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VCE Mathematical Methods ¾
SAC 1 Revision V [0.20]
Workshop Solutions

Error Logbook:

New Ideas/Concepts	Didn't Read Question
Pg / Q #: ____________________ Notes:	Pg / Q #: ____________________ Notes:
Algebraic/Arithmetic/ Calculator Input Mistake	**Working Out Not Detailed Enough**
Pg / Q #: ____________________ Notes:	Pg / Q #: ____________________ Notes:

Section A: SAC 1 Success

Welcome to the fifth SAC 1 workshop!

Context: SAC 1 Workshops

- SAC 1-50% of SACs, 20% of the study score.
- Will be running all the way till mid-June.
- After that, the workshop will turn into SAC 2 Workshop (Integration).
- Make sure to complete the SAC $1 \sim 8$.

Successful SAC

$$\text{Study Score} = \text{How much you know} \times \text{How much you show}$$

- Answer everything you know.
- Answer without mistakes.
- Time management is **key!**

Head Tutor's Comment: Explain how even if you know everything, if you cannot show in the SAC, you will get 0.

Analogy: Skipping Questions

- Let's say if you were to fight them and win, you get assigned marks.

- Who would you fight first?
- Skip the hard question with little marks if it doesn't make sense during the reading time.

SAC Proficiency List

Before the SAC

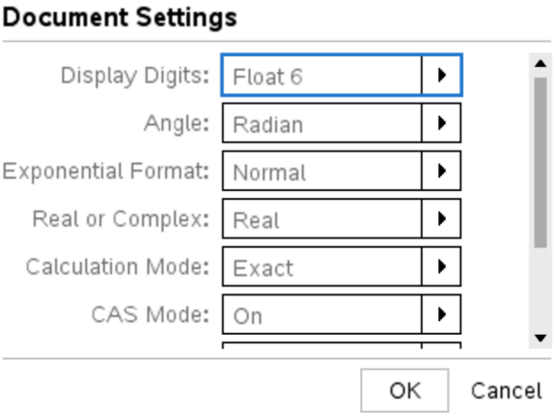
☐ Prepare your stationery including a ruler, eraser and your mechanical pencil lead.

☐ Skim through the bound reference (if applicable).

☐ Do not speak to other people and lock in.

☐ TI & Mathematica Only: Check your Contour UDFS.

☐ TI Only: Check technology settings.



Document Settings

Display Digits: Float 6

Angle: Radian

Exponential Format: Normal

Real or Complex: Real

Calculation Mode: Exact

CAS Mode: On

Reading Time

☐ **Detailed strategy** on how to exactly solve the question on your technology – Don't just **read**, think about how to solve it and use what technology commands.

☐ Identify questions to **skip.**

For difficult SACs, it's not necessarily about getting the 100%. *It's about getting better than everyone else. While others are stuck on a hard 1-marker question, you can do an easy 3-marker first.*

☐ Identify questions to **start first** – You don't have to start from $Q1$!

☐ Look for potential pitfalls – **Units, domain restriction of the unknown, variable and function meaning.**

Writing Time

☐ **Circle what the question is asking** for in the question.

☐ Spend the first **50**% of the time on all the **easy questions** you identified.

☐ Spend the next **25**% of doing the **difficult questions** you left blank.

☐ Spend the last $\mathbf{25}\%$ of the time on **checking your answers**.

☐ Check your answer by reading the question again and see if you answered the question.

☐ Check in the order of:

Domino effect (check a, b, c first) $>$ ***Questions with high marks*** $(\mathbf{3}+)$ $>$ *Hard Questions*

☐ $\mathbf{TI}$ ONLY: Use new document - **doc $\mathbf{4}$, $\mathbf{1}$.**

After the SAC

☐ Think about how each mark loss can be prevented using this proficiency list.

☐ Think about the big picture and improve the marks -

Instead of spending 10 *minutes on* 10*c) (*1 *mark), I should have checked* 5*a) (*3 *marks)*

Space for Personal Notes

Section B: SAC Questions - Tech-Active (52 Marks)

INSTRUCTION:

- 52 Marks. 15 Minutes Reading, 75 Minutes Writing.

Question 1 (9 marks)

Zack Mewton is conducting an experimental synthesis in pursuit of the Philosopher's Stone - a theoretical substance capable of catalysing the transmutation of lead into gold. The reaction chamber begins at a temperature of 24°C and is heated at a controlled rate of 2°C per minute using a calibrated thermal source.

a.

i. Derive an equation for the temperature T, in °C, t minutes after the heat source is activated. (1 mark)

$\boxed{T = 24 + 2t. \quad [\mathbf{1A}]}$

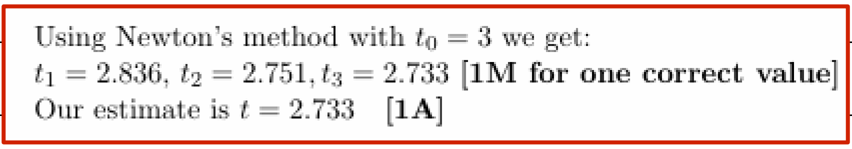
ii. The initial reagent — a purified suspension of metallic salts — must be added when the temperature reaches 32°C. After how many minutes should this be done? (1 mark)

$\boxed{t = 4 \text{ minutes}. \quad [\mathbf{1A}]}$

b. The addition of the metallic salts induces a non-linear thermal fluctuation. The resulting temperature of the system is given by:

$$T_1(t) = T(t) + R(t), \text{ where } R(t) = 8 - 2e^{4t-8}$$

i. Perform three iterations of Newton's method with $t_0 = 3$ to find an approximate solution to the equation $T_1(t) = 0$. Give your answer correct to three decimal places. (2 marks)



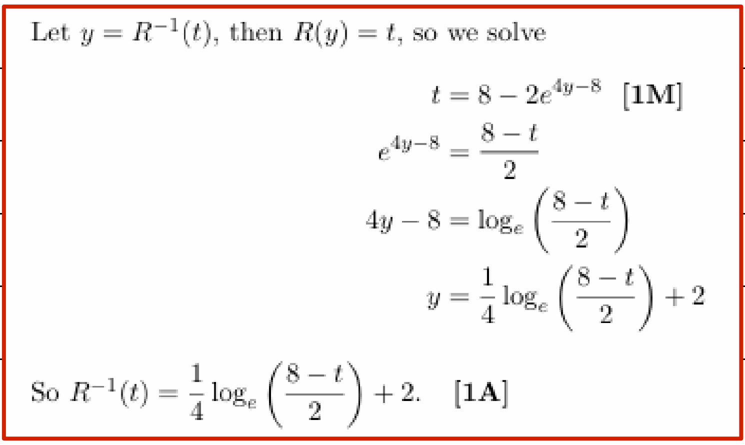
Using Newton's method with $t_0 = 3$ we get:

$t_1 = 2.836$, $t_2 = 2.751, t_3 = 2.733$ **[1M for one correct value]**

Our estimate is $t = 2.733$ **[1A]**

ii. To stabilise the system, Zack introduces a compensating agent with inverse behaviour.

Determine the equation of the inverse function $R^{-1}(t)$. (2 marks)



Let $y = R^{-1}(t)$, then $R(y) = t$, so we solve

$$\begin{aligned} t &= 8 - 2e^{4y-8} \quad \textbf{[1M]} \\ e^{4y-8} &= \frac{8-t}{2} \\ 4y - 8 &= \log_e\left(\frac{8-t}{2}\right) \\ y &= \frac{1}{4}\log_e\left(\frac{8-t}{2}\right) + 2 \end{aligned}$$

So $R^{-1}(t) = \frac{1}{4}\log_e\left(\frac{8-t}{2}\right) + 2$. **[1A]**

c. For precision calibration, the reaction is transferred to a digitally stabilised platform. However, this platform introduces time-dependent perturbations in temperature, described by:

$$C(x) = -3x^2 + 12x - 5$$

where x is the time in minutes since the stabiliser was engaged.

i. Determine the largest value of a such that the domain restriction $[0, a]$ allows $C(x)$ to possess an inverse. (1 mark)

Turning point at $(2, 7)$ so $a = 2$. **[1M]**

ii. Find the expression for the inverse function $C^{-1}(x)$ over the restricted domain $[0, a]$. (2 marks)

Let $y = C^{-1}(x)$. Then we solve $C(y) = x$.

This yields $y = 2 \pm \sqrt{\frac{1}{3}(7 - x)}$. **[1M]**

But the range must be $[0, 2]$ so we have

$$C^{-1}(x) = 2 - \sqrt{\frac{1}{3}(7 - x)} \quad \textbf{[1A]}$$

Space for Personal Notes

Question 2 (22 marks)

Consider the lines L_1 and L_2 defined by the equations:

$$L_1: \ kx - 6y \ = k + 2m \text{ and } L_2: x + (k+5)y = 2,$$

where k and m are real constants.

a. State the value of k if one of the lines is a vertical line. (1 mark)

Vertical line is of the form $x = a$.
This is only possible if $k = -5$. **[1A]**

b. Find the value(s) of k and m if:

i. The lines L_1 and L_2 have a unique point of intersection. (3 marks)

The gradient of the two lines must differ for a unique point of intersection. **[1M]**

Gradients are the same if $\frac{k}{-6} = \frac{1}{k+5} \implies k = -3, -2$ **[1M]**

So there is a unique point of intersection if

$$k \in \mathbb{R} \setminus \{-3, -2\}, \ m \in \mathbb{R} \ \ \textbf{[1A]}$$

ii. The equations defining L_1 and L_2 have no solution. (2 marks)

No solution if the lines are parallel and not the same.

If $k = -3$ then if $m = -\frac{3}{2}$ the lines are the same.

So no solution if $k = -3, \quad m \in \mathbb{R} \setminus \left\{-\frac{3}{2}\right\}$ **[1A]**

If $k = -2$ then if $m = -1$ the lines are the same.

So no solution if $k = -2, \quad m \in \mathbb{R} \setminus \{-1\}$ **[1A]**

iii. The equations defining L_1 and L_2 have an infinite number of solutions. (1 mark)

$k = -3$ and $m = -\frac{3}{2}$
OR $k = -2$ and $m = -1$. **[1A]**

c. Find the value(s) of k and m if:

i. The lines L_1 and L_2 are perpendicular to each other. (2 marks)

Perpendicular if product of the gradients is -1.
Thus $\frac{k}{6} \times \frac{-1}{k+5} = -1$ **[1M]** $\implies k = -6$.
$k = -6, \quad m \in \mathbb{R}$. **[1A]**

ii. The acute angle between L_1 and L_2 is equal to 50°. Give your answer(s) correct to three decimal places. (2 marks)

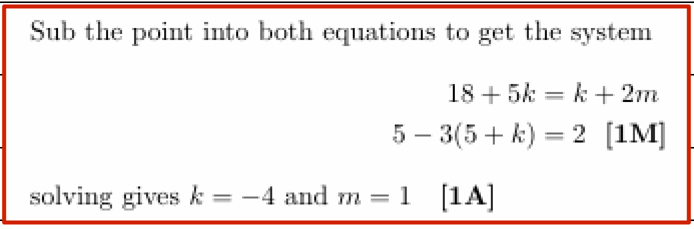
We require that

$$\left| \frac{\frac{k}{6} + \frac{1}{k+5}}{1 + \frac{k}{6}\left(\frac{-1}{k+5}\right)} \right| = \tan(50^\circ) \quad \textbf{[1M]}$$

$\implies k = -4.996, k = 5.955$
So $k = -4.996, \quad m \in \mathbb{R} \quad$ or $\quad k = 5.955, \quad m \in \mathbb{R}$. **[1A]**

d. The lines L_1 and L_2 intersect at the point $(5, -3)$.

i. Find the value of k and the value of m. (2 marks)

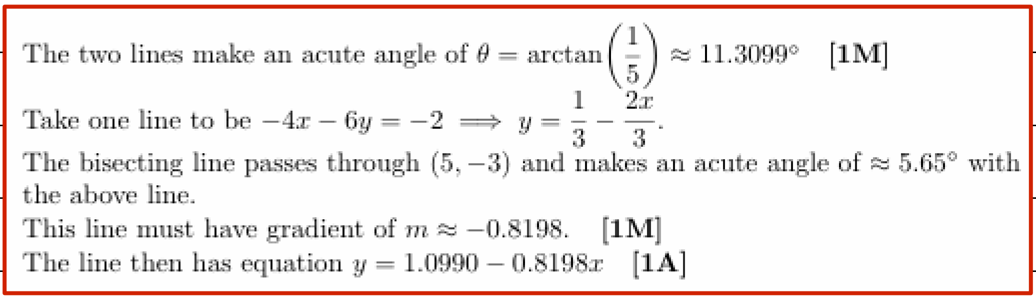


Sub the point into both equations to get the system

$$18 + 5k = k + 2m$$
$$5 - 3(5 + k) = 2 \quad \textbf{[1M]}$$

solving gives $k = -4$ and $m = 1$ **[1A]**

ii. Find the equation of the line that **bisects** the acute angle made between L_1 and L_2 when they intersect at $(5, -3)$. Give your answer in the form $y = mx + c$ where m and c are given correct to four decimal places. (3 marks)



The two lines make an acute angle of $\theta = \arctan\left(\frac{1}{5}\right) \approx 11.3099°$ **[1M]**

Take one line to be $-4x - 6y = -2 \implies y = \frac{1}{3} - \frac{2x}{3}$.

The bisecting line passes through $(5, -3)$ and makes an acute angle of $\approx 5.65°$ with the above line.

This line must have gradient of $m \approx -0.8198$. **[1M]**

The line then has equation $y = 1.0990 - 0.8198x$ **[1A]**

e. Find the value(s) of k and m if:

i. The line L_1 is tangent to the curve $y = \frac{1}{x} + x^2$ at the point where $x = 1$. (3 marks)

Let $f(x) = \dfrac{1}{x} + x^2$. Then $f'(1) = 1$.
So line L_1 has gradient of 1 and passes through $(1, 2)$. **[1M]**

$$y = 1 + x \quad \textbf{[1M]}$$

so now $\dfrac{k}{6} = 1$ and $\dfrac{k + 2m}{-6} = 1 \implies k = 6, m = -6$ **[1A]**

ii. The line L_1 passes through the point $(2, -9)$, has a negative x-axis intercept, and is tangent to the curve $y = \frac{1}{x} + x^2$. (3 marks)

Tangent to $f(x) = \dfrac{1}{x} + x^2$ at $x = a$ is given by $y = -\dfrac{a^4 - 2a^3x - 2a + x}{a^2}$ **[1M]**
This line passes through $(2, -9)$ so we get the equation

$$-\frac{a^4 - 4a^3 - 2a + 2}{a^2} = -9$$
$$\implies a = -1, 3 \pm \sqrt{7} \quad \textbf{[1M]}$$

Only $a = -1$ will give a negative x-axis intercept.
Thus the line is $y = -3 - 3x$.
So $k = -18$ and $m = 18$. **[1A]**

Space for Personal Notes

Question 3 (21 marks)

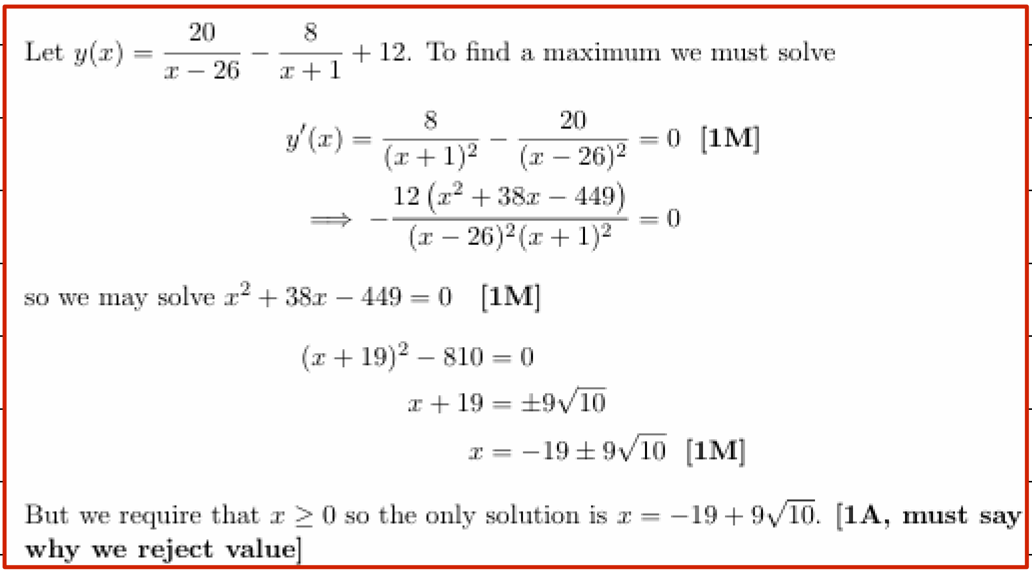
A medical researcher is studying the diffusion of a drug from two injection sites, A and B, placed 24 millimetres apart in a tissue sample. The concentration of the drug, y, in mg/L at a point x millimetres from site A along the straight line between the two sites is modelled by:

$$y = \frac{p}{x-26} - \frac{q}{x+1} + 12, \qquad 0 \le x \le 24, \qquad y \ge 0$$

where p and q are positive constants.

a. For a particular formulation of the drug, the parameters are $p = 20$ and $q = 8$.

i. Solve an appropriate equation using algebra to determine the location x (in mm from site A) at which the drug concentration is at a maximum. All working must be shown. (4 marks)



Let $y(x) = \frac{20}{x-26} - \frac{8}{x+1} + 12$. To find a maximum we must solve

$$y'(x) = \frac{8}{(x+1)^2} - \frac{20}{(x-26)^2} = 0 \quad \textbf{[1M]}$$

$$\implies -\frac{12\left(x^2+38x-449\right)}{(x-26)^2(x+1)^2} = 0$$

so we may solve $x^2 + 38x - 449 = 0$ **[1M]**

$$(x+19)^2 - 810 = 0$$

$$x + 19 = \pm 9\sqrt{10}$$

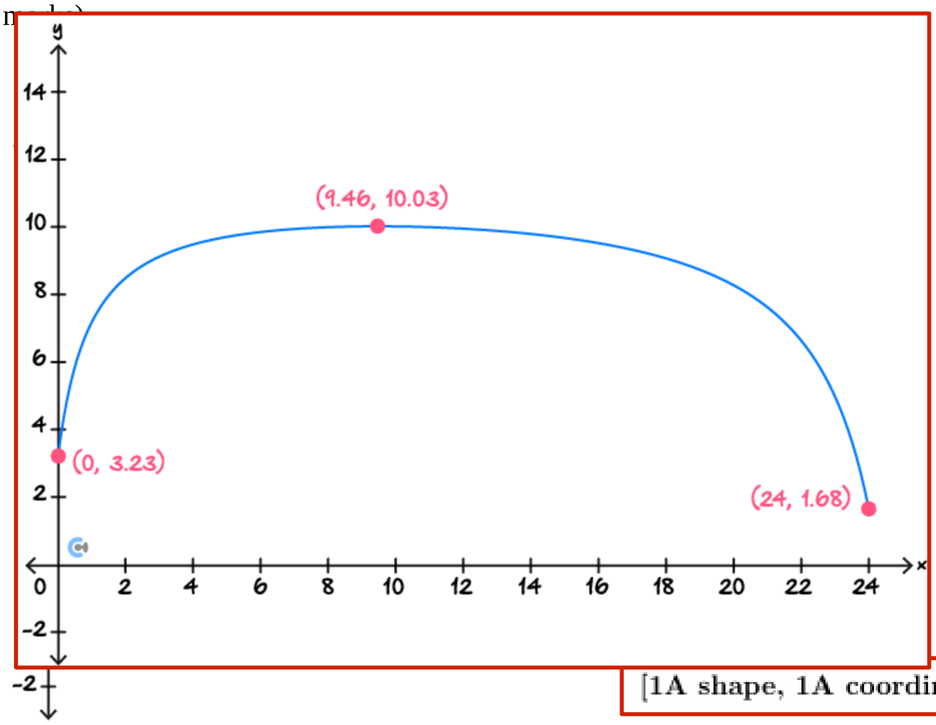
$$x = -19 \pm 9\sqrt{10} \quad \textbf{[1M]}$$

But we require that $x \ge 0$ so the only solution is $x = -19 + 9\sqrt{10}$. **[1A, must say why we reject value]**

ii. Sketch the graph of the function:

$$y = \frac{20}{x-26} - \frac{8}{x+1} + 12$$

for $0 \leq x \leq 24$. Label the endpoints and the turning point with their coordinates, correct to two decimal places. (2 m



[1A shape, 1A coordinates]

iii. At what distance(s) from the injection site B is the drug concentration equal to 7 mg/L? Give your answer in millimetres, correct to three decimal places (2 marks)

Solve $y(x) = 7 \implies x = 0.903, 21.697$. These are distances from site A. [1M]
Therefore distance froms site B are 23.097 and 2.303 millimetres. [1A]

iv. Determine, correct to one decimal place, the percentage of the distance between A and B over which the concentration y is below $8\ mg/L$. (2 marks)

We solve $y(x) = 8 \implies x = 1.51317, 20.4868.$ **[1M]**

So the percentage is $\dfrac{1.51317 + (24 - 20.4868)}{24} \times 100 = 20.9\%$ **[1A]** .

b. The researcher fixes $p = 20$ and considers how varying the drug with different values of q affects the concentration model.

i. Determine the possible values of q for which the model is defined on the interval $0 \leq x \leq 24$. (2 marks)

We require that $y(0) \geq 0$ and that $y(24) \geq 0$. **[1M]**

Also $q > 0$ since we said that it is a **positve** constant.

So $0 < q \leq \dfrac{146}{13}$. **[1A]**

ii. Find, in terms of q, the x-coordinate of the turning point of the function y, when it exists. (2 marks)

We solve $y'(x) = 0 \implies x = 26 - \dfrac{54\left(\sqrt{5q} - 10\right)}{q - 20}, \dfrac{54\left(\sqrt{5q} + 10\right)}{q - 20} + 26$ **[1M]**

But we need the value that is between 0 and 24 on the domain of q.

So $x = 26 - \dfrac{54\left(\sqrt{5q} - 10\right)}{q - 20}$ or equivalent. **[1A]**

iii. Identify the values of q for which the function y does **not** have an inverse. (2 marks)

No inverse if the function has a turning point in the interval $0 < x < 24$.
Note we already have the restriction $0 < q \leq \frac{146}{13}$
Using above part b.ii we also solve $0 < 26 - \frac{54\left(\sqrt{5q}-10\right)}{q-20} \implies q > \frac{5}{169}$ **[1M]**
Combining our restrictions gives no inverse if

$$\frac{5}{169} < q \leq \frac{146}{13} \quad \textbf{[1A]}$$

iv. Determine the value(s) of q for which the **minimum** drug concentration occurs when $x = 24$. (2 marks)

We will require that $y(0) \geq y(24)$ **[1M]**
Thus $0 < q \leq \frac{125}{13}$ **[1A]** .

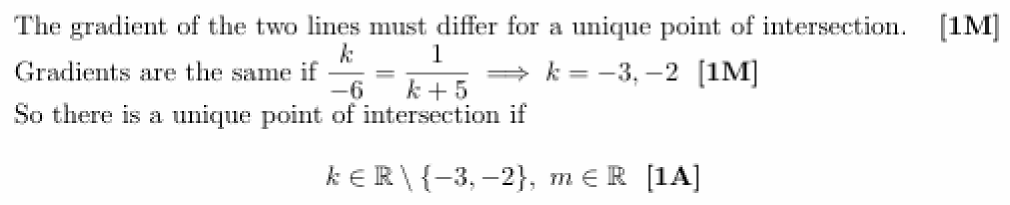
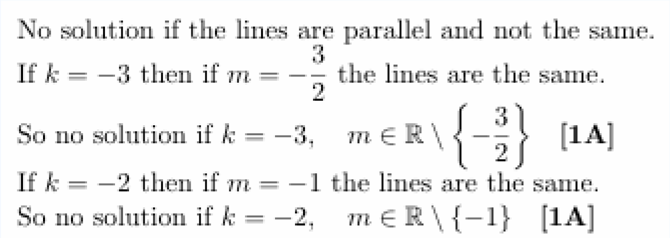
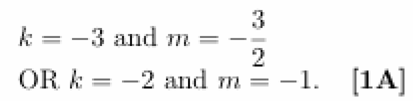
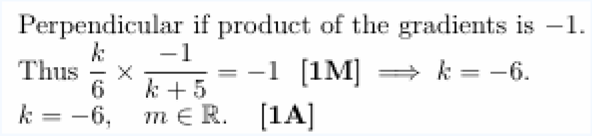
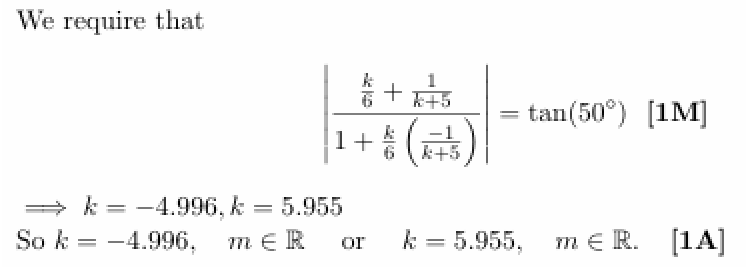
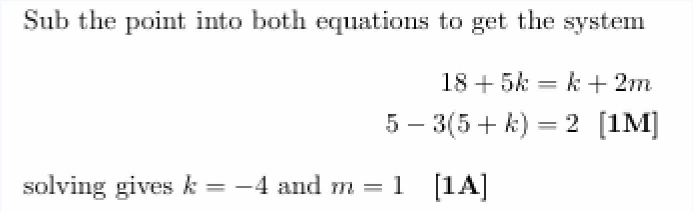
c. The research team designs a formulation such that the drug reaches a maximum concentration of exactly 10 mg/L at the midpoint between the injection sites A and B. Determine the corresponding values of p and q that achieve this. (3 marks)

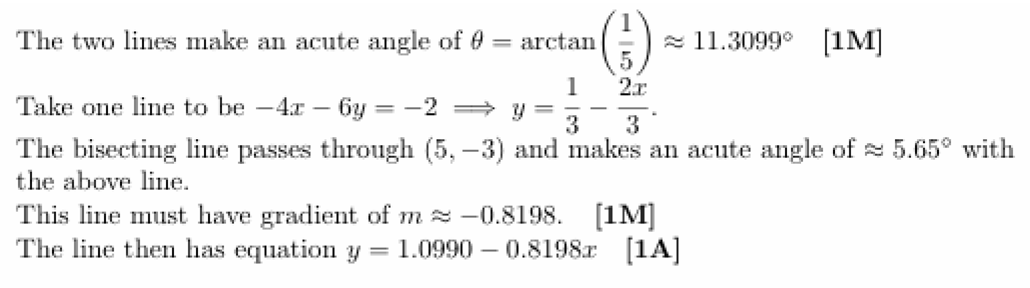
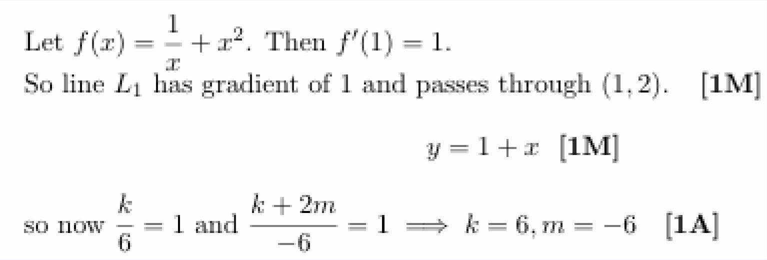
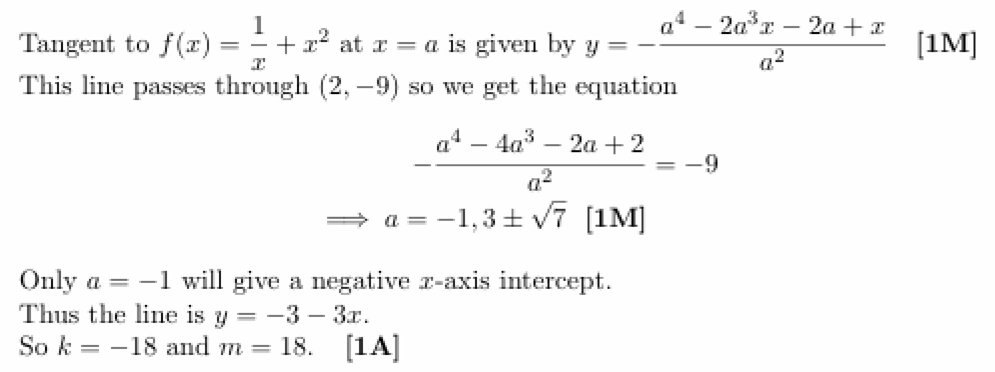
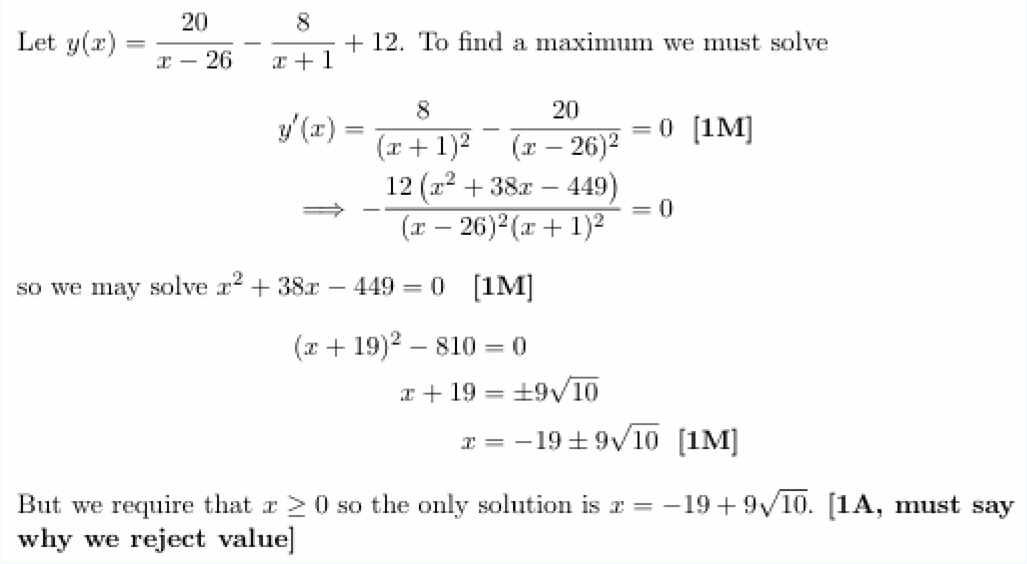
We must have a max of 10 when $x = 12$. So must have $\frac{dy}{dx} = 0$ when $x = 12$. **[1M]**
We solve $y'(12) = 0$ and $y(12) = 10$ simultaneously to get
$p = \frac{392}{27}$ **[1M]** and $q = \frac{338}{27}$ **[1A]** .

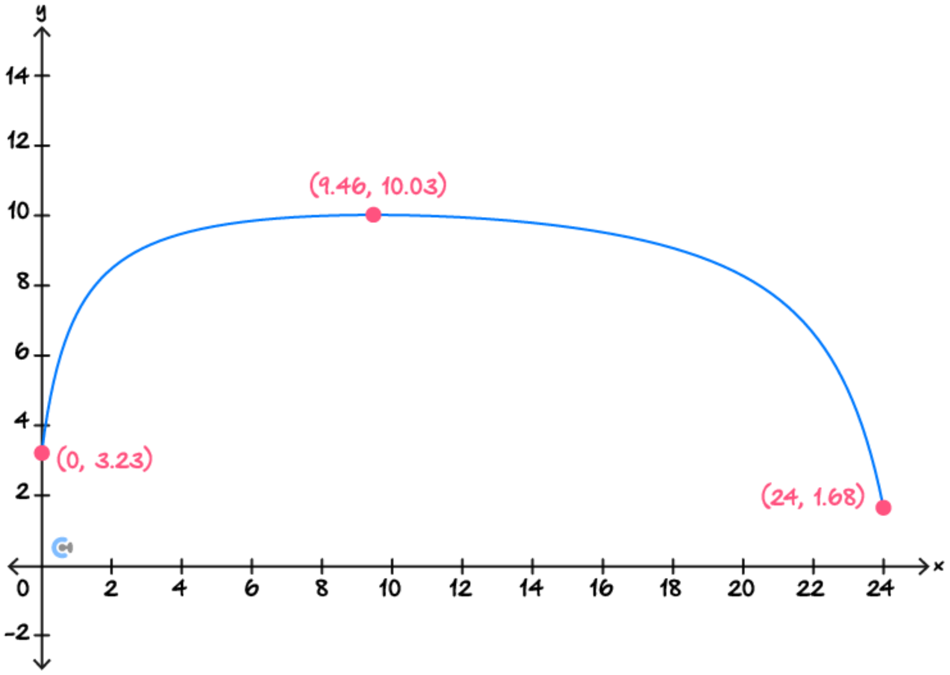
Space for Personal Notes

Section C: Marking Scheme

Question Number	Solutions
1 a i	$T = 24 + 2t.$ **[1A]**
1 a ii	$t = 4$ minutes. **[1A]**
1 b i	Using Newton's method with $t_0 = 3$ we get: $t_1 = 2.836,\ t_2 = 2.751, t_3 = 2.733$ **[1M for one correct value]** Our estimate is $t = 2.733$ **[1A]**
1 b ii	Let $y = R^{-1}(t)$, then $R(y) = t$, so we solve $t = 8 - 2e^{4y-8}$ **[1M]** $e^{4y-8} = \frac{8-t}{2}$ $4y - 8 = \log_e\left(\frac{8-t}{2}\right)$ $y = \frac{1}{4}\log_e\left(\frac{8-t}{2}\right) + 2$ So $R^{-1}(t) = \frac{1}{4}\log_e\left(\frac{8-t}{2}\right) + 2.$ **[1A]**
1 c i	Turning point at $(2, 7)$ so $a = 2.$ **[1M]**
1 c ii	Let $y = C^{-1}(x)$. Then we solve $C(y) = x$. This yields $y = 2 \pm \sqrt{\frac{1}{3}(7-x)}.$ **[1M]** But the range must be $[0, 2]$ so we have $C^{-1}(x) = 2 - \sqrt{\frac{1}{3}(7-x)}$ **[1A]**
2 a	Vertical line is of the form $x = a$. This is only possible if $k = -5.$ **[1A]**

2 b i	 The gradient of the two lines must differ for a unique point of intersection. [1M] Gradients are the same if $\frac{k}{-6} = \frac{1}{k+5} \implies k = -3, -2$ [1M] So there is a unique point of intersection if $$k \in \mathbb{R} \setminus \{-3, -2\},\ m \in \mathbb{R}$$ [1A]
2 b ii	 No solution if the lines are parallel and not the same. If $k = -3$ then if $m = -\frac{3}{2}$ the lines are the same. So no solution if $k = -3, \quad m \in \mathbb{R} \setminus \left\{-\frac{3}{2}\right\}$ [1A] If $k = -2$ then if $m = -1$ the lines are the same. So no solution if $k = -2, \quad m \in \mathbb{R} \setminus \{-1\}$ [1A]
2 b iii	 $k = -3$ and $m = -\frac{3}{2}$ OR $k = -2$ and $m = -1.$ [1A]
2 c i	 Perpendicular if product of the gradients is -1. Thus $\frac{k}{6} \times \frac{-1}{k+5} = -1$ [1M] $\implies k = -6.$ $k = -6, \quad m \in \mathbb{R}.$ [1A]
2 c ii	 We require that $$\left\lvert \frac{\frac{k}{6} + \frac{1}{k+5}}{1 + \frac{k}{6}\left(\frac{-1}{k+5}\right)} \right\rvert = \tan(50^\circ)$$ [1M] $\implies k = -4.996, k = 5.955$ So $k = -4.996, \quad m \in \mathbb{R}$ or $k = 5.955, \quad m \in \mathbb{R}.$ [1A]
2 d i	 Sub the point into both equations to get the system $$18 + 5k = k + 2m$$ $$5 - 3(5 + k) = 2$$ [1M] solving gives $k = -4$ and $m = 1$ [1A]

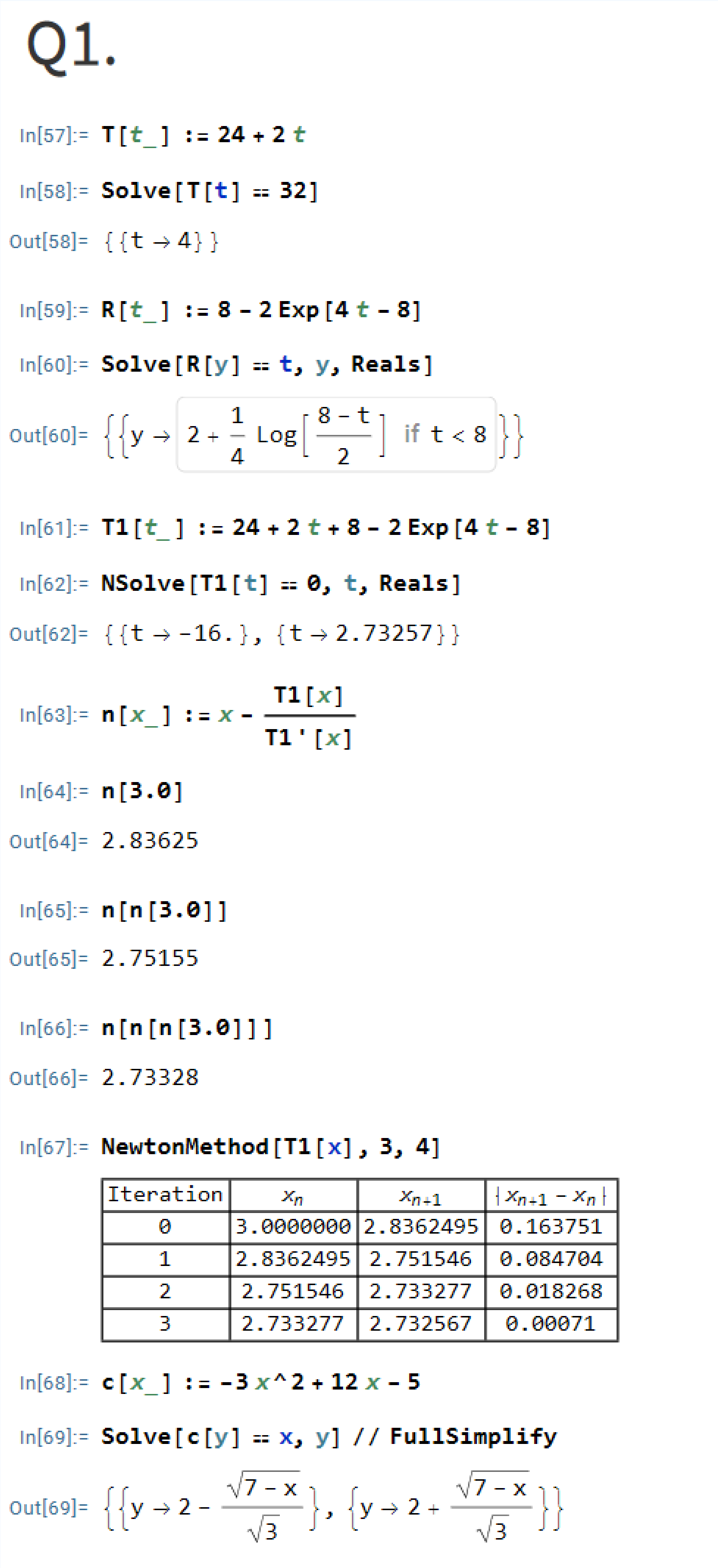
2 d ii	 The two lines make an acute angle of $\theta = \arctan\left(\frac{1}{5}\right) \approx 11.3099°$ **[1M]** Take one line to be $-4x - 6y = -2 \implies y = \frac{1}{3} - \frac{2x}{3}$. The bisecting line passes through $(5, -3)$ and makes an acute angle of $\approx 5.65°$ with the above line. This line must have gradient of $m \approx -0.8198$. **[1M]** The line then has equation $y = 1.0990 - 0.8198x$ **[1A]**
2 e i	 Let $f(x) = \frac{1}{x} + x^2$. Then $f'(1) = 1$. So line L_1 has gradient of 1 and passes through $(1, 2)$. **[1M]** $y = 1 + x$ **[1M]** so now $\frac{k}{6} = 1$ and $\frac{k+2m}{-6} = 1 \implies k = 6, m = -6$ **[1A]**
2 e ii	 Tangent to $f(x) = \frac{1}{x} + x^2$ at $x = a$ is given by $y = -\frac{a^4 - 2a^3x - 2a + x}{a^2}$ **[1M]** This line passes through $(2, -9)$ so we get the equation $-\frac{a^4 - 4a^3 - 2a + 2}{a^2} = -9$ $\implies a = -1, 3 \pm \sqrt{7}$ **[1M]** Only $a = -1$ will give a negative x-axis intercept. Thus the line is $y = -3 - 3x$. So $k = -18$ and $m = 18$. **[1A]**
3 a i	 Let $y(x) = \frac{20}{x-26} - \frac{8}{x+1} + 12$. To find a maximum we must solve $y'(x) = \frac{8}{(x+1)^2} - \frac{20}{(x-26)^2} = 0$ **[1M]** $\implies -\frac{12\left(x^2 + 38x - 449\right)}{(x-26)^2(x+1)^2} = 0$ so we may solve $x^2 + 38x - 449 = 0$ **[1M]** $(x+19)^2 - 810 = 0$ $x + 19 = \pm 9\sqrt{10}$ $x = -19 \pm 9\sqrt{10}$ **[1M]** But we require that $x \geq 0$ so the only solution is $x = -19 + 9\sqrt{10}$. **[1A, must say why we reject value]**

3 a ii	 **[1A shape, 1A coordinates]**
3 a iii	Solve $y(x) = 7 \implies x = 0.903, 21.697$. These are distances from site A. **[1M]** Therefore distance froms site B are 23.097 and 2.303 millimetres. **[1A]**
3 a iv	We solve $y(x) = 8 \implies x = 1.51317, 20.4868$. **[1M]** So the percentage is $\frac{1.51317 + (24 - 20.4868)}{24} \times 100 = 20.9\%$ **[1A]** .
3 b i	We require that $y(0) \geq 0$ and that $y(24) \geq 0$. **[1M]** Also $q > 0$ since we said that it is a **positve** constant. So $0 < q \leq \frac{146}{13}$. **[1A]**
3 b ii	We solve $y'(x) = 0 \implies x = 26 - \frac{54\left(\sqrt{5q} - 10\right)}{q - 20}, \frac{54\left(\sqrt{5q} + 10\right)}{q - 20} + 26$ **[1M]** But we need the value that is between 0 and 24 on the domain of q. So $x = 26 - \frac{54\left(\sqrt{5q} - 10\right)}{q - 20}$ or equivalent. **[1A]**

3 b iii	No inverse if the function has a turning point in the interval $0 < x < 24$. Note we already have the restriction $0 < q \le \frac{146}{13}$ Using above part b.ii we also solve $0 < 26 - \frac{54\left(\sqrt{5q}-10\right)}{q-20} \implies q > \frac{5}{169}$ **[1M]** Combining our restrictions gives no inverse if $\frac{5}{169} < q \le \frac{146}{13}$ **[1A]**
3 b iv	We will require that $y(0) \ge y(24)$ **[1M]** Thus $0 < q \le \frac{125}{13}$ **[1A]** .
3 c	We must have a max of 10 when $x = 12$. So must have $\frac{dy}{dx} = 0$ when $x = 12$. **[1M]** We solve $y'(12) = 0$ and $y(12) = 10$ simultaneously to get $p = \frac{392}{27}$ **[1M]** and $q = \frac{338}{27}$ **[1A]** .

Space for Personal Notes

Section D: Mathematica Solutions

Question Number	Solutions
1	

Q1.

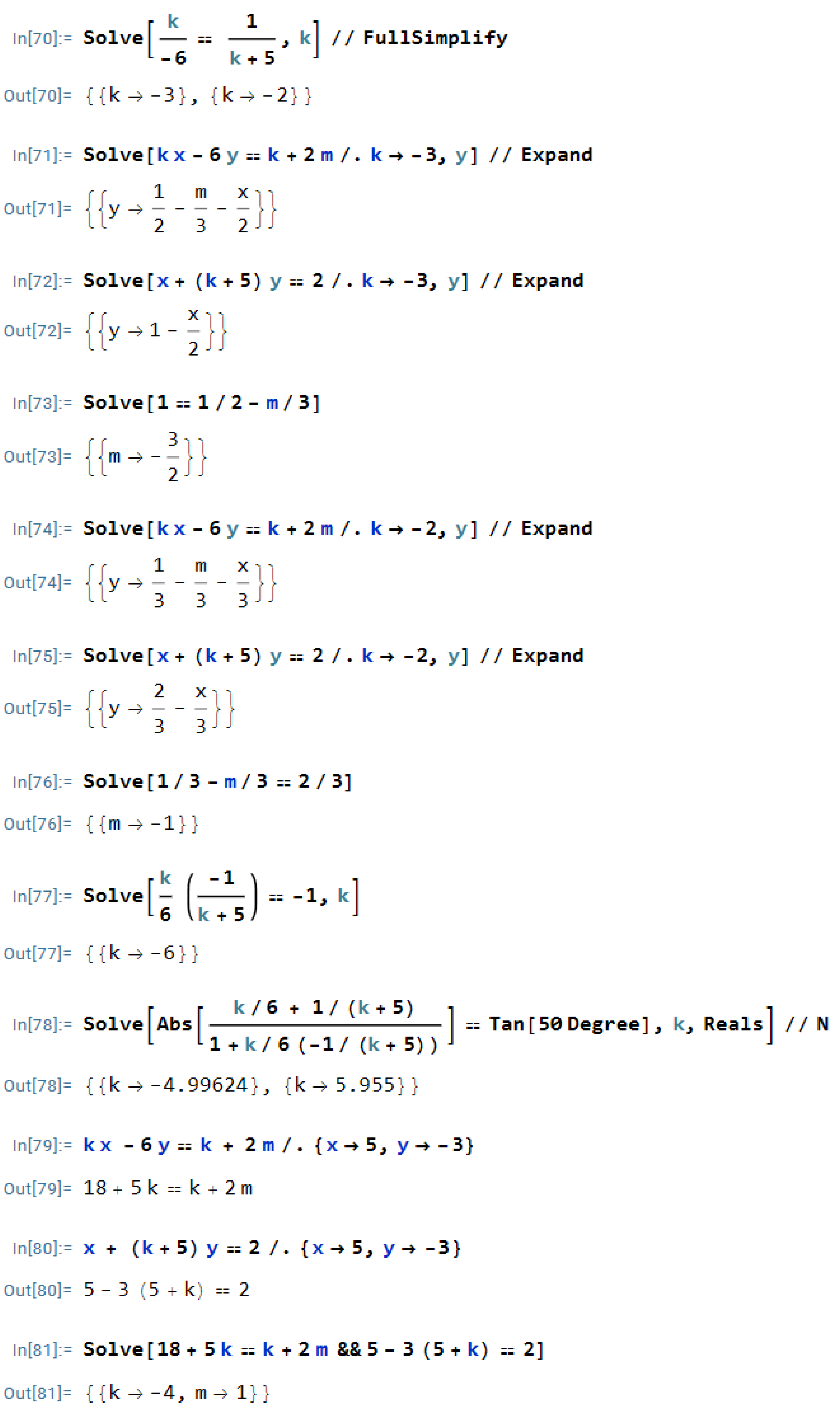
```
In[57]:= T[t_] := 24 + 2 t

In[58]:= Solve[T[t] == 32]

Out[58]= {{t → 4}}

In[59]:= R[t_] := 8 - 2 Exp[4 t - 8]

In[60]:= Solve[R[y] == t, y, Reals]

Out[60]= {{y → 2 + 1/4 Log[(8 - t)/2]   if t < 8}}

In[61]:= T1[t_] := 24 + 2 t + 8 - 2 Exp[4 t - 8]

In[62]:= NSolve[T1[t] == 0, t, Reals]

Out[62]= {{t → -16.}, {t → 2.73257}}

In[63]:= n[x_] := x - T1[x]/T1'[x]

In[64]:= n[3.0]

Out[64]= 2.83625

In[65]:= n[n[3.0]]

Out[65]= 2.75155

In[66]:= n[n[n[3.0]]]

Out[66]= 2.73328

In[67]:= NewtonMethod[T1[x], 3, 4]
```

Iteration	x_n	x_{n+1}	$\lvert x_{n+1} - x_n \rvert$
0	3.0000000	2.8362495	0.163751
1	2.8362495	2.751546	0.084704
2	2.751546	2.733277	0.018268
3	2.733277	2.732567	0.00071

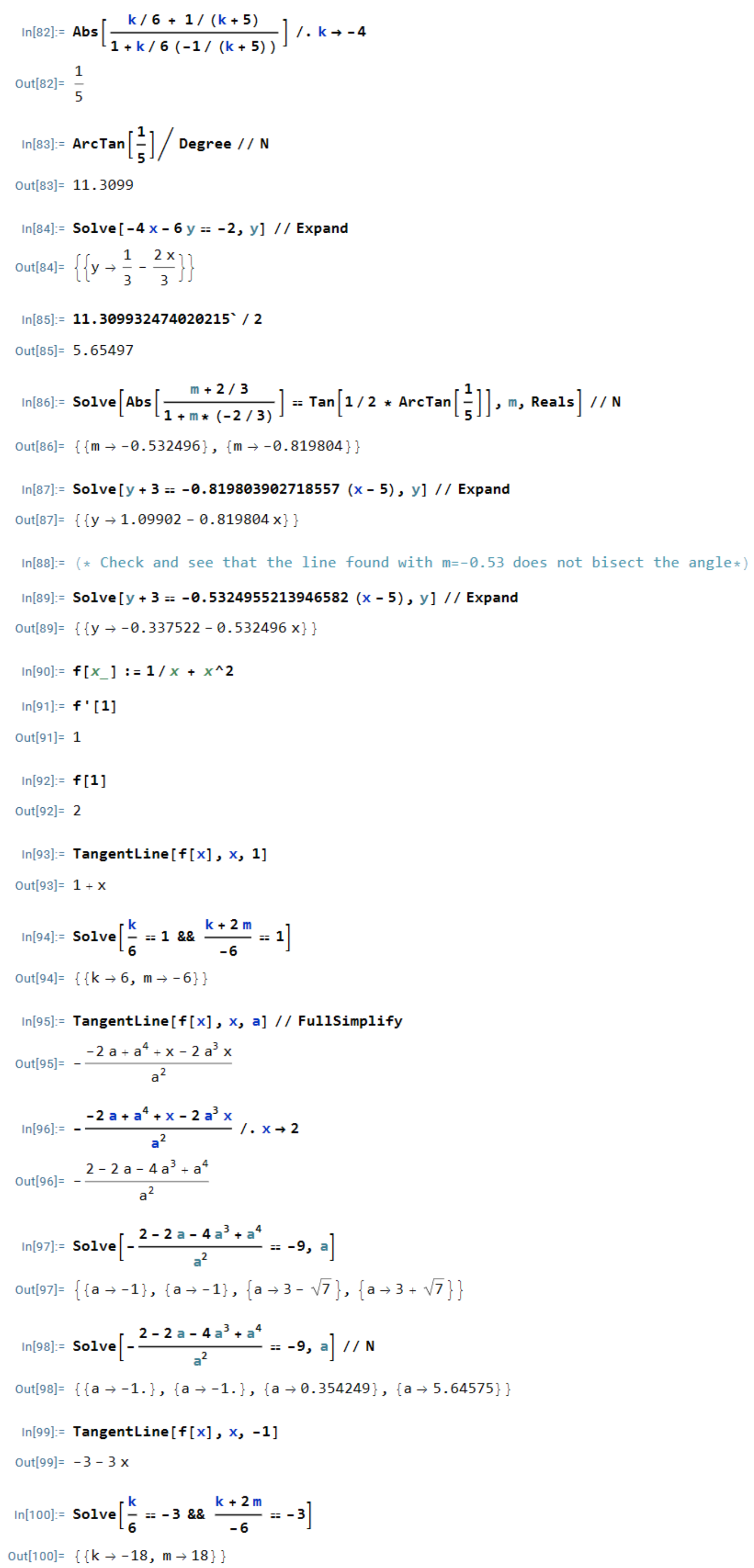
```
In[68]:= c[x_] := -3 x^2 + 12 x - 5

In[69]:= Solve[c[y] == x, y] // FullSimplify

Out[69]= {{y → 2 - √(7 - x)/√3}, {y → 2 + √(7 - x)/√3}}
```

2

Q2.

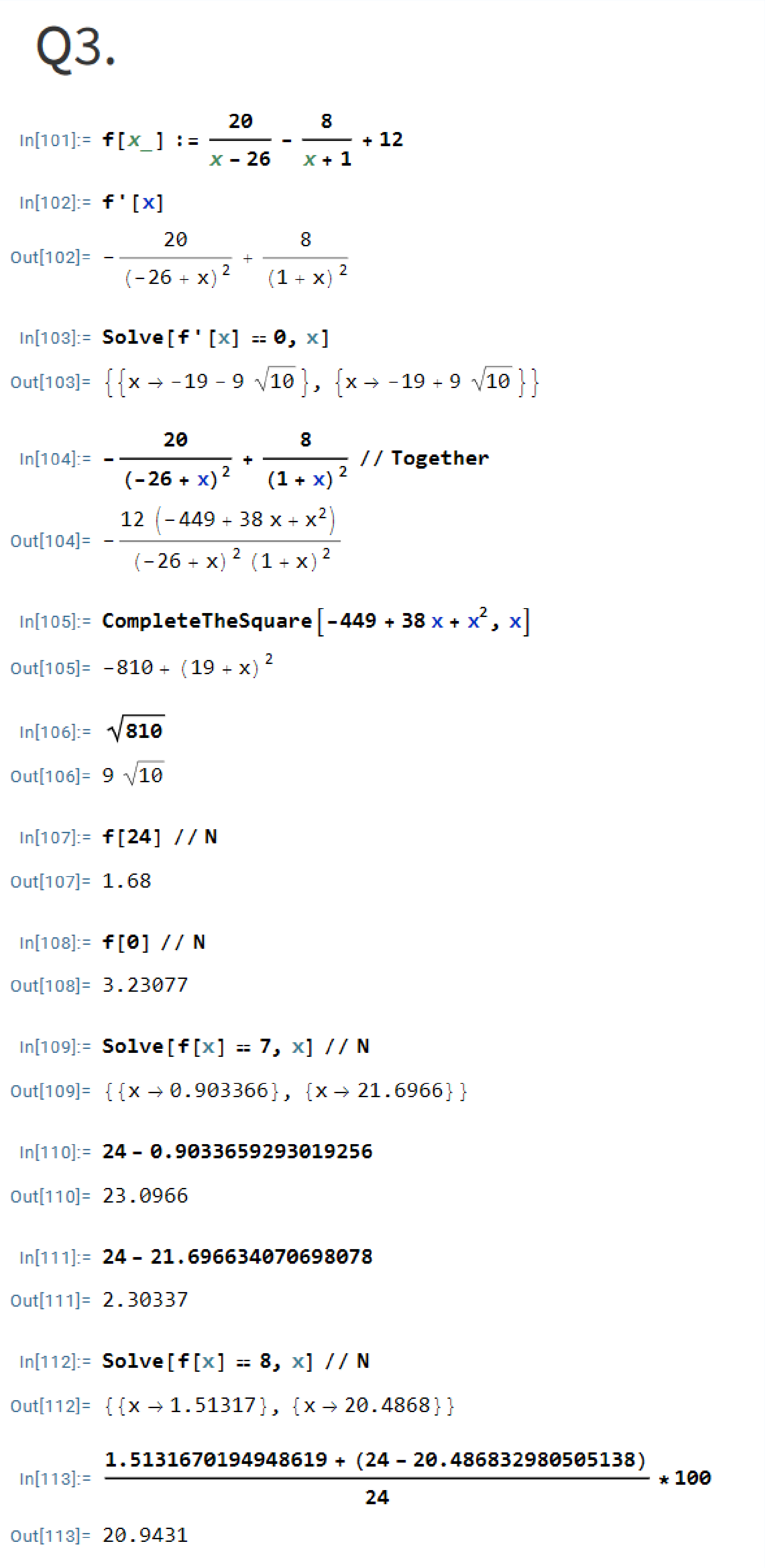


```
In[70]:= Solve[k/(-6) == 1/(k + 5), k] // FullSimplify
Out[70]= {{k → -3}, {k → -2}}

In[71]:= Solve[k x - 6 y == k + 2 m /. k → -3, y] // Expand
Out[71]= {{y → 1/2 - m/3 - x/2}}

In[72]:= Solve[x + (k + 5) y == 2 /. k → -3, y] // Expand
Out[72]= {{y → 1 - x/2}}

In[73]:= Solve[1 == 1 / 2 - m / 3]
Out[73]= {{m → -3/2}}

In[74]:= Solve[k x - 6 y == k + 2 m /. k → -2, y] // Expand
Out[74]= {{y → 1/3 - m/3 - x/3}}

In[75]:= Solve[x + (k + 5) y == 2 /. k → -2, y] // Expand
Out[75]= {{y → 2/3 - x/3}}

In[76]:= Solve[1 / 3 - m / 3 == 2 / 3]
Out[76]= {{m → -1}}

In[77]:= Solve[k/6 (-1/(k + 5)) == -1, k]
Out[77]= {{k → -6}}

In[78]:= Solve[Abs[(k / 6 + 1 / (k + 5))/(1 + k / 6 (-1 / (k + 5)))] == Tan[50 Degree], k, Reals] // N
Out[78]= {{k → -4.99624}, {k → 5.955}}

In[79]:= k x - 6 y == k + 2 m /. {x → 5, y → -3}
Out[79]= 18 + 5 k == k + 2 m

In[80]:= x + (k + 5) y == 2 /. {x → 5, y → -3}
Out[80]= 5 - 3 (5 + k) == 2

In[81]:= Solve[18 + 5 k == k + 2 m && 5 - 3 (5 + k) == 2]
Out[81]= {{k → -4, m → 1}}
```



```
In[82]:= Abs[(k/6 + 1/(k+5))/(1 + k/6 (-1/(k+5)))] /. k → -4
Out[82]= 1/5

In[83]:= ArcTan[1/5]/Degree // N
Out[83]= 11.3099

In[84]:= Solve[-4 x - 6 y == -2, y] // Expand
Out[84]= {{y → 1/3 - (2 x)/3}}

In[85]:= 11.309932474020215` / 2
Out[85]= 5.65497

In[86]:= Solve[Abs[(m + 2/3)/(1 + m*(-2/3))] == Tan[1/2 * ArcTan[1/5]], m, Reals] // N
Out[86]= {{m → -0.532496}, {m → -0.819804}}

In[87]:= Solve[y + 3 == -0.819803902718557 (x - 5), y] // Expand
Out[87]= {{y → 1.09902 - 0.819804 x}}

In[88]:= (* Check and see that the line found with m=-0.53 does not bisect the angle*)

In[89]:= Solve[y + 3 == -0.5324955213946582 (x - 5), y] // Expand
Out[89]= {{y → -0.337522 - 0.532496 x}}

In[90]:= f[x_] := 1/x + x^2

In[91]:= f'[1]
Out[91]= 1

In[92]:= f[1]
Out[92]= 2

In[93]:= TangentLine[f[x], x, 1]
Out[93]= 1 + x

In[94]:= Solve[k/6 == 1 && (k + 2 m)/-6 == 1]
Out[94]= {{k → 6, m → -6}}

In[95]:= TangentLine[f[x], x, a] // FullSimplify
Out[95]= -(-2 a + a^4 + x - 2 a^3 x)/a^2

In[96]:= -(-2 a + a^4 + x - 2 a^3 x)/a^2 /. x → 2
Out[96]= -(2 - 2 a - 4 a^3 + a^4)/a^2

In[97]:= Solve[-(2 - 2 a - 4 a^3 + a^4)/a^2 == -9, a]
Out[97]= {{a → -1}, {a → -1}, {a → 3 - √7}, {a → 3 + √7}}

In[98]:= Solve[-(2 - 2 a - 4 a^3 + a^4)/a^2 == -9, a] // N
Out[98]= {{a → -1.}, {a → -1.}, {a → 0.354249}, {a → 5.64575}}

In[99]:= TangentLine[f[x], x, -1]
Out[99]= -3 - 3 x

In[100]:= Solve[k/6 == -3 && (k + 2 m)/-6 == -3]
Out[100]= {{k → -18, m → 18}}
```

3



Q3.

In[101]:= $f[x\_] := \frac{20}{x-26} - \frac{8}{x+1} + 12$

In[102]:= $f'[x]$

Out[102]= $-\frac{20}{(-26+x)^2} + \frac{8}{(1+x)^2}$

In[103]:= $\text{Solve}[f'[x] == 0, x]$

Out[103]= $\{\{x \to -19 - 9\sqrt{10}\}, \{x \to -19 + 9\sqrt{10}\}\}$

In[104]:= $-\frac{20}{(-26+x)^2} + \frac{8}{(1+x)^2}$ // Together

Out[104]= $-\frac{12\left(-449 + 38x + x^2\right)}{(-26+x)^2 (1+x)^2}$

In[105]:= $\text{CompleteTheSquare}\left[-449 + 38x + x^2, x\right]$

Out[105]= $-810 + (19+x)^2$

In[106]:= $\sqrt{810}$

Out[106]= $9\sqrt{10}$

In[107]:= f[24] // N

Out[107]= 1.68

In[108]:= f[0] // N

Out[108]= 3.23077

In[109]:= Solve[f[x] == 7, x] // N

Out[109]= {{x → 0.903366}, {x → 21.6966}}

In[110]:= 24 - 0.9033659293019256

Out[110]= 23.0966

In[111]:= 24 - 21.696634070698078

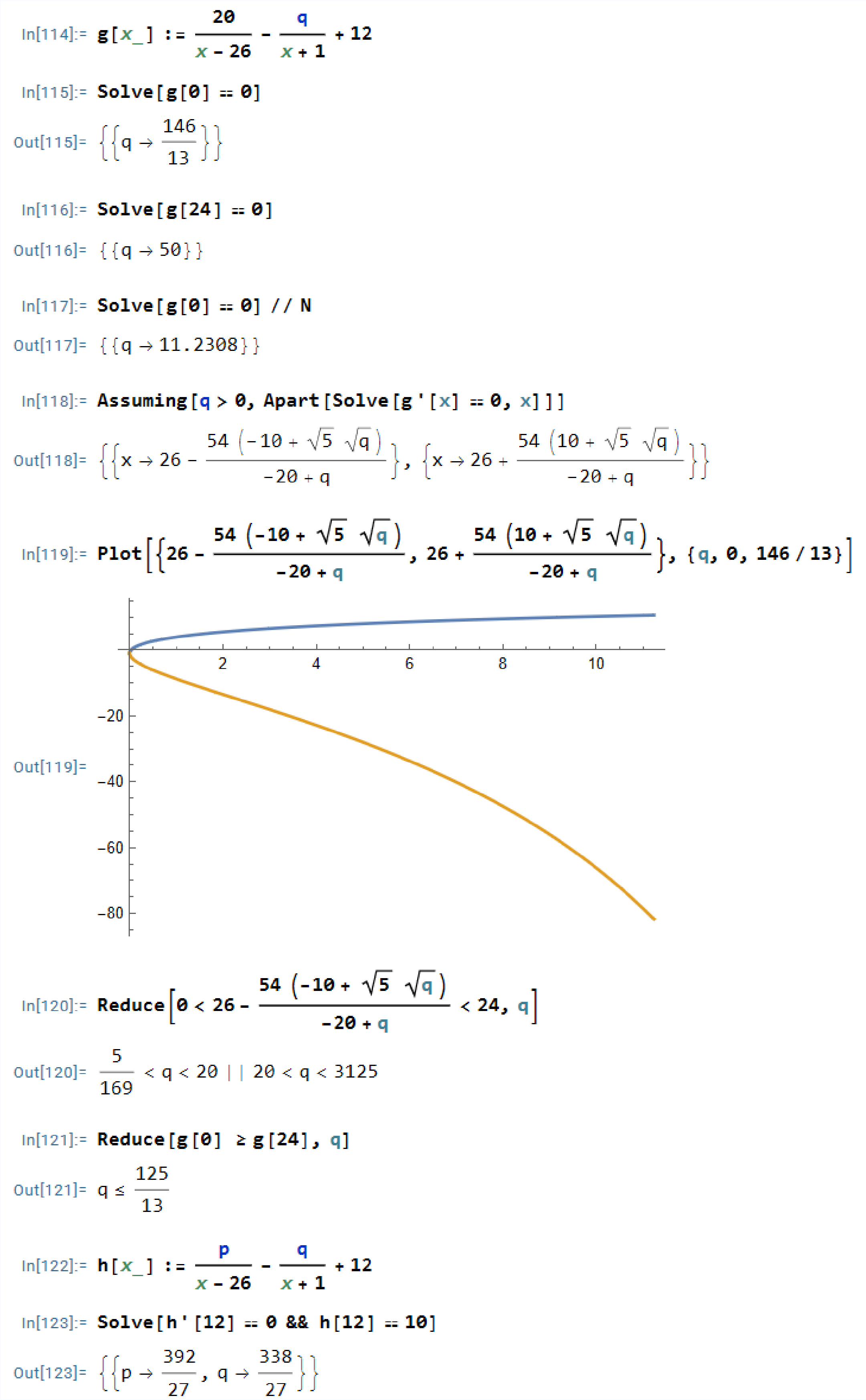
Out[111]= 2.30337

In[112]:= Solve[f[x] == 8, x] // N

Out[112]= {{x → 1.51317}, {x → 20.4868}}

In[113]:= $\frac{1.5131670194948619 + (24 - 20.486832980505138)}{24} * 100$

Out[113]= 20.9431



In[114]:= $g[x\_] := \dfrac{20}{x-26} - \dfrac{q}{x+1} + 12$

In[115]:= `Solve[g[0] == 0]`

Out[115]= $\left\{\left\{q \to \dfrac{146}{13}\right\}\right\}$

In[116]:= `Solve[g[24] == 0]`

Out[116]= `{{q → 50}}`

In[117]:= `Solve[g[0] == 0] // N`

Out[117]= `{{q → 11.2308}}`

In[118]:= `Assuming[q > 0, Apart[Solve[g'[x] == 0, x]]]`

Out[118]= $\left\{\left\{x \to 26 - \dfrac{54\left(-10+\sqrt{5}\sqrt{q}\right)}{-20+q}\right\}, \left\{x \to 26 + \dfrac{54\left(10+\sqrt{5}\sqrt{q}\right)}{-20+q}\right\}\right\}$

In[119]:= $\text{Plot}\left[\left\{26 - \dfrac{54\left(-10+\sqrt{5}\sqrt{q}\right)}{-20+q}, 26 + \dfrac{54\left(10+\sqrt{5}\sqrt{q}\right)}{-20+q}\right\}, \{q, 0, 146/13\}\right]$

Out[119]=

In[120]:= $\text{Reduce}\left[0 < 26 - \dfrac{54\left(-10+\sqrt{5}\sqrt{q}\right)}{-20+q} < 24, q\right]$

Out[120]= $\dfrac{5}{169} < q < 20 \,||\, 20 < q < 3125$

In[121]:= `Reduce[g[0] ≥ g[24], q]`

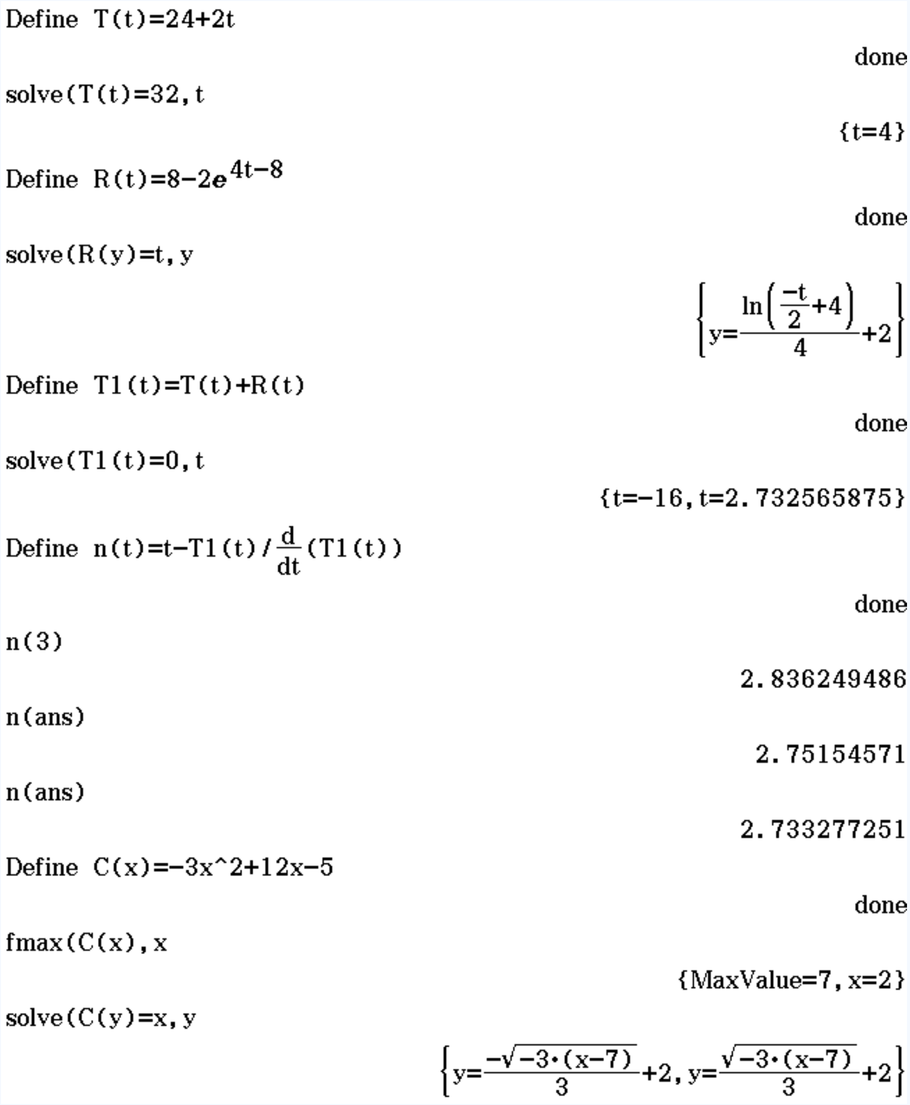
Out[121]= $q \le \dfrac{125}{13}$

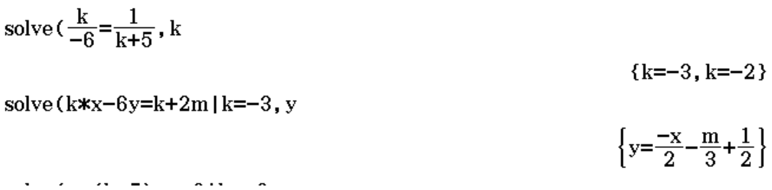
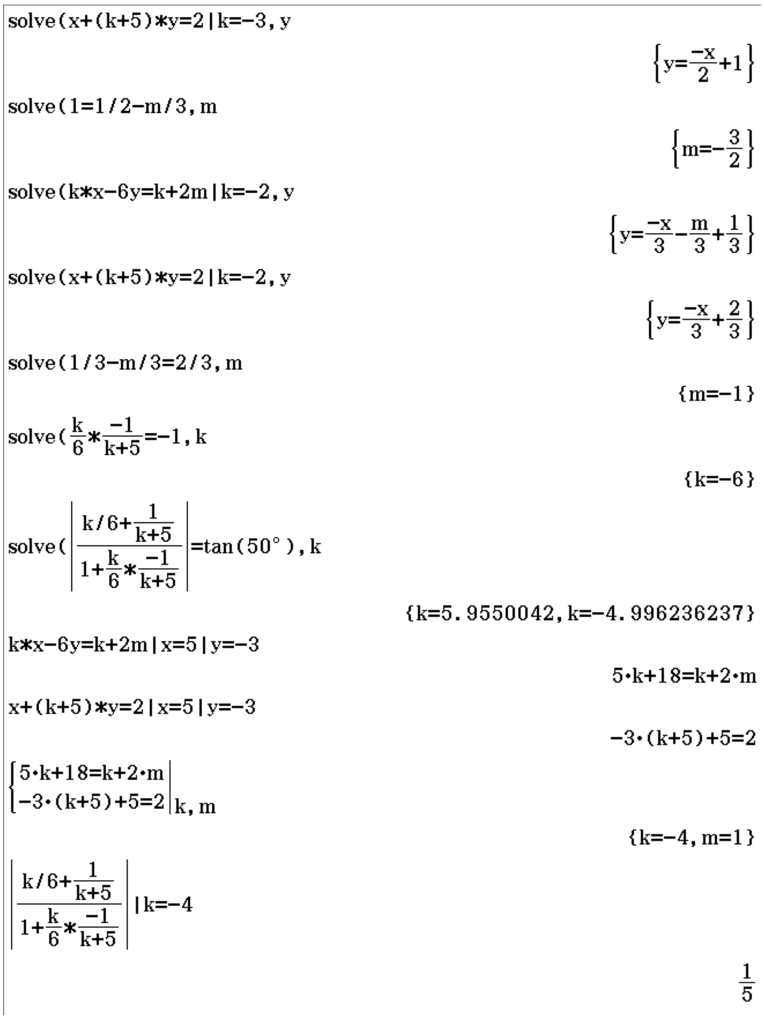
In[122]:= $h[x\_] := \dfrac{p}{x-26} - \dfrac{q}{x+1} + 12$

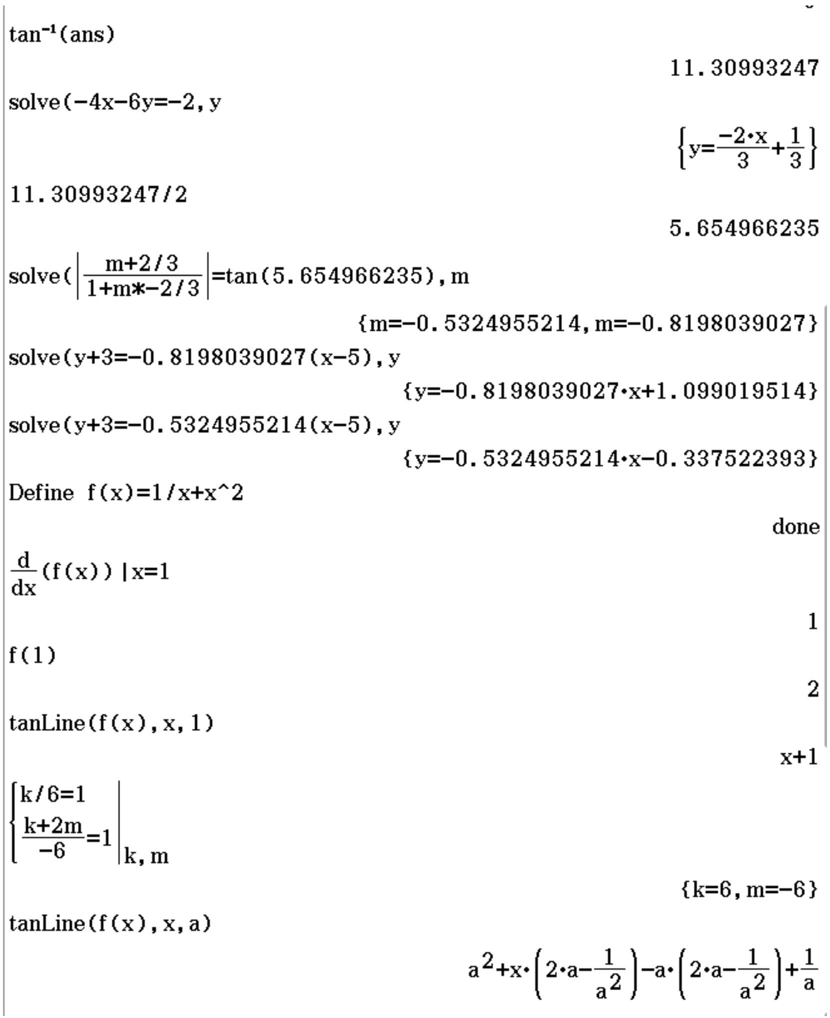
In[123]:= `Solve[h'[12] == 0 && h[12] == 10]`

Out[123]= $\left\{\left\{p \to \dfrac{392}{27}, q \to \dfrac{338}{27}\right\}\right\}$

Section E: Casio Solutions

Question Number	Solutions
1	 Define T(t)=24+2t done solve(T(t)=32,t {t=4} Define $R(t)=8-2e^{4t-8}$ done solve(R(y)=t,y $\left\{y=\frac{\ln\left(\frac{-t}{2}+4\right)}{4}+2\right\}$ Define T1(t)=T(t)+R(t) done solve(T1(t)=0,t {t=−16, t=2.732565875} Define $n(t)=t-T1(t)/\frac{d}{dt}(T1(t))$ done n(3) 2.836249486 n(ans) 2.75154571 n(ans) 2.733277251 Define C(x)=−3x^2+12x−5 done fmax(C(x),x {MaxValue=7, x=2} solve(C(y)=x,y $\left\{y=\frac{-\sqrt{-3\cdot(x-7)}}{3}+2, y=\frac{\sqrt{-3\cdot(x-7)}}{3}+2\right\}$

2	 $\text{solve}\left(\frac{k}{-6}=\frac{1}{k+5},k\right.$ $\{k=-3,k=-2\}$ $\text{solve}(k*x-6y=k+2m\mid k=-3,y$ $\left\{y=\frac{-x}{2}-\frac{m}{3}+\frac{1}{2}\right\}$  $\text{solve}(x+(k+5)*y=2\mid k=-3,y$ $\left\{y=\frac{-x}{2}+1\right\}$ $\text{solve}(1=1/2-m/3,m$ $\left\{m=-\frac{3}{2}\right\}$ $\text{solve}(k*x-6y=k+2m\mid k=-2,y$ $\left\{y=\frac{-x}{3}-\frac{m}{3}+\frac{1}{3}\right\}$ $\text{solve}(x+(k+5)*y=2\mid k=-2,y$ $\left\{y=\frac{-x}{3}+\frac{2}{3}\right\}$ $\text{solve}(1/3-m/3=2/3,m$ $\{m=-1\}$ $\text{solve}\left(\frac{k}{6}*\frac{-1}{k+5}=-1,k\right.$ $\{k=-6\}$ $\text{solve}\left(\left\|\frac{k/6+\frac{1}{k+5}}{1+\frac{k}{6}*\frac{-1}{k+5}}\right\|=\tan(50°),k\right.$ $\{k=5.9550042,k=-4.996236237\}$ $k*x-6y=k+2m\mid x=5\mid y=-3$ $5\cdot k+18=k+2\cdot m$ $x+(k+5)*y=2\mid x=5\mid y=-3$ $-3\cdot(k+5)+5=2$ $\left\{\begin{array}{l}5\cdot k+18=k+2\cdot m\\-3\cdot(k+5)+5=2\end{array}\right\|_{k,m}$ $\{k=-4,m=1\}$ $\left\|\frac{k/6+\frac{1}{k+5}}{1+\frac{k}{6}*\frac{-1}{k+5}}\right\|\mid k=-4$ $\frac{1}{5}$



$\tan^{-1}(\text{ans})$

11.30993247

solve(−4x−6y=−2, y

$\left\{y=\frac{-2\cdot x}{3}+\frac{1}{3}\right\}$

11.30993247/2

5.654966235

$\text{solve}\left(\left|\frac{m+2/3}{1+m*-2/3}\right|=\tan(5.654966235),m\right.$

{m=−0.5324955214, m=−0.8198039027}

solve(y+3=−0.8198039027(x−5), y

{y=−0.8198039027·x+1.099019514}

solve(y+3=−0.5324955214(x−5), y

{y=−0.5324955214·x−0.337522393}

Define f(x)=1/x+x^2

done

$\frac{d}{dx}(f(x))|x=1$

1

f(1)

2

tanLine(f(x), x, 1)

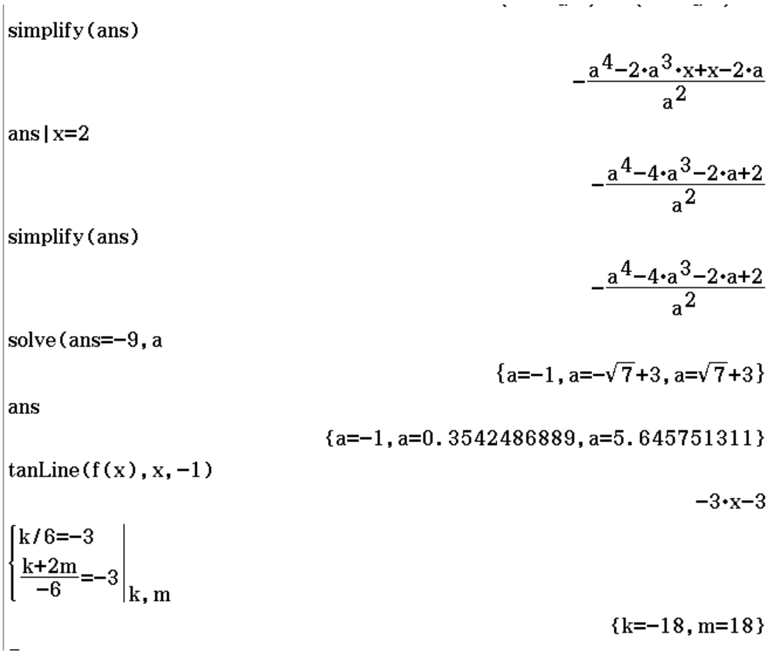
x+1

$\begin{cases} k/6=1 \\ \frac{k+2m}{-6}=1 \end{cases}\Bigg|k,m$

{k=6, m=−6}

tanLine(f(x), x, a)

$a^2+x\cdot\left(2\cdot a-\frac{1}{a^2}\right)-a\cdot\left(2\cdot a-\frac{1}{a^2}\right)+\frac{1}{a}$



simplify(ans)

$-\frac{a^4-2\cdot a^3\cdot x+x-2\cdot a}{a^2}$

ans|x=2

$-\frac{a^4-4\cdot a^3-2\cdot a+2}{a^2}$

simplify(ans)

$-\frac{a^4-4\cdot a^3-2\cdot a+2}{a^2}$

solve(ans=−9, a

$\{a=-1, a=-\sqrt{7}+3, a=\sqrt{7}+3\}$

ans

{a=−1, a=0.3542486889, a=5.645751311}

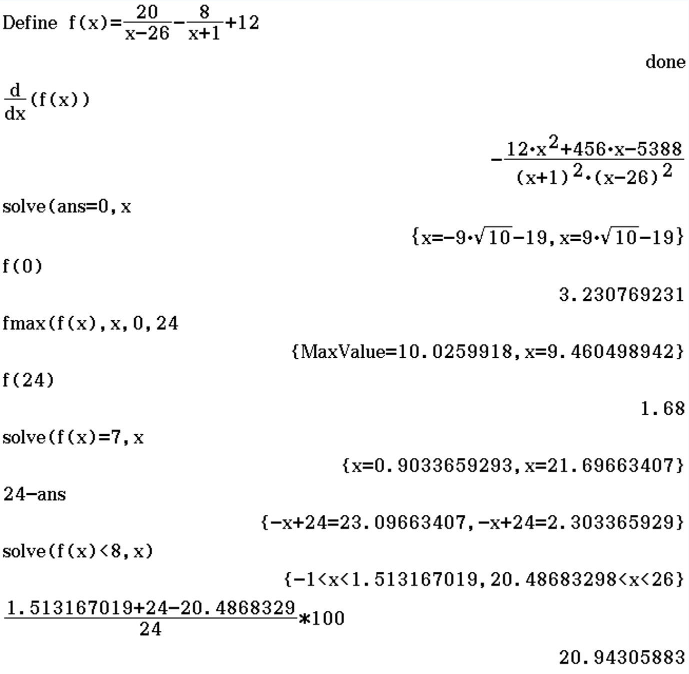
tanLine(f(x), x, −1)

−3·x−3

$\begin{cases} k/6=-3 \\ \frac{k+2m}{-6}=-3 \end{cases}\Bigg|k,m$

{k=−18, m=18}

3



Define $f(x)=\frac{20}{x-26}-\frac{8}{x+1}+12$

done

$\frac{d}{dx}(f(x))$

$-\frac{12\cdot x^2+456\cdot x-5388}{(x+1)^2\cdot(x-26)^2}$

solve(ans=0, x

$\{x=-9\cdot\sqrt{10}-19, x=9\cdot\sqrt{10}-19\}$

f(0)

3.230769231

fmax(f(x), x, 0, 24

{MaxValue=10.0259918, x=9.460498942}

f(24)

1.68

solve(f(x)=7, x

{x=0.9033659293, x=21.69663407}

24−ans

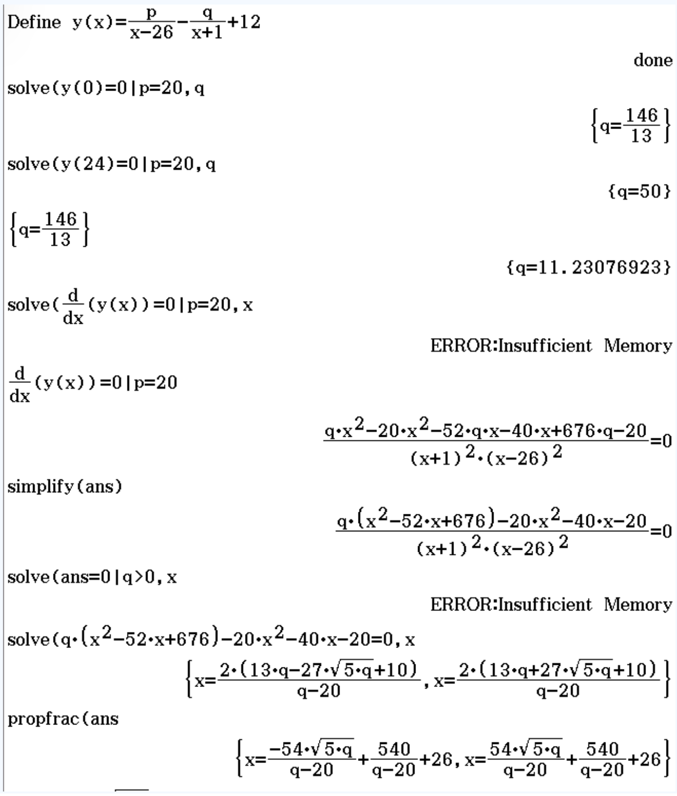
{−x+24=23.09663407, −x+24=2.303365929}

solve(f(x)<8, x)

{−1<x<1.513167019, 20.48683298<x<26}

$\frac{1.513167019+24-20.4868329}{24}*100$

20.94305883



Define $y(x)=\frac{p}{x-26}-\frac{q}{x+1}+12$

done

solve(y(0)=0 | p=20, q

$\left\{q=\frac{146}{13}\right\}$

solve(y(24)=0 | p=20, q

{q=50}

$\left\{q=\frac{146}{13}\right\}$

{q=11.23076923}

solve($\frac{d}{dx}$(y(x))=0 | p=20, x

ERROR:Insufficient Memory

$\frac{d}{dx}$(y(x))=0 | p=20

$\frac{q\cdot x^2-20\cdot x^2-52\cdot q\cdot x-40\cdot x+676\cdot q-20}{(x+1)^2\cdot(x-26)^2}=0$

simplify(ans)

$\frac{q\cdot\left(x^2-52\cdot x+676\right)-20\cdot x^2-40\cdot x-20}{(x+1)^2\cdot(x-26)^2}=0$

solve(ans=0 | q>0, x

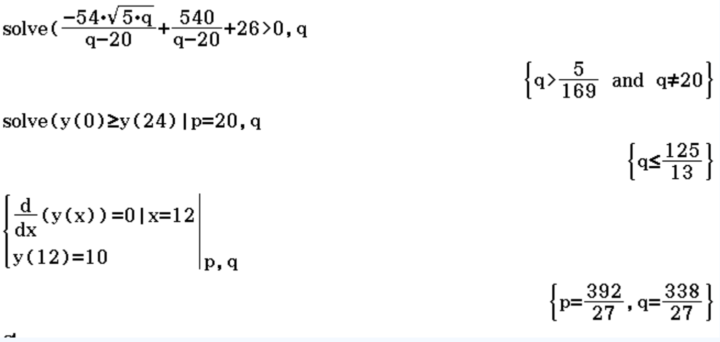
ERROR:Insufficient Memory

solve($q\cdot\left(x^2-52\cdot x+676\right)-20\cdot x^2-40\cdot x-20=0$, x

$\left\{x=\frac{2\cdot(13\cdot q-27\cdot\sqrt{5\cdot q}+10)}{q-20}, x=\frac{2\cdot(13\cdot q+27\cdot\sqrt{5\cdot q}+10)}{q-20}\right\}$

propfrac(ans

$\left\{x=\frac{-54\cdot\sqrt{5\cdot q}}{q-20}+\frac{540}{q-20}+26, x=\frac{54\cdot\sqrt{5\cdot q}}{q-20}+\frac{540}{q-20}+26\right\}$



solve($\frac{-54\cdot\sqrt{5\cdot q}}{q-20}+\frac{540}{q-20}+26>0$, q

$\left\{q>\frac{5}{169} \text{ and } q\neq 20\right\}$

solve(y(0)≥y(24) | p=20, q

$\left\{q\leq\frac{125}{13}\right\}$

$\left\{\begin{array}{l}\frac{d}{dx}(y(x))=0 \mid x=12\\ y(12)=10\end{array}\right|_{p,q}$

$\left\{p=\frac{392}{27}, q=\frac{338}{27}\right\}$

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VCE Mathematical Methods ¾

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