CONTOUREDUCATION

VCE Mathematical Methods ¾
Coordinate Geometry Exam Skills [0.6]
Workshop

Error Logbook:

Mistake/Misconception #1		Mistake/Misconception #2	
Question #:	Page #:	Question #:	Page #:
Notes:		Notes:	
Mistake/Misconception #3		Mistake/Misconception #4	
Question #:	Page #:	Question #:	Page #:
Notes:		Notes:	

CONTOUREDUCATION

VCE Methods ¾ Questions? Message +61 440 138 726

Section A: Recap

Midpoint
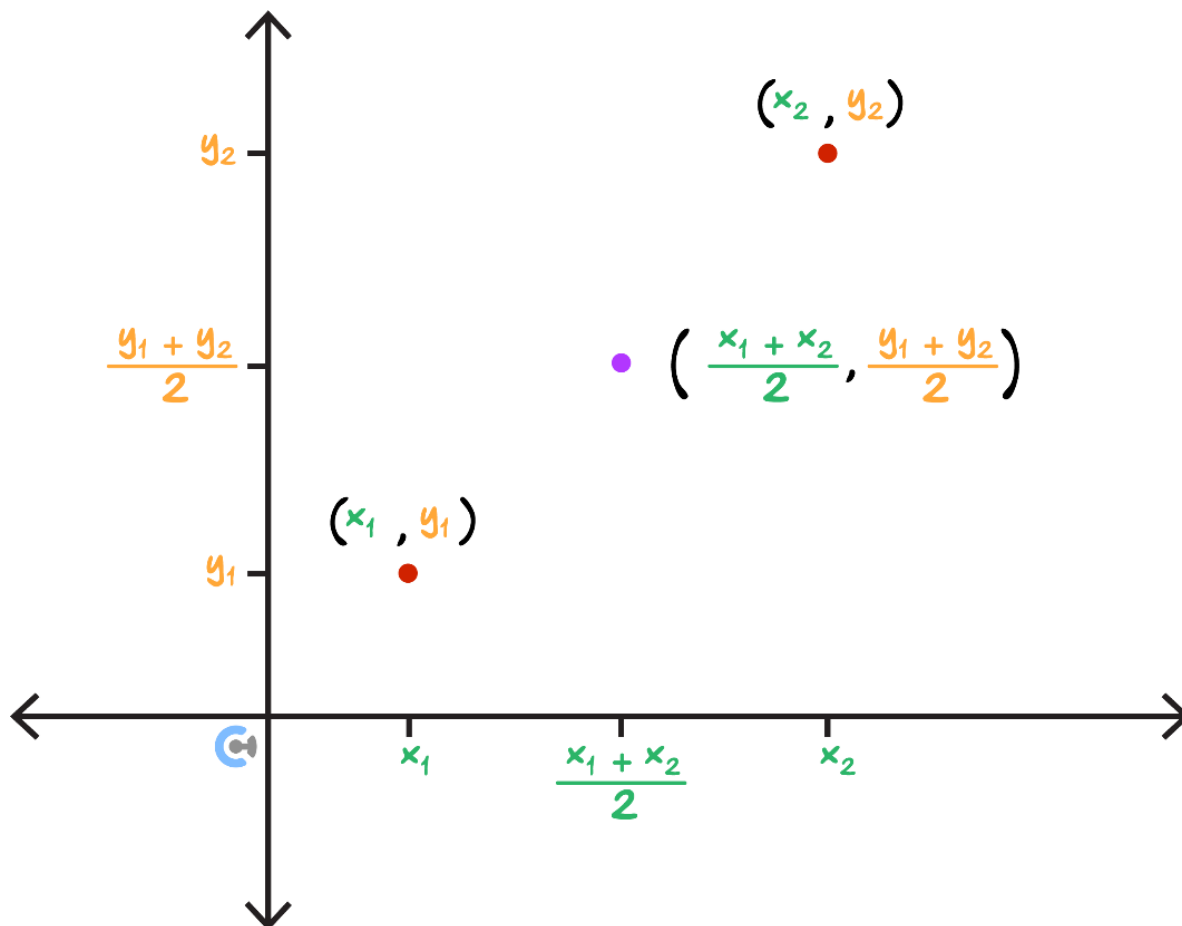
➤ The midpoint, M, of two points A and B is simply the point halfway between A and B.

$$M(x_m, y_m) = \left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}\right)$$

➤ The midpoint can be found by taking the average of the x-coordinate and y-coordinate of the two points.
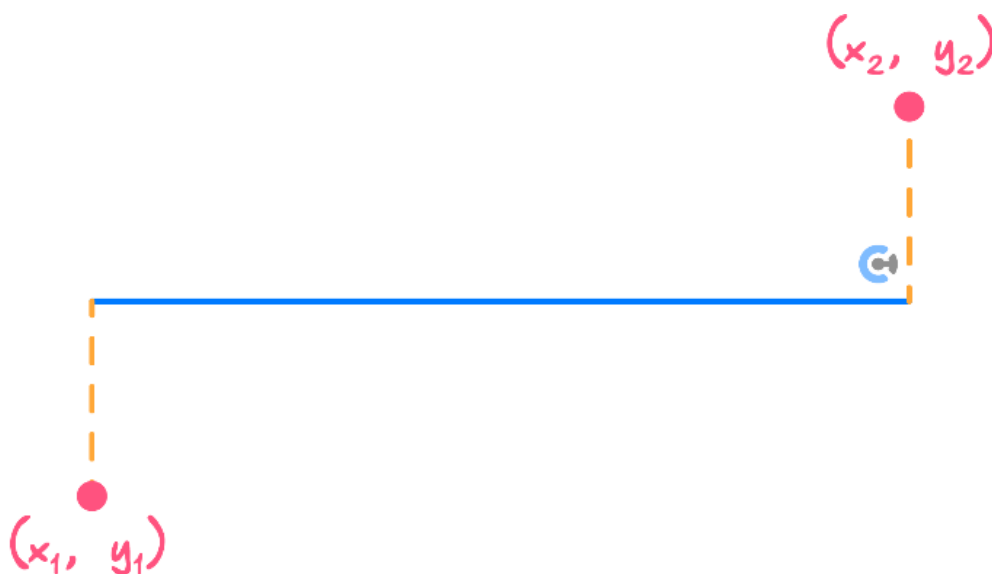
Distance Between Two Points

➤ The distance between two points (x_1, x_2) and (y_1, y_2) can be found using Pythagoras' theorem:
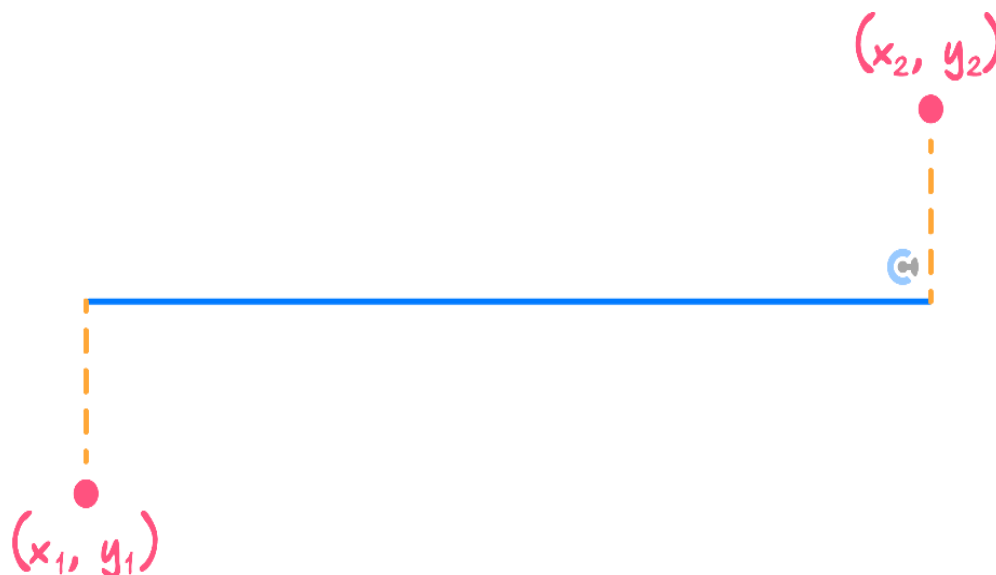
$$Distance = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

CONTOUREDUCATION

VCE Methods ¾ Questions? Message +61 440 138 726

Horizontal Distance

$$Horizontal\ Distance = x_2 - x_1, where\ x_2 > x_1$$

➤ Find the difference between their x-values.

Vertical Distance

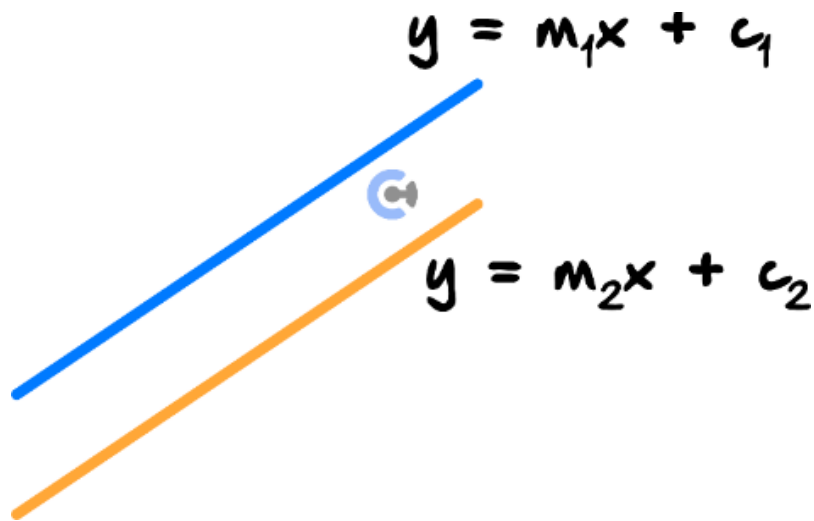$$Vertical\ Distance = y_2 - y_1, where\ y_2 > y_1$$

➤ Find the difference between their y-values.

CONTOUREDUCATION

VCE Methods ¾ Questions? Message +61 440 138 726

Parallel Lines

$$y = m_1 x + c_1$$

$$y = m_2 x + c_2$$

➤ Parallel lines have the same gradient.

$$m_1 = m_2$$

Perpendicular Lines

$$y = m_1 x + c_1$$

$$y = m_2 x + c_2$$
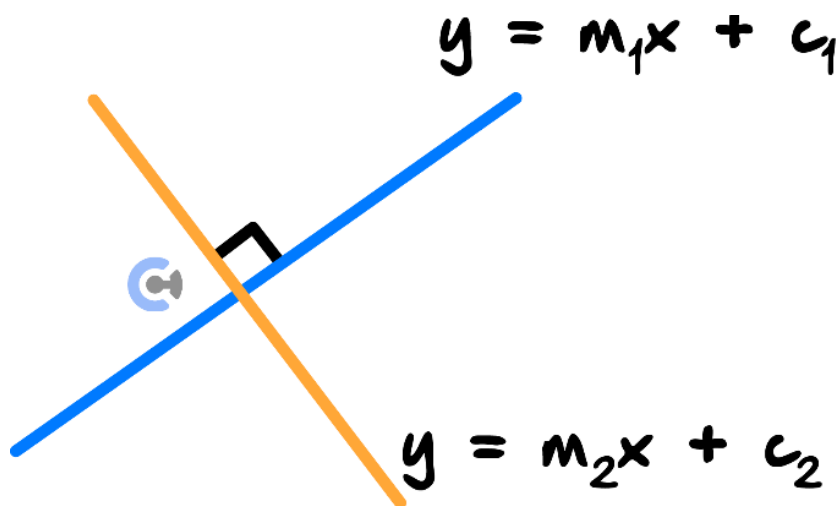
➤ A line that is perpendicular to another line has a gradient which is the negative reciprocal of the gradient of the first line.

$$m_\perp = -\frac{1}{m}$$

Angle Between a Line and the x-axis
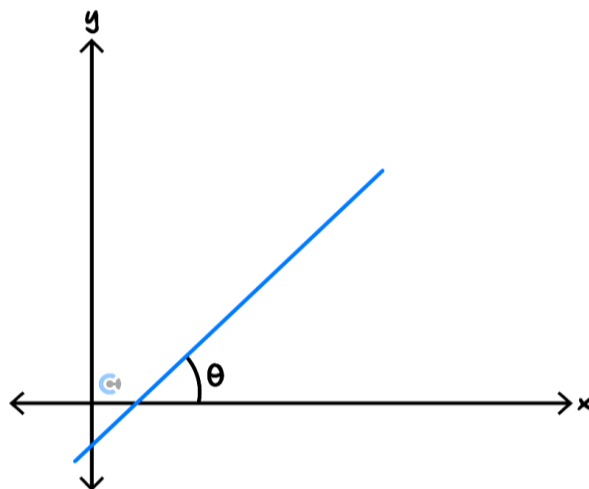
➤ The angle between a line and the **positive direction of the x-axis** (anticlockwise) is given by:

$$\tan(\theta) = m$$

Acute Angle Between Two Lines

$$\theta = |\tan^{-1}(m_1) - \tan^{-1}(m_2)|$$

➤ Alternatively:

$$\tan(\theta) = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

CONTOUREDUCATION

VCE Methods ¾ Questions? Message +61 440 138 726

Simultaneous Linear Equations

➤ Elimination Method.

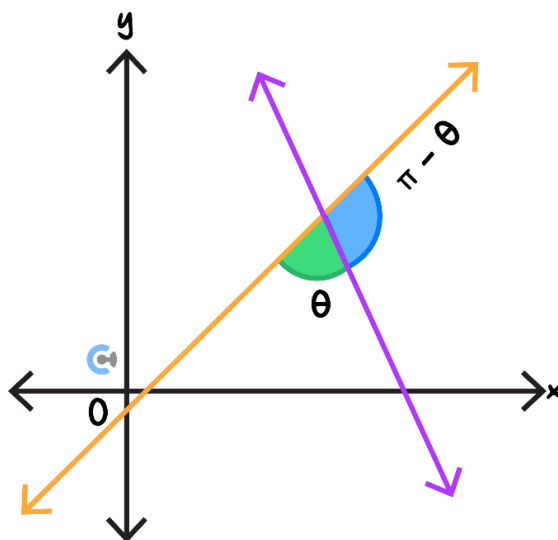➤ Substitution Method.

General Solutions of Simultaneous Linear Equations

➤ Two linear equations are either:

  ↻ The same line, expressed in a different form. In this case, they have infinite solutions.

  ↻ Unique lines which are **parallel**. In this case, they have NO solutions.

  ↻ Unique lines which are not parallel. In this case, they have exactly one solution.

Solving Systems of Linear Equations with Parameters

➤ Occurs when solving for three variables with two equations. We simply,

$$Let\ x = k,\ or$$

$$Let\ y = k,\ or$$

$$Let\ z = k$$

➤ And solve simultaneously.

Space for Personal Notes

CONTOUREDUCATION

VCE Methods ¾ Questions? Message +61 440 138 726

Addition of Ordinates

➤ **Definition:**

- Technique used to graph the sum/difference of two functions.

$$\text{E.g., } y = \sin(x) + \sqrt{x}$$
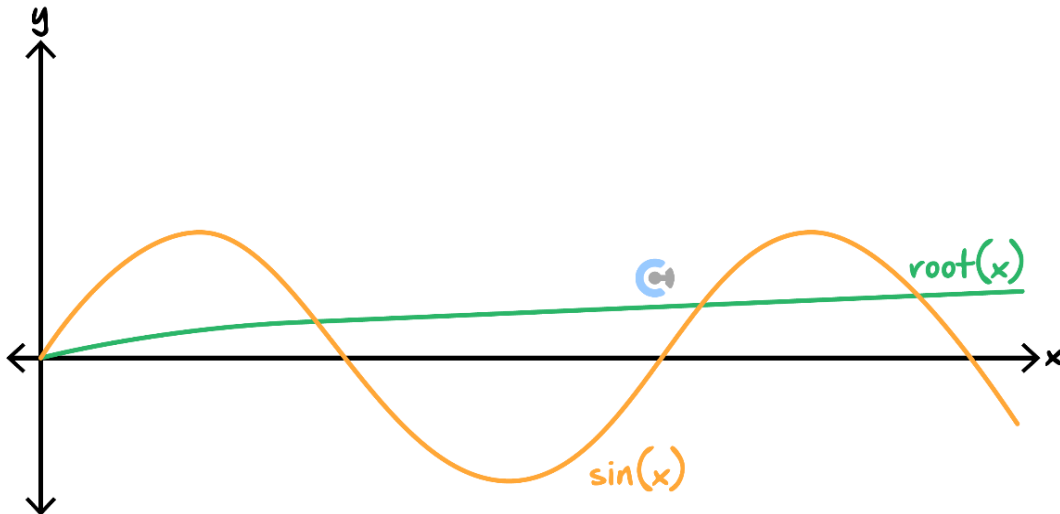
➤ The addition of ordinates involves adding the y-values of two functions.

➤ **Steps to sketching $f(x) + g(x)$:**

1. Sketch $f(x)$ and $g(x)$ on the same axes.

2. Plot points for $f(x) + g(x)$ by adding the **y-values** of $f(x)$ and $g(x)$.

   ➤ At x-intercepts, the sum equals to the other function.

   ➤ At intersections, the sum equals to the y-value.

   ➤ When functions are equidistant from x-axis, the sum equals to 0.

3. Join the plotted points.

Space for Personal Notes

CONTOUREDUCATION

VCE Methods ¾ Questions? Message +61 440 138 726

Reflection of a Point Around a Vertical/Horizontal Line

Definition

$\bullet \ (x', y')$

——————————————— $y = 3$

$\bullet \ (x, y)$

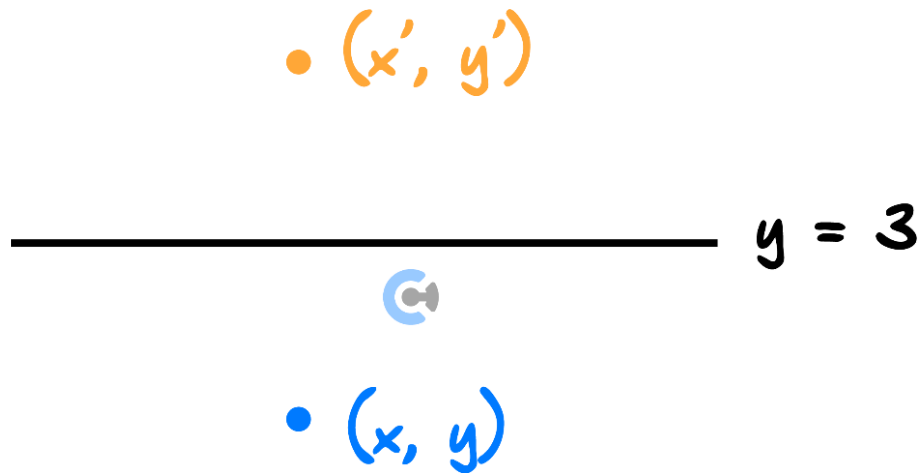➤ Midpoint must be on the line of reflection.

Finding the Reflection of a Point In a Line

Definition

A
$\bullet$

$y = mx + c$

$\bullet B$
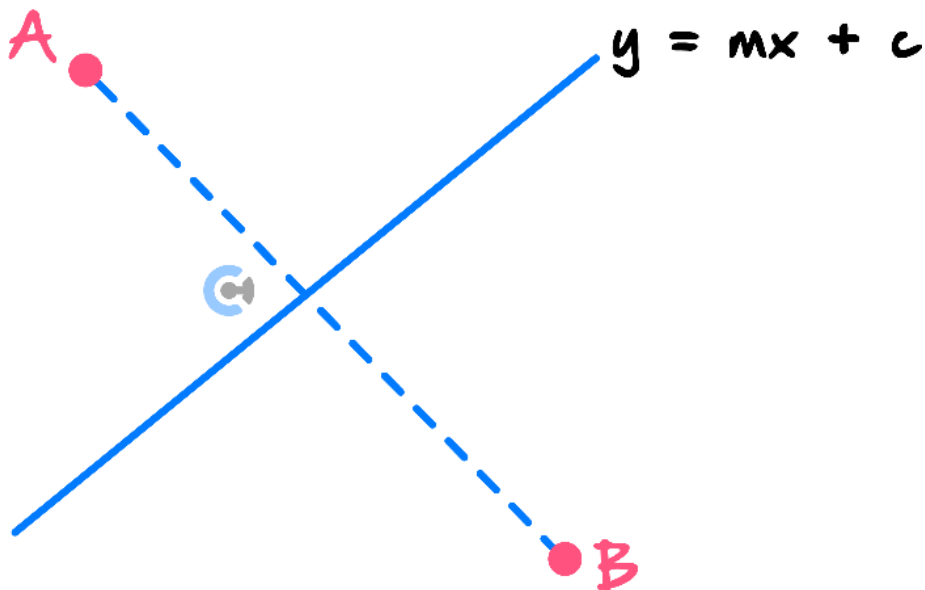
➤ Steps:

1. Find the perpendicular line passing through the point.

2. Find the intersection between the original line and the perpendicular line.

3. Find the reflected point (x, y) by treating the intersection from **step (2)** as the midpoint between the original and reflected point.

CONTOUREDUCATION

VCE Methods ¾ Questions? Message +61 440 138 726

Section B: Warm-Up

INSTRUCTION: 5 Minutes Writing.

Question 1

Consider the line segment AB with coordinates $A(1, 2)$ and $B(5, 6)$.

a. Find the midpoint of AB.

_____

_____

_____

b. Find the equation of the line segment AB.

_____

_____

_____

_____

c. Show that the equation of the perpendicular bisector of AB is $y = -x + 7$.

_____

_____

_____

_____

_____

d. The point $A(2, 5)$ is reflected in the line $y = x + 1$ to become the point A'. Find the coordinates of A'.

_____

_____

_____

_____

_____

_____

Space for Personal Notes

Section C: Exam 1 Questions (28 Marks)

Question 2 (5 marks)

Consider the points $A(2, 3)$ and $B(5, 1)$.

a. Find the distance between points A and B. (1 mark)

_____

_____

_____

_____

b. The distance between point A and a point $C(5, k)$ is 4. Find the possible value(s) of k. (2 marks)

_____

_____

_____

_____

_____

c. Find the coordinates of the point A' obtained by reflecting A in the line $x = -1$ and then in the line $y = 2$. (2 marks)

_____

_____

_____

_____

Space for Personal Notes

CONTOUREDUCATION

VCE Methods ¾ Questions? Message +61 440 138 726

Question 3 (5 marks)

Consider the linear equations:

$$(k + 2)x + 4y = 6$$

$$3x + 2(k - 3)y = k - 1$$

a. For what value(s) of k, will the system have a unique solution? (2 marks)

_____

_____

_____

_____

_____

_____

_____

b. For what value of k, will the system have infinitely many solutions? (2 marks)

_____

_____

_____

c. For what value of k, will the system have no solutions? (1 mark)

_____

_____

_____

CONTOUREDUCATION

VCE Methods ¾ Questions? Message +61 440 138 726

Question 4 (3 marks)

It is known that the acute angle made by the lines $y = 3x + 3$ and $y = mx - 4$ is 45°. Find the possible value(s) of m.

_____

_____

_____

_____

_____

_____

_____

_____

Space for Personal Notes

CONTOUREDUCATION

VCE Methods ¾ Questions? Message +61 440 138 726

Question 5 (2 marks)

The graphs of the functions f and g are sketched on the axes below. Sketch the graph of $f + g$ on the same set of axes and label its axis intercepts with coordinates.
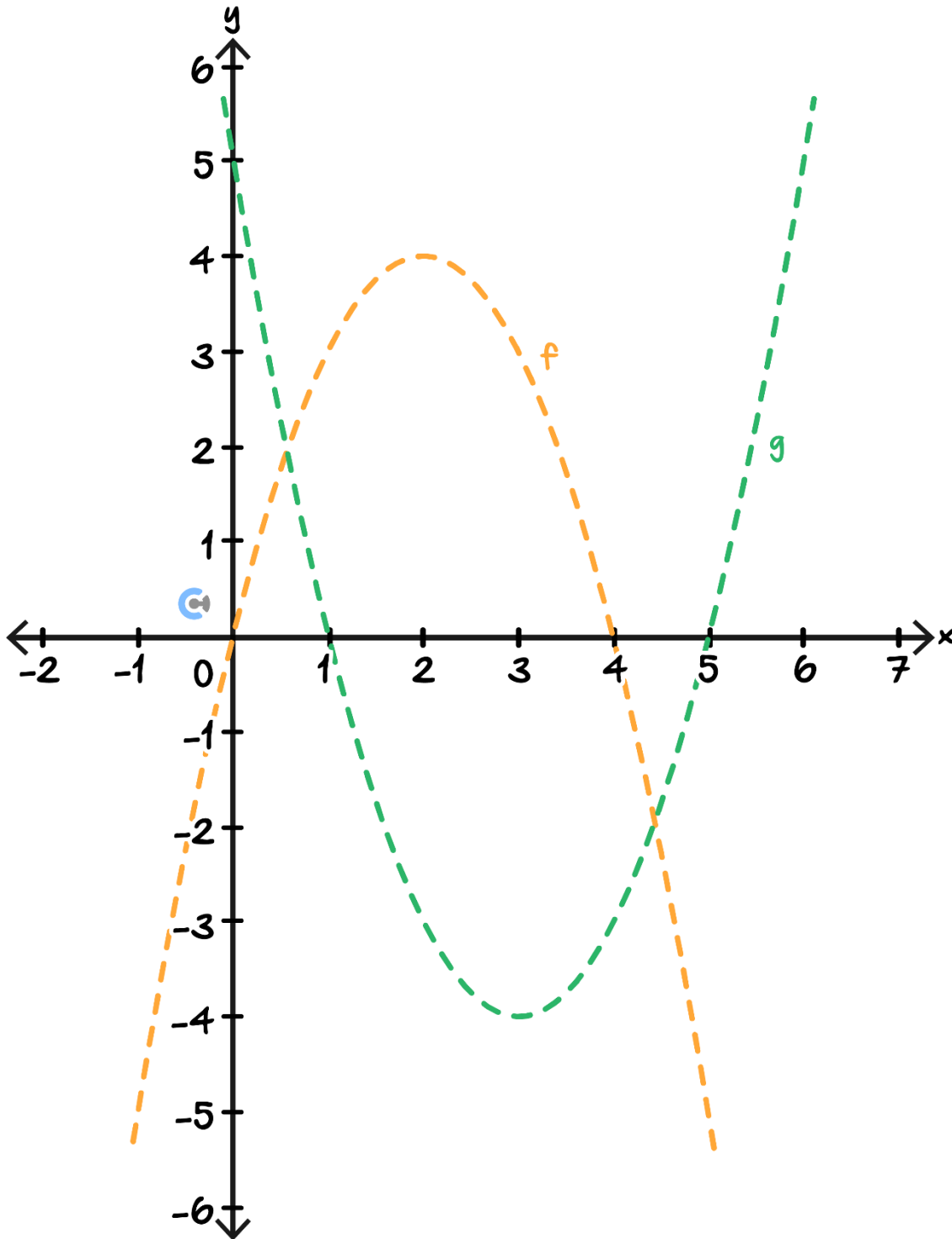
Question 6 (8 marks)

James is leaving the gym located at the point $J(3, 4)$. He decides to go for a walk to warm down.

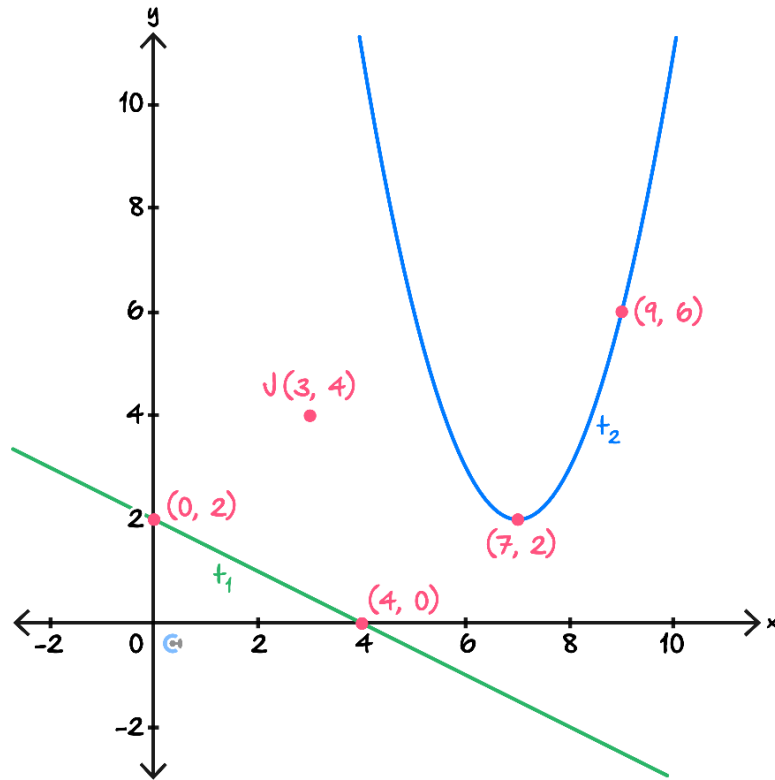James's location and two walking trails t_1 and t_2 are shown in the diagram below. The walking trail t_1 is modelled by a linear function and the trail t_2 is modelled by a quadratic function.

a. Show that t_1 can be modelled by the line $y = -\frac{1}{2}x + 2$. (1 mark)

_____

_____

_____

b. Show that t_2 can be modelled by the quadratic $y = (x - 7)^2 + 2$. (1 mark)

_____

_____

_____

_____

CONTOUREDUCATION

VCE Methods ¾ Questions? Message +61 440 138 726

c. Find the distance from J to t_2 when $x = 5$. (2 marks)

_____

_____

_____

_____

d. Find the shortest horizontal distance between t_1 and t_2 when $y = 2$. (1 mark)

_____

_____

_____

e.

    i. Find the equation of the line perpendicular to t_1 that passes through $J(3, 4)$. (1 mark)

_____

_____

_____

_____

_____

    ii. Hence, find the shortest distance between J and t_1. (2 marks)

_____

_____

_____

_____

CONTOUREDUCATION

VCE Methods ¾ Questions? Message +61 440 138 726

Question 7 (5 marks)

A gardener is designing a symmetrical flower bed. One pair of flowers are located at the points $F_1(2, 2)$ and $F_2(10, 6)$ respectively.

a. Find the equation of the line that F_1 is reflected in to map to F_2. (2 marks)

_____

_____

_____

_____

_____

_____

b. The gardener plants another pair of flowers. He plants the first flower at the point $A(2, 6)$. The location of the second flower is obtained by reflecting the point A in the line $y = 2x - 8$. Find the coordinates of where the second flower is planted. (3 marks)

_____

_____

_____

_____

_____

_____

_____

_____

_____

Space for Personal Notes

CONTOUR**EDUCATION**

VCE Methods ¾ Questions? Message +61 440 138 726

Section D: Tech Active Exam Skills

Calculator Commands: Simultaneous Equations on CAS

System of Linear Equations

Example:

The simultaneous linear equations $ax - 3y = 5$ and $3x - ay = 8 - a$ have no solution for:

A. $a = 3$

B. $a = -3$

C. Both $a = 3$ and $a = -3$.

D. $a \in R\backslash\{3\}$

E. $a \in R\backslash[-3, 3]$

$$\text{system\_solve}(a \cdot x - 3 \cdot y = 5, 3 \cdot x - a \cdot y = 8 - a, a)$$

▸ Solving: $\begin{bmatrix} a \cdot x - 3 \cdot y = 5 \\ 3 \cdot x - a \cdot y = 8 - a \end{bmatrix}$

▸ Unique Solution: $a \ne -3$ and $a \ne 3$

▸ No Solutions: $a = -3$

▸ Infinite Solutions: $a = 3$

Overview:

This program takes two linear equations and a parameter and finds the parameter values for the system to obtain a unique solution, no solution, or infinite solutions.

Input:

$$\text{system\_solve}(< equation\ 1 >, < equation\ 2 >, < parameter >)$$

Other Notes:

The program can only handle one parameter.

➤ Or menu $- 3 - 7$.

CONTOUREDUCATION

VCE Methods ¾ Questions? Message +61 440 138 726

➤ UDF line functions:

Normal Line

$normal\_line\left(x^3-x,x,2\right)$

▶ Derivative: $3 \cdot x^2 - 1$
▶ Gradient: 11
▶ Perpendicular Gradient: $\dfrac{-1}{11}$
▶ Passes Through: $\begin{bmatrix} 2 & 6 \end{bmatrix}$
▶ x –Intercept: $\begin{bmatrix} 68 & 0 \end{bmatrix}$
▶ Vertical Intercept: $\begin{bmatrix} 0 & \dfrac{68}{11} \end{bmatrix}$
▶ Normal Line:
$$\dfrac{68}{11} - \dfrac{x}{11}$$

Overview:

This program will find all the necessary information related to a normal line at a point on a function, which includes:

➤ The derivative.

➤ The gradient and perpendicular gradient.

➤ The point on the function the normal line passes through.

➤ The axis intercepts of the normal line.

➤ The equation of the normal line.

Input:

$$normal\_line(< function >, < variable >, < x\,point >)$$

Tangent Line

$tangent\_line\left(x^3-x,x,2\right)$

▶ Derivative: $3 \cdot x^2 - 1$
▶ Gradient: 11
▶ Passes Through: $\begin{bmatrix} 2 & 6 \end{bmatrix}$
▶ x –Intercept: $\begin{bmatrix} \dfrac{16}{11} & 0 \end{bmatrix}$
▶ Vertical Intercept: $\begin{bmatrix} 0 & -16 \end{bmatrix}$
▶ Tangent Line:
$11 \cdot x - 16$

Overview:

This program will find all the necessary information related to a tangent line at a point on a function, which includes:

➤ The derivative.

➤ The gradient of the tangent line.

➤ The point on the function the tangent line passes through.

➤ The axis intercepts of the tangent line.

➤ The equation of the tangent line.

Input:

$$tangent\_line(< function >, < variable >, < x\,point >)$$

CONTOUR**EDUCATION**

VCE Methods ¾ Questions? Message +61 440 138 726

Calculator Commands: Finding the Angle Between Two Lines

➤ The angle between two lines with gradients m_1 and m_2 respectively is:

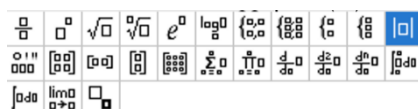$$\theta = |\tan^{-1}(m_1) - \tan^{-1}(m_2)|$$

➤ **Mathematica**

⟲ Use the Abs[] function.

```
In[126]:= Abs[ArcTan[2] - ArcTan[1]] / Degree // N
Out[126]= 18.4349
```
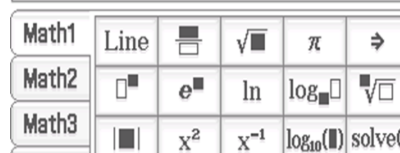
➤ **TI-Nspire**

⟲ Find the modulus sign.

$|\tan^{-1}(2) - \tan^{-1}(1)|$ 18.4349

➤ **Casio Classpad**

⟲ Modulus sign under Math1.

$|\tan^{-1}(2) - \tan^{-1}(1)|$
                    18.43494882

Calculator Commands: Finding the Gradients of Lines Given the Angle They Make

➤ If we know the angle and one of the gradients m_1 or m_2 then, we can find the other gradient by solving,

$$\tan(\theta) = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

➤ E.g., find the gradient of the line that makes an angle of $60°$ with $y = -x$.

➤ **Mathematica**

```
Solve[Tan[60 Degree] == Abs[(m1 + 1)/(1 - m1)], m1]
```

⋯ Solve: Inverse functions are being used by Solve, so s
Reduce for complete solution information. ⓘ

$\{\{m1 \to 2 - \sqrt{3}\}, \{m1 \to 2 + \sqrt{3}\}\}$

➤ **TI-Nspire**

⟲ Find the modulus sign.

$\text{solve}\left(\tan(60) = \left|\frac{m1+1}{1-m1}\right|, m1\right)$
$m1 = -(\sqrt{3} - 2)$ or $m1 = \sqrt{3} + 2$

➤ **Casio Classpad**

⟲ Modulus sign under Math1.

$\text{solve}\left(\tan(60) = \left|\frac{m1+1}{1-m1}\right|, m1\right)$
$\left\{m1 = \frac{\sqrt{3}-1}{\sqrt{3}+1}, m1 = \frac{\sqrt{3}+1}{\sqrt{3}-1}\right\}$

CONTOUREDUCATION

VCE Methods ¾ Questions? Message +61 440 138 726

Section E: Exam 2 Questions (25 Marks)

Question 8 (1 mark)

Consider the following pair of simultaneous equations:

$$x + ay = 3$$

$$(a - 3)x - 2y = 1 - 2a$$

For what value(s) of a, do the equations have infinitely many solutions?

A. $a = 1$

B. $a = 2$

C. $a = 1, 2$

D. $a \in \mathbb{R} \backslash \{1, 2\}$

Question 9 (1 mark)

The distance between the point $(2, 5)$ and the line $y = 5 - x$ when $x = a$ is $\sqrt{2}$. The possible value(s) of a are:

A. $0, 2$

B. $0, 3$

C. 1 only.

D. $2, 3$

Question 10 (1 mark)

The point $(1, 3)$ is reflected about a line $y = mx + c$ to become the point $(6, 4)$. The values of m and c are:

A. $m = 5, c = -16$

B. $m = 5, c = 22$

C. $m = -5, c = 21$

D. $m = -5, c = 16$

Question 11 (1 mark)

When $x = a$ the vertical distance between the functions $f(x) = (x - 2)^2 + 3$ and $g(x) = x + 2$ is 1. All possible values of a are:

A. $a = 2, 3$

B. $a = 3, 4$

C. $a = 1, 2, 3, 4$

D. $a = 2, 3, 4$

Question 12 (1 mark)

The **obtuse** angle made by the lines $y = 3x - 1$ and $y = -\frac{1}{2}x + 2$ is closest to:

A. 98°

B. 108°
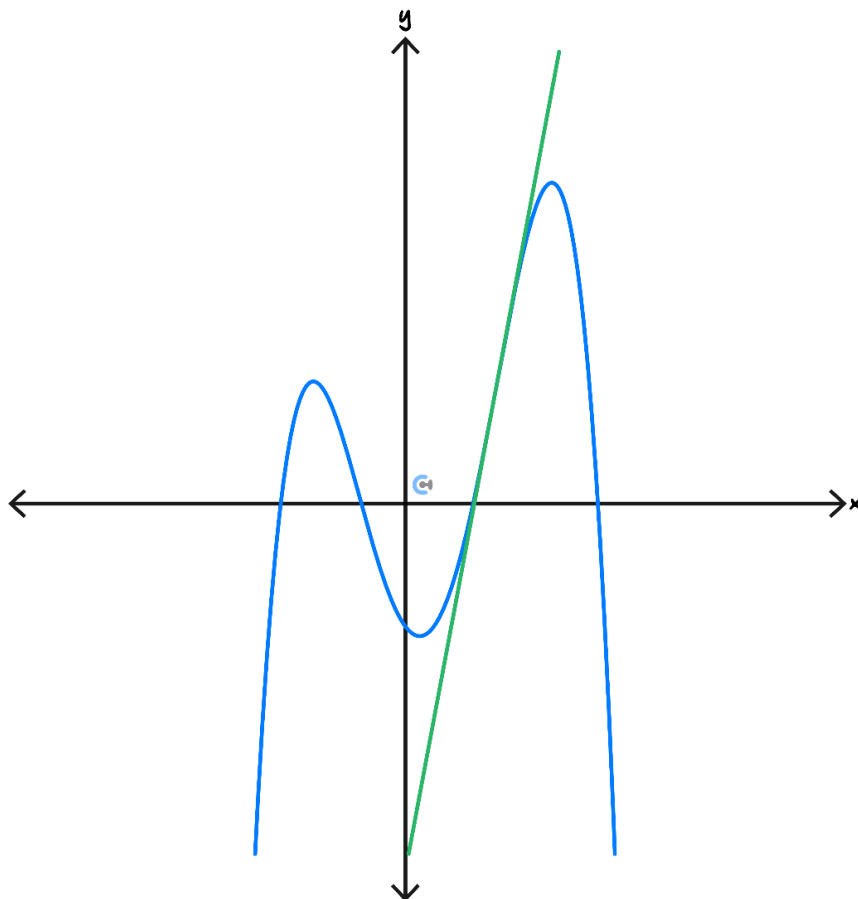
C. 118°

D. 158°

Question 13 (1 mark)

The shortest distance between the lines $y = 3x + 2$ and $y = 3x + 12$ is:

A. 3

B. $\sqrt{10}$

C. $\sqrt{13}$

D. $\sqrt{15}$

Space for Personal Notes

Question 14 (9 marks)

Consider the function $f(x) = 2 - 2(x + 1)^2(x - 2)(x - 1)$. The graph of $y = f(x)$ and the tangent line to f when $x = 1$ is shown below.

a. The tangent line has a y-intercept of $(0, -6)$. Find the equation of this tangent. (2 marks)

_____

_____

_____

_____

b. State the angle that this tangent with the positive x-axis. Give your answer correct to the nearest degree. (1 mark)

_____

_____

CONTOUREDUCATION

VCE Methods ¾ Questions? Message +61 440 138 726

c. Find the equation of the line that is perpendicular to this tangent. (2 marks)

_____

_____

_____

_____

d. Another tangent line, with the same gradient as the other tangent, can be drawn to the graph when $x = a$ and $a < 0$. Use the fact that the shortest distance between the two tangent lines $\frac{2187}{128\sqrt{65}}$ to find the value of a. (4 marks)

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

Space for Personal Notes

Question 15 (10 marks)

City planners are using a Cartesian plane to model locations for their new park.

It has been decided that the park will be in the shape of an equilateral triangle.

The vertex $A(2, 2)$ is the vertex of the triangle that has the smallest x- and y-coordinates.

The two sides of the triangle that are adjacent to A are modelled by lines that are inverse functions of each other.

a. Show that two sides of the triangle can be modelled by the lines. (3 marks)

$$y = \left(2 - \sqrt{3}\right)x + 2\sqrt{3} - 2 \text{ and,}$$

$$y = \left(2 + \sqrt{3}\right)x - 2 - 2\sqrt{3}$$

_____

_____

_____

_____

_____

_____

_____

_____

_____

CONTOUREDUCATION

VCE Methods ¾ Questions? Message +61 440 138 726

b.

    i. If the triangle has another vertex at the point $B\left(5, 8 + 3\sqrt{3}\right)$, find the location of the final vertex C and the equation of the line that passes through B and C. (2 marks)

_____

_____

_____

_____

_____

_____

_____

_____

_____

    ii. Hence, find the area of this triangle ABC. (2 marks)

    Note: The formula $\frac{1}{2}bc\sin(A)$ may be helpful.

_____

_____

_____

_____

_____

c. Find the locations of the other two vertices if the triangle has an area of 100. Give your answer correct to three decimal places. (3 marks)

_____

_____

_____

_____

_____

_____

_____

_____

_____

Space for Personal Notes

CONTOUREDUCATION

VCE Methods ¾ Questions? Message +61 440 138 726

Section F: Extension Exam 1 (15 Marks)

Question 16 (6 marks)

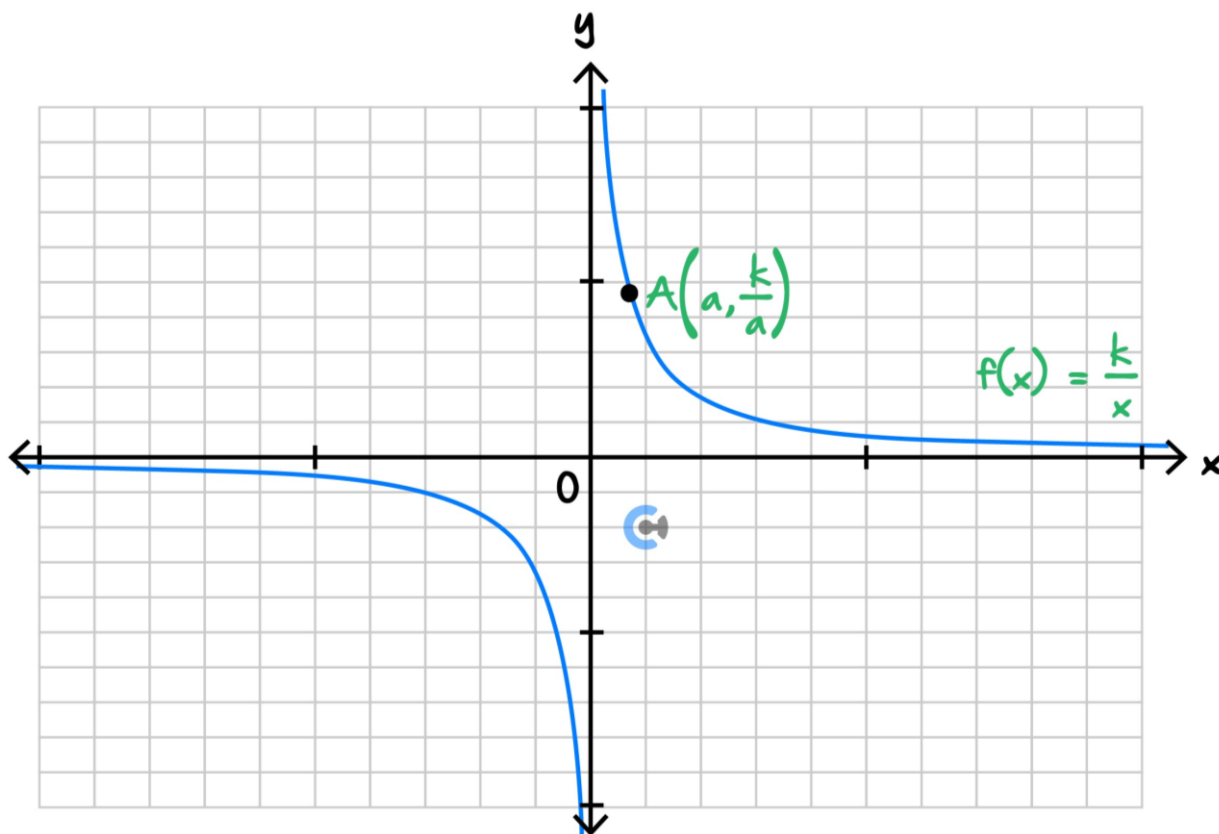**a.** Reflect the point (a, b) in the line $y = x - c$. (2 marks)

_____

_____

_____

_____

b. Find the rule for the transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ that reflects a point (a, b) in the line $y = \frac{1}{2}x + 6$. (4 marks)

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

Space for Personal Notes

Question 17 (6 marks)

The graph of the general hyperbola $f(x) = \frac{k}{x}$, where $k > 0$, is shown below. The point A is located on the graph with coordinates $\left(a, \frac{k}{a}\right)$, where $a > 0$.

Point A is reflected around two axes to point A' where their mid point is located at $(0, 0)$.

a. State the coordinates of A' and plot its coordinates relative to A on the graph above. (1 mark)

_____

_____

_____

_____

b. Show that the distance between the points A and A' can be expressed as $\frac{2\sqrt{a^4+k^2}}{a}$. (2 marks)

_____

_____

_____

_____

_____

_____

_____

_____

_____

c. Find the minimum distance between the two branches. Give your answer in terms of k. (3 marks)

_____

_____

_____

_____

_____

_____

_____

_____

_____

Space for Personal Notes

CONTOUREDUCATION

VCE Methods ¾ Questions? Message +61 440 138 726

Question 18 (3 marks)

The area of the triangle enclosed by the lines $y = 2x - 2$, $y = -\frac{1}{2}x + c$ and the x-axis is 5 square units. Find the possible values of c.

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

Space for Personal Notes

CONTOUREDUCATION

VCE Methods ¾ Questions? Message +61 440 138 726

Section G: Extension Exam 2 (15 Marks)

Question 19 (1 mark)

The lines $y = -ax + 6$ and $y = bx - 2$ make an angle of $45°$ when they intersect. The relationship between a and b is:

A. $a = \dfrac{b+1}{b-1}$

B. $a = \dfrac{1-b}{1+b}$ or $a = \dfrac{b+1}{b-1}$.

C. $a = \dfrac{b-1}{1-b}$

D. $a = \dfrac{b+1}{1-b}$ or $a = \dfrac{b-1}{b+1}$.

Question 20 (1 mark)

The transformation that describes a point being reflected in the line $y = -2x + 3$ is:

A. $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x,y) = \left(\dfrac{12-3x-2y}{5}, \dfrac{6-2x+3y}{5}\right)$

B. $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x,y) = \left(\dfrac{12-3x-4y}{5}, \dfrac{6-4x+3y}{5}\right)$

C. $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x,y) = \left(\dfrac{24-3x-2y}{10}, \dfrac{12-2x+3y}{10}\right)$

D. $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x,y) = \left(\dfrac{12-3x-2y}{10}, \dfrac{6-2x+3y}{10}\right)$

Question 21 (1 mark)

The area of the triangle formed by the lines $y = \dfrac{4}{5}x - \dfrac{8}{5}$, $y = 32 - 4x$ and the x-axis is:

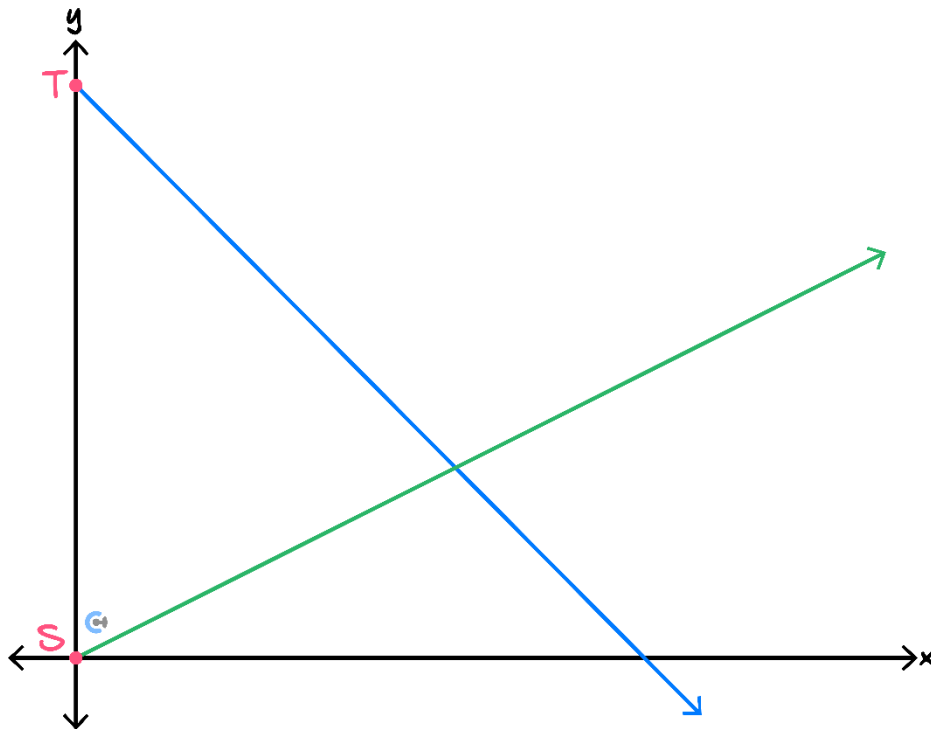A. 8

B. 10

C. 12

D. 14

Question 22 (12 marks)

Sam is running through the countryside.

He starts his run from the origin and runs in a straight line. The path he takes sees him cross a set of train tracks somewhere in the first quadrant.

The train tracks are modelled by the line $y = 600 - x$.

All measurements are in metres and the diagram below shows the situation.

a. Find the shortest distance that Sam can run to reach the train tracks. (2 marks)

_____

_____

_____

_____

_____

b. Sam runs a total of $400\sqrt{26}$ metres before he stops. If he runs three times further after crossing the train tracks than before crossing, find the possible equations of the line he runs along. (2 marks)

_____

_____

_____

_____

_____

_____

_____

c. Sam runs along a line that makes an angle of θ with the positive x-axis. Find the value of θ given that Sam's line makes an angle of $70°$ with the train tracks and that he crosses the train tracks at a y-value less than 300. Give an exact answer in degrees. (2 marks)

_____

_____

_____

_____

_____

The front of a train is located at $(0, 600)$. Sam starts his run from the origin at the same time that the train starts moving at a constant speed. Sam runs at a constant speed of $4\ m/s$.

d. Sam runs along the line that makes an angle of $30°$ with the positive x-axis. Find the speed that the train must travel at for Sam and the front of the train to meet at the same point. Give your answer in m/s correct to two decimal places. (3 marks)

_____

_____

_____

_____

_____

_____

_____

e. It is now known that the train travels at a constant speed of $5\ m/s$. If Sam runs along the line $y = mx$, where $m > 0$. Find the values of m such that Sam crosses the train tracks before the train has reached his crossing point. (3 marks)

_____

_____

_____

_____

_____

Space for Personal Notes

CONTOUREDUCATION

VCE Mathematical Methods ¾

Free 1-on-1 Support

Be Sure to Make The Most of These (Free) Services!

➤ Experienced Contour tutors (45+ raw scores, 99+ ATARs).
➤ For fully enrolled Contour students with up-to-date fees.
➤ After school weekdays and all-day weekends.

1-on-1 Video Consults	Text-Based Support
➤ Book via bit.ly/contour-methods-consult-2025 (or QR code below). ➤ One active booking at a time (must attend before booking the next).	➤ Message +61 440 138 726 with questions. ➤ Save the contact as "Contour Methods".

Booking Link for Consults
bit.ly/contour-methods-consult-2025

Number for Text-Based Support
+61 440 138 726