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VCE Mathematical Methods ¾
SAC 1 Revision VI [0.21]
Workshop Solutions

Error Logbook:

New Ideas/Concepts	Didn't Read Question
Pg / Q #: ____________ Notes:	Pg / Q #: ____________ Notes:
Algebraic/Arithmetic/ Calculator Input Mistake	**Working Out Not Detailed Enough**
Pg / Q #: ____________ Notes:	Pg / Q #: ____________ Notes:

Section A: SAC 1 Success

Welcome to the sixth SAC 1 workshop!

Context: SAC 1 Workshops

- SAC 1 - 50% of SACs, 20% of the study score.
- Will be running all the way till mid-June.
- After that, the workshop will turn into SAC 2 Workshop (Integration).
- Make sure to complete the SAC $1 \sim 8$.

Successful SAC

Definition

Study Score = How much you know × How much you show

- Answer everything you know.
- Answer without mistakes.
- Time Management is **key!**

Tutor's Comment: Explain how even if you know everything, if you cannot show in the SAC, you will get 0.

Analogy: Skipping Questions

- Let's say if you were to fight them and win, you get the assigned marks.

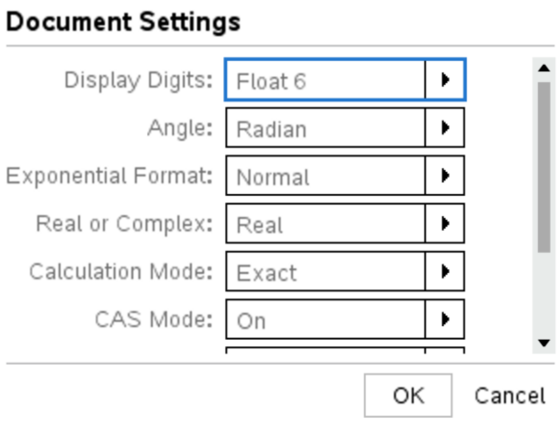
- Who would you fight first?
- Skip the hard questions with little marks if it doesn't make sense during the reading time.

Definition

SAC Proficiency List

Before the SAC:

- ☐ Prepare your stationery including a ruler, eraser, and your mechanical pencil lead.
- ☐ Skim through the bound reference (if applicable).
- ☐ Do not speak to other people and lock in.
- ☐ TI & Mathematica Only: Check your Contour UDFS.
- ☐ TI Only: Check technology settings.



Reading Time:

- ☐ **Detailed strategy** on how to exactly solve the question on your technology – Don't just **read**, think about how to solve it and using what technology commands.
- ☐ Identify questions to **skip.**

  For difficult SACs, it's not necessarily about getting the 100%. It's about getting better than everyone else. While others are stuck on a hard 1-marker question, you can do an easy 3-marker first.

- ☐ Identify questions to **start first** – You don't have to start from Q1!
- ☐ Look for potential pitfalls – **Units, Domain restriction of the unknown, variable and function meaning.**

Writing Time:

- ☐ **Circle what the question is asking** for in the question.
- ☐ Spend the first **50%** of the time on all the **easy questions** you identified.
- ☐ Spend the next **25%** of doing the **difficult questions** you left blank.

- [ ] Spend the last 25% of the time on **checking your answers**.
- [ ] Check your answer by reading the question again and see if you answered the question.
- [ ] Check in the order of:

  Domino Effect (check a, b, c first) > ***Questions with High Marks*** $(3+)$ > *Hard Questions*

- [ ] $\mathbf{TI}$ ONLY: Use new document - **doc** $\mathbf{4, 1.}$

After the SAC:

- [ ] Think about how each mark loss can be prevented using this proficiency list.
- [ ] Think about the big picture and improve the marks -

  Instead of spending 10 minutes on 10c. (1 mark), I should have checked 5a. (3 marks).

Space for Personal Notes

Section B: SAC Questions - Tech Active (53 Marks)

INSTRUCTION:

- 53 Marks. 15 Minutes Reading, 75 Minutes Writing.

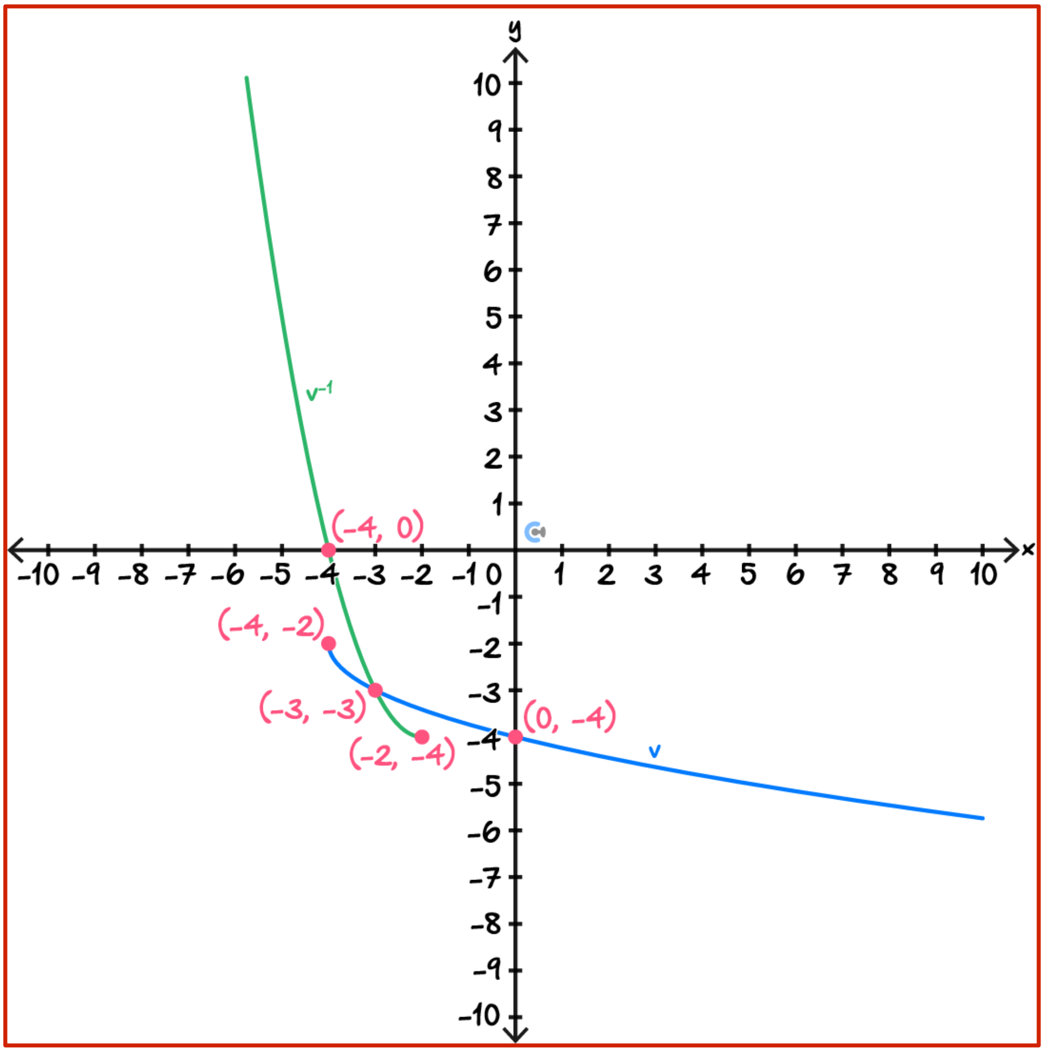
Question 1 (17 marks)

Two exploration drones, Vega and Orion, are traversing a crater basin on Mars. Their programmed movement follows synchronised but inverse flight paths. Each unit on the coordinate grid corresponds to 100 metres. Mission control is located at the origin (0,0), and the route of drone Vega is modelled by the function:

$$v : [-4, \infty) \to \mathbb{R}, v(x) = -2 - \sqrt{x+4}$$

Drone Orion's path is modelled by the inverse function $v^{-1}(x)$.

a. Sketch the path taken by drone Orion. Label all axis intercepts and endpoints with coordinates on the set of axes provided. (2 marks) [1A shape, 1A endpoint and y-intercept labelled]



b.

i. Find the rule, domain, and range of the path followed by drone Orion. (3 marks)

To find the rule for the inverse we solve $x = -2 - \sqrt{y+4} \implies y = x^2 + 4x$.
$v^{-1}(x) = x^2 + 4x$. **[1A]**
dom $v^{-1} = (-\infty, -2]$ **[1A]**
ran $v^{-1} = [-4, \infty)$ **[1A]**

ii. Find the function $v\left(v^{-1}(x)\right)$ and state its domain. (2 marks)

$v(v^{-1}(x)) = x$. **[1A]**
The domain is dom $v^{-1} = (-\infty, -2]$

c.

i. Find the point of intersection of the paths of the two drones. (2 marks)

They will intersect when $v(x) = v^{-1}(x)$ **[1M]**
Solving gives $x = -3$. The point of intersection is $(-3, -3)$. **[1A]**
(could also solve $v(x) = x$)

ii. On the same set of axes from **part a.**, sketch the graph of the path for drone Orion. Label all axis intercepts, the point of intersection, and any endpoints. (2 marks) [**1A shape, 1A endpoints and intersection**]

iii. Find the straight-line distance from the intersection point to mission control. (2 marks)

$d = \sqrt{3^2 + 3^2} = 3\sqrt{2}$. [**1M**]
So distance is $300\sqrt{2}$ metres. [**1A units required, remember we specified 1 unit = 100 metres**]

iv. Find the equation of the line connecting the intersection point to mission control. (1 mark)

$y = x$. **[1A]**

d. The drones are racing back to mission control to transmit critical geological data. Both drones take a path that is 540 m in length. Vega, detecting Orion's superior acceleration, switches to jet propulsion partway through. They both arrive at the origin simultaneously. Orion flies the whole path at a constant speed of 2.4 m/s. Vega begins at a speed of 1.8 m/s, then switches to jet propulsion which gives it a speed of 4.2 m/s. (Note: m/s is metres per second.)

i. How long, in seconds, did the drones take to reach mission control? (1 mark)

$$\frac{540}{2.4} = 225 \text{ seconds.}$$

ii. How far did Vega travel at 1.8 m/s before activating jet propulsion? (2 marks)

Let Vega fly at 1.8 m/s for s seconds and fly at 4.2 m/s for j seconds.
The total time is $t = s + j \implies 225 = s + j \implies j = 225 - s$
[1M for some correct equation linking variables]
Then we also have $540 = 1.8s + 4.2j \implies 540 = 1.8s + 4.2(225 - s) \implies s = 168.75.$

Therefore travels $168.75 \times 1.8 = 303.75$ metres before activating jet propulsion. **[1A]**

Space for Personal Notes

Question 2 (13 marks)

While venturing into the Forbidden Forest, Ron Weaselby is bitten by the giant spider, Aragog. Aragog's venom begins to spread through Ron's bloodstream. However, due to prior potions training, Ron's magical resistance slows down the spread after an initial surge. The concentration of venom in his bloodstream can be modelled by the function:

$$P(t) = ate^{\frac{10-kt}{4}}, 0 \leq t \leq 10$$

where, P is the number of venom units in the bloodstream, and t is the time in hours since being bitten.

After 3 hours, there are $15e$ units of venom in his bloodstream. After 5 hours, the venom level is measured at 25 units.

a. Use algebra to show that $a = 5$ and $k = 2$. (2 marks)

We have that $P(3) = 15e$ and $P(5) = 25$. This gives the simultaneous equations

$$3ae^{\frac{1}{4}(10-3k)} = 15e \quad (1)$$
$$5ae^{\frac{1}{4}(10-5k)} = 25 \quad (2)$$

[1M for simultaneous equations] We divide (1) by (2) to get

$$\frac{3}{5}e^{k/2} = \frac{15e}{25} \implies k = 2$$

then put $k = 2$ into (2) to get

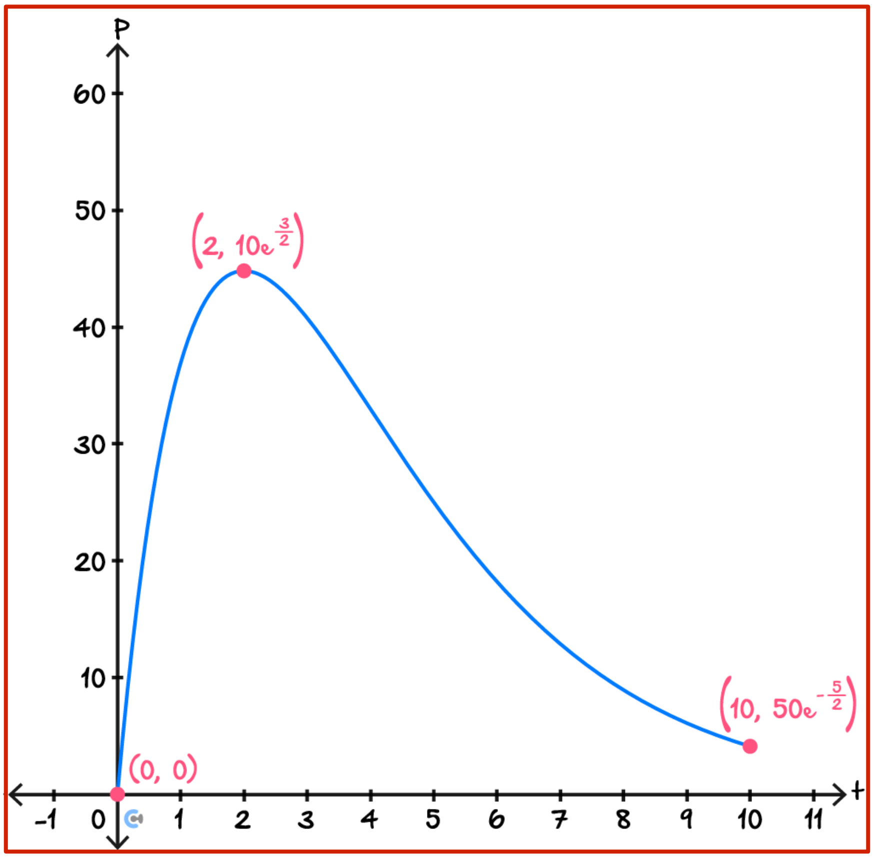
$$5a = 25 \implies a = 5$$

So we have $k = 2$ and $a = 5$. **[1A correct algebraic working]**

b. Find the maximum concentration of venom (to two decimal places), and the time (to the nearest minute) when this occurs. (2 marks)

We solve $P'(t) = 0$ **[1M]**
This yields $t = 2$ and $P(2) = 10e^{3/2} \approx 44.82$.
So max concentration of 44.82 when $t = 2$. **[1A]**

c. Sketch the graph of $P(t) = 5te^{\frac{10-2t}{4}}$, for $0 \leq t \leq 10$, labelling all endpoints and turning points with **exact coordinates.** (3 marks)



[1A shape, 1A endpoints, 1A maximum, can get 2 marks if all correct but used decimals rather than exact]

d. It is known that the venom becomes fatal if its concentration remains above 35 units for more than 3 hours.

i. Show that the venom is not fatal according to this model. (2 marks)

We solve $P(t) = 35 \implies t = 0.902, 3.754$ **[1M]**
Then by the shape of the graph it is above 35 for $3.754 - 0.902 = 2.85$ hours. Thus it is not fatal. **[1A]**

ii. If the concentration of venom in Ron's bloodstream now followed by the model:

$$P_1(t) = bte^{\frac{10-2t}{4}}, 0 \leq t \leq 10$$

where, $b \in \mathbb{R}$. Find the minimum value of b, correct to two decimal places, such that the venom is fatal to Ron. (2 marks)

Let the two times where it is at 35 be t_1 and t_2 where $t_2 > t_1$.
We must then solve the system of equations

$$P_1(t_1) = 35 \quad \text{and} \quad P_1(t_2) = 35 \quad \text{and} \quad t_2 - t_1 = 3 \quad \textbf{[1M]}$$

This gives $b = 5.13$ **[1A]**

Four hours after the initial bite, Ron is bitten again. The updated venom concentration is given by the piecewise function:

$$Q(t) = \begin{cases} P(t), & 0 \le t \le 4 \\ P(t) + P(t-4), & 4 < t \le 10 \end{cases}$$

e. Determine whether Ron will survive. Justify your answer. (2 marks)

From **part d.i** we know that it is not fatal for $0 \le t \le 4$.
So we may just consider the function $q(t) = P(t) + P(t-4)$ for $4 < t \le 10$.
Solve $q(t) = 35 \implies t = 4.03934, 8.61879$. **[1M]**
Therefore it is above 35 for 4.57945 hours and so is fatal. **[1A]**

Space for Personal Notes

Question 3 (16 marks)

An alien gemstone recovered from a meteorite impact site in the Atacama Desert exhibits oscillatory energy activity, which scientists model using the function:

$$A(t) = a\cos(bt + c) + d$$

where $a, b, c,$ and d are real constants, and t represents the number of hours after midnight.

Researchers note that the gemstone's activity diminishes in cool, low-light conditions. Archived experimental logs give the following parameter estimates:

$$a = 6$$
$$b = \frac{\pi}{12}$$
$$c = \frac{1}{6}$$
$$d = 9$$

Minimum activity corresponds to the gemstone's safest state. According to safety protocols, researchers are only permitted to access the gemstone when its activity level is below 3.6.

a.

i. Using the given values of a, b, c, d, determine the minimum possible value of the activity level A. (1 mark)

Minimum is $9 - 6 = 3$ **[1A]**

ii. Determine the time between midnight and 11:59 PM when the gemstone reaches minimum activity. Express the time correct to the nearest minute. (2 marks)

Solve $A(t) = 3 \implies t = 11.3634$ **[1M]**
So the time is 11 : 22 am. **[1A]**

iii. Determine the number of hours per day, when researchers are permitted to access the gemstone. Give your answer correct to two decimal places. (2 marks)

We solve $A(t) = 3.6 \implies t = 9.64058, 13.0862.$ [1M]
Then by the shape of the graph our answer is $13.0862 - 9.64058 = 3.45$ hours.

Recent studies suggest this specific gemstone displays a sensitivity to terrestrial environmental conditions. It is now known that:

- The amplitude parameter a is affected by relative humidity.
- The parameters b and c are affected by local light exposure.
- The value of d varies randomly.

A local cooling system helps regulate humidity. After being switched off, the humidity, and hence, the parameter a, decays according to the model:

$$a(w) = e^{-\frac{w}{2}} + 6, \qquad w \in [0,2.5]$$

where, w is the number of hours since the cooling system was deactivated.

b. The system was turned off at 3: 30 PM. If the current time is 5: 20 PM, evaluate the current value of a, correct to two decimal places. (1 mark)

$$a\left(1 + 50/60\right) = 6.40$$

c. This model may also be expressed in terms of t, the number of hours since midnight:

$$a(t) = e^{-\frac{(t-h)}{2}} + 6, \qquad t \in [15.5,18]$$

Determine the value of h. (1 mark)

3:30pm is 15.5 hours after midnight.
Thus $h = 15.5.$ [1A]

Light exposure data for the gemstone is recorded as a function of time t, where the intensity $L(t)$, measured in lumens, is defined piecewise as follows:

$$L(t) = \begin{cases} -\frac{t^2}{20} + \frac{13t}{10} - 6 & \text{for } 6 \le t < 8 \text{ or } 18 \le t < 20 \\ \frac{1}{2}\sin\left(\frac{2\pi t}{5}\right) + H & \text{for } 8 \le t < 18 \\ 0 & \text{otherwise} \end{cases}$$

d.

i. Determine the **exact** value of H that ensures **continuity** of $L(t)$. (2 marks)

Let $L_1(t) = -\frac{t^2}{20} + \frac{13t}{10} - 6$ and $L_2(t) = \frac{1}{2}\sin\left(\frac{2\pi t}{5}\right) + H$.
Note that L_2 has a period of 5 so $t = 18$ is exactly 2 periods away from $t = 8$
So, to be continuous we just require that $L_1(8) = L_2(8)$ **[1M]**
$H = \frac{1}{8}\sqrt{10 - 2\sqrt{5}} + \frac{6}{5}$ [**1A or equivalent** $H \approx 1.49389$]

ii. Identify all values of t for which light intensity $L(t)$ is maximised. (2 marks)

There will be max when $\sin\left(\frac{2\pi t}{5}\right) = 1$ and $8 < t < 18$. **[1M]**
$t = \frac{45}{4}, \frac{65}{4}$. **[1A]**

iii. Evaluate the maximum light intensity, correct to three decimal places. (1 mark)

$L\left(\frac{45}{4}\right) = 1.994$ **[1A]**

Based on the answers to **part c.** and **part d. i.** and using an approximate value of $d = 7$, an updated activity model is now adopted,

$$A_1(t) = a(t)\cos(L(t)\cdot t) + d, t \in [15.5, 18]$$

e.

i. Calculate the value of $A_1(t)$ at 5:20 PM. Round the result to two decimal places. (2 marks)

5:20pm is when $t = 17 + 20/60 = \frac{52}{3}$ [1M]

$a\left(\frac{52}{3}\right) = 6.40.$

$L\left(\frac{52}{3}\right) = 1.59785$

So

$$A_1\left(\frac{52}{3}\right) = 6.40\cos\left(1.59785 \cdot \frac{52}{3}\right) + 7 = 1.64 \quad [1A]$$

ii. Safety protocols have been updated and now gemstone contact is restricted if $A_1(t) > 2.2$. Between 3:30 PM and 6 PM, determine the values of t when gemstone contact is **restricted**. Give your answer correct to three decimal places. (2 marks)

We have that

$$A_1(t) = \left(e^{-\frac{t-15.5}{2}} + 6\right)\cos\left(t\left(\frac{1}{2}\sin\left(\frac{2\pi t}{5}\right) + \frac{1}{8}\sqrt{10 - 2\sqrt{5}} + \frac{6}{5}\right)\right)$$

We solve $A_1(t) > 2.2 \implies 15.6495 < t < 17.183$ or $17.3492 < t < 17.8692$.
So our answer is

$$15.650 < t < 17.183 \quad \text{or} \quad 17.349 < t < 17.869.$$

[1M, 2 correct values written anywhere, 1A both intervals correctly specified]

Space for Personal Notes

Question 4 (7 marks)

A snowboarder is sliding down a hill shaped by the function $f(x) = x^3 - 3x^2 + 3$. One unit on the graph corresponds to 100 metres.

At a certain point, he fires a laser beam that follows the tangent to the curve, and then rides down to x-value where the beam hits the x-axis — the predicted root of f using Newton's Method — then fires the laser again from this new location.

a.

i. Find the equation of the tangent to f when $x = 1$. (1 mark)

We have $f(1) = 1$ and $f'(1) = -3$.
Thus the equation of the tangent is $y = -3x + 4$.

ii. Newton's method is used to find a root to $f(x)$ with initial value $x_0 = 1$. Use your answer to **part a.i** to find the value of x_1. (2 marks)

Our value for x_1 will be the x-intercept of the tangent drawn at $x = x_0$.
We solve $-3x + 4 = 0 \implies x = \frac{4}{3}$.
[1M must use previous part cant just use newton formula for method mark]
So $x_1 = \frac{4}{3}$ **[1A]**

The snowboarder first fires his laser when he is at point (1,1) and he will stop firing the laser when the next predicted x-intercept of the laser differs by less than **1 metre** from the previous prediction. A computer scientist, Anuk, is modeling this behavior in a pseudocode implementation of Newton's method as shown below. (Recall that one unit on the graph corresponds to 100 metres).

define $f(x)$
    return $x^3 - 3x^2 + 3$

define $f'(x)$
    return $3x^2 - 6x$

$xnext \leftarrow 1$
$xprev \leftarrow 50$
while $|xnext - xprev| \geq$ 0.01
    $xprev \leftarrow xnext$
    $xnext \leftarrow xprev - \dfrac{f(xprev)}{f'(xprev)}$
    print $xnext$
end while

b. Fill in the **two** blanks in the pseudocode above. (2 marks)

c. How many times will the snowboarder fire the laser? (2 marks)

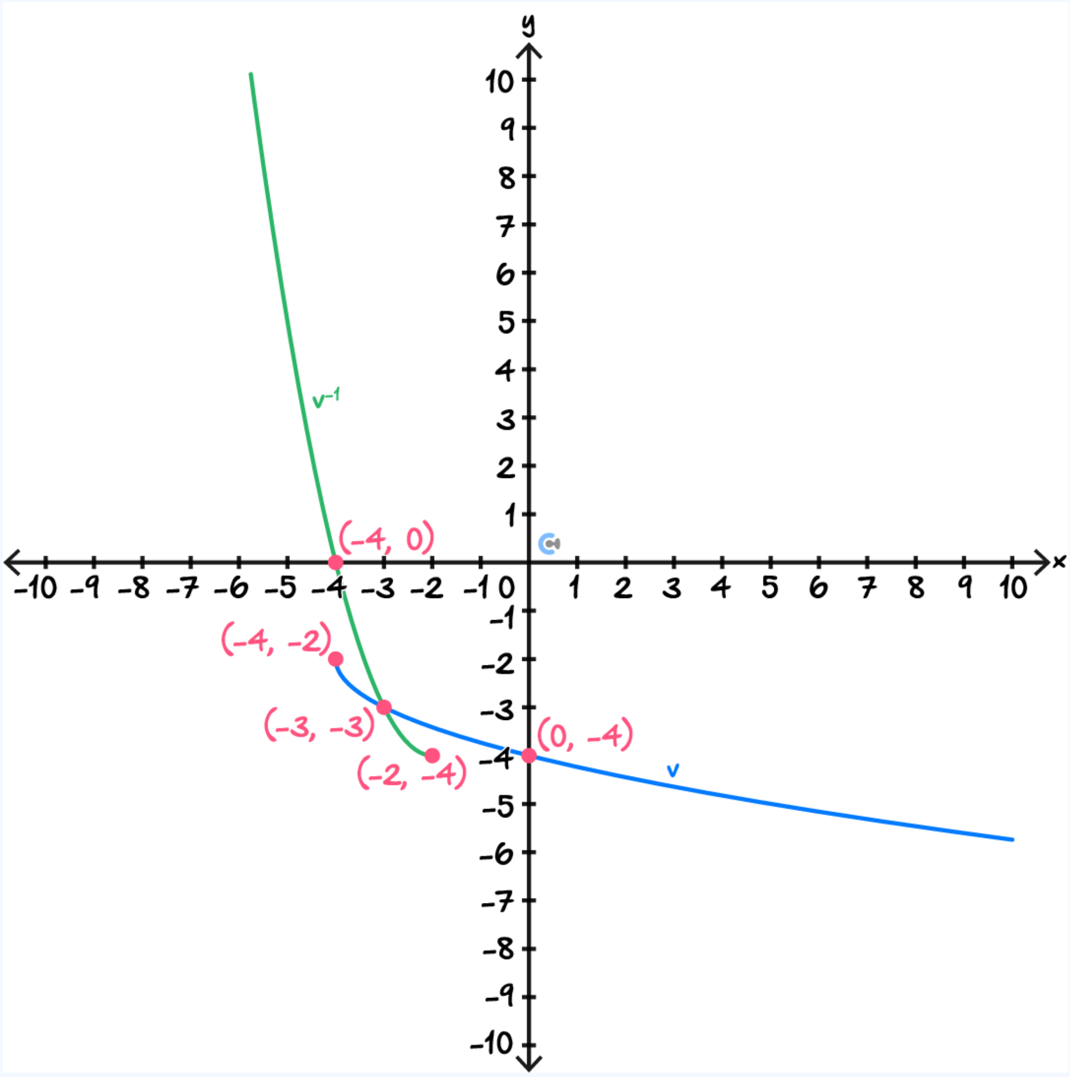
We perform Newton's method starting with $x_0 = 1$, until $|x_{n+1} - x_n| < 0.01$:

$$x_0 = 1$$
$$x_1 = 1.3333$$
$$x_2 = 1.34722$$
$$x_3 = 1.3473 \quad [1M]$$

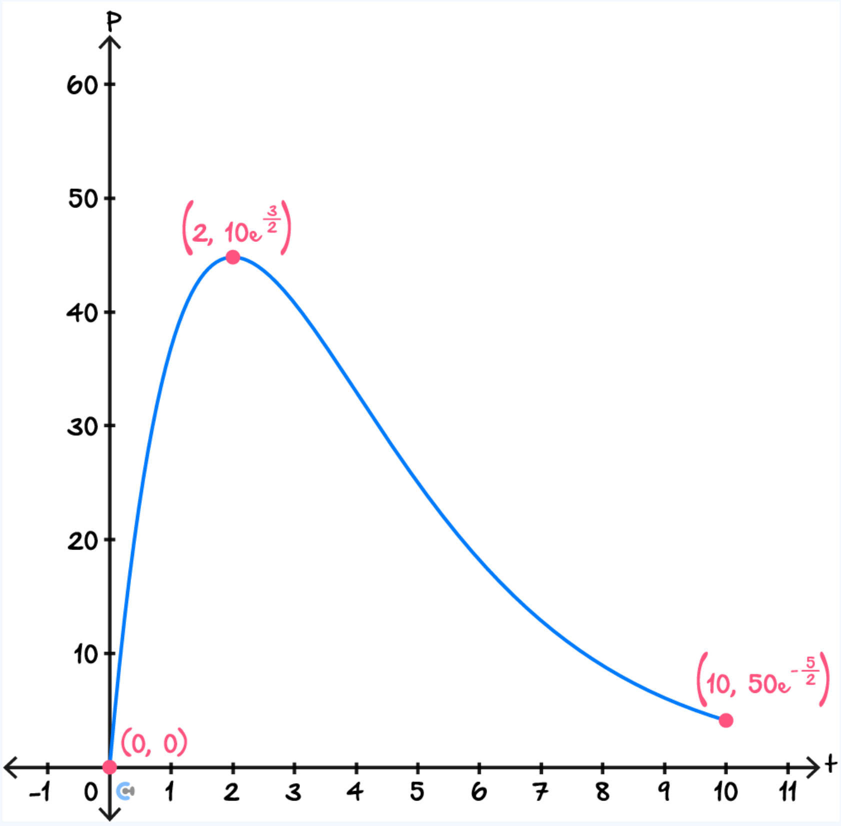
So the snowboarder fires the laser 3 times. **[1A]**

Space for Personal Notes

Section C: Marking Scheme

Question Number	Solutions
1. a.	
1. b. i	To find the rule for the inverse we solve $x = -2 - \sqrt{y+4} \implies y = x^2 + 4x$. $v^{-1}(x) = x^2 + 4x$. **[1A]** $\text{dom } v^{-1} = (-\infty, -2]$ **[1A]** $\text{ran } v^{-1} = [-4, \infty)$ **[1A]**
1. b. ii	$v(v^{-1}(x)) = x$. **[1A]** The domain is $\text{dom } v^{-1} = (-\infty, -2]$

1. c. i	They will intersect when $v(x) = v^{-1}(x)$ **[1M]** Solving gives $x = -3$. The point of intersection is $(-3, -3)$. **[1A]** (could also solve $v(x) = x$)
1. c. ii	**[1A shape, 1A endpoints and intersection]**
1. c. iii	$d = \sqrt{3^2 + 3^2} = 3\sqrt{2}$. **[1M]** So distance is $300\sqrt{2}$ metres. **[1A units required, remember we specified 1 unit = 100 metres]**
1. c. iv	$y = x$. **[1A]**
1. d. i	$\frac{540}{2.4} = 225$ seconds.
1. d. ii	Let Vega fly at 1.8 m/s for s seconds and fly at 4.2 m/s for j seconds. The total time is $t = s + j \implies 225 = s + j \implies j = 225 - s$ **[1M for some correct equation linking variables]** Then we also have $540 = 1.8s + 4.2j \implies 540 = 1.8s + 4.2(225 - s) \implies s = 168.75$. Therefore travels $168.75 \times 1.8 = 303.75$ metres before activating jet propulsion. **[1A]**
2. a.	We have that $P(3) = 15e$ and $P(5) = 25$. This gives the simultaneous equations $3ae^{\frac{1}{4}(10-3k)} = 15e \quad (1)$ $5ae^{\frac{1}{4}(10-5k)} = 25 \quad (2)$ **[1M for simultaneous equations]** We divide (1) by (2) to get $\frac{3}{5}e^{k/2} = \frac{15e}{25} \implies k = 2$ then put $k = 2$ into (2) to get $5a = 25 \implies a = 5$ So we have $k = 2$ and $a = 5$. **[1A correct algebraic working]**

2. b.	We solve $P'(t) = 0$ **[1M]** This yields $t = 2$ and $P(2) = 10e^{3/2} \approx 44.82$. So max concentration of 44.82 when $t = 2$. **[1A]**
2. c.	
2. d. i	We solve $P(t) = 35 \implies t = 0.902, 3.754$ **[1M]** Then by the shape of the graph it is above 35 for $3.754 - 0.902 = 2.85$ hours. Thus it is not fatal. **[1A]**
2. d. ii	Let the two times where it is at 35 be t_1 and t_2 where $t_2 > t_1$. We must then solve the system of equations $P_1(t_1) = 35 \quad \text{and} \quad P_1(t_2) = 35 \quad \text{and} \quad t_2 - t_1 = 3$ **[1M]** This gives $b = 5.13$ **[1A]**
2. e.	From **part d.i** we know that it is not fatal for $0 \le t \le 4$. So we may just consider the function $q(t) = P(t) + P(t-4)$ for $4 < t \le 10$. Solve $q(t) = 35 \implies t = 4.03934, 8.61879$. **[1M]** Therefore it is above 35 for 4.57945 hours and so is fatal. **[1A]**

3. a. i	Minimum is $9 - 6 = 3$ **[1A]**
3. a. ii	Solve $A(t) = 2 \implies t = 11.3634$ **[1M]** So the time is 11 : 22 am. **[1A]**
3. a. iii	We solve $A(t) = 3.6 \implies t = 9.64058, 13.0862.$ **[1M]** Then by the shape of the graph our answer is $13.0862 - 9.64058 = 3.45$ hours.
3. b.	$a\,(1 + 50/60) = 6.40$
3. c.	3:30pm is 15.5 hours after midnight. Thus $h = 15.5$. **[1A]**
3. d. i	Let $L_1(t) = -\frac{t^2}{20} + \frac{13t}{10} - 6$ and $L_2(t) = \frac{1}{2}\sin\left(\frac{2\pi t}{5}\right) + H$. Note that L_2 has a period of 5 so $t = 18$ is exactly 2 periods away from $t = 8$ So, to be continuous we just require that $L_1(8) = L_2(8)$ **[1M]** $H = \frac{1}{8}\sqrt{10 - 2\sqrt{5}} + \frac{6}{5}$ **[1A or equivalent** $H \approx 1.49389$**]**
3. d. ii	There will be max when $\sin\left(\frac{2\pi t}{5}\right) = 1$ and $8 < t < 18$. **[1M]** $t = \frac{45}{4}, \frac{65}{4}$. **[1A]**
3. d. iii	$L\left(\frac{45}{4}\right) = 1.994$ **[1A]**

3. e. i	5:20pm is when $t = 17 + 20/60 = \frac{52}{3}$ **[1M]** $a\left(\frac{52}{3}\right) = 6.40.$ $L\left(\frac{52}{3}\right) = 1.59785$ So $$A_1\left(\frac{52}{3}\right) = 6.40\cos\left(1.59785 \cdot \frac{52}{3}\right) + 7 = 1.64 \quad \textbf{[1A]}$$
3. e. ii	We have that $$A_1(t) = \left(e^{-\frac{t-15.5}{2}} + 6\right)\cos\left(t\left(\frac{1}{2}\sin\left(\frac{2\pi t}{5}\right) + \frac{1}{8}\sqrt{10-2\sqrt{5}} + \frac{6}{5}\right)\right)$$ We solve $A_1(t) > 2.2 \implies 15.6495 < t < 17.183$ or $17.3492 < t < 17.8692$. So our answer is $$15.650 < t < 17.183 \quad \text{or} \quad 17.349 < t < 17.869.$$ **[1M, 2 correct values written anywhere, 1A both intervals correctly specified]**
4. a. i	We have $f(1) = 1$ and $f'(1) = -3$. Thus the equation of the tangent is $y = -3x + 4$.
4. a. ii	Our value for x_1 will be the x-intercept of the tangent drawn at $x = x_0$. We solve $-3x + 4 = 0 \implies x = \frac{4}{3}$. **[1M must use previous part cant just use newton formula for method mark]** So $x_1 = \frac{4}{3}$ **[1A]**
4. b.	The blanks are: $\boxed{3x^2 - 6x}$ for the derivative $\boxed{0.01}$ for the stopping condition (1 metre = 0.01 units)
4. c.	We perform Newton's method starting with $x_0 = 1$, until $\lvert x_{n+1} - x_n \rvert < 0.01$: $x_0 = 1$ $x_1 = 1.3333$ $x_2 = 1.34722$ $x_3 = 1.3473$ **[1M]** So the snowboarder fires the laser 3 times. **[1A]**

Section D: Mathematica Solutions

Question Number | **Solutions**

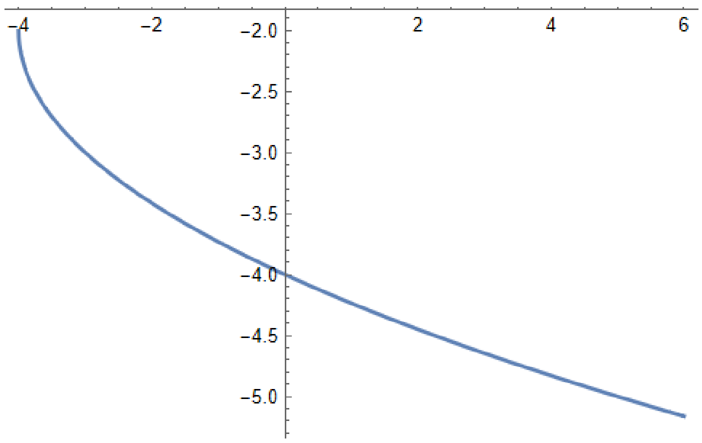
1

Q1.

In[105]:= v[x_] := -2 - $\sqrt{x+4}$

In[106]:= Plot[v[x], {x, -4, 6}]

Out[106]=



In[107]:= Solve[v[y] == x, y]

Solve: There may be values of the parameters for which some or all solutions are not valid.

Out[107]= $\{\{y \to 4x + x^2\}\}$

In[108]:= $\sqrt{3\text{^}2 + 3\text{^}2}$

Out[108]= $3\sqrt{2}$

In[109]:= 540 / 2.4

Out[109]= 225.

In[110]:= Solve[540 == 1.8 s + 4.2 (225 - s)]

Out[110]= {{s → 168.75}}

In[111]:= 168.75 * 1.8

Out[111]= 303.75

In[112]:= 540 - 303.75

Out[112]= 236.25

In[113]:= $\frac{303.75}{1.8} + \frac{236.25}{4.2}$

Out[113]= 225.

Q2.

```
In[114]:= P[t_] := a t Exp[(10 - k t)/4]

In[115]:= P[3]
```

Out[115]= $3\,a\,e^{\frac{1}{4}(10-3k)}$

```
In[116]:= P[5]
```

Out[116]= $5\,a\,e^{\frac{1}{4}(10-5k)}$

```
In[117]:= P[3]/P[5] // FullSimplify
```

Out[117]= $\frac{3\,e^{k/2}}{5}$

```
In[118]:= Solve[P[3] == 15 E && P[5] == 25, Reals]
Out[118]= {{a → 5, k → 2}}

In[119]:= p[t_] := 5 t Exp[(10 - 2 t)/4]

In[120]:= Solve[p'[t] == 0]
Out[120]= {{t → 2}}

In[121]:= p[2]
```

Out[121]= $10\,e^{3/2}$

2

In[122]:= N[$10\,e^{3/2}$]

```
Out[122]= 44.8169

In[123]:= p[10]
```

Out[123]= $\frac{50}{e^{5/2}}$

```
In[124]:= NSolve[p[t] == 35, t, Reals]
Out[124]= {{t → 0.902084}, {t → 3.75362}}

In[125]:= 3.7536223914736877` - 0.902083648217979`
Out[125]= 2.85154

In[126]:= p1[t_] := b t Exp[(10 - 2 t)/4]

In[127]:= NSolve[p1[t1] == 35 && p1[t2] == 35 && t2 - t1 == 3]
Out[127]= {{b → 5.12986, t1 → 0.861651, t2 → 3.86165}}

In[128]:= p[t] + p[t - 4] // FullSimplify
```

Out[128]= $5\,e^{\frac{5}{2}-\frac{t}{2}}\left(e^{2}\,(-4+t)+t\right)$

In[129]:= q[t_] := $5\,e^{\frac{5}{2}-\frac{t}{2}}\left(e^{2}\,(-4+t)+t\right)$

```
In[130]:= Solve[q[t] == 35, t] // N
```

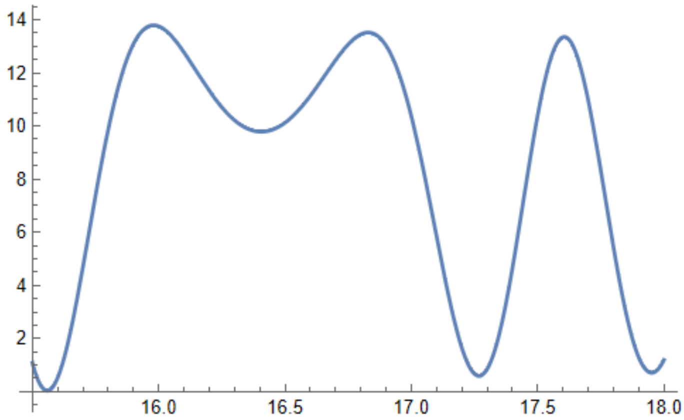
Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

```
Out[130]= {{t → 4.03934}, {t → 8.61879}}

In[131]:= 8.618792026402854 - 4.039344678948651
Out[131]= 4.57945
```

3	**Q3.**

```
In[132]:= A[t_] := 6 Cos[Pi / 12 t + 1 / 6] + 9

In[133]:= Solve[A[t] == 3 && 0 ≤ t ≤ 24] // N
Out[133]= {{t → 11.3634}, {t → 11.3634}}

In[134]:= .3634 * 60
Out[134]= 21.804

In[135]:= Solve[A[t] == 3.6 && 0 ≤ t ≤ 24]
  Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a co
Out[135]= {{t → 9.64058}, {t → 13.0862}}

In[136]:= 13.086175745176893 - 9.640584710087943
Out[136]= 3.44559

In[137]:= d[x_] := 8 x / (x^2 - 4 x + 8) + 9

In[138]:= d[8 / 12]
Out[138]= 129/13

In[139]:= N[129/13]
Out[139]= 9.92308

In[140]:= a[w_] := Exp[-w / 2] + 6

In[141]:= a[1 + 50 / 60] // N
Out[141]= 6.39985

In[142]:= L1[t_] := -t^2 / 20 + 13 t / 10 - 6

In[143]:= L2[t_] := 1 / 2 Sin[2 Pi t / 5] + H

In[144]:= L1[8]
Out[144]= 6/5

In[145]:= Solve[L2[8] == 6 / 5, H] // FullSimplify
Out[145]= {{H → 6/5 + 1/8 Sqrt[10 - 2 Sqrt[5]]}}

In[146]:= Solve[L2[8] == 6 / 5, H] // N
Out[146]= {{H → 1.49389}}

In[147]:= Solve[Sin[2 Pi t / 5] == 1 && 8 < t < 18]
Out[147]= {{t → 45/4}, {t → 45/4}, {t → 65/4}, {t → 65/4}}

In[148]:= L2[45 / 4] /. H → 1.4938926261462369
Out[148]= 1.99389
```

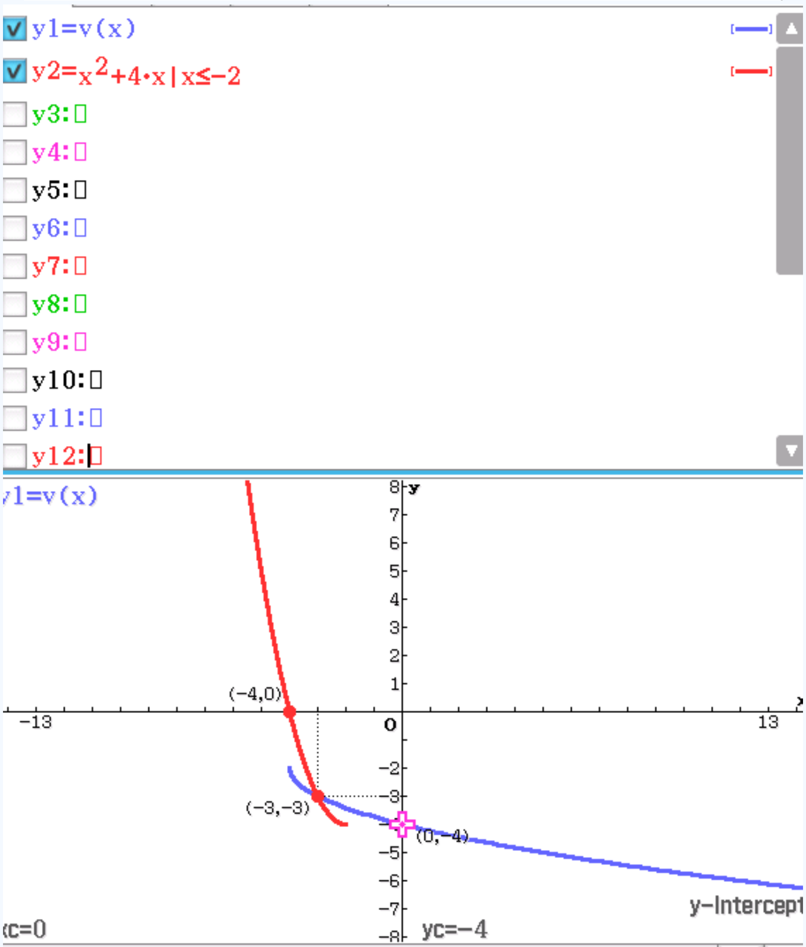
```
In[149]:= 17 + 20 / 60

Out[149]= 52/3

In[150]:= L2[52 / 3] /. H → 1.4938926261462369

Out[150]= 1.59785

In[151]:= 6.399849654344847 * Cos[1.5978484715551164 * 52 / 3] + 7 // N

Out[151]= 1.64079

In[152]:= A1[t_] := (Exp[-(t - 15.5)/2] + 6) * Cos[t * (1 / 2 Sin[2 Pi t / 5] + 6/5 + 1/8 Sqrt[10 - 2 Sqrt[5]])] +

In[153]:= A1[52 / 3]

Out[153]= 1.64079

In[154]:= Plot[A1[t], {t, 15.5, 18}]

Out[154]=
```




```
In[155]:= Reduce[A1[t] > 2.2 && 15.5 ≤ t ≤ 18]

Reduce: Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a cc

Out[155]= 15.6495 < t < 17.183 || 17.3492 < t < 17.8692
```

4	**Q4.** In[158]:= `f[x_] := x^3 - 3 x^2 + 3` In[159]:= `TangentLine[f[x], x, 1]` Out[159]= $4 - 3x$ In[160]:= `Solve[4 - 3 x == 0, x]` Out[160]= $\left\{\left\{x \to \frac{4}{3}\right\}\right\}$ In[161]:= `n[x_] := x - f[x]/f'[x]` In[162]:= `n[1.0]` Out[162]= 1.33333 In[163]:= `n[n[1.0]]` Out[163]= 1.34722 In[164]:= `n[n[n[1.0]]]` Out[164]= 1.3473

Section E: Casio Solutions

Question Number	Solutions
1	Define v(x)=−2−$\sqrt{x+4}$ done solve(v(y)=x, y {y=x^2+4·x} solve(v(x)=x, x {x=−3} $\sqrt{3^2+3^2}$ 3·$\sqrt{2}$ 540/2.4 225 solve(540=1.8s+4.2(225−s), s {s=168.75} 168.75*1.8 303.75 540−ans 236.25 303.75/1.8+236.25/4.2 225 

2

Define $p(t)=a*t*e^{\frac{10-k*t}{4}}$

done

simplify(p(3))

$3\cdot a\cdot e^{\frac{-3\cdot k}{4}+\frac{5}{2}}$

simplify(p(5))

$5\cdot a\cdot e^{\frac{-5\cdot k}{4}+\frac{5}{2}}$

simplify(p(3)/p(5))

$\frac{3\cdot e^{\frac{k}{2}}}{5}$

$\begin{cases} p(3)=15e^{1} \\ p(5)=25 \end{cases}\Big|_{a,k}$

{a=5, k=2}

Define $p(t)=a*t*e^{\frac{10-k*t}{4}}$ | {a=5, k=2}

done

fmax(p(t), t, 0, 10

$\left\{MaxValue=10\cdot e^{\frac{3}{2}}, t=2\right\}$

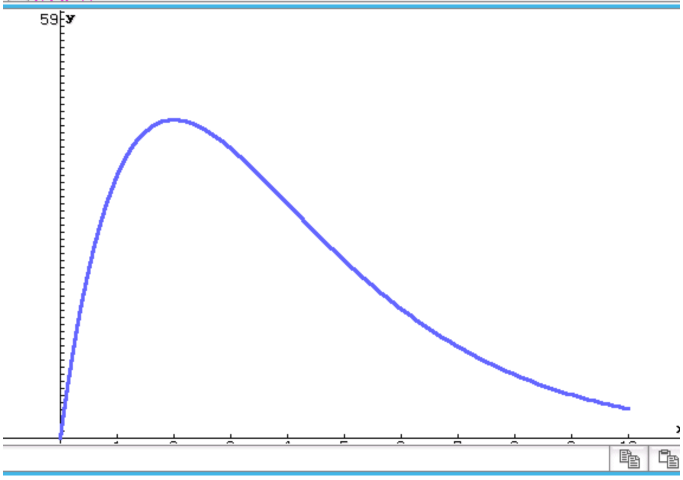
solve($\frac{d}{dt}$(p(t))=0, t

{t=2}

p(2)

44.8168907

y1=p(x) | 0≤x≤10
y2:
y3:
y4:
y5:
y6:
y7:
y8:
y9:
y10:
y11:
y12:
y13:



p(0)

$$0$$

p(10)

$$50\cdot e^{-\frac{5}{2}}$$

solve(p(t)=35,t

{t=0.9020836482,t=3.753622391}

3.753622391−0.9020836482

2.851538743

$$\text{Define } p1(t)=b*t*e^{\frac{10-2t}{4}}$$

done

$$\begin{cases} p1(t1)=35 \\ p1(t2)=35 \\ t2-t1=3 \end{cases}\Bigg|b,t1,t2$$

{b=5.129861645,t1=0.8616507504,t2=3.86165075}

solve(p(t)+p(t−4)=35,t

{t=4.039344679,t=8.618792026}

8.618792026−4.039344679

4.579447347

3	Define $a(t)=6\cos(\frac{\pi}{12}*t+1/6)+9$ done fMin(a(t),t,0,24) {MinValue=3, t=11.36338023} solve(a(t)=3|0≤t≤24,t {t=11.36338023} solve(a(t)=3.6|0≤t≤24,t {t=9.64058471, t=13.08617575} 13.08617575−9.64058471 3.44559104 Define $a(w)=e^{-w/2}+6$ done a(1+50/60) 6.399849654 Define l1(t)=−t^2/20+13t/10−6 done Define l2(t)=1/2sin(2πt/5)+h done l1(8) $\frac{6}{5}$ solve(l1(8)=l2(8),h $\left\{h=\frac{\sqrt{2\cdot(-\sqrt{5}+5)}}{8}+\frac{6}{5}\right\}$ solve(sin(2πt/5)=1|8<t<18,t $\left\{t=\frac{45}{4}, t=\frac{65}{4}\right\}$

$$l2\left(\frac{45}{4}\right) \mid \left\{h=\frac{\sqrt{2\cdot(-\sqrt{5}+5)}}{8}+\frac{6}{5}\right\}$$

1.993892626

17+20/60

$\frac{52}{3}$

$$l2(52/3) \mid \left\{h=\frac{\sqrt{2\cdot(-\sqrt{5}+5)}}{8}+\frac{6}{5}\right\}$$

1.597848472

a(1+50/60)*cos(1.597848472*52/3)+7

1.640788142

$$\text{Define } a1(t)=\left(e^{\frac{-(t-15.5)}{2}}+6\right)*\cos\left(t*\left(1/2\sin(2\pi t/5)+\frac{\sqrt{2\cdot(-\sqrt{5}+5)}}{8}+\frac{6}{5}\right)\right)+7$$

done

a1(52/3)

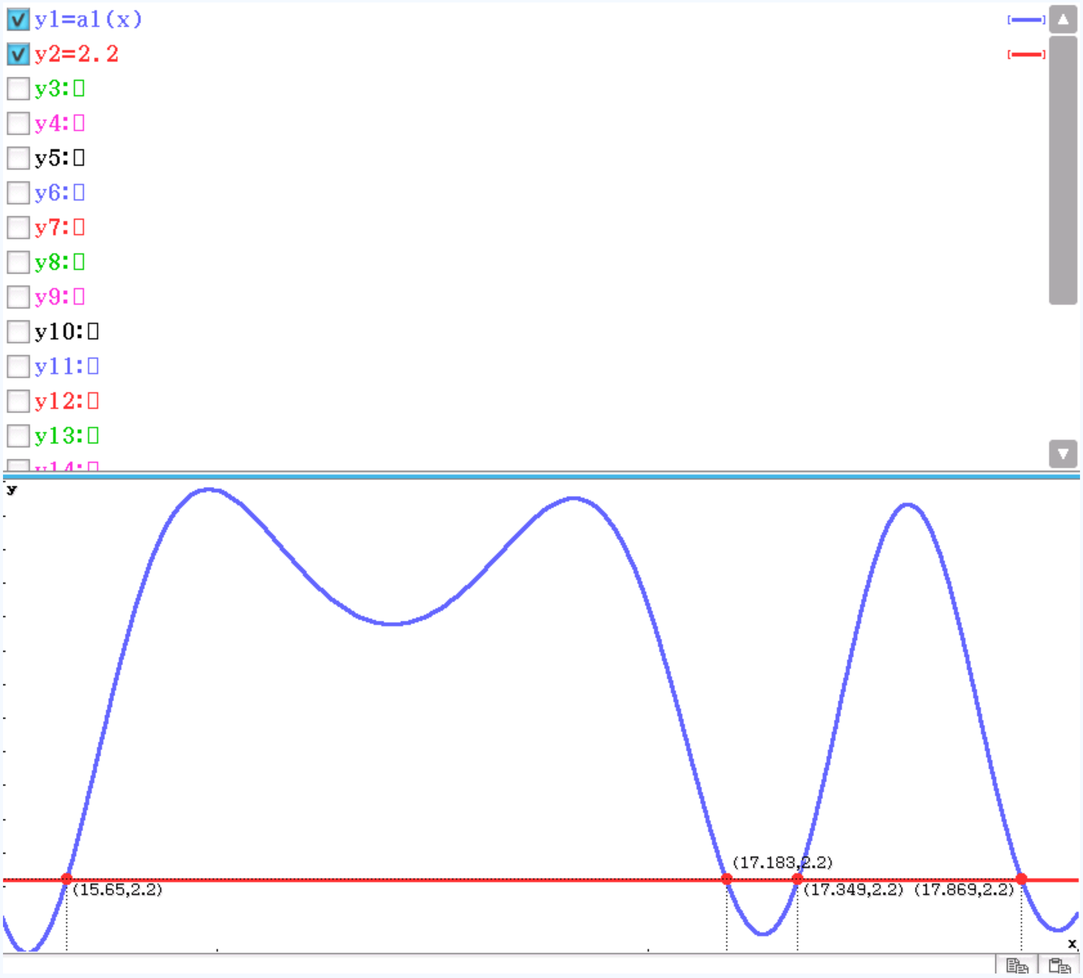
1.640788169

solve(a1(t)=2.2,t,0,15.5,18)

{t=15.64954649,t=17.18296186,t=17.3491649,t=17.86917506}

Note: Use "numerical solve" on CASIO – Interactive, advanced, solve, solve numerically

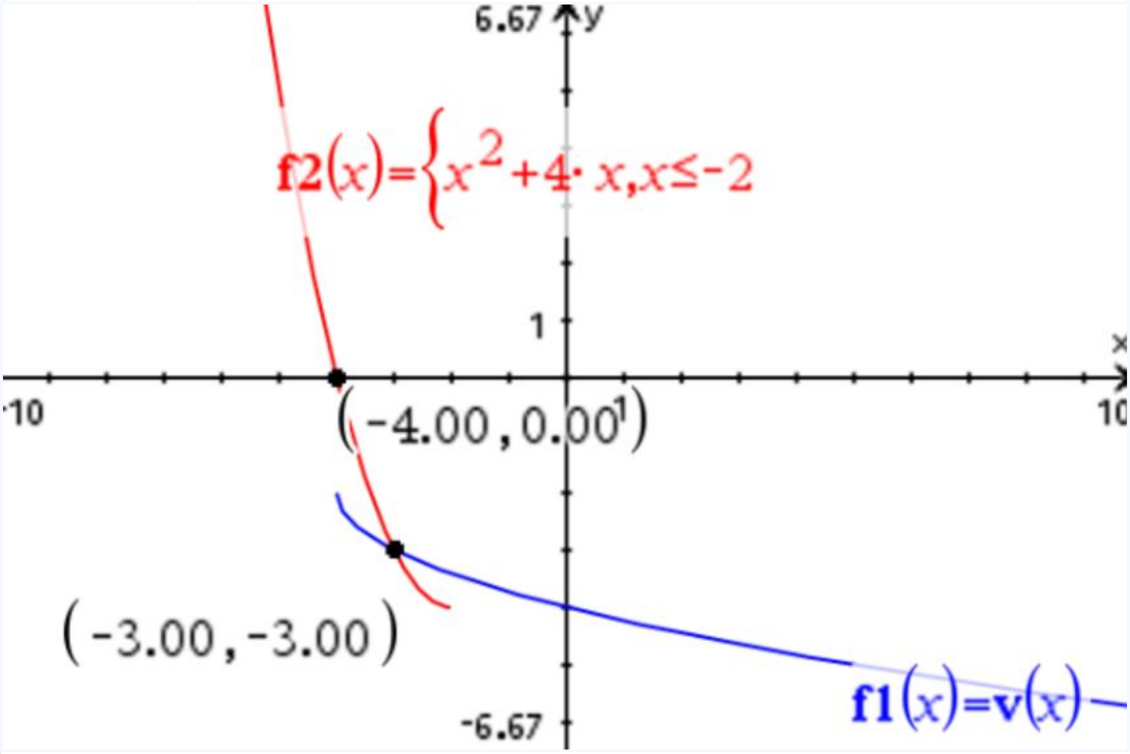
Or, use graph & table to find intersection

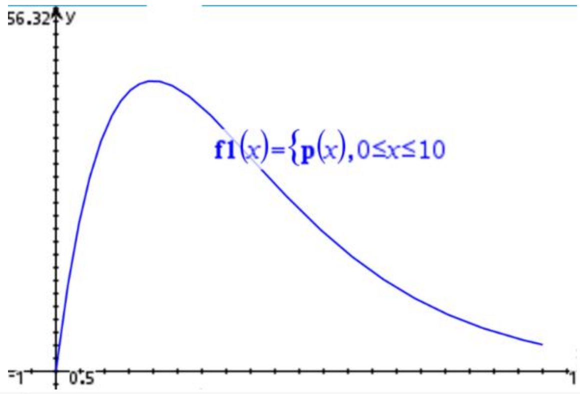


Set zoom settings:
15.5<=x<=18
0<=y<=14

4	Define f(x)=x^3−3x^2+3 done tanLine(f(x), x, 1) $-3\cdot x+4$ solve(ans=0, x $\left\{x=\frac{4}{3}\right\}$ Define $n(x)=x-\frac{f(x)}{\frac{d}{dx}(f(x))}$ done n(1) 1.333333333 n(ans) 1.347222222 n(ans) 1.347296353

Section F: TI Solutions

Question Number	Solutions
1	Define $v(x)=^-2-\sqrt{x+4}$ — Done $\text{solve}(v(y)=x,y)$ — $y=x^2+4\cdot x$ and $x+2\le 0$ $\text{solve}(x+2\le 0,x)$ — $x\le{}^-2$ $\text{solve}(v(x)=x,x)$ — $x=^-3$  $\sqrt{3^2+3^2}$ — $3\cdot\sqrt{2}$ $\frac{540}{2.4}$ — 225. $\text{solve}(540=1.8\cdot s+4.2\cdot(225-s),s)$ — $s=168.75$ $168.75\cdot 1.8$ — 303.75 $540-303.75$ — 236.25 $\frac{303.75}{1.8}+\frac{236.25}{4.2}$ — 225.

2	Define $p(t)=a\cdot t\cdot e^{\frac{10-k\cdot t}{4}}$ — *Done* $p(3)$ — $3\cdot a\cdot e^{\frac{5}{2}-\frac{3\cdot k}{4}}$ $p(5)$ — $5\cdot a\cdot e^{\frac{5}{2}-\frac{5\cdot k}{4}}$ $\frac{p(3)}{p(5)}$ — $\frac{3\cdot e^{\frac{k}{2}}}{5}$ solve$(p(3)=15\cdot e$ and $p(5)=25,a,k)$ — $a=5$ and $k=2$ Define $p(t)=a\cdot t\cdot e^{\frac{10-k\cdot t}{4}}\|a=5$ and $k=2$ — *Done* solve$\left(\frac{d}{dt}(p(t))=0 \text{ and } y=p(t),t\right)$ — $t=2$ and $y=10\cdot e^{\frac{3}{2}}$ fMax$(p(t),t,0,10)$ — $t=2$ $p(2)$ — $10\cdot e^{\frac{3}{2}}$ $p(2)$ — 44.81689 $p(0)$ — 0 $p(10)$ — $50\cdot e^{\frac{-5}{2}}$  solve$(p(t)=35,t)$ — $t=0.9020836$ or $t=3.753622$ $3.753622-0.9020836$ — 2.851538 Define $p1(t)=b\cdot t\cdot e^{\frac{10-2\cdot t}{4}}$ — *Done* solve$(p1(t1)=35$ and $p1(t2)=35$ and $t2-t1=3,b,t1,t2)$ $b=5.129862$ and $t1=0.8616508$ and $t2=3.861651$ solve$(p(t)+p(t-4)=35,t)$ — $t=4.039345$ or $t=8.618792$ $8.618792-4.039345$ — 4.579447

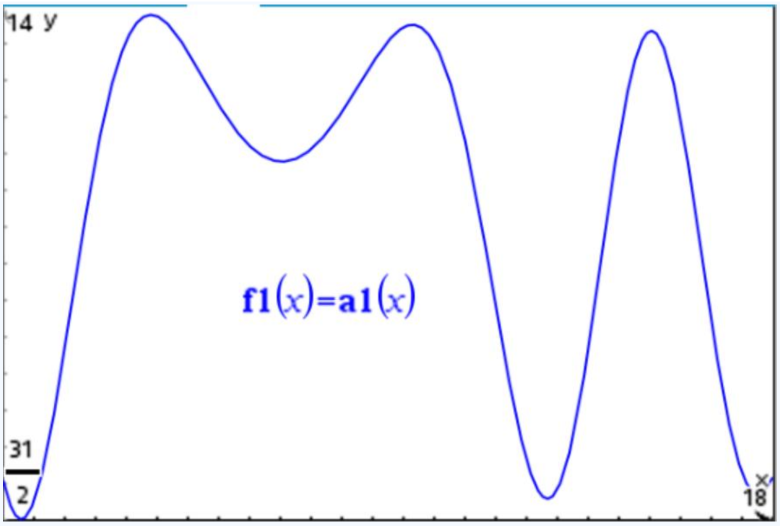
3	Define $a(t)=6\cdot\cos\left(\frac{\pi}{12}\cdot t+\frac{1}{6}\right)+9$ — *Done* $\text{fMin}(a(t),t,0,24)$ — $t=11.36338$ $a(11.36338)$ — $3.$ $\text{solve}(a(t)=3,t)\|0\le t\le 24$ — $t=11.36338$ $0.36338\cdot 60$ — 21.8028 $\text{solve}(a(t)=3.6,t)\|0\le t\le 24$ — $t=9.640585$ or $t=13.08618$ $13.08618-9.640585$ — 3.445595 Define $a(w)=e^{\frac{-w}{2}}+6$ — *Done* $a\left(1+\frac{50}{60}\right)$ — 6.39985 Define $l1(t)=\frac{-t^2}{20}+\frac{13\cdot t}{10}-6$ — *Done* Define $l2(t)=\frac{1}{2}\cdot\sin\left(\frac{2\cdot\pi\cdot t}{5}\right)+h$ — *Done* $l1(8)$ — $\frac{6}{5}$ $\text{solve}(l1(8)=l2(8),h)$ — $h=\frac{\sqrt{-2\cdot(\sqrt{5}-5)}}{8}+\frac{6}{5}$ $\text{solve}\left(\sin\left(\frac{2\cdot\pi\cdot t}{5}\right)=1,t\right)\|8<t<18$ — $t=\frac{45}{4}$ or $t=\frac{65}{4}$ $l2\left(\frac{45}{4}\right)\|h=\frac{\sqrt{-2\cdot(\sqrt{5}-5)}}{8}+\frac{6}{5}$ — 1.993893 $17+\frac{20}{60}$ — $\frac{52}{3}$ $l2\left(\frac{52}{3}\right)\|h=\frac{\sqrt{-2\cdot(\sqrt{5}-5)}}{8}+\frac{6}{5}$ — 1.597848 $a\left(1+\frac{50}{60}\right)\cdot\cos\left(1.5978484715558\cdot\frac{52}{3}\right)+7$ — 1.640788

Define $a1(t)=\left(e^{\frac{-(t-15.5)}{2}}+6\right)\cdot\cos\left(t\cdot\left(\frac{1}{2}\cdot\sin\left(\frac{2\cdot\pi\cdot t}{5}\right)+\frac{\sqrt{-2\cdot(\sqrt{5}-5)}}{8}+\frac{6}{5}\right)\right)+7$ *Done*

$a1\left(\frac{52}{3}\right)$ 1.640788

solve$(a1(t)=2.2,t)|15.5\leq t\leq 18$

$t=15.64955$ or $t=17.18296$ or $t=17.34916$ or $t=17.86918$



4

Define $f(x)=x^3-3\cdot x^2+3$ *Done*

tangentLine$(f(x),x,1)$ $4-3\cdot x$

solve$(4-3\cdot x=0,x)$ $x=\frac{4}{3}$

methods_diffcalc\newtons_method$(f(x),x,1)$

▶ Derivative: $3\cdot x^2-6\cdot x$

▶ Iterative Formula: $\frac{2\cdot x^3-3\cdot x^2-3}{3\cdot x\cdot(x-2)}$

▶ Number of Iterations: 3

"n"	"xn"	"|xn−xn-1|"	"f(xn)"	"f'(xn)"
0.	1.	_	1.	-3.
1.	1.333333	0.3333333	0.037037	-2.666667
2.	1.347222	0.0138889	0.0001956	-2.63831
3.	1.347296	0.0000741	5.7248E-9	-2.638156

Done

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