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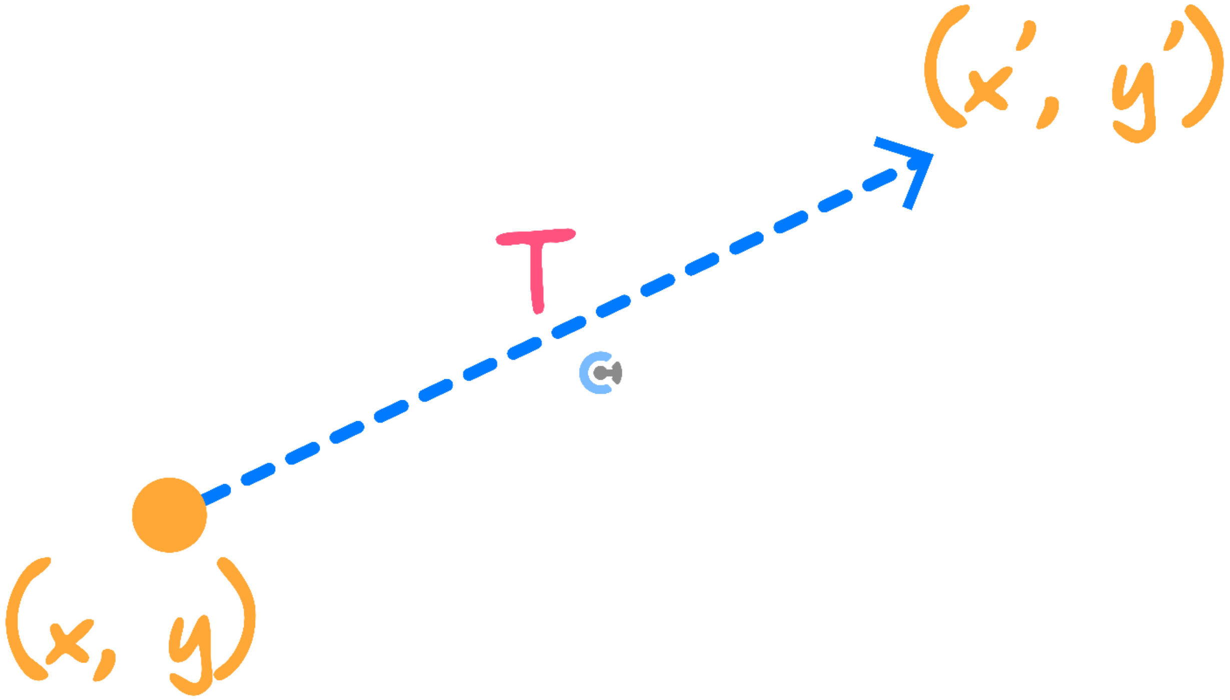
VCE Mathematical Methods ¾
Transformations Exam Skills [0.4]
Workshop Solutions

Section A: Recap

Sub-Section: Image and Pre-Image

Image and Pre-Image



- The original coordinate is called the $\boxed{\text{pre-image}}$.
- The transformed coordinate is called the $\boxed{\text{image}}$.

Pre-Image: (x, y)

Image: (x', y')

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Sub-Section: Dilation

Dilation

Dilation by a factor a from the x-axis: $y' = ay$

Dilation by a factor b from the y-axis: $x' = bx$

NOTE: We are applying the transformations on (x, y) not (x', y').

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Sub-Section: Reflection

Reflection

Reflection in the x-axis: $y' = -y$

Reflection in the y-axis: $x' = -x$

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Sub-Section: Translation

Translation

Translation by c units in the positive direction of the x-axis: $x' = x + c$

Translation by d units in the positive direction of the y-axis: $y' = y + d$

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Sub-Section: The Order of Transformations

The Order of Transformation

$$\text{Order} = \text{BODMAS Order}$$

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Sub-Section: Interpreting the Transformation of Points

Interpretation of Transformations

- When the new variables x' and y' are the subjects, we can read the transformation directly.

$$x' = x + 5 \rightarrow 5 \text{ right}$$

- When the original variables x and y are the subjects instead, we must read the transformation in the opposite way.
- This includes the order of transformation!

$$x = x' - 5 \rightarrow 5 \text{ right}$$

NOTE: This includes the order of transformation!

TIP: It is best to make x' and y' the subject before you interpret the transformations.

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Sub-Section: Applying Transformations to Functions

Transformation of Functions

- The aim is to get rid of the old variables, x and y, and have the new variables, x' and y', instead.

$$y = f(x) \rightarrow y' = f(x')$$

- Steps:

1. Transform the points.
2. Make x and y the subjects.
3. Substitute them into the function.

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Sub-Section: Finding the Applied Transformations

Now, let's go backwards!

Reverse Engineering

- Steps:

1. Add the dashes (') back to the transformed function.
2. Make $f(\ )$ the subject.
3. Equate the LHS of the original and transformed functions to the RHS of the original and transformed functions.
4. Make x' and y' the subjects and interpret the transformations.

Quick Method

- The transformation of x in the function is represented in the opposite way in the final function.
- For applying transformation in a quick method,

Apply everything for x in the opposite direction.
Including the order!

- For interpreting transformation in a quick method,

Read everything for x in the opposite direction.
Including the order!

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Finding Opposite Transformations

- Order is reversed.
- All transformations are opposed.

Finding Domain, Range, Points, and Tangents of Transformed Functions

- Everything moves together as a function.
- **Steps:**
  1. Find the transformations between two functions.
  2. Apply the same transformations to domain, range, points, and tangents.

Finding Transformation of Inverse Functions

$$f(x) \to f(x-2): 2 \text{ Right}$$

$$f^{-1}(x) \to f^{-1}(x) + 2: 2 \text{ Up}$$

- **Steps:**
  1. Find the transformation between two original functions.
  2. Inverse the transformations found in $\mathbf{1}$.

Multiple Pathways.

- Same transformations can be done differently by either putting it in or out of the $f()$.
- Commonly, look for basic algebra, index and $\log$ laws.

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Definition

<u>Manipulating the Function to Find Appropriate Transformations</u>

- **Steps:**

1. Identify the region of x.

2. Identify the region of y.

3. Manipulate the function so that all the changes are within the region of x or y.

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Section B: Warmup

Question 1

a. Apply the following sequence of transformations to $y = \cos(x)$.

- A dilation by factor 2 from the x-axis.
- A dilation by factor 3 from the y-axis.
- A reflection in the y-axis.
- A translation 2 units left.
- A translation 4 units down.

Solution: $y = 2\cos\left(-\frac{1}{3}(x+2)\right) - 4$

b. State the transformations required to transform $y = \cos(x)$ into $y = -3\cos(2x + \pi) - 2$ in the order DRT.

- A dilation by factor 3 from the x-axis
- A dilation by factor $\frac{1}{2}$ from the y-axis
- A reflection in the x-axis
- A translation $\frac{\pi}{2}$ units left
- A translation 2 units down

c. The graph of $y = f(x)$ has a tangent line $y = 3x - 2$ when $x = 2$.

Find the equation of the tangent to the graph of $y = 2f\left(\frac{x}{3}\right) + 1$ when $x = 6$.

Solution: Let $t(x) = 3x - 2$. Then the required tangent is

$$y = 2t\left(\frac{x}{3}\right) + 1 = 2x - 3$$

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Section C: Exam 1 (21 Marks)

INSTRUCTION: 21 Marks. 28 Minutes Writing.

Question 2 (4 marks)

a. Describe a sequence of transformations that map the function $f(x) = 2(x-1)^2 + 3$ to $g(x) = 4x^2 - 24x + 43$. (2 marks)

Solution: $g(x) = 4(x-3)^2 + 7$. Therefore,

- A dilation by factor 2 from the x-axis
- A translation 2 units to the right
- A translation 1 unit up

b. Hence, describe a sequence of transformations that map $g(x)$ to $f(x)$. (2 marks)

Solution:

- A translation 1 unit down
- A translation 2 units to the left
- A dilation by factor $\frac{1}{2}$ from the x-axis

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Question 3 (5 marks)

It is known that the function $f(x)$ has a domain of $[1,6)$ and a range of $[-4,12)$.

$f(x)$ is transformed to become the function $g(x) = -2f\left(\frac{x}{3} + 6\right) + 2$.

a. Describe a sequence of transformations that map $f(x)$ to $g(x)$. (2 marks)

Solution: $g(x) = -2f\left(\frac{1}{3}(x+18)\right) + 2$

- A dilation by factor 2 from the x-axis
- A dilation by factor 3 from the y-axis
- A reflection in the x-axis
- A translation 18 units left
- A translation 2 units up

b. State the domain of $g(x)$. (1 mark)

Solution: $[3, 18)$

c. State the range of $g(x)$. (2 marks)

Solution: $(-22, 10]$

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Question 4 (5 marks)

It is known that the function $f(x)$ has an x-intercept at the $(4,0)$ and the tangent line of $y = 3x - 2$ when $x = 4$.

The function f is transformed to become the function g, where $g(x) = 2f(3x - 2)$.

a. Describe a sequence of transformations that map $f(x)$ to $g(x)$. (2 marks)

Solution: We have $g(x) = 2f\left(3\left(x - \frac{2}{3}\right)\right)$

- A dilation by factor 2 from the x-axis
- A dilation by factor $\frac{1}{3}$ from the y-axis
- A translation $\frac{2}{3}$ units to the right

b. State the x-intercept of $g(x)$. (1 mark)

$(3, 0)$

c. Find the tangent to the graph $y = g(x)$ when $x = 2$. (2 marks)

Let $t(x) = 3x - 2$, then the tangent line when $x = 2$ is

$$y = 2t(3x - 2) = 2(3(3x - 2) - 2) = 18x - 16$$

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Question 5 (4 marks)

Consider the functions $f(x) = 2\log_2(x-3)+1$ and $g(x) = 4\log_2(x+1)+3$.

a. Describe a sequence of transformations that map the function f to the function g. (2 marks)

- A dilation by factor 2 from the x-axis
- A translation 4 units to the left
- A translation 1 unit up

b. Hence, describe a sequence of transformations that map the inverse function f^{-1} to the inverse function g^{-1}. (1 mark)

- A dilation by factor 2 from the y-axis
- A translation 4 units down
- A translation 1 unit right

c. Hence, describe a sequence of transformations that map the inverse function f to the inverse function g^{-1}. (1 mark)

- A reflection in the line $y = x$
- A dilation by factor 2 from the y-axis
- A translation 4 units down
- A translation 1 unit right

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Question 6 (3 marks)

The image of the graph $y = 2\sqrt{(x-1)^2 + 4}$ and a transformation T, has the equation $y = 6\sqrt{4x^2 + 12x + 13}$.

Describe the sequence of transformations that make up T, with dilations and reflections before translations.

- A dilation by factor 3 from the x-axis
- A dilation by factor $\frac{1}{2}$ from the y-axis
- A reflection in the y-axis
- A translation 1 unit to the left.

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Section D: Tech Active Exam Skills

Calculator Tip: Finding Transformed Functions

- Save the function as $f(x)$.
- Substitute the x and y in terms of x' and y'.
- Solve for y!
- Can also apply the transformations directly to $f(x)$. Must make sure you interpret the transformations correctly or you can easily make a mistake doing this.

Mathematica UDF:

- ApplyTransformList[]

ApplyTransformList[*f[x] , {x, y}, list of transforms*]
Applies the list of transforms to f[x] in the chronological order.

```
ApplyTransformList[x^2, {x, y}, {x - 1, 2 x, y + 3}]
```

$$4 + x + \frac{x^2}{4}$$

```
ApplyTransformInvList[f[x], {x, y}, {x - 1, 2 x, y + 3}]
```

```
-3 + f[2 (-1 + x)]
```

```
ApplyTransformInvList[Sin[x], {x, y}, {x - π / 2, 2 y, y - 1 }]
```

$$\mathrm{Sin}\left[\frac{x}{2}\right]^2$$

- ApplyTransformInvList[]

ApplyTransformInvList[*f[x]* , *{x, y}*, *list of transforms*]
Applies the list of transforms to f[x] in reverse order and as the inverse to the transforms of *ApplyTransformList*.

```
In[∘]:= ApplyTransformInvList[x^2, {x, y}, {x - 1, 2 * x, y + 3}]
Out[∘]= 1 - 8 x + 4 x^2

In[∘]:= ApplyTransformInvList[f[x], {x, y}, {x - 1, 2 * x, y + 3}]
Out[∘]= -3 + f[2 (-1 + x)]

In[∘]:= ApplyTransformInvList[2 * Cos[x] - 1, {x, y}, {x - Pi / 2, 2 * y, y - 1 }]
Out[∘]= Sin[x]
```

TI UDF:

- Transform()

Transform a Function

$$transform\left(\sin(x),x,\left\{x-\frac{\pi}{2},2\cdot y,y-1\right\}\right)$$

- Translation $\frac{\pi}{2}$ units along the neg. x−dir.

$\cos(x)$

- Dilation by factor of 2 from the x−axis

$2\cdot\cos(x)$

- Translation −1 unit along the neg. y−dir.

$2\cdot\cos(x)-1$

Overview:

Apply any sequence of transformations to a function. The program will display the transformed function after each step.

Input:

```
transform(<function>, <variable>,
      <list of transformations>)
```

Other notes:

- The list of transformations can either be presented in a (horizontal or vertical) matrix of expressions or a list of expressions

- Transform_inv()

Invert a Transformation

$transform\_inv\left(x^2,x,\{x-1,2\cdot x,y+3\}\right)$

▸ Inverted Transformations:

$\left\{y-3,\frac{x}{2},x+1\right\}$

▸ Translation -3 units along the neg. y-dir.

x^2-3

▸ Dilation by factor of $\frac{1}{2}$ from the y-axis

$4\cdot x^2-3$

▸ Translation 1 unit along the pos. x-dir.

$4\cdot x^2-8\cdot x+1$

Overview:

Find the preimage of a function under a list of transformations. The program will display the list of inverted transformations and the transformed function after each step.

Input:

```
transform_inv(<function>, <variable>,
      <list of transformations>)
```

Other notes:

- The list of transformations can either be presented in a row or column matrix, or a list of expressions

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Section E: Exam 2 (29 Marks)

INSTRUCTION: 29 Marks. 37 Minutes Writing.

Question 7 (1 mark)

The graph of the function f passes through the point $(3, -4)$. If $h(x) = 4f(x-3)$, then the graph of the function h must pass through the point:

A. $(0, -4)$

B. $(3, -16)$

C. $(6, -16)$

D. $(6, -4)$

Question 8 (1 mark)

The graph of the function $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = 3^x - 2$, is reflected in the y-axis and then translated 3 units to the right and 1 unit up. Which one of the following is the rule of the transformed graph?

A. $y = 3^{-x} + 1$

B. $y = 3^{-x+3} + 1$

C. $y = \left(\frac{1}{3}\right)^{x-3} - 1$

D. $y = \frac{1}{3} . 3^{-x+3} + 1$

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Question 9 (1 mark)

The graph of the function g is obtained from the graph of the function:

$$f: [-1,2] \to \mathbb{R}, f(x) = 3x^2 - 6x + 6,$$

by a dilation of factor 3 from the y-axis, followed by a dilation of factor $\frac{1}{3}$ from the x-axis, followed by a reflection in the x-axis, and finally followed by a translation of 2 units in the positive direction of the y-axis. The domain and range of g are respectively:

A. $[-6,3]$ and $[0,3]$

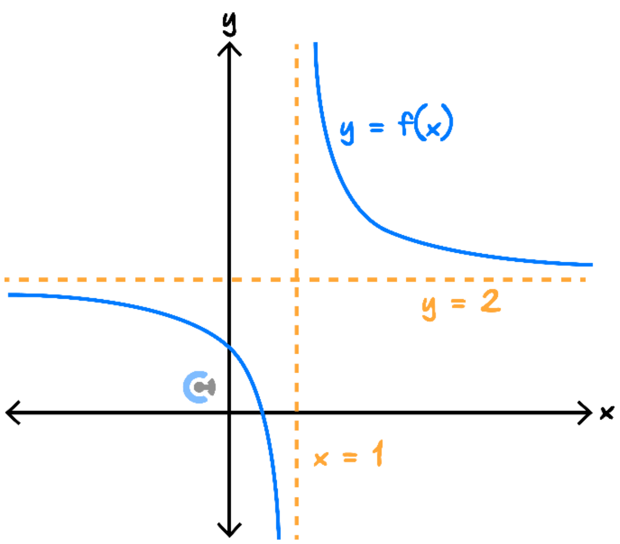
B. $[-3,6]$ and $[-3,0]$

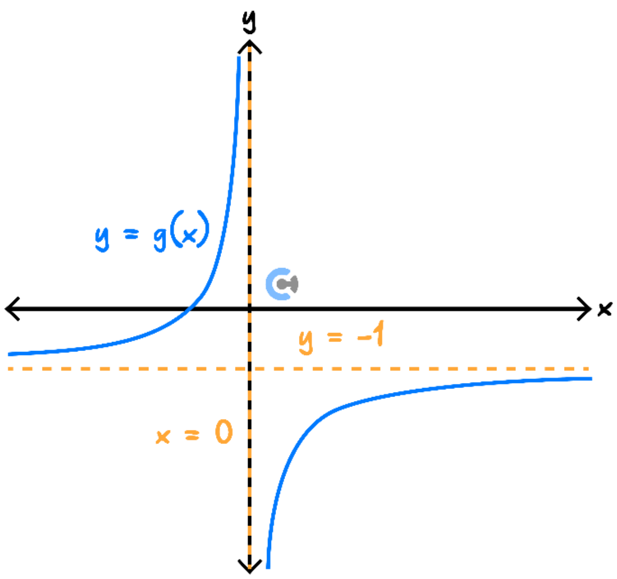
C. $[-3,6]$ and $[-3,1]$

D. $[-6,3]$ and $[-3,3]$

Question 10 (1 mark)

Consider the graphs of f and g, which have the same scale.





A. $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2, T(x,y) = (1-x, y-3)$

$$\boxed{f(x) = \frac{1}{x-1} + 2 \text{ and } g(x) = -\frac{1}{x} - 1}$$

B. $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2, T(x,y) = (x-1, y-3)$

C. $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2, T(x,y) = (x-1, 3-y)$

D. $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2, T(x,y) = (1-x, 3-y)$

Question 11 (1 mark)

If the graphs of $y = f(x)$ and $y = g(x)$ intersect at (a, b), then the graphs of $y = 3f\left(\frac{x}{2}\right)$ and $y = 3g\left(\frac{x}{2}\right)$ intersect at:

A. $\left(2a, \frac{3b}{2}\right)$

B. $\left(\frac{a}{2}, 3b\right)$

C. $(2a, 3b)$

D. $\left(3a, \frac{b}{2}\right)$

Question 12 (1 mark)

The line $y = -\frac{1}{2}x + 6$ is tangent to the graph of f when $x = 2$. The following sequence of transformations maps the graph of f onto the graph of g:

1. A dilation by a factor of 3 from the y-axis, followed by,
2. A translation of 4 units in the positive direction of the y-axis.

Which of the following statements is true?

A. The line $y = -\frac{1}{6}x + 8$ is tangent to g at the point $(2,10)$.

B. The line $y = -\frac{1}{6}x + 10$ is tangent to g at the point $(6,9)$.

C. The line $y = -\frac{1}{4}x + 8$ is tangent to g at the point $(6,10)$.

D. The line $y = -\frac{1}{4}x + 10$ is tangent to g at the point $(2,8)$.

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Question 13 (1 mark)

The image of the function $g(x) = 3x^5$ is $y = -\frac{4}{5}\left(\frac{x}{2} - 2\right)^5$. The transformations that could have been applied are:

A. Reflection in the x-axis, then translation in the positive direction of the x-axis by 2 units, followed by a dilation from the y-axis by a factor of $\frac{1}{2}$.

B. Reflection in the x-axis, then translation in the negative direction of the x-axis by 2 units, followed by a dilation from the x-axis by a factor of 5.

C. Reflection in the x-axis, then a dilation from the x-axis by a factor of $\frac{4}{15}$, followed by a translation in the positive direction of the x-axis by 2 units, and finally, a dilation from the y-axis by a factor of 2.

D. Reflection in the x-axis, then a dilation from the y-axis by a factor of $\frac{4}{5}$, followed by a translation in the negative direction of the x-axis by 3 units, and finally, a dilation from the x-axis by a factor of $\frac{1}{2}$.

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Question 14 (12 marks)

Consider the functions,

$$f: \mathbb{R} \longrightarrow \mathbb{R}, f(x) = \frac{1}{2}x^3 - \frac{3}{2}x + 1$$

$$g: \mathbb{R} \longrightarrow \mathbb{R}, g(x) = (2x+1)^2(x-1)$$

a.

i. Find the coordinates of the axial intercepts of f. (1 mark)

Solution: Solve $f(x) = 0$ to get the coordinates of the x-axis intercepts of $(-2, 0)$ and $(1, 0)$.
Evaluate $f(0)$ to get the coordinates of the y-axis intercepts of $(0, 1)$.

ii. Hence or otherwise, describe a sequence of **reflections and dilations** that map the graph of f onto the graph of g. (2 marks)

Solution: g has x-axis intercepts $\left(-\frac{1}{2}, 0\right)$ and $(1, 0)$ and y-axes intercept $(0, -1)$.
This helps us realise that the appropriate transformations are

- A dilation by factor $\frac{1}{2}$ from the y-axis
- A reflection in the x-axis
- A reflection in the y-axis.

iii. Describe a sequence of **dilations and translations** that map the graph of f onto the graph of g. (2 marks)

Solution: We note that $g(x) = f(2x) - 2$. Therefore,

- A dilation by factor $\frac{1}{2}$ from the y-axis
- A translation of 2 units down.

b. The equation to the tangent of g at $x = 1$ is $y = 9x - 9$. Use this to find the equation of the tangent to f when $x = -2$. (2 marks)

$f(x) = -g\left(-\frac{1}{2}x\right)$. Therefore,

$$y = -\left(-\frac{9}{2}x - 9\right)$$
$$y = \frac{9}{2}x + 9$$

Consider the following transformations:

$$T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2, T(x, y) = (-x - 1, 3y + 4)$$

$$S: \mathbb{R}^2 \longrightarrow \mathbb{R}^2, S(x, y) = (2x + 1, 2y - 3)$$

c. Find the rule for the image of g after it has undergone the transformation T followed by the transformation S. (3 marks)

Solution: Transformation $T : g_1(x) = 3g(-1 - x) + 4 = -12x^3 - 36x^2 - 27x - 2$

Then transformation $S : g_2(x) = 2g_1\left(\frac{1}{2}(x - 1)\right) - 3 = 5 - 9x^2 - 3x^3$

So the image of g is

$$g_2(x) = -3x^3 - 9x^2 + 5$$

d. Find the coordinates of the point $P(u, v)$, if the image of the point P under T and S is the same. (2 marks)

Solution: Solve the equations

$$-x - 1 = 2x + 1$$
$$3y + 4 = 2y - 3$$

$x = -\dfrac{2}{3}$ and $y = -7$. Therefore, $P\left(-\dfrac{2}{3}, -7\right)$

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Question 15 (10 marks)

Consider the function $f: (-2,2) \to \mathbb{R}, f(x) = (x-1)^2(2x+3)$.

a. State the range of f. (1 mark)

From the graph of f we see that the range is $(-9,7)$.

b. The following sequence of transformations, T, map the graph of f onto the graph of g.

- A dilation by a factor of 2 from the x-axis, followed by,
- A translation of 3 units down and 2 units left, followed by,
- A reflection in the y-axis.

i. State the rule of g. (2 marks)

$g(x) = 2f(-x+2) - 3 = -4x^3 + 22x^2 - 32x + 11$

ii. State the domain of g. (1 mark)

Solution: We apply the transformation $x \mapsto -(x-2)$ onto the interval $(-2,2)$ to get the domain of g.
Thus the domain of g is $(0,4)$.

iii. State the range of g. (1 mark)

Solution: We apply the transformation $y \mapsto 2y-3$ onto the interval $(-9,7)$ to get the range of g.
Thus the range of g is $(-21,11)$

c. The tangent to the graph of f at the point $A(-1,4)$ is given by the equation,

$$y = 4x + 8$$

i. Find the point B, the image of A under T. (1 mark)

$$\boxed{B = (3,5)}$$

ii. Find the equation of the tangent to the graph of g at point B. (1 mark)

Solution: Apply the tranformation T to the line

$$y = 29 - 8x$$

d. A transformation $S: \mathbb{R}^2 \longrightarrow \mathbb{R}^2, S(x,y) = (a - x, b - y)$ maps the graph of f onto itself. Determine the values of a and b. (3 marks)

Solution: $x' = a - x \implies x = a - x'$
$y' = b - y \implies y = b - y'$

$$b - y = f(a - x) \implies y = b - f(a - x)$$
$$y = -2a^3 + (6a^2 - 2a - 4)x + a^2 + (1 - 6a)x^2 + 4a + b + 2x^3 - 3$$

and the original is

$$y = 3 - 4x - x^2 + 2x^3$$

We compare coefficient to conclude that

$$a = \frac{1}{3} \quad \text{and} \quad b = \frac{125}{27}$$

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Section F: Extension Exam 1 (17 Marks)

INSTRUCTION: 17 Marks. 22 Minutes Writing.

Question 16 (5 marks)

Let $f: \mathbb{R} \to \mathbb{R}$, where $f(x) = 4x^3 + 1$, and let $g: \mathbb{R} \to \mathbb{R}$, where $g(x) = 2 - 2x$.

a.

i. Find $g\big(f(x)\big)$. (1 mark)

$$\boxed{g(f(x)) = -8x^3.}$$

ii. Find $f\big(g(x)\big)$ and express it in the form $k - m(x-d)^3$, where m, k, and d are integers. (2 marks)

$$\boxed{\begin{aligned} f(g(x)) &= 4(2(1-x))^3 + 1 \\ &= 32(1-x)^3 + 1 \\ &= 1 - 32(x-1)^3. \end{aligned}}$$

b. The transformation $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ with rule $T(x, y) = (x + b, ay + c)$ where a, b, and c are integers, maps the graph of $y = g(f(x))$ onto the graph of $y = f(g(x))$.

Find the values of a, b, and c. (2 marks)

Solution: We can obtain $f(g(x))$ from $g(f(x))$ by

- A dilation by factor 4 from the x-axis
- A translation 1 unit to the right
- A translation 1 unit up.

Therefore, $b = 1, a = 4$ and $c = 1$.

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Question 17 (3 marks)

Consider the functions,

$$f: [-4, \infty) \to \mathbb{R}, f(x) = x^2 + 4x + 2$$

$$g: (-\infty, 2] \to \mathbb{R}, g(x) = 4(2x - 3)^2 + 3$$

Describe a sequence of dilation followed by two translations and, lastly, a reflection that maps the graph of f onto the graph of g.

Solution: Looking at the domain of f and g, our reflection must be in the y-axis. Have that

$$f(x) = (x + 2)^2 - 2$$

Since we have 1 dilation, we can bring the 4 into the square in the rule of g to get

$$g(x) = (4x - 6)^2 + 3 = (6 - 4x)^2 + 3$$

Now we have that $x + 2 = 6 - 4x' \implies 4x' = 4 - x \implies x' = -\frac{1}{4}x' + 1$
We check and confirm that this also maps the domain $[-4, \infty)$ to $(-\infty, 2]$
Hence our transformations are,

- A dilation by factor $\frac{1}{4}$ from the y-axis
- A translation of 1 unit right
- A translation of 5 units up
- A reflection in the y-axis.

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Question 18 (4 marks)

Consider the function $f(x) = \sqrt{4x + c} - 1$ defined on its maximal domain and where c is a real number.

a. State the translation that maps the function $g(x) = \sqrt{4x} - 1$ to the function $f(x)$. (1 mark)

A translation $\frac{c}{4}$ units to the left.

b. Find the values of c for which the graphs of $y = f(x)$ and $y = f^{-1}(x)$ intersect twice. (3 marks)

Solution: Intersect on the line $y = x$. Solve

$$\sqrt{4x + c} - 1 = x \implies x = 1 \pm \sqrt{c}$$

So it must be that $c > 0$ for two solutions.
Now f has domain $\left[-\frac{c}{4}, \infty\right)$ and f^{-1} has domain $[-1, \infty)$
One of the solutions will not be valid if $1 - \sqrt{c} < -1 \implies c > 4$
Therefore, intersect twice for $0 < c < 4$

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Question 19 (5 marks)

Let $f: (-\infty, 2] \to \mathbb{R}, f(x) = 3x^2 - 12x + 20$ and $g: [-1, \infty) \to \mathbb{R}, g(x) = 2\sqrt{x+1} + 3$.

a. Describe a sequence of transformations that map the graph of f onto the graph of g^{-1}. (3 marks)

Solution: We first find that

$$g^{-1}(x) = \frac{1}{4}(x-3)^2 - 1$$

with dom $g^{-1} = [3, \infty)$.
Now we note that

$$f(x) = 3(x-2)^2 + 8$$

Note that to map the domain $(-\infty, 2]$ to $[3, \infty)$ we reflect in the y-axis and then shift 5 units right.
We see that $g^{-1}(x) = \frac{1}{12}f(-x+5) - \frac{5}{3}$
A sequence of transformations is

- A dilation by factor $\frac{1}{12}$ from the x-axis
- A reflection in the y-axis
- A translation 5 units right
- A translation $\frac{5}{3}$ units down.

b. Hence or otherwise, describe a sequence of transformations that map the graph of g onto the graph of f^{-1}. (2 marks)

Solution: Consider transforming g^{-1} to f then we swap x and y to get the transformation of g to f^{-1}
The required transformations are:

- A translation $\frac{5}{3}$ units right
- A translation 5 units down
- A reflection in the x-axis
- A dilation by factor 12 from the y-axis

Section G: Extension Exam 2 (16 Marks)

INSTRUCTION: 17 Marks. 22 Minutes Writing.

Question 20 (1 mark)

The transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$, which maps the graph of $y = 5 - \log_e\left(\frac{x+3}{4}\right)$ onto the graph of $y = \log_e(x)$ has the rule:

A. $T(x,y) = \left(\frac{x+3}{4}, 5-y\right)$

B. $T(x,y) = (4x-3, y-5)$

C. $T(x,y) = (4x-3, 5-y)$

D. $T(x,y) = \left(\frac{x+3}{4}, y+5\right)$

Question 21 (1 mark)

The image of the parabola $y = 2x^2 - 4x + 5$ after it is reflected in the line $x = p$ and then in the line $y = q$ is $y = -2x^2 + 20x - 51$.
The values of p and q are:

A. $p = 3$ and $q = -1$

B. $p = -3$ and $q = 1$

C. $p = 3$ and $q = 1$

D. $p = 1$ and $q = 3$

Space for Personal Notes

Question 22 (1 mark)

The mapping $T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = \left(\frac{12 - 3x - 4y}{5}, \frac{6 - 4x + 3y}{5}\right)$ reflects a point (x, y) in the line:

A. $y = 2x + 3$

B. $y = -2x + 3$

C. $y = -2x - 3$

D. $y = 2x - 3$

Question 23 (1 mark)

The image of the curve $y = \sqrt{x^2 + 4}$ under the transformation T, the equation $y = \frac{3}{2}\sqrt{x^2 + 6x + 25}$.

The transformation T could be described as:

A. A dilation by factor 3 from the y-axis followed by a dilation by factor 2 from the x-axis and a translation 3 units to the right.

B. A dilation by factor $\frac{1}{3}$ from the y-axis followed by a dilation by factor 3 from the x-axis and a translation 2 units to the right.

C. A dilation by factor 2 from the y-axis followed by a dilation by factor 3 from the x-axis and a translation 3 units to the left.

D. A dilation by factor 3 from the y-axis followed by a dilation by 3 from the x-axis and a translation 2 units to the left.

Space for Personal Notes

Question 24 (1 mark)

The area bounded by a quadratic function $f(x)$ and the x-axis is 12 square units.

Determine the area bounded by the function $\frac{1}{2}f\left(\frac{x}{4}\right)$ and the x-axis.

A. 12

B. 24

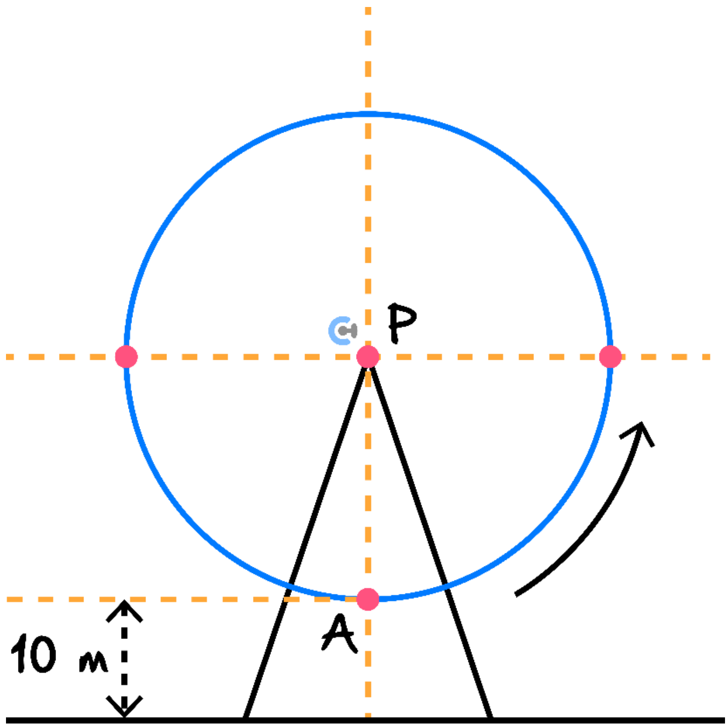
C. 48

D. 96

Space for Personal Notes

Question 25 (11 marks)

The following diagram represents a Ferris wheel, with its centre at point P. Passengers are seated in pods, which are carried around as the wheel turns. The wheel moves anticlockwise with constant speed and completes one full rotation every 20 minutes. When a pod is at the lowest point of the wheel (point A), it is 10 metres above the ground. The wheel has a radius of 40 metres.



The height of the bottom of a pod that was originally situated at the point A, t minutes after the start of a trip is given by,

$$h(t) = 50 - 40\cos\left(\frac{\pi t}{10}\right)$$

a. Describe a sequence of transformations, without using any reflections, that map the graph of $y = \sin(t)$ onto the graph of $y = h(t)$. (2 marks)

Solution: Observe that $\sin\left(t - \frac{\pi}{2}\right) = -\cos(t)$.
Thus our sequence of transformations is:

- A translation $\frac{\pi}{2}$ units to the right
- A dilation by factor $\frac{10}{\pi}$ from the y-axis
- A dilation by factor 40 from the x-axis
- A translation 50 units up

b. The horizontal displacement, d from the bottom of the pod to the centre of the Ferris Wheel t minutes after the start of a trip is,

$$d(t) = 40\sin\left(\frac{\pi t}{10}\right)$$

The transformation $T(t, y) = (t + a, y + b)$ maps the graph of d onto the graph of h.

i. Find b. (1 mark)

$$\boxed{b = 50}$$

ii. Find all possible values of a. (2 marks)

Solution: We require $40\sin\left(\frac{\pi(t-a)}{10}\right) = -40\cos\left(\frac{\pi t}{10}\right)$

Since $\sin\left(t - \frac{\pi}{2} + 2n\pi\right) = -\cos(t)$, where $n \in \mathbb{Z}$ we require that

$$-\frac{\pi a}{10} = -\frac{\pi}{2} + 2n\pi$$

Thus $a = 5 - 20n$ where $n \in \mathbb{Z}$.

c. 10 minutes into a trip on the Ferris Wheel, the Ferris Wheel malfunctions. This causes the Ferris Wheel to stop for 5 minutes before starting again at half speed. The height of the Ferris wheel in this trip, $h_1: [0, r] \to \mathbb{R}$ is given by the following function:

$$h_1(t) = \begin{cases} h(t) & 0 \le t < 10 \\ k & 10 \le t < 15 \\ h(pt + q) & 15 \le t \le r \end{cases}$$

The Ferris Wheel stops once a full rotation has been completed.

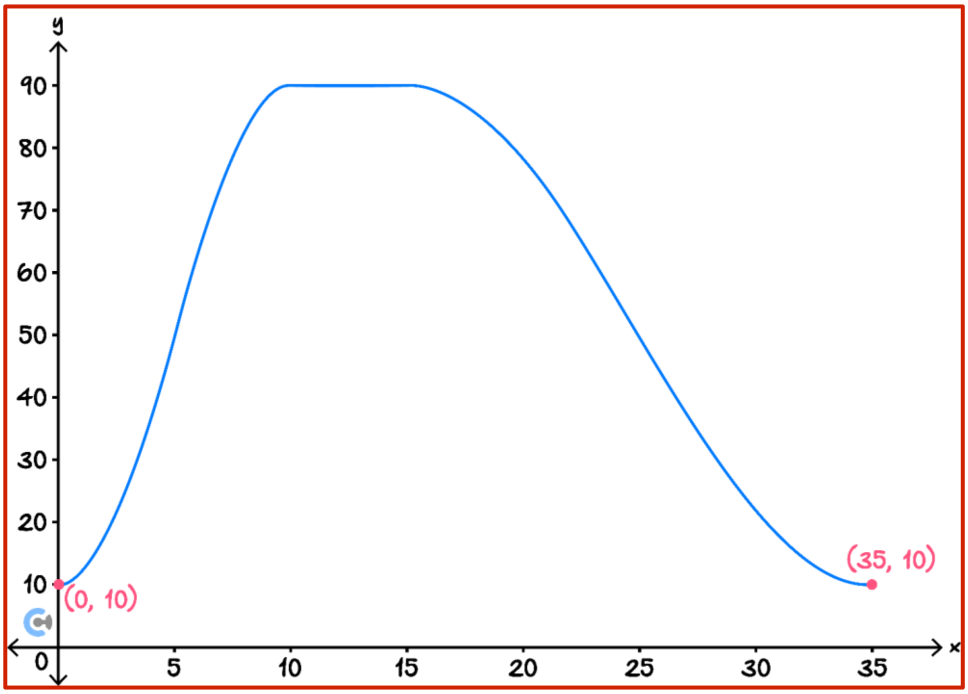
Find a set of possible values of p, q, k and r. (3 marks)

Solution: We know that $k = h(10) = 90$.
The Ferris wheel has completed half of its trip at the point where it stops. Thus once it starts again it takes 20 minutes to complete the trip since it is at half speed.

$$r = 15 + 20 = 35$$

Since we are going at half speed, after the crash we see that $p = \frac{1}{2}$. Now we require that $h\left(\frac{1}{2} \times 15 + q\right) = 90 \implies q = \frac{5}{2}$

d. Draw the graph of h_1 labelling endpoints with their coordinates. (3 marks)



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