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VCE Mathematical Methods ¾
Coordinate Geometry [0.5]
Workshop Solutions

Error Logbook:

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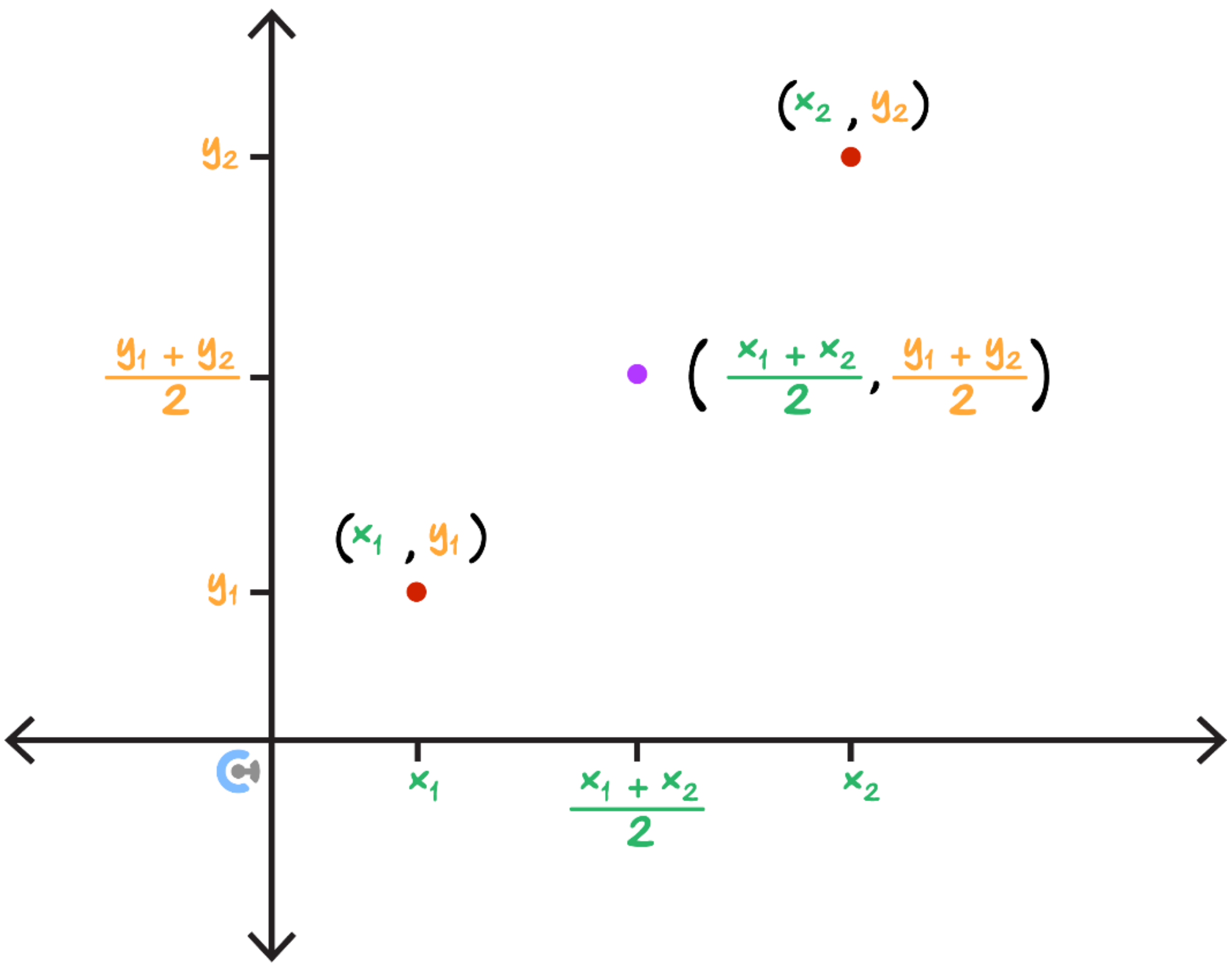
Section A: Recap

Midpoint

Definition



- The midpoint, M, of two points A and B is simply the point halfway between A and B.

$$M(x_m, y_m) = \left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}\right)$$

- The midpoint can be found by taking the average of the x-coordinate and y-coordinate of the two points.

Distance Between Two Points

Definition

- The distance between two points (x_1, x_2) and (y_1, y_2) can be found using Pythagoras' theorem:

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Horizontal Distance

Horizontal Distance $= x_2 - x_1$, where $x_2 > x_1$

- Find the difference between their x-values.

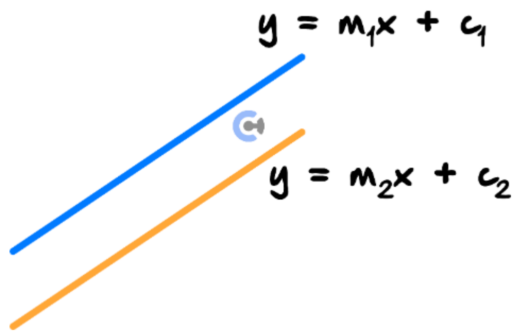
Vertical Distance

Vertical Distance $= y_2 - y_1$, where $y_2 > y_1$

- Find the difference between their y-values.

Definition

Parallel Lines

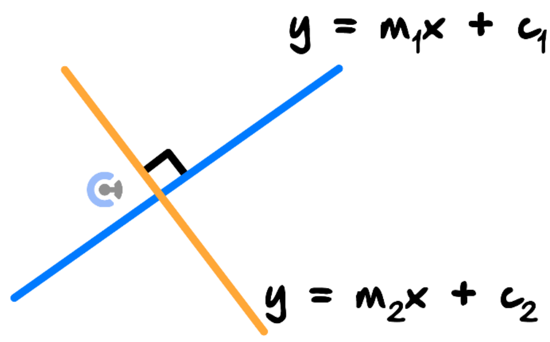


- Parallel lines have the same gradient.

$$m_1 = m_2$$

Definition

Perpendicular Lines



- A line that is perpendicular to another line has a gradient which is the negative reciprocal of the gradient of the first line.

$$m_{\perp} = -\frac{1}{m}$$

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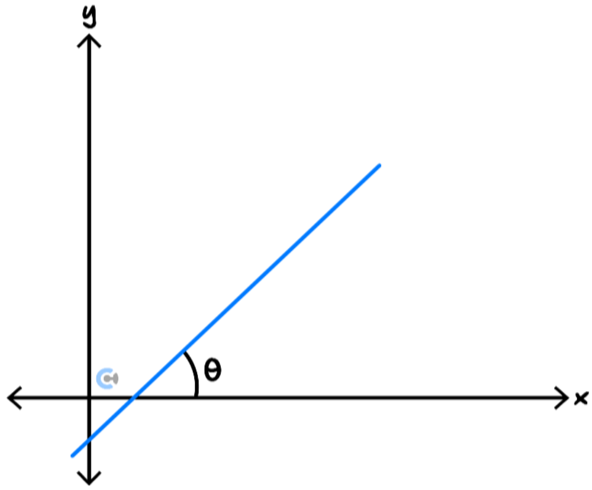
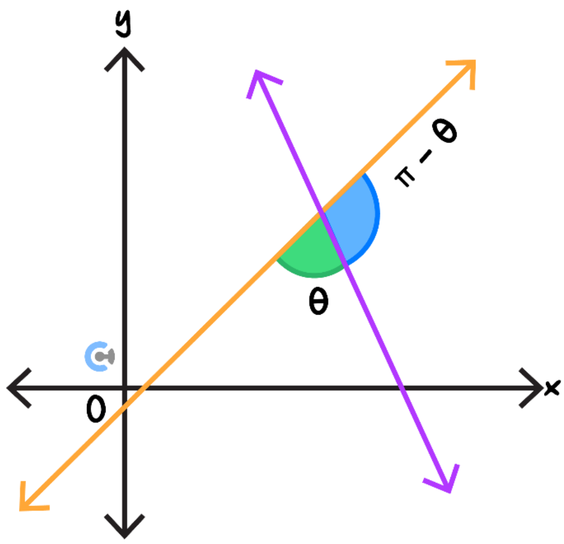
Angle Between a Line and the x-axis



- The angle between a line and the **positive direction of the x-axis** (anticlockwise) is given by:

$$\tan(\theta) = m$$

Acute Angle Between Two Lines



$$\theta = |\tan^{-1}(m_1) - \tan^{-1}(m_2)|$$

- **Alternatively:**

$$\tan(\theta) = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

Simultaneous Linear Equations

- Elimination Method
- Substitution Method

General Solutions of Simultaneous Linear Equations

- Two linear equations are either:
  - The same line, expressed in a different form. In this case, they have infinite solutions.
  - Unique lines that are **parallel**. In this case, they have NO solutions.
  - Unique lines which are not parallel. In this case, they have exactly one solution.

Solving Systems of Linear Equations with Parameters

- Occurs when solving for three variables with two equations. We simply,

$$\text{Let } x = k, \text{ or}$$

$$\text{Let } y = k, \text{ or}$$

$$\text{Let } z = k$$

- And solve simultaneously.

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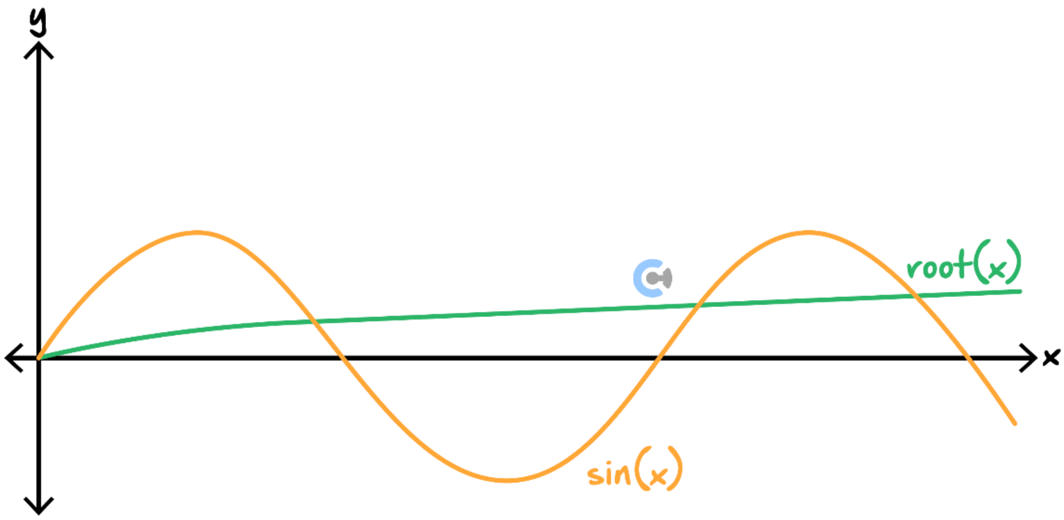
Definition

Addition of Ordinates

- **Definition:**
  - Technique used to graph the sum/difference of two functions.

$$\text{e.g. } y = \sin(x) + \sqrt{x}$$



- The addition of ordinates involves adding the y-values of two functions.
- **Steps to sketching $f(x) + g(x)$:**

1. Sketch $f(x)$ and $g(x)$ on the same axes.
2. Plot points for $f(x) + g(x)$ by adding the **y-values** of $f(x)$ and $g(x)$.
   - At x-intercepts, the sum equals to the other function.
   - At intersections, the sum equals to the y-value.
   - When functions are equidistant from x-axis, the sum equals to 0.
3. Join the plotted points.

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Section B: Warm Up

Question 1

a. Find the midpoint of the points $(2, 5)$ and $(-4, 3)$.

$(-1, 4)$

b. Find the distance between the points $(3, 5)$ and $(-4, 2)$.

$\sqrt{58}$

c. Find the equation of the line parallel to $y = 2x + 3$ that passes through $(1, 3)$.

$y = 2x + 1$

d. Find the perpendicular bisector of the line through the points $(2, 4)$ and $(6, 2)$.

$y = 2x - 5$

e. The distance between a point (a, b) that lies on the line $y = x - 2$ and the point $(2, 4)$ is $2\sqrt{2}$. Find the point (a, b).

$(a, b) = (4, 2)$

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Section C: Exam 1 (23 Marks)

Question 2 (5 marks)

Two linear equations can be written in the form:

$$2mx + (2m - 1)y = 2m + 3$$

$$-(2 - 3m)x + 3y = 4m - 1$$

Where m is a real constant.

a. Find the value(s) of m such that the graphs of the two lines have a unique solution. (2 marks)

$m_1 \neq m_2$

$\frac{2m}{2m-1} \neq \frac{-(2-3m)}{3}$

$6m \neq (3m-2)(2m-1)$

$6m \neq 6m^2 - 7m + 2$

$0 \neq 6m^2 - 13m + 2$

$0 \neq (6m-1)(m-2)$

$m \neq \frac{1}{6}, 2$

b. Find the value(s) of m such that the graphs of the two lines have no solution. (2 marks)

$m_1 = m_2$

$m = \frac{1}{6}, 2.$

$c_1 \neq c_2$

$\frac{2m-1}{2m+3} \neq \frac{3}{4m-1}$

$(2m-1)(4m-1) \neq 6m+9$

$8m^2 - 6m + 1 \neq 6m + 9$

$8m^2 - 12m - 8 \neq 0$

$2m^2 - 3m - 2 \neq 0$

$(2m+1)(m-2) \neq 0$

$m \neq -\frac{1}{2}, 2.$

$\therefore m = \frac{1}{6}$

c. Find the value(s) of m such that the graphs of the two lines have infinite solutions. (1 mark)

$m_1 = m_2$ $\qquad$ $c_1 = c_2$

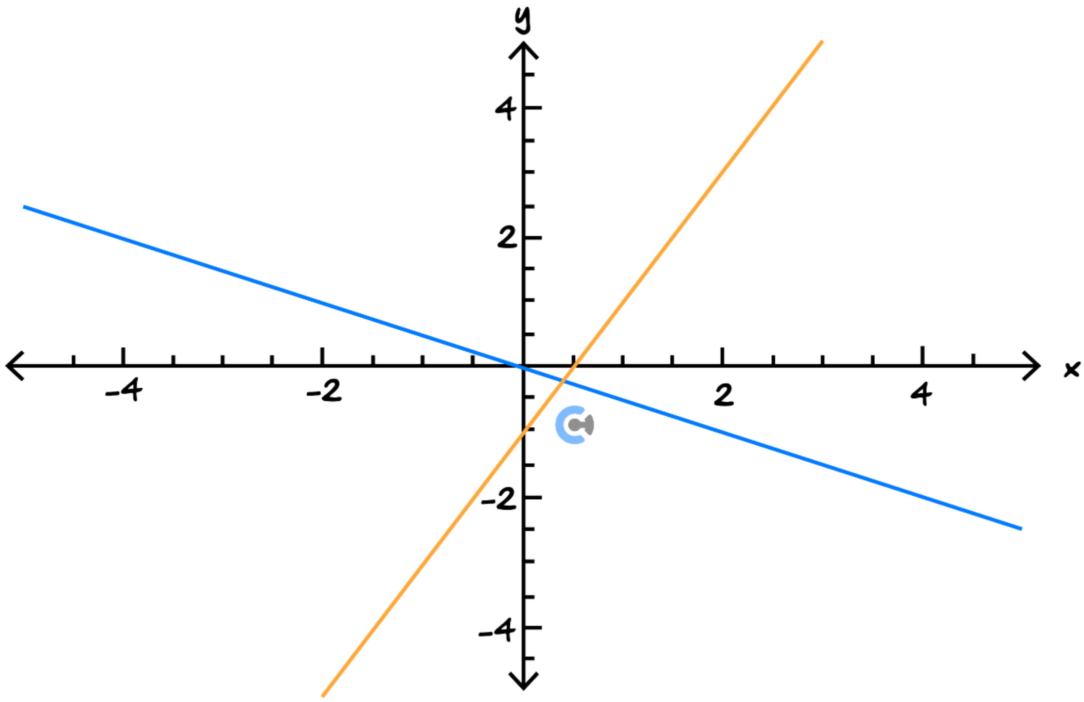
$m = \frac{1}{6}, 2$ $\qquad$ $m = -\frac{1}{2}, 2$

$m = 2$

Question 3 (6 marks)

Consider two lines:

$$f(x) = kx, \text{ where } k \text{ is a constant.}$$
$$g(x) = 2x - 1$$



a. Solve for the intersection between $f(x)$ and $g(x)$ in terms of k. (2 marks)

$\left(\frac{1}{2-k}, \frac{k}{2-k}\right)$

$kx = 2x - 1$

$(k-2)x = -1$

$x = \frac{-1}{k-2}$

$y = k \cdot \left(\frac{-1}{k-2}\right)$

$\therefore \left(\frac{-1}{k-2}, \frac{-k}{k-2}\right)$

b. Find the rule for $d(k)$, the distance between the intersection found in **part a.**, and the origin, in terms of k. (1 mark)

$$d(k) = \sqrt{\left(\frac{-1}{k-2}\right)^2 + \left(\frac{-k}{k-2}\right)^2}$$

c. Solve for the minimum value of $d(k)$ and the value of k. (3 marks)

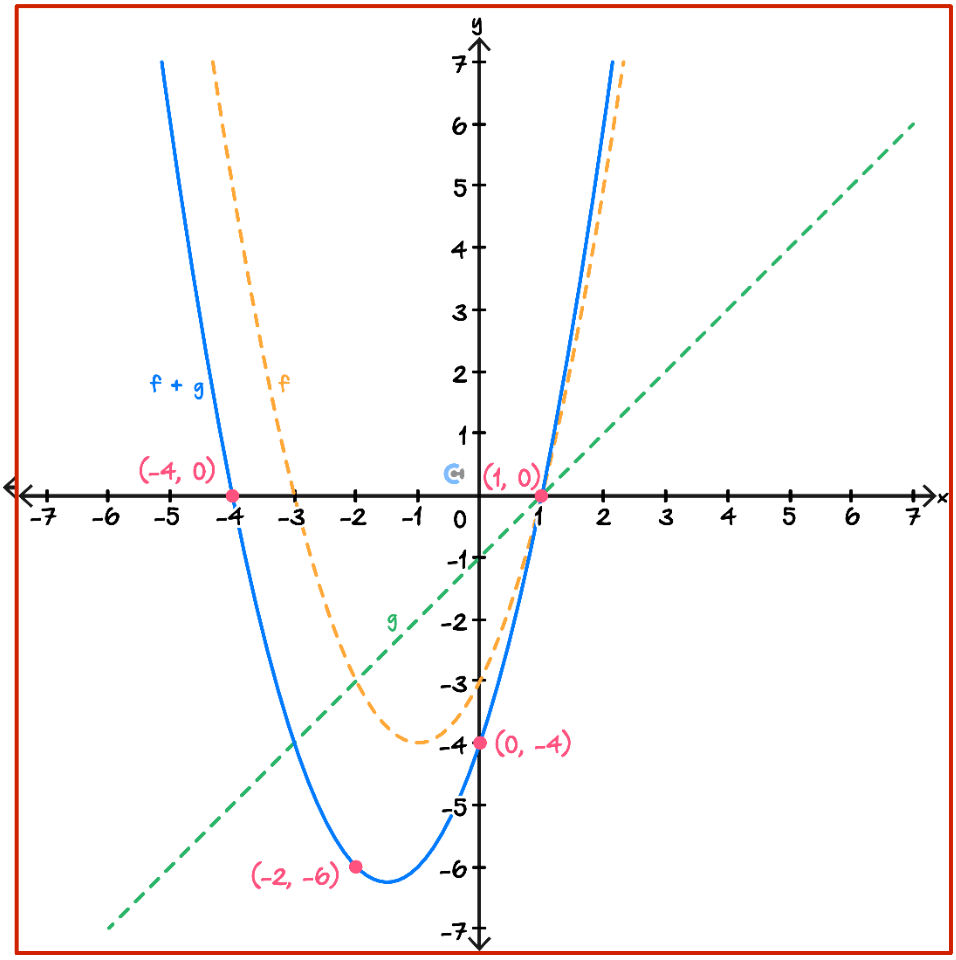
Min occurs when $k = -\frac{1}{2}$ (Perpendicular gradient)

$\therefore d\left(-\frac{1}{2}\right) = \sqrt{\left(\frac{-1}{-\frac{1}{2}-2}\right)^2 + \left(\frac{\frac{1}{2}}{-\frac{1}{2}-2}\right)^2}$

$= \sqrt{\left(\frac{2}{5}\right)^2 + \left(-\frac{1}{5}\right)^2} = \sqrt{\frac{4}{25} + \frac{1}{25}} = \sqrt{\frac{1}{5}} = \frac{1}{\sqrt{5}}$

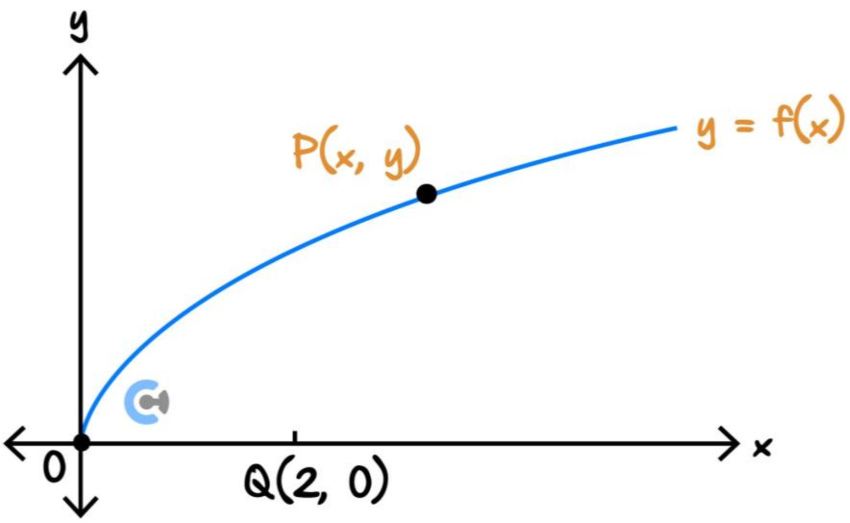
Question 4 (3 marks)

The graphs of the functions f and g are sketched on the axes below. Sketch the graph of $f + g$ on the same set of axes and label its axis intercepts with coordinates.



Question 5 (3 marks)

Let $f: [0, \infty) \to R, f(x) = \sqrt{x}$. The graph of f is shown below.



The point P lies on the graph of f.
The point $Q(2, 0)$ lies on the x-axis.

It is known that the function f has a gradient of $\frac{1}{2\sqrt{a}}$ at the point where $x = a$.

Find the minimum distance from P to Q. Express your answer in the form $\frac{\sqrt{a}}{b}$ where a and b are positive integers.

```
In[239]:= f[x_] := √x

In[240]:= Solve[(f[a] - 0)/(a - 2) * 1/(2 √a) == -1, a]

Out[240]= {{a → 3/2}}

In[241]:= EuclideanDistance[{3 / 2, f[3 / 2]}, {2, 0}]

Out[241]= √7/2
```

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Question 6 (2 marks)

Consider the following function:

$$f: \mathbb{R} \to \mathbb{R}, f(x) = kx$$
$$\text{where } k \in \mathbb{R}^+$$

Find the value(s) of k for which the acute angle between the $f(x)$ and $f^{-1}(x)$ is 30°.

$$\therefore k = \sqrt{3} \text{ or } \frac{1}{\sqrt{3}}$$

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Question 7 (4 marks)

a. Find the minimum distance between the lines $y = -2x$ and $y = -2x + 5$. (2 marks)

Solution: Perp line: $y = \frac{1}{2}x$. Two lines intersect at $(2, 1)$.
Min distance is $\sqrt{5}$

b. The line $y = -2x + 5$ runs tangent to the parabola $y = (x - 3)^2$ when $x = 2$. Find the minimum distance between the point $(-2, -1)$ and the parabola $y = (x - 3)^2$. (2 marks)

Solution: Min distance between $(0, 0)$ and parabola is $\sqrt{5}$ from part a. since $(2, 1)$ is on the line $y = \frac{1}{2}x$ and on the parabola. The distance from $(-2, -1)$ to origin is $\sqrt{5}$. Therefore distance from $(-2, -1)$ to parabola is $2\sqrt{5}$.

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Section D: Tech-Active Exam Skills

Calculator Commands: Simultaneous Equations on CAS

System of Linear Equations

Example:

The simultaneous linear equations $ax - 3y = 5$ and $3x - ay = 8 - a$ have no solution for:

A. $a = 3$

B. $a = -3$

C. Both $a = 3$ and $a = -3$.

D. $a \in R\backslash\{3\}$

E. $a \in R\backslash[-3, 3]$

$system\_solve(a\cdot x-3\cdot y=5, 3\cdot x-a\cdot y=8-a, a)$

- Solving: $\begin{bmatrix} a\cdot x-3\cdot y=5 \\ 3\cdot x-a\cdot y=8-a \end{bmatrix}$
- Unique Solution: $a\neq-3$ and $a\neq3$
- No Solutions: $a=-3$
- Infinite Solutions: $a=3$

Overview:

This program takes two linear equations and a parameter and finds the parameter values for the system to obtain a unique solution, no solution, or infinite solutions.

Input:

$system\_solve(< equation\ 1 >, < equation\ 2 >, < parameter >)$

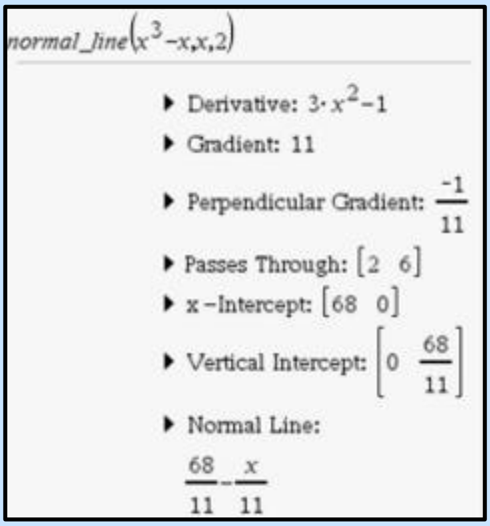
Other Notes:

The program can only handle one parameter.

- Or menu $- 3 - 7$.

- UDF line functions:

Normal Line



Overview:

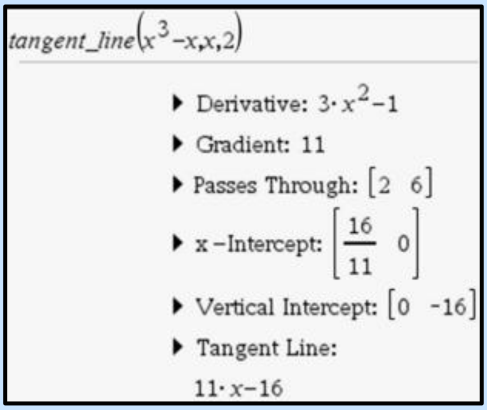
This program will find all the necessary information related to a normal line at a point on a function, which includes:

- The derivative.
- The gradient and perpendicular gradient.
- The point on the function the normal line passes through.
- The axis intercepts of the normal line.
- The equation of the normal line.

Input:

$\boldsymbol{normal\_line}(<\ \boldsymbol{function}\ >,<\ \boldsymbol{variable}\ >, <\ \boldsymbol{x\,point}\ >)$

Tangent Line



Overview:

This program will find all the necessary information related to a tangent line at a point on a function, which includes:

- The derivative.
- The gradient of the tangent line.
- The point on the function the tangent line passes through.
- The axis intercepts of the tangent line.
- The equation of the tangent line.

Input:

$\boldsymbol{tangent\_line}(<\ \boldsymbol{function}\ >,<\ \boldsymbol{variable}\ >, <\ \boldsymbol{x\,point}\ >)$

Calculator Commands: Finding the Angle Between Two Lines

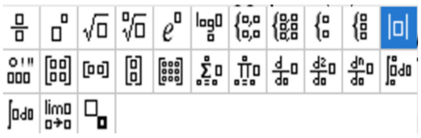
- The angle between two lines with gradients m_1 and m_2 respectively is:

$$\theta = |\tan^{-1}(m_1) - \tan^{-1}(m_2)|$$

- **Mathematica**
  - Use the Abs[] function.

```
In[126]:= Abs[ArcTan[2] - ArcTan[1]] / Degree // N
Out[126]= 18.4349
```

- **TI-Nspire**
  - Find the modulus sign.

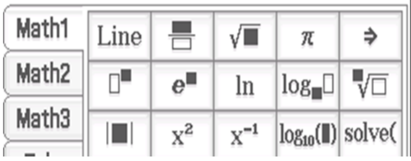


$|\tan^{-1}(2)-\tan^{-1}(1)|$ 18.4349

- **Casio Classpad**
  - Modulus sign under Math1.

$|\tan^{-1}(2)-\tan^{-1}(1)|$

18.43494882



Calculator Commands: Finding the Gradients of Lines Given the Angle They Make

- If we know the angle and one of the gradients m_1 or m_2 then we can find the other gradient by solving,

$$\tan(\theta) = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

- e.g. Find the gradient of the line that makes an angle of $60°$ with $y = -x$.

- **Mathematica**

```
Solve[Tan[60 Degree] == Abs[(m1 + 1)/(1 - m1)], m1]
Solve: Inverse functions are being used by Solve, so s
Reduce for complete solution information.
{{m1 → 2 - √3}, {m1 → 2 + √3}}
```

- **TI-Nspire**
  - Find the modulus sign.

$\text{solve}\left(\tan(60)=\left|\frac{m1+1}{1-m1}\right|,m1\right)$

$m1=-(\sqrt{3}-2)$ or $m1=\sqrt{3}+2$

- **Casio Classpad**
  - Modulus sign under Math1.

$\text{solve}\left(\tan(60)=\left|\frac{m1+1}{1-m1}\right|,m1\right)$

$\left\{m1=\frac{\sqrt{3}-1}{\sqrt{3}+1},\ m1=\frac{\sqrt{3}+1}{\sqrt{3}-1}\right\}$

Section E: Exam 2 Questions (31 Marks)

Question 8 (1 mark)

The simultaneous linear equations $2y + (m-1)x = 2$ and $my + 3x = k$ have infinitely many solutions for:

A. $m = 3$ and $k = 2$

B. $m = 3$ and $k = 3$

C. $m = -2$ and $k = 2$

D. $m = -2$ and $k = 3$

Question 9 (1 mark)

The simultaneous linear equations $ax - 3y = 5$ and $3x - ay = 6 - a$ have **no solution** for:

A. $a = 3$

B. $a = -3$

C. Both $a = 3$ and $a = -3$

D. $a \in R \backslash \{3\}$

Question 10 (1 mark)

Which of the following lines is **NOT** parallel to the rest?

A. Line joining $(2, 3)$ and $(5, -3)$.

B. Perpendicular bisector of the line segment joining $(-10, 4)$ and $(0, 9)$.

C. The shortest path between $(2, 0)$ and $2y = x - 4$.

D. $(k-2)x + (k-1)y = 10$ where $k = 3$.

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Question 11 (1 mark)

The length of the line segment that joins $(-1, 3)$ to $(2, -3)$ is:

A. $\sqrt{3}$

B. $5\sqrt{3}$

C. $3\sqrt{5}$

D. $\sqrt{37}$

Question 12 (1 mark)

It is known that exactly one point on the line $y = -x + k$ has a distance of $2\sqrt{2}$ from the point $(2, 2)$. Find the value(s) of k.

A. $k = 0$ and 8

B. $k = 2$ and 4

C. $k = 8$

D. $k = 2$

Question 13 (1 mark)

Himalaya is standing on top of Mt. Everest at $(1, 3)$ and wants to take the shortest path to a straight road defined by the relation $3x - 4y = 2$. Find the shortest distance Himalaya can travel to reach the road.

A. $\frac{11}{5}$

B. $\frac{7}{5}$

C. 2

D. $\frac{7}{10}$

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Question 14 (1 mark)

The linear function $f: [-1,3] \rightarrow R, f(x) = mx + 2$ has a range of $[-7, 5]$.

The value of m is:

A. -3

B. -1

C. 1

D. 3

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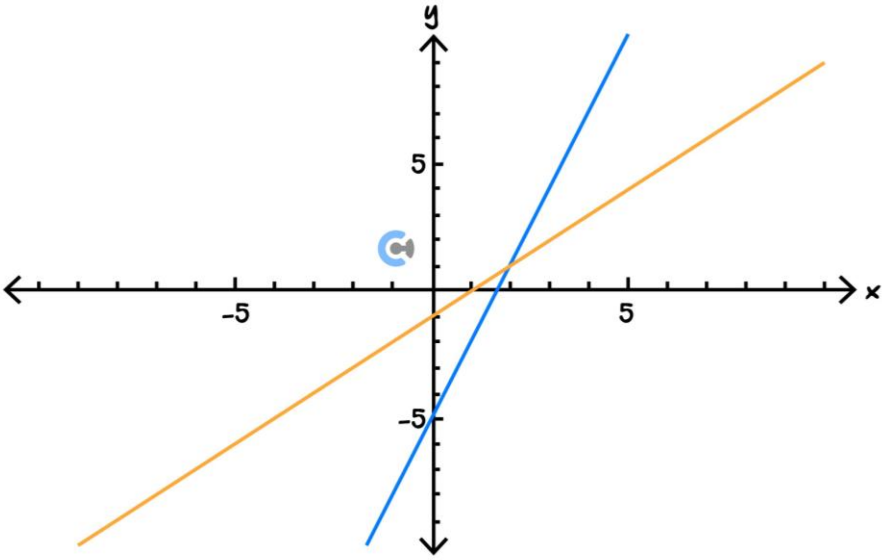
Question 15 (13 marks)

Part of the graphs of $f: \mathbb{R} \to \mathbb{R}, f(x) = 3x - 5$ and $g: \mathbb{R} \to \mathbb{R}, g(x) = x - 1$ are shown below.



a.

i. Find the coordinates of the point of intersection. (1 mark)

$(2, 1)$

ii. Find the distance between the origin and the intersection. (1 mark)

$\sqrt{5}$

iii. Find the size of the acute angle between f and g at the intersection point, in degrees correct to 2 decimal places. (2 marks)

$\theta = |\tan^{-1}(1) - \tan^{-1}(3)|$

$\theta = \tan^{-1}\left(\left|\frac{1-3}{1+3}\right|\right)$

$26.57°$

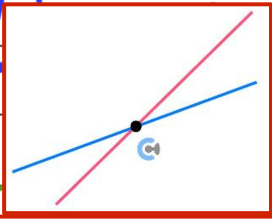
Consider another function $h: \mathbb{R} \to \mathbb{R}, h(x) = \frac{1}{k^2}x - \frac{1}{k}$ where $k \in \mathbb{R}^+$.

b. Find the value(s) of k such that $f(x)$ and $h(x)$ have a unique solution. (2 marks)

$\frac{1}{k^2} \neq 3$

$k \neq \pm\frac{1}{\sqrt{3}}$.

$k \in \mathbb{R}^+ \setminus \{\frac{1}{\sqrt{3}}\}$



c. Find the coordinates of the point of intersection between $y = f(x)$ and $y = h(x)$ in terms of k. (2 marks)

$\left(\frac{5k^2-1}{3k^2-1}, -5 + \frac{3k(5k-1)}{3k^2-1}\right)$

Consider another function $l: [-k, k] \to R, l(x) = \frac{1}{k^2}x - \frac{1}{k}$ where $k \in \mathbb{R}^+$.

d.

i. Find the value(s) of k for which $l(x) = f(x)$ has a unique solution. (3 marks)

Make sure that intersection is within $[-k, k]$

$$-k \le \frac{5k^2-1}{3k^2-1} \le k$$

$k \in \mathbb{R}^+$

$k \in (0, \frac{1}{3}] \cup [\frac{5}{3}, \infty)$

ii. Hence, could the intersection between $l(x)$ and $f(x)$, and the origin have a distance between them of 30 units? (2 marks)

Hence, could the intersection between $l(x)$ and $f(x)$, and the origin have a distance between them of 30 units? (2 marks)

$$\sqrt{\left(\frac{5k^2-1}{3k^2-1} - 0\right)^2 + \left(-5 + \frac{3k(5k-1)}{3k^2-1}\right)^2} = 30$$

$k = 0.542, 0.609$

$k \in (0, 0.333] \cup [1.666..., \infty)$

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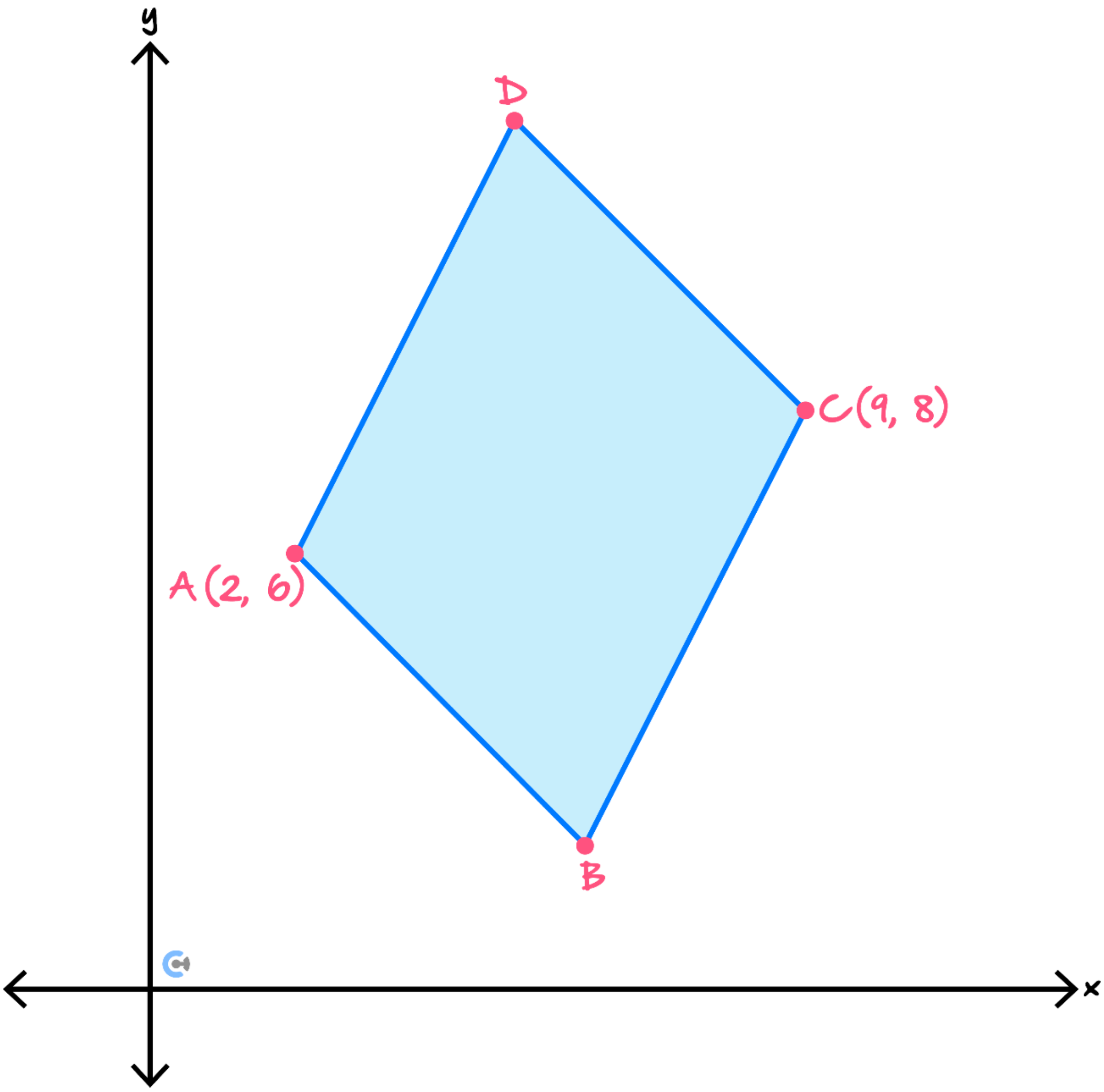
NO. It is impossible

(* The values of k are not within the values of k which gives us the \
intersection point in the first place. It is impossible to \
have a distance of 30 *)

Question 16 (11 marks)

The parallelogram $ABCD$ with two known points $A(2, 6)$ and $C(9, 8)$ is shown in the diagram below.

It is known that the line segment AB has gradient -1 and that the line segment AD has gradient 2.



a. Find the distance between A and C. (1 mark)

$\sqrt{53}$

b.

i. Find the Cartesian equation of the line segment AB. (1 mark)

$$y = -x + 8$$

ii. Find the Cartesian equation of the line segment BC. (1 mark)

$$y = 2x - 10$$

iii. Hence, find the coordinates of B and D. (2 marks)

$$B(6, 2) \text{ and } D(5, 12)$$

c. Find the angle $\angle ABC$. Give your answer correct to two decimal places. (1 mark)

$$\angle ABC = 71.57^{\circ}$$

d. Find the shortest distance between the line segments AB and DC. (3 marks)

Solution: The shortest distance will be on the line with gradient 1 through A. This line is

$$y = x + 4$$

The segment DC follows the line $y = 17 - x$. These two lines intersect at $I = \left(\frac{13}{2}, \frac{21}{2}\right)$. So our shorest distance is the distance from A to I

$$d = \frac{9}{\sqrt{2}}$$

e. Hence, or otherwise, find the area of $ABCD$. (2 marks)

Solution: $b = |AB| = 4\sqrt{2}$ and $h = \frac{9}{\sqrt{2}}$

Therefore Area $= bh = 36$

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