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VCE Mathematical Methods $\frac{3}{4}$
Coordinate Geometry [0.5]
Workshop

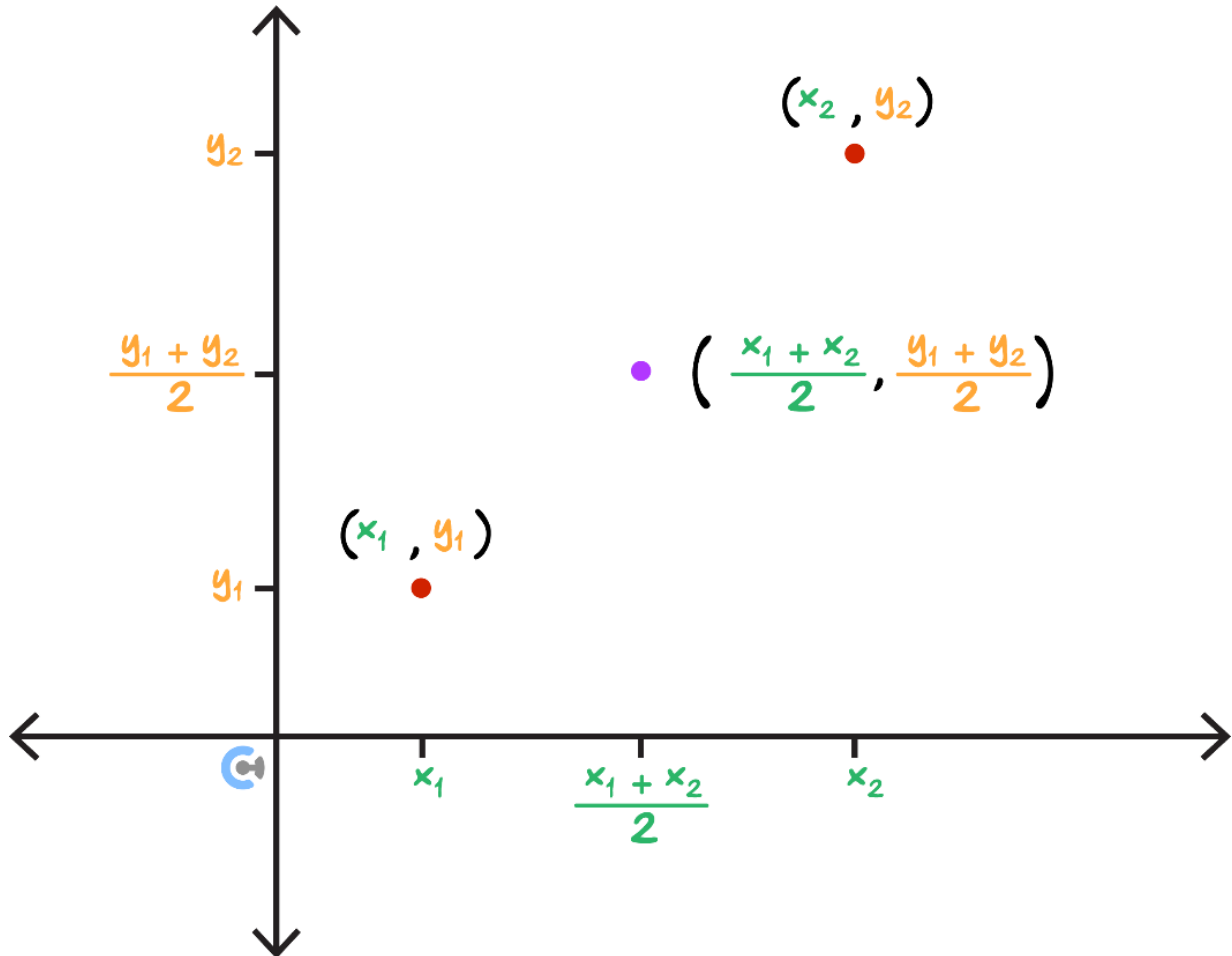
Error Logbook:



Mistake/Misconception #1		Mistake/Misconception #2	
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Section A: Recap

Midpoint



- The midpoint, M , of two points A and B is simply the point halfway between A and B .

$$M(x_m, y_m) = \left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2} \right)$$

- The midpoint can be found by taking the average of the x -coordinate and y -coordinate of the two points.

Distance Between Two Points

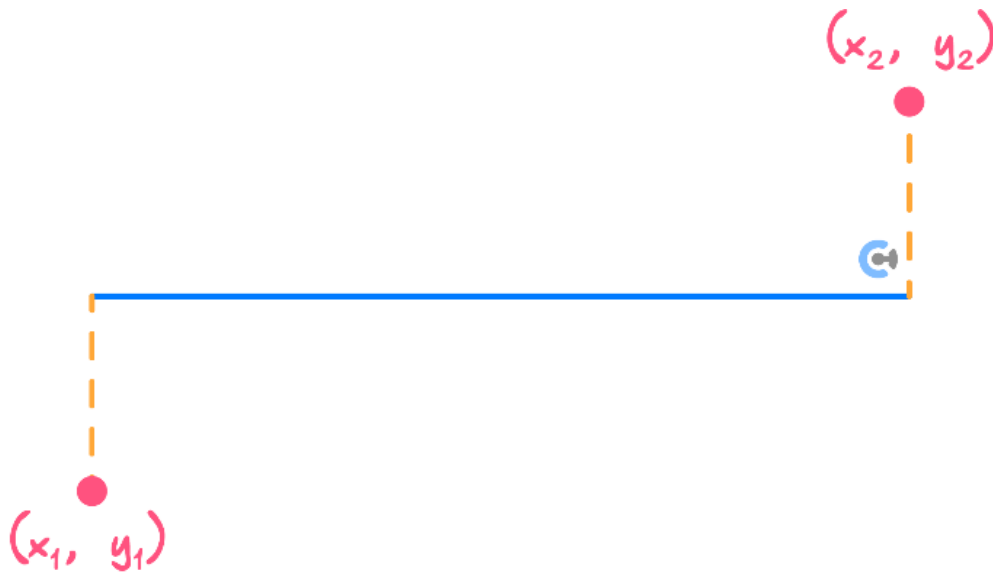


- The distance between two points (x_1, x_2) and (y_1, y_2) can be found using Pythagoras' theorem:

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Horizontal Distance

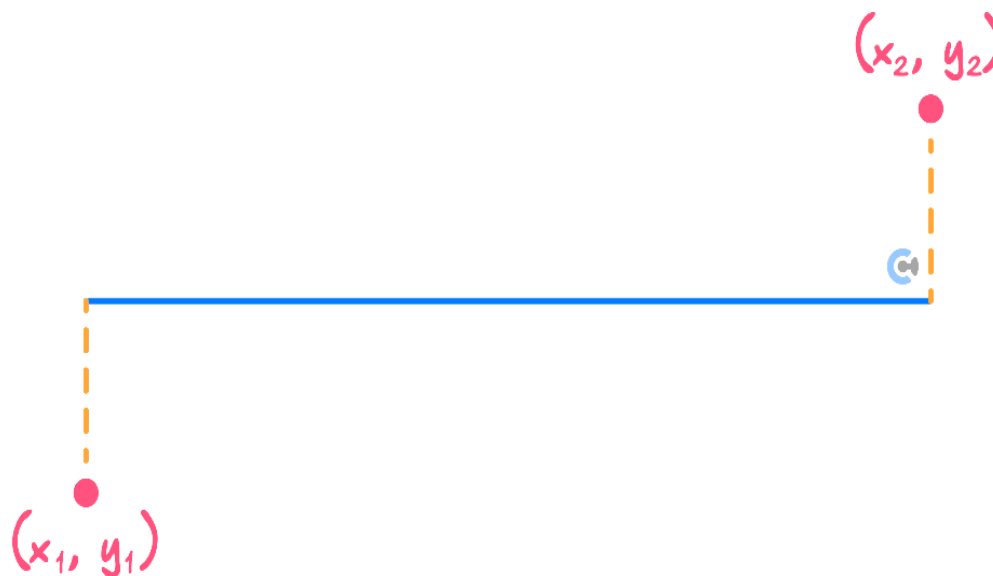


$$\text{Horizontal Distance} = x_2 - x_1, \text{ where } x_2 > x_1$$

- Find the difference between their x -values.



Vertical Distance

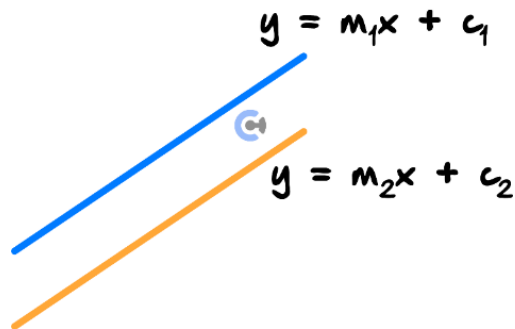


$$\text{Vertical Distance} = y_2 - y_1, \text{ where } y_2 > y_1$$

- Find the difference between their y -values.



Parallel Lines

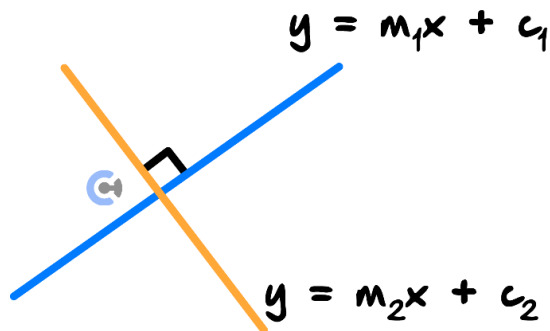


- Parallel lines have the same gradient.

$$m_1 = m_2$$



Perpendicular Lines



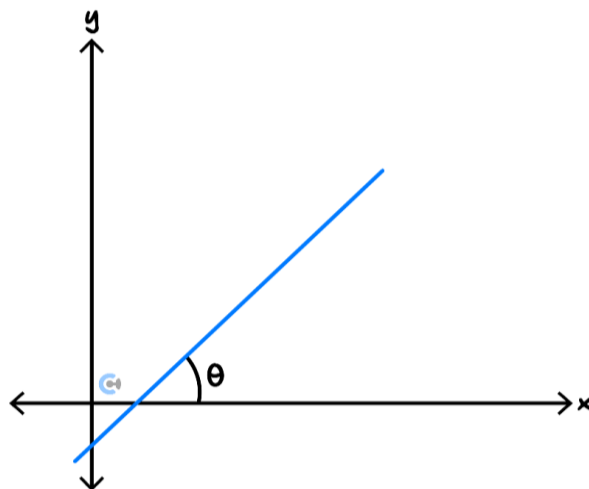
- A line that is perpendicular to another line has a gradient which is the negative reciprocal of the gradient of the first line.

$$m_{\perp} = -\frac{1}{m}$$

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Angle Between a Line and the x -axis

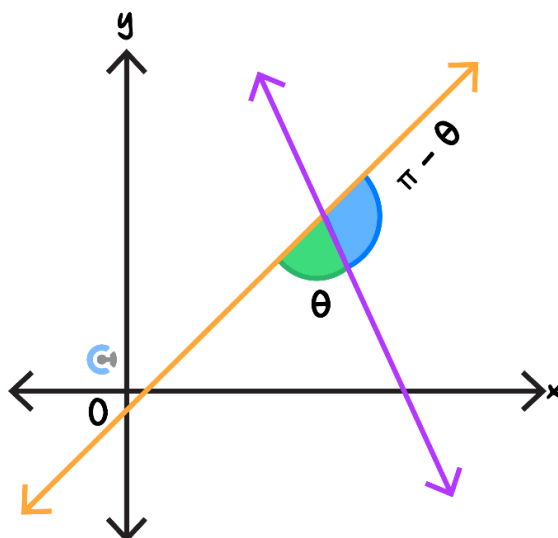


➤ The angle between a line and the **positive direction of the x -axis** (anticlockwise) is given by:

$$\tan(\theta) = m$$



Acute Angle Between Two Lines



$$\theta = |\tan^{-1}(m_1) - \tan^{-1}(m_2)|$$

➤ Alternatively:

$$\tan(\theta) = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$



Simultaneous Linear Equations

- Elimination Method
- Substitution Method



General Solutions of Simultaneous Linear Equations

- Two linear equations are either:
 - 🌀 The same line, expressed in a different form. In this case, they have infinite solutions.
 - 🌀 Unique lines that are **parallel**. In this case, they have NO solutions.
 - 🌀 Unique lines which are not parallel. In this case, they have exactly one solution.



Solving Systems of Linear Equations with Parameters

- Occurs when solving for three variables with two equations. We simply,

Let $x = k$, or

Let $y = k$, or

Let $z = k$

- And solve simultaneously.

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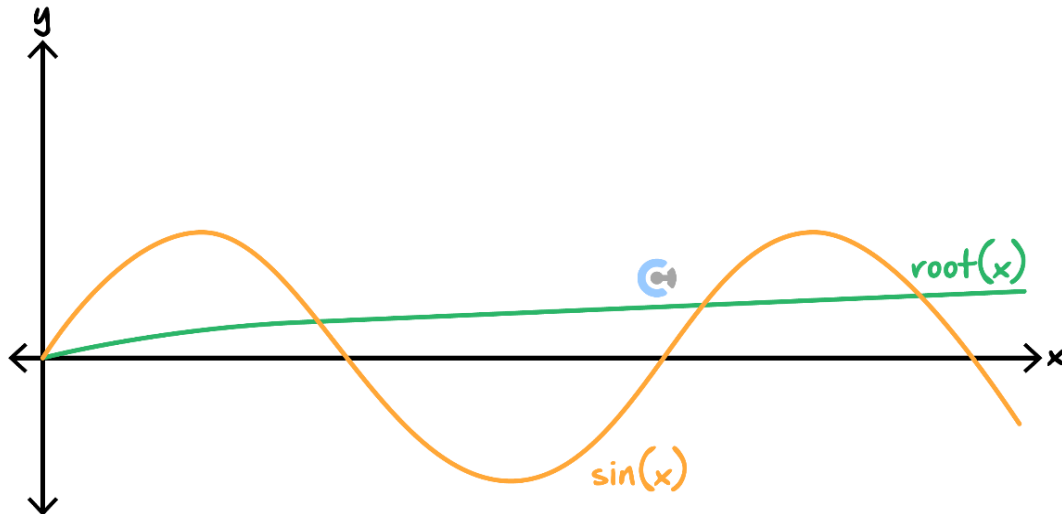


Addition of Ordinates

➤ Definition:

🔗 Technique used to graph the sum/difference of two functions.

e.g. $y = \sin(x) + \sqrt{x}$



➤ The addition of ordinates involves adding the y -values of two functions.

➤ Steps to sketching $f(x) + g(x)$:

1. Sketch $f(x)$ and $g(x)$ on the same axes.
2. Plot points for $f(x) + g(x)$ by adding the **y -values** of $f(x)$ and $g(x)$.
 - At x -intercepts, the sum equals to the other function.
 - At intersections, the sum equals to the y -value.
 - When functions are equidistant from x -axis, the sum equals to 0.
3. Join the plotted points.

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Section B: Warm Up**Question 1**

- a. Find the midpoint of the points $(2, 5)$ and $(-4, 3)$.

- b. Find the distance between the points $(3, 5)$ and $(-4, 2)$.

- c. Find the equation of the line parallel to $y = 2x + 3$ that passes through $(1, 3)$.

- d. Find the perpendicular bisector of the line through the points $(2, 4)$ and $(6, 2)$.

- e. The distance between a point (a, b) that lies on the line $y = x - 2$ and the point $(2, 4)$ is $2\sqrt{2}$. Find the point (a, b) .

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Section C: Exam 1 (23 Marks)**Question 2 (5 marks)**

Two linear equations can be written in the form:

$$2mx + (2m - 1)y = 2m + 3$$

$$-(2 - 3m)x + 3y = 4m - 1$$

Where m is a real constant.

- a. Find the value(s) of m such that the graphs of the two lines have a unique solution. (2 marks)

- b. Find the value(s) of m such that the graphs of the two lines have no solution. (2 marks)

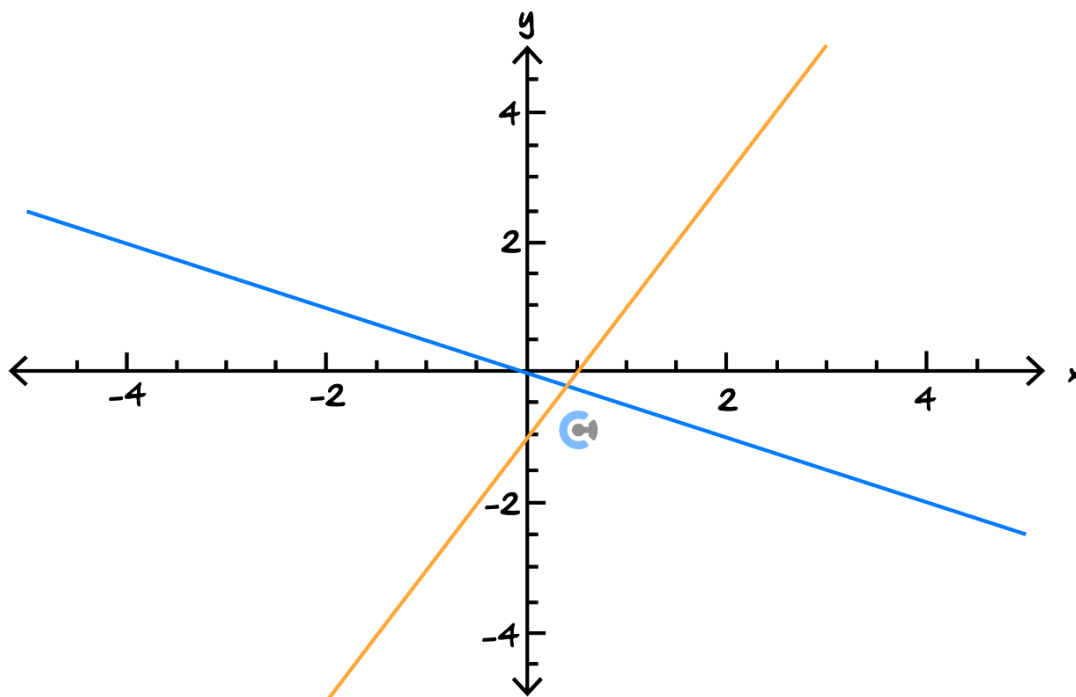
- c. Find the value(s) of m such that the graphs of the two lines have infinite solutions. (1 mark)

Question 3 (6 marks)

Consider two lines:

$$f(x) = kx, \text{ where } k \text{ is a constant.}$$

$$g(x) = 2x - 1$$



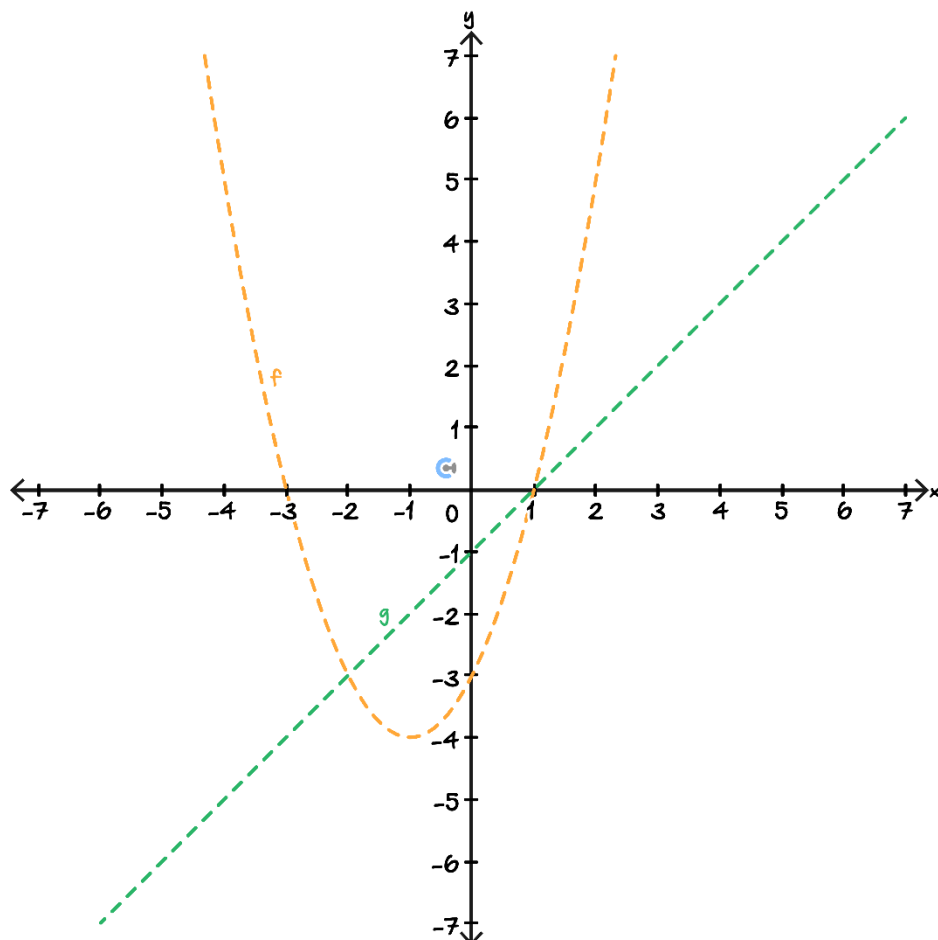
- a.** Solve for the intersection between $f(x)$ and $g(x)$ in terms of k . (2 marks)

- b.** Find the rule for $d(k)$, the distance between the intersection found in **part a.**, and the origin, in terms of k . (1 mark)

c. Solve for the minimum value of $d(k)$ and the value of k . (3 marks)

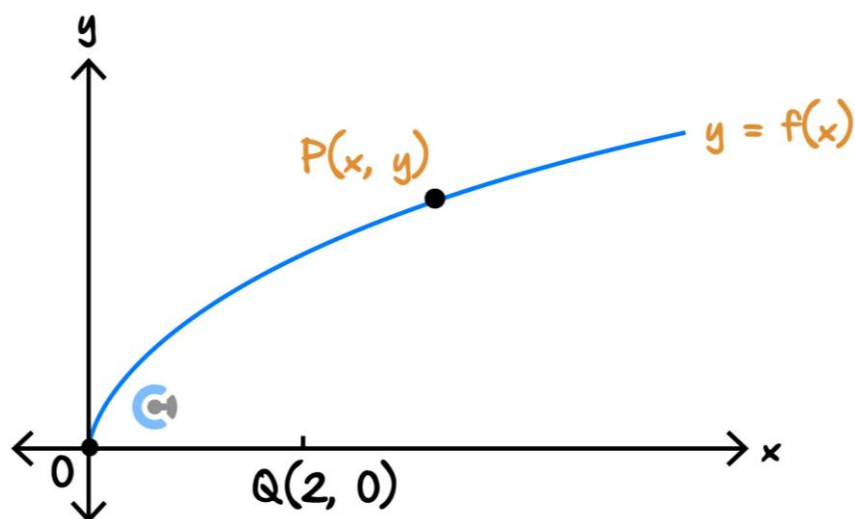
Question 4 (3 marks)

The graphs of the functions f and g are sketched on the axes below. Sketch the graph of $f + g$ on the same set of axes and label its axis intercepts with coordinates.



Question 5 (3 marks)

Let $f: [0, \infty) \rightarrow \mathbb{R}, f(x) = \sqrt{x}$. The graph of f is shown below.



The point P lies on the graph of f .
The point $Q(2, 0)$ lies on the x -axis.

It is known that the function f has a gradient of $\frac{1}{2\sqrt{a}}$ at the point where $x = a$.

Find the minimum distance from P to Q . Express your answer in the form $\frac{\sqrt{a}}{b}$ where a and b are positive integers.

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Question 6 (2 marks)

Consider the following function:

$$f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = kx$$

where $k \in \mathbb{R}^+$

Find the value(s) of k for which the acute angle between the $f(x)$ and $f^{-1}(x)$ is 30° .

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Question 7 (4 marks)

- a. Find the minimum distance between the lines $y = -2x$ and $y = -2x + 5$. (2 marks)

- b. The line $y = -2x + 5$ runs tangent to the parabola $y = (x - 3)^2$ when $x = 2$. Find the minimum distance between the point $(-2, -1)$ and the parabola $y = (x - 3)^2$. (2 marks)

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Section D: Tech-Active Exam Skills



Calculator Commands: Simultaneous Equations on CAS

System of Linear Equations

Example:

The simultaneous linear equations $ax - 3y = 5$ and $3x - ay = 8 - a$ have no solution for:

- A. $a = 3$
- B. $a = -3$
- C. Both $a = 3$ and $a = -3$.
- D. $a \in \mathbb{R} \setminus \{3\}$
- E. $a \in \mathbb{R} \setminus [-3, 3]$

`system_solve(a*x-3*y=5,3*x-a*y=8-a,a)`

- ▶ Solving: $\begin{bmatrix} a \cdot x - 3 \cdot y = 5 \\ 3 \cdot x - a \cdot y = 8 - a \end{bmatrix}$
- ▶ Unique Solution: $a \neq -3$ and $a \neq 3$
- ▶ No Solutions: $a = -3$
- ▶ Infinite Solutions: $a = 3$

Overview:

This program takes two linear equations and a parameter and finds the parameter values for the system to obtain a unique solution, no solution, or infinite solutions.

Input:

`system_solve(< equation 1 >, < equation 2 >, < parameter >)`

Other Notes:

The program can only handle one parameter.

➤ Or menu $-3 - 7$.

➤ UDF line functions:

Normal Line

```
normal_line(x^3-x,x,2)

▶ Derivative: 3·x2-1
▶ Gradient: 11
▶ Perpendicular Gradient:  $-\frac{1}{11}$ 
▶ Passes Through: [2 6]
▶ x-Intercept: [68 0]
▶ Vertical Intercept:  $\left[0 \frac{68}{11}\right]$ 
▶ Normal Line:

$$\frac{68}{11} - \frac{x}{11}$$

```

Overview:

This program will find all the necessary information related to a normal line at a point on a function, which includes:

- The derivative.
- The gradient and perpendicular gradient.
- The point on the function the normal line passes through.
- The axis intercepts of the normal line.
- The equation of the normal line.

Input:

normal_line(< function >, < variable >, < x point >)

Tangent Line

```
tangent_line(x^3-x,x,2)

▶ Derivative: 3·x2-1
▶ Gradient: 11
▶ Passes Through: [2 6]
▶ x-Intercept:  $\left[\frac{16}{11} 0\right]$ 
▶ Vertical Intercept: [0 -16]
▶ Tangent Line:

$$11 \cdot x - 16$$

```

Overview:

This program will find all the necessary information related to a tangent line at a point on a function, which includes:

- The derivative.
- The gradient of the tangent line.
- The point on the function the tangent line passes through.
- The axis intercepts of the tangent line.
- The equation of the tangent line.

Input:

tangent_line(< function >, < variable >, < x point >)



Calculator Commands: Finding the Angle Between Two Lines

- The angle between two lines with gradients m_1 and m_2 respectively is:

$$\theta = |\tan^{-1}(m_1) - \tan^{-1}(m_2)|$$

➤ Mathematica

- Use the Abs[] function.

```
In[126]:= Abs[ArcTan[2] - ArcTan[1]] / Degree // N
Out[126]:= 18.4349
```

➤ TI-Nspire

- Find the modulus sign.



TI-Nspire calculator interface showing the calculation of the angle between two lines with gradients 2 and 1. The result is 18.4349.

➤ Casio Classpad

- Modulus sign under Math1.

Casio Classpad calculator interface showing the calculation of the angle between two lines with gradients 2 and 1. The result is 18.43494882.

Calculator Commands: Finding the Gradients of Lines Given the Angle They Make

- If we know the angle and one of the gradients m_1 or m_2 then we can find the other gradient by solving,

$$\tan(\theta) = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

- e.g. Find the gradient of the line that makes an angle of 60° with $y = -x$.

➤ Mathematica

```
Solve[Tan[60 Degree] == Abs[m1 - m2 / (1 + m1 m2)], m1]
```

*** Solve: Inverse functions are being used by Solve, so s
Reduce for complete solution information. ⓘ

```
{m1 -> 2 - sqrt(3)}, {m1 -> 2 + sqrt(3)}
```

➤ TI-Nspire

- Find the modulus sign.



TI-Nspire calculator interface showing the calculation of the gradient of the line that makes an angle of 60° with $y = -x$. The result is $m_1 = -(\sqrt{3} - 2)$ or $m_1 = \sqrt{3} + 2$.

➤ Casio Classpad

- Modulus sign under Math1.

Casio Classpad calculator interface showing the calculation of the gradient of the line that makes an angle of 60° with $y = -x$. The result is $m_1 = \frac{\sqrt{3}-1}{\sqrt{3}+1}$ or $m_1 = \frac{\sqrt{3}+1}{\sqrt{3}-1}$.

Section E: Exam 2 Questions (31 Marks)**Question 8** (1 mark)

The simultaneous linear equations $2y + (m - 1)x = 2$ and $my + 3x = k$ have infinitely many solutions for:

- A. $m = 3$ and $k = 2$
- B. $m = 3$ and $k = 3$
- C. $m = -2$ and $k = 2$
- D. $m = -2$ and $k = 3$

Question 9 (1 mark)

The simultaneous linear equations $ax - 3y = 5$ and $3x - ay = 6 - a$ have **no solution** for:

- A. $a = 3$
- B. $a = -3$
- C. Both $a = 3$ and $a = -3$
- D. $a \in R \setminus \{3\}$

Question 10 (1 mark)

Which of the following lines is **NOT** parallel to the rest?

- A. Line joining $(2, 3)$ and $(5, -3)$.
- B. Perpendicular bisector of the line segment joining $(-10, 4)$ and $(0, 9)$.
- C. The shortest path between $(2, 0)$ and $2y = x - 4$.
- D. $(k - 2)x + (k - 1)y = 10$ where $k = 3$.

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Question 11 (1 mark)

The length of the line segment that joins $(-1, 3)$ to $(2, -3)$ is:

- A. $\sqrt{3}$
- B. $5\sqrt{3}$
- C. $3\sqrt{5}$
- D. $\sqrt{37}$

Question 12 (1 mark)

It is known that exactly one point on the line $y = -x + k$ has a distance of $2\sqrt{2}$ from the point $(2, 2)$. Find the value(s) of k .

- A. $k = 0$ and 8
- B. $k = 2$ and 4
- C. $k = 8$
- D. $k = 2$

Question 13 (1 mark)

Himalaya is standing on top of Mt. Everest at $(1, 3)$ and wants to take the shortest path to a straight road defined by the relation $3x - 4y = 2$. Find the shortest distance Himalaya can travel to reach the road.

- A. $\frac{11}{5}$
- B. $\frac{7}{5}$
- C. 2
- D. $\frac{7}{10}$

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Question 14 (1 mark)

The linear function $f: [-1, 3] \rightarrow \mathbb{R}$, $f(x) = mx + 2$ has a range of $[-7, 5]$.

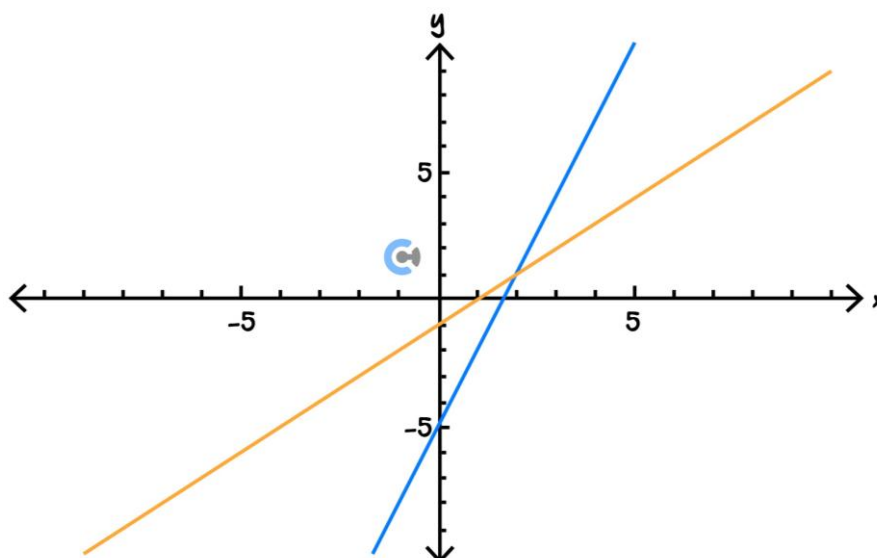
The value of m is:

- A. -3
- B. -1
- C. 1
- D. 3

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Question 15 (13 marks)

Part of the graphs of $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = 3x - 5$ and $g: \mathbb{R} \rightarrow \mathbb{R}, g(x) = x - 1$ are shown below.



- a.
- i. Find the coordinates of the point of intersection. (1 mark)
-
-
- ii. Find the distance between the origin and the intersection. (1 mark)
-
-
- iii. Find the size of the acute angle between f and g at the intersection point, in degrees correct to 2 decimal places. (2 marks)
-
-
-
-

Consider another function $h: \mathbb{R} \rightarrow \mathbb{R}, h(x) = \frac{1}{k^2}x - \frac{1}{k}$ where $k \in \mathbb{R}^+$.

- b. Find the value(s) of k such that $f(x)$ and $h(x)$ have a unique solution. (2 marks)

- c. Find the coordinates of the point of intersection between $y = f(x)$ and $y = h(x)$ in terms of k . (2 marks)

Consider another function $l: [-k, k] \rightarrow \mathbb{R}, l(x) = \frac{1}{k^2}x - \frac{1}{k}$ where $k \in \mathbb{R}^+$.

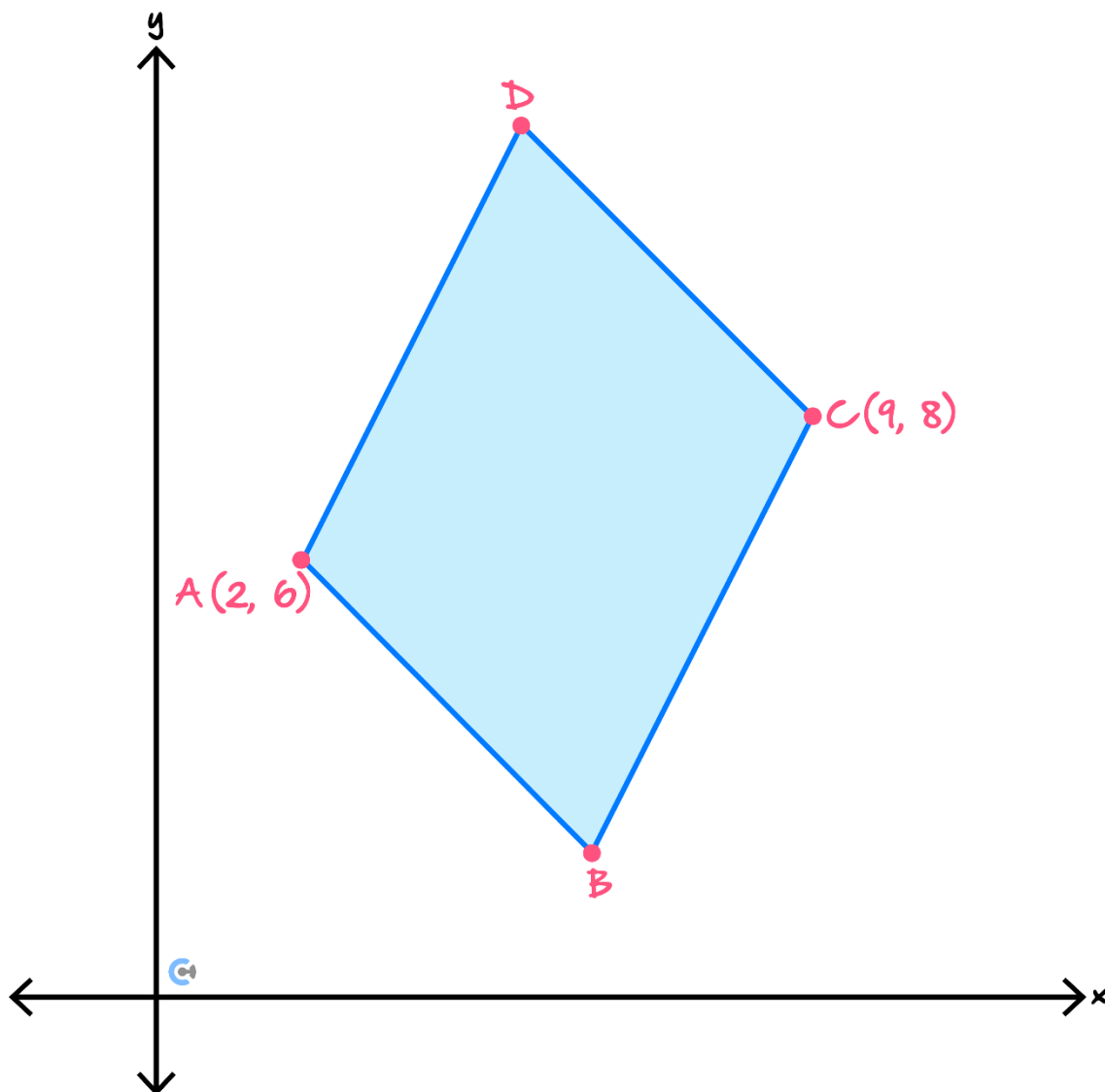
- d.**
- i.** Find the value(s) of k for which $l(x) = f(x)$ has a unique solution. (3 marks)
- _____
- _____
- _____
- _____
- _____
- ii.** Hence, could the intersection between $l(x)$ and $f(x)$, and the origin have a distance between them of 30 units? (2 marks)
- _____
- _____
- _____
- _____
- _____

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Question 16 (11 marks)

The parallelogram $ABCD$ with two known points $A(2, 6)$ and $C(9, 8)$ is shown in the diagram below.

It is known that the line segment AB has gradient -1 and that the line segment AD has gradient 2 .



- a. Find the distance between A and C . (1 mark)

b.

- i.** Find the Cartesian equation of the line segment AB . (1 mark)

- ii.** Find the Cartesian equation of the line segment BC . (1 mark)

- iii.** Hence, find the coordinates of B and D . (2 marks)

- c.** Find the angle $\angle ABC$. Give your answer correct to two decimal places. (1 mark)

d. Find the shortest distance between the line segments AB and DC . (3 marks)

e. Hence, or otherwise, find the area of $ABCD$. (2 marks)

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