient and perpendicular gradient.
- The point on the function the normal line passes through.
- The axis intercepts of the normal line.
- The equation of the normal line.

Input:

$$\boldsymbol{normal\_line}(<\ \boldsymbol{function}\ >,<\ \boldsymbol{variable}\ >, <\ \boldsymbol{x\,point}\ >)$$

Tangent Line



Overview:

This program will find all the necessary information related to a tangent line at a point on a function, which includes:

- The derivative.
- The gradient of the tangent line.
- The point on the function the tangent line passes through.
- The axis intercepts of the tangent line.
- The equation of the tangent line.

Input:

$$\boldsymbol{tangent\_line}(<\ \boldsymbol{function}\ >,<\ \boldsymbol{variable}\ >, <\ \boldsymbol{x\,point}\ >)$$

Calculator Commands: Finding the Angle Between Two Lines

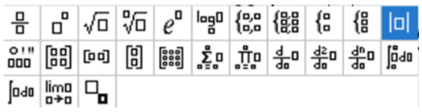
- The angle between two lines with gradients m_1 and m_2 respectively is:

$$\theta = |\tan^{-1}(m_1) - \tan^{-1}(m_2)|$$

- **Mathematica**
  - Use the Abs[] function.

```
In[126]:= Abs[ArcTan[2] - ArcTan[1]] / Degree // N
Out[126]= 18.4349
```

- **TI-Nspire**
  - Find the modulus sign.

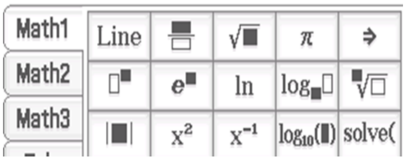


$|\tan^{-1}(2)-\tan^{-1}(1)|$ 18.4349

- **Casio Classpad**
  - Modulus sign under Math1.

$|\tan^{-1}(2)-\tan^{-1}(1)|$

18.43494882



Calculator Commands: Finding the Gradients of Lines Given the Angle They Make

- If we know the angle and one of the gradients $m_1$or m_2 then, we can find the other gradient by solving,

$$\tan(\theta) = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

- E.g., find the gradient of the line that makes an angle of 60° with $y = -x$.

- **Mathematica**

Solve$\left[\text{Tan}[60\,\text{Degree}] == \text{Abs}\left[\frac{m1+1}{1-m1}\right], m1\right]$

Solve: Inverse functions are being used by Solve, so s

Reduce for complete solution information.

$\{\{m1 \to 2-\sqrt{3}\}, \{m1 \to 2+\sqrt{3}\}\}$

- **TI-Nspire**
  - Find the modulus sign.

solve$\left(\tan(60)=\left|\frac{m1+1}{1-m1}\right|, m1\right)$

$m1=-(\sqrt{3}-2)$ or $m1=\sqrt{3}+2$

- **Casio Classpad**
  - Modulus sign under Math1.

solve$\left(\tan(60)=\left|\frac{m1+1}{1-m1}\right|, m1\right)$

$\left\{m1=\frac{\sqrt{3}-1}{\sqrt{3}+1}, m1=\frac{\sqrt{3}+1}{\sqrt{3}-1}\right\}$

Section E: Exam 2 Questions (25 Marks)

Question 8 (1 mark)

Consider the following pair of simultaneous equations:

$$x + ay = 3$$

$$(a - 3)x - 2y = 1 - 2a$$

For what value(s) of a, do the equations have infinitely many solutions?

A. $a = 1$

B. $a = 2$

C. $a = 1,2$

D. $a \in \mathbb{R}\backslash\{1, 2\}$

Question 9 (1 mark)

The distance between the point $(2, 5)$ and the line $y = 5 - x$ when $x = a$ is $\sqrt{2}$. The possible value(s) of a are:

A. 0,2

B. 0,3

C. 1 only.

D. 2,3

Question 10 (1 mark)

The point $(1, 3)$ is reflected about a line $y = mx + c$ to become the point $(6, 4)$. The values of m and c are:

A. $m = 5, c = -16$

B. $m = 5, c = 22$

C. $m = -5, c = 21$

D. $m = -5, c = 16$

Question 11 (1 mark)

When $x = a$ the vertical distance between the functions $f(x) = (x-2)^2 + 3$ and $g(x) = x + 2$ is 1. All possible values of a are:

A. $a = 2, 3$

B. $a = 3, 4$

C. $a = 1, 2, 3, 4$

D. $a = 2, 3, 4$

Question 12 (1 mark)

The **obtuse** angle made by the lines $y = 3x - 1$ and $y = -\frac{1}{2}x + 2$ is closest to:

A. 98^o

B. 108^o

C. 118^o

D. 158^o

Question 13 (1 mark)

The shortest distance between the lines $y = 3x + 2$ and $y = 3x + 12$ is:

A. 3

B. $\sqrt{10}$

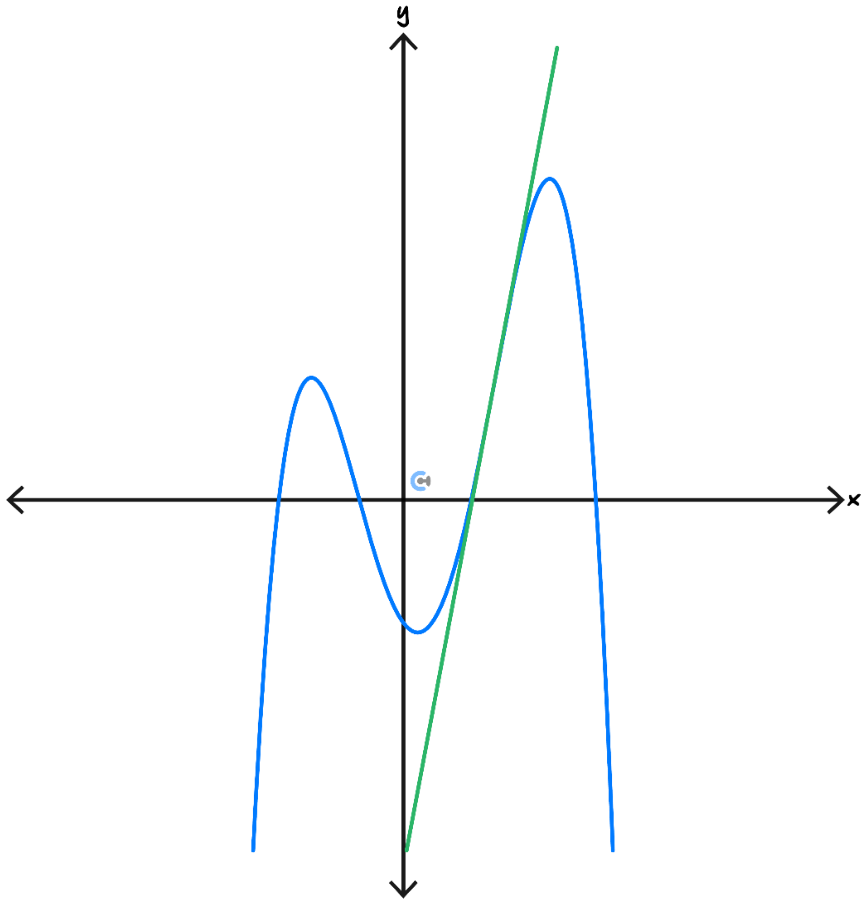
C. $\sqrt{13}$

D. $\sqrt{15}$

Space for Personal Notes

Question 14 (9 marks)

Consider the function $f(x) = 2 - 2(x+1)^2(x-2)(x-1)$. The graph of $y = f(x)$ and the tangent line to f when $x = 1$ is shown below.



a. The tangent line has a y-intercept of $(0, -6)$. Find the equation of this tangent. (2 marks)

Line through $(0, -6)$ and $(1, 2)$.

$y = 8x - 6$

b. State the angle that this tangent with the positive x-axis. Give your answer correct to the nearest degree. (1 mark)

$\arctan(8) = 83°$

c. Find the equation of the line that is perpendicular to this tangent. (2 marks)

Line with gradient $-\frac{1}{8}$ and through $(1,2)$.

$$y = -\frac{1}{8}x + \frac{17}{8}$$

d. Another tangent line, with the same gradient as the other tangent, can be drawn to the graph when $x = a$ and $a < 0$. Use the fact that the shortest distance between the two tangent lines $\frac{2187}{128\sqrt{65}}$ to find the value of a. (4 marks)

Solution: The second tangent has equation $y = 8x + c$. Intersection of the normal line $y = -\frac{1}{8}x + \frac{17}{8}$ and this tangent is at

$$A = \left(\frac{17-8c}{65}, \frac{136+c}{65}\right)$$

It must be that the distance between $(1,2)$ and A is $\frac{2187}{128\sqrt{65}}$. So we solve

$$\sqrt{\left(\frac{17-8c}{65} - 1\right)^2 + \left(\frac{136+c}{65} - 2\right)^2} = \frac{2187}{128\sqrt{65}}$$

$$\implies c = -\frac{2955}{128}, \frac{1419}{128}$$

by inspecting graphs we see that $c = \frac{1419}{128}$ is the correct solution.

We now solve $f(x) = 8x + \frac{1419}{128}$ to find $x = -\frac{5}{4}$.

Therefore $a = -\frac{5}{4}$.

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Question 15 (10 marks)

City planners are using a Cartesian plane to model locations for their new park.

It has been decided that the park will be in the shape of an equilateral triangle.

The vertex $A(2,2)$ is the vertex of the triangle that has the smallest x- and y-coordinates.

The two sides of the triangle that are adjacent to A are modelled by lines that are inverse functions of each other.

a. Show that two sides of the triangle can be modelled by the lines. (3 marks)

$$y = (2-\sqrt{3})x + 2\sqrt{3} - 2 \text{ and,}$$

$$y = (2+\sqrt{3})x - 2 - 2\sqrt{3}$$

Solution: We want the equation of inverse functions that make an angle of 60° and meet at the point $(2,2)$. Further note that both gradients must be positive since A has the smallest x and y coordinates of the triangle.
Therefore we solve

$$\left|\frac{m - \frac{1}{m}}{1+1}\right| = \sqrt{3} \text{ and } m > 0$$

and so $m = 2 \pm \sqrt{3}$. So one line has gradient $2-\sqrt{3}$ and the other has gradient $2-\sqrt{3}$. Now we just find the equation of the lines through $(2,2)$ with these gradients to get our answer

$$y = (2-\sqrt{3})x + 2\sqrt{3} - 2 \text{ and}$$
$$y = (2+\sqrt{3})x - 2 - 2\sqrt{3}$$

b.

i. If the triangle has another vertex at the point $B(5, 8+3\sqrt{3})$, find the location of the final vertex C and the equation of the line that passes through B and C. (2 marks)

Solution: We find the equation of the line that makes an acute angle of $60°$ with $y = (2+\sqrt{3})x - 2 - 2\sqrt{3}$ and passes through the point B.
The line has gradient -1 and passes through B. Therefore

$$y = 13 + 3\sqrt{3} - x$$

is the equation of the line that passes through B and C.
Now we find the intersection of $y = 13 + 3\sqrt{3} - x$ and $y = (2-\sqrt{3})x + 2\sqrt{3} - 2$ to get the coordinates C $(8+3\sqrt{3}, 5)$
NOTE: Because the lines are inverse functions we could have immediately gotten C and then used the two points to find the equation of the line.

ii. Hence, find the area of this triangle ABC. (2 marks)

Note: The formula $\frac{1}{2}bc\sin(A)$ may be helpful.

Solution: Equilateral triangle with side length $6\sqrt{2+\sqrt{3}}$
$\text{Area} = \frac{1}{2} \times \left(6\sqrt{2+\sqrt{3}}\right)^2 \times \sin(60°) = 27 + 18\sqrt{3}$

c. Find the locations of the other two vertices if the triangle has an area of 100. Give your answer correct to three decimal places. (3 marks)

Solution: We know that area $= \frac{1}{2} \times \frac{\sqrt{3}}{2} \times s^2$, where s is one of the side lengths.

Since area $= 100$ it must be that $s^2 = \frac{400}{\sqrt{3}}$

We find that the other vertices are at $(5.933, 16.679)$ and $(16.679, 5.933)$

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Section F: Extension Exam 1 (15 Marks)

Question 16 (6 marks)

a. Reflect the point (a, b) in the line $y = x - c$. (2 marks)

Solution: Equivalent to reflecting the point $(a - c, b + c)$ in the line $y = x$.
Therefore, $(b + c, a - c)$

b. Find the rule for the transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ that reflects a point (a, b) in the line $y = \frac{1}{2}x + 6$. (4 marks)

Solution: Perpendicular line through (a, b): $y = -2x + 2a + b$

Intersection of the two lines: $\left(\frac{2}{5}(2a + b - 6), \frac{1}{5}(2a + b + 24)\right)$

Let (x, y) be the reflected point.

$$\frac{a + x}{2} = \frac{2}{5}(2a + b - 6) \implies \frac{1}{5}(3a + 4b - 24)$$

and

$$\frac{a + y}{2} = \frac{1}{5}(2a + b + 24)$$

Therefore the transformation that reflects a point in the line $y = \frac{1}{2}x + 6$ is

$$T: \mathbb{R}^2 \to \mathbb{R}^2,\ T(x, y) = \left(\frac{1}{5}(3x + 4y - 24), \frac{1}{5}(4x - 3y + 48)\right)$$

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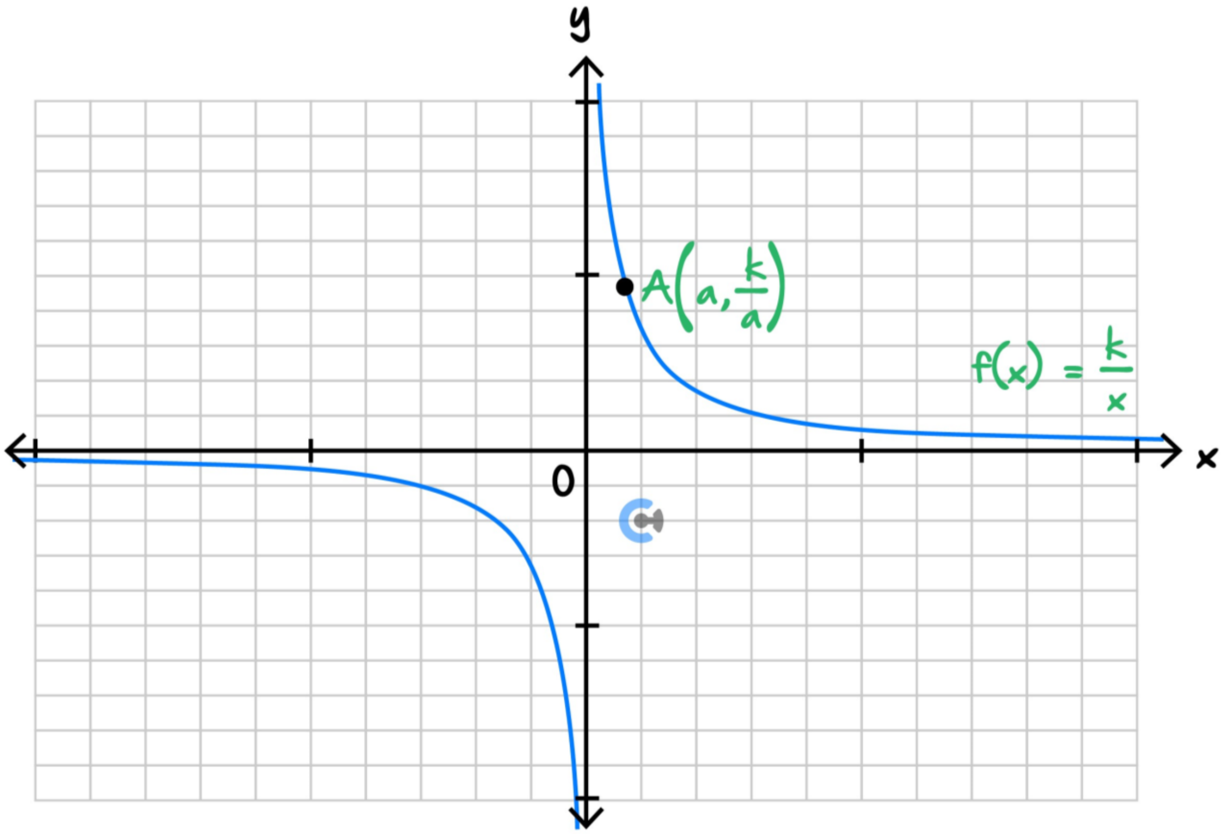
Question 17 (6 marks)

The graph of the general hyperbola $f(x) = \frac{k}{x}$, where $k > 0$, is shown below. The point A is located on the graph with coordinates $\left(a, \frac{k}{a}\right)$, where $a > 0$.



Point A is reflected around two axes to point A' where their mid point is located at $(0, 0)$.

a. State the coordinates of A' and plot its coordinates relative to A on the graph above. (1 mark)

$A': \left(-a, \frac{-k}{a}\right)$

b. Show that the distance between the points A and A' can be expressed as $\frac{2\sqrt{a^4+k^2}}{a}$. (2 marks)

$$\sqrt{(a--a)^2+\left(\frac{k}{a}--\frac{k}{a}\right)^2}$$

$$=\sqrt{(2a)^2+\left(\frac{2k}{a}\right)^2}$$

$$=\sqrt{4a^2+\frac{4k^2}{a^2}}$$

$$=\sqrt{\frac{4a^4+4k^2}{a^2}}$$

$$=\frac{2}{a}\sqrt{a^4+k^2}$$

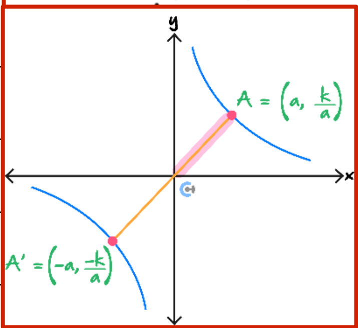
$$=\frac{2\sqrt{a^4+k^2}}{a}$$

c. Find the minimum distance between the two branches. Give your answer in terms of k. (3 marks)

= min distance btw A & A'

$\frac{k}{x} = k \cdot x^{-1}$

c. Find the minimum distance between the two branches of the hyperbola given by $y = f(x)$. Give your answe in terms of k. (3 marks)



$-k \cdot x^{-2} = -\frac{k}{x^2}$

$$\left(\frac{\frac{k}{a}-0}{a-0}\right)\cdot\left(\frac{-k}{a^2}\right) = -1$$

$$\frac{k}{a^2} \times \frac{-k}{a^2} = -1$$

$$k^2 = a^4 \rightarrow a = \sqrt{k}$$

$$\therefore \text{dist} = \frac{2\sqrt{k^2+k^2}}{\sqrt{k}} = \frac{2\sqrt{2k^2}}{\sqrt{k}}$$

$$= \frac{2\sqrt{2}k}{\sqrt{k}} = 2\sqrt{2k} = 2\sqrt{2}\sqrt{k}$$

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Question 18 (3 marks)

The area of the triangle enclosed by the lines $y = 2x - 2$, $y = -\frac{1}{2}x + c$ and the x-axis is 5 square units. Find the possible values of c.

Solution: Lines intersect at $\left(\frac{2}{5}(c+2), \frac{2}{5}(2c-1)\right)$

x-intercepts at $x = 1$ and $x = 2c$.

Area of triangle given by $\frac{1}{2}(2c-1)\frac{2}{5}(2c-1) = \frac{1}{5}(2c-1)^2$

Solve

$$\frac{1}{5}(2c-1)^2 = 5$$
$$(2c-1)^2 = 25$$
$$2c - 1 = \pm 5$$

$c = -2$ or $c = 3$

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Section G: Extension Exam 2 (15 Marks)

Question 19 (1 mark)

The lines $y = -ax + 6$ and $y = bx - 2$ make an angle of 45° when they intersect. The relationship between a and b is:

A. $a = \frac{b+1}{b-1}$

B. $a = \frac{1-b}{1+b}$ or $a = \frac{b+1}{b-1}$.

C. $a = \frac{b-1}{1-b}$

D. $a = \frac{b+1}{1-b}$ or $a = \frac{b-1}{b+1}$.

Question 20 (1 mark)

The transformation that describes a point being reflected in the line $y = -2x + 3$ is:

A. $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = \left(\frac{12-3x-2y}{5}, \frac{6-2x+3y}{5}\right)$

B. $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = \left(\frac{12-3x-4y}{5}, \frac{6-4x+3y}{5}\right)$

C. $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = \left(\frac{24-3x-2y}{10}, \frac{12-2x+3y}{10}\right)$

D. $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = \left(\frac{12-3x-2y}{10}, \frac{6-2x+3y}{10}\right)$

Question 21 (1 mark)

The area of the triangle formed by the lines $y = \frac{4}{5}x - \frac{8}{5}, y = 32 - 4x$ and the x-axis is:

A. 8

B. 10

C. 12

D. 14

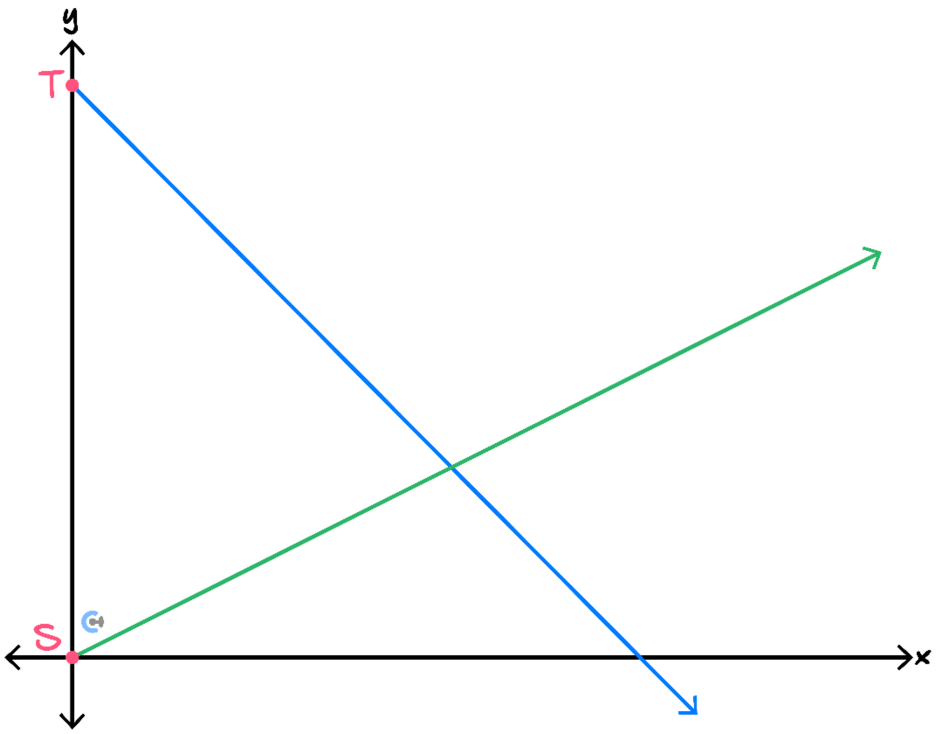
Question 22 (12 marks)

Sam is running through the countryside.

He starts his run from the origin and runs in a straight line. The path he takes sees him cross a set of train tracks somewhere in the first quadrant.

The train tracks are modelled by the line $y = 600 - x$.

All measurements are in metres and the diagram below shows the situation.



a. Find the shortest distance that Sam can run to reach the train tracks. (2 marks)

$300\sqrt{2}$

b. Sam runs a total of $400\sqrt{26}$ metres before he stops. If he runs three times further after crossing the train tracks than before crossing, find the possible equations of the line he runs along. (2 marks)

Solution: He must reach the train tracks after running $100\sqrt{26}$ metres. We solve

$$\sqrt{x^2+y^2} = 100\sqrt{26} \text{ and } y = 600 - x$$
$$\implies x = 100, 500$$

Intersection with train tracks could be at $(100, 500)$ or $(500, 100)$. Therefore the possible equations of the line he runs along are

$$y = 5x \text{ or } y = \frac{1}{5}x$$

c. Sam runs along a line that makes an angle of θ with the positive x-axis. Find the value of θ given that Sam's line makes an angle of 70° with the train tracks and that he crosses the train tracks at a y-value less than 300. Give an exact answer in degrees. (2 marks)

$\theta = 25°$

The front of a train is located at $(0, 600)$. Sam starts his run from the origin at the same time that the train starts moving at a constant speed. Sam runs at a constant speed of $4\ m/s$.

d. Sam runs along the line that makes an angle of 30° with the positive x-axis. Find the speed that the train must travel at for Sam and the front of the train to meet at the same point. Give your answer in m/s correct to two decimal places. (3 marks)

Solution: Intersection of $y = \frac{1}{\sqrt{3}}x$ and $y = 600 - x$ is at $I(900 - 300\sqrt{3}, 300\sqrt{3} - 300)$.
Therefore Sam has run $600\sqrt{4 - 2\sqrt{3}}$ metres which takes him $150\sqrt{4 - 2\sqrt{3}}$ seconds.
The train travels a distance of $900\sqrt{2} - 300\sqrt{6}$ metres (distance from $(0, 600)$ to I).
Therefore the trains speed is $\frac{900\sqrt{2} - 300\sqrt{6}}{150\sqrt{4 - 2\sqrt{3}}} \approx 4.90$ m/s.

e. It is now known that the train travels at a constant speed of $5\ m/s$. If Sam runs along the line $y = mx$, where $m > 0$. Find the values of m such that Sam crosses the train tracks before the train has reached his crossing point. (3 marks)

Solution: If Sam runs along line $y = mx$, he intersects track at $I\left(\frac{600}{1+m}, \frac{600m}{1+m}\right)$.
It takes him

$$\frac{1}{4}\sqrt{\left(\frac{600}{m+1}\right)^2 + \left(\frac{600m}{m+1}\right)^2} = 150\sqrt{\frac{1+m^2}{(1+m)^2}}$$

seconds to reach this point I.
The train travels a distance of

$$\sqrt{\left(\frac{600}{m+1}\right)^2 + \left(600 - \frac{600m}{m+1}\right)^2} = \frac{600\sqrt{2}}{1+m}$$

metres to reach I and therefore takes $\frac{120\sqrt{2}}{1+m}$ seconds.
We want all times where Sam crosses I before the train. Therefore we solve

$$150\sqrt{\frac{1+m^2}{(1+m)^2}} < \frac{120\sqrt{2}}{1+m}, m > 0$$

$$\implies 0 < m < \frac{\sqrt{7}}{5}$$

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