[Qing]

CONTOUREDUCATION

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VCE Mathematical Methods ¾
SAC 1 Revision V [0.20]
Workshop

Error Logbook:

New Ideas/Concepts	Didn't Read Question
Pg / Q #: ____________________ Notes:	Pg / Q #: ____________________ Notes:
Algebraic/Arithmetic/ Calculator Input Mistake	**Working Out Not Detailed Enough**
Pg / Q #: ____________________ Notes:	Pg / Q #: ____________________ Notes:

Section A: SAC 1 Success

Welcome to the fifth SAC 1 workshop!

Context: SAC 1 Workshops

- SAC 1-50% of SACs, 20% of the study score.
- Will be running all the way till mid-June.
- After that, the workshop will turn into SAC 2 Workshop (Integration).
- Make sure to complete the SAC $1 \sim 8$.

Successful SAC

$$\text{Study Score} = \text{How much you know} \times \text{How much you show}$$

2 marks

As much as you can

- Answer everything you know.
- Answer without mistakes.
- Time management is **key!**

Analogy: Skipping Questions

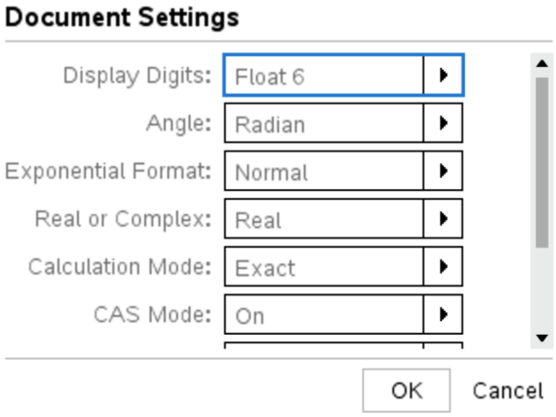
- Let's say if you were to fight them and win, you get assigned marks.

- Who would you fight first?
- Skip the hard question with little marks if it doesn't make sense during the reading time.

SAC Proficiency List

Before the SAC

- ☐ Prepare your stationery including a ruler, eraser and your mechanical pencil lead.
- ☐ Skim through the bound reference (if applicable).
- ☐ Do not speak to other people and lock in.
- ☐ TI & Mathematica Only: Check your Contour UDFS.
- ☐ TI Only: Check technology settings.



Reading Time

- ☐ **Detailed strategy** on how to exactly solve the question on your technology – Don't just **read**, think about how to solve it and use what technology commands.
- ☐ Identify questions to **skip.**

part b)......

part c)..

For difficult SACs, it's not necessarily about getting the 100%. *It's about getting better than everyone else. While others are stuck on a hard 1-marker question, you can do an easy 3-marker first.*

- ☐ Identify questions to **start first** – You don't have to start from $Q1$!
- ☐ Look for potential pitfalls – **Units, domain restriction of the unknown, variable and function meaning.**

Writing Time

- ☐ **Circle what the question is asking** for in the question.
- ☐ Spend the first $\mathbf{50}\%$ of the time on all the **easy questions** you identified.
- ☐ Spend the next $\mathbf{25}\%$ of doing the **difficult questions** you left blank.

- Spend the last 25% of the time on **checking your answers**.
- Check your answer by reading the question again and see if you answered the question.
- Check in the order of:

  Domino effect (check a, b, c first) $>$ ***Questions with high marks*** $(3+)$ $>$ *Hard Questions*

- $\mathbf{TI}$ ONLY: Use new document - **doc $\mathbf{4, 1.}$**

After the SAC

- Think about how each mark loss can be prevented using this proficiency list.
- Think about the big picture and improve the marks -

  Instead of spending 10 *minutes on* 10*c) (*1 *mark), I should have checked* 5*a) (*3 *marks)*

Space for Personal Notes

Section B: SAC Questions - Tech-Active (52 Marks)

INSTRUCTION:

- 52 Marks. 15 Minutes Reading, 75 Minutes Writing.

Question 1 (9 marks)

Zack Mewton is conducting an experimental synthesis in pursuit of the Philosopher's Stone - a theoretical substance capable of catalysing the transmutation of lead into gold. The reaction chamber begins at a temperature of 24°C and is heated at a controlled rate of 2°C per minute using a calibrated thermal source.

$t=0,\ T=24$ $m=2$

a.

i. Derive an equation for the temperature T, in °C, t minutes after the heat source is activated. (1 mark)

$T = 2t + 24$

ii. The initial reagent — a purified suspension of metallic salts — must be added when the temperature reaches 32°C. After how many minutes should this be done? (1 mark)

$T = 32$

$32 = 2t + 24$

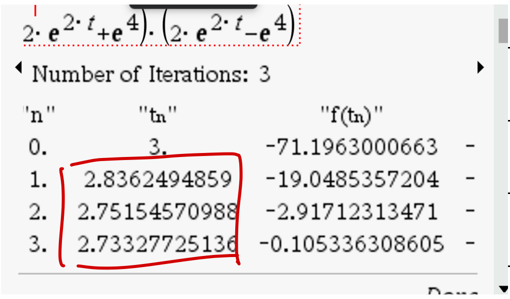
$2t = 8,\quad t = 4$

After 4 minutes reaches $32^\circ C$

b. The addition of the metallic salts induces a non-linear thermal fluctuation. The resulting temperature of the system is given by: $T(t) = 2t + 24$

$$T_1(t) = T(t) + R(t), \text{ where } R(t) = 8 - 2e^{4t-8}$$

i. Perform three iterations of Newton's method with $t_0 = 3$ to find an approximate solution to the equation $T_1(t) = 0$. Give your answer correct to three decimal places. (2 marks)


$2\cdot e^{2\cdot t}+e^{4})\cdot(2\cdot e^{2\cdot t}-e^{4})$

Number of Iterations: 3

"n"	"tn"	"f(tn)"	
0.	3.	-71.1963000663	-
1.	2.8362494859	-19.0485357204	-
2.	2.75154570988	-2.91712313471	-
3.	2.73327725136	-0.105336308605	-

$t_{n+1} = t_n - \frac{T_1(t_n)}{T_1'(t_n)}$

$t_1 = 2.836$

$t_2 = 2.752$

$t_3 = 2.733$

ii. To stabilise the system, Zack introduces a compensating agent with inverse behaviour.

Determine the equation of the inverse function $R^{-1}(t)$. (2 marks)

$y = 8 - 2e^{4t-8}$

swap y & t

$t = 8 - 2e^{4y-8}$

$e^{4y-8} = \frac{t-8}{-2}$

$e^{4y-8} = \frac{8-t}{2}$

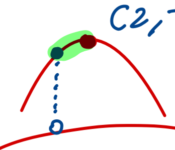
$4y - 8 = \ln\left(\frac{8-t}{2}\right)$

$y = \frac{1}{4}\left(\ln\left(\frac{8-t}{2}\right) + 8\right)$

$y = \frac{1}{4}\ln\left(\frac{8-t}{2}\right) + 2$

$R^{-1}(t) = \frac{1}{4}\ln\left(\frac{8-t}{2}\right) + 2$

c. For precision calibration, the reaction is transferred to a digitally stabilised platform. However, this platform introduces time-dependent perturbations in temperature, described by:



$$C(x) = -3x^2 + 12x - 5$$

mew → 7 → 5 compl. the square

where x is the time in minutes since the stabiliser was engaged.

i. Determine the largest value of a such that the domain restriction $[0, a]$ allows $C(x)$ to possess an inverse. (1 mark)

1−1

$C(x) = -3(x-2)^2 + 7$

T.P: $(2, 7)$

$\boxed{a = 2}$

ii. Find the expression for the inverse function $C^{-1}(x)$ over the restricted domain $[0, a]$. (2 marks)

let $y = -3(x-2)^2 + 7$

swap x & y

$x = -3(y-2)^2 + 7$

$(y-2)^2 = \frac{x-7}{-3}$

$y - 2 = \pm\sqrt{\frac{x-7}{-3}}$

$y = 2 \pm \sqrt{\frac{x-7}{-3}}$

Ran C^{-1} = Dom C = $[0, 2]$

$C^{-1}(x) = 2 - \sqrt{\frac{x-7}{-3}}$

Space for Personal Notes

Question 2 (22 marks)

Consider the lines L_1 and L_2 defined by the equations:

$L_1: y = \frac{k}{6}x - \frac{(k+2m)}{6}$

$L_2: y = \frac{-1}{k+5}x + \frac{2}{k+5}$

$$L_1:\ kx - 6y = k + 2m \text{ and } L_2: x + (k+5)y = 2,$$

where k and m are real constants.

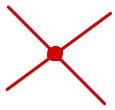
a. State the value of k if one of the lines is a vertical line. (1 mark)

$x = a$

$k = -5$

b. Find the value(s) of k and m if:

i. The lines L_1 and L_2 have a unique point of intersection. (3 marks)



Diff gradients

$\frac{k}{6} \neq \frac{-1}{k+5}$

$k(k+5) \neq -6$

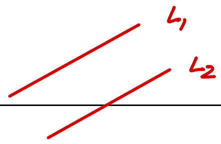
$k^2 + 5k + 6 \neq 0$

$(k+2)(k+3) \neq 0$

$k \neq -2, -3$

$\boxed{k \in \mathbb{R} \setminus \{-2, -3\} \quad m \in \mathbb{R}.}$

ii. The equations defining L_1 and L_2 have no solution. (2 marks)



Same gradient

$\frac{k}{6} = \frac{-1}{k+5}$

$\boxed{k = -2, -3}$

Diff c-value

$-\frac{(k+2m)}{6} \neq \frac{2}{k+5}$

$k = -2:$

$-\frac{(-2+2m)}{6} \neq \frac{2}{3}$

$\frac{2-2m}{6} \neq \frac{2}{3}$

$2 - 2m \neq 4$

$\boxed{m \neq -1}$

$k = -3:$

$\frac{3-2m}{6} \neq \frac{2}{2}$

$3 - 2m \neq 6$

$2m \neq -3$

$m \neq -\frac{3}{2}$

$\begin{cases} k = -2 \\ m \neq -1 \end{cases}$

$\begin{cases} k = -3 \\ m \neq -\frac{3}{2} \end{cases}$

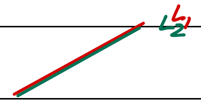
$L_1: \ y = \frac{k}{6}x - \frac{(k+2m)}{6}$

$L_2: \ y = \frac{-1}{k+5}x + \frac{2}{k+5}$

iii. The equations defining L_1 and L_2 have an infinite number of solutions. (1 mark)

same gradient — same c-value

$k = -2, -3$ — $m = -1, -\frac{3}{2}$



$\left(\begin{cases} k = -2 \\ m = -1 \end{cases} \quad \begin{cases} k = -3 \\ m = -\frac{3}{2} \end{cases} \right)$

c. Find the value(s) of k and m if:

i. The lines L_1 and L_2 are perpendicular to each other. (2 marks)

$\frac{k}{6} \cdot \frac{-1}{k+5} = -1$

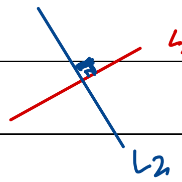
$\frac{k}{6(k+5)} = 1$

$k = 6k + 30$

$-5k = 30$

$k = -6$

$m \in \mathbb{R}$.



ii. The acute angle between L_1 and L_2 is equal to 50°. Give your answer(s) correct to three decimal places. (2 marks)

$\tan(\theta) = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

$\tan(50°) = \left| \frac{\frac{k}{6} - \left(\frac{-1}{k+5}\right)}{1 - \frac{k}{6(k+5)}} \right|$

solve for k:

$k \approx -4.996, \ 5.955$

$m \in \mathbb{R}$ — $m \in \mathbb{R}$.

d. The lines L_1 and L_2 intersect at the point $(5, -3)$.

i. Find the value of k and the value of m. (2 marks)

sub in $(5,-3)$ into both

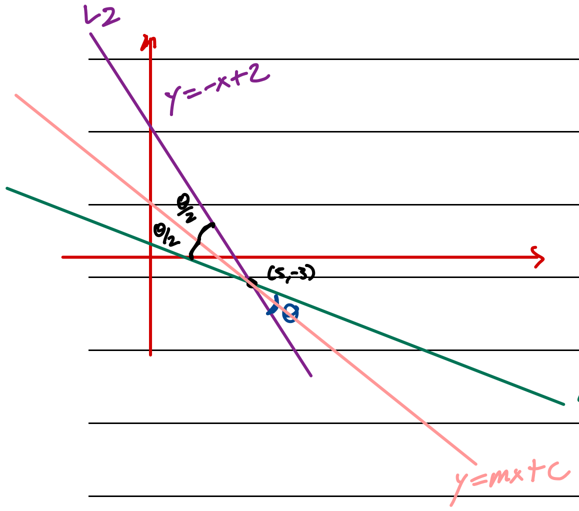
$L_1: 5k+18 = k+2m$
$L_2: 5-3(k+5)=2$
solve for k & m:

$k=-4$
$m=1$

cuts Angle into 2

ii. Find the equation of the line that **bisects** the acute angle made between L_1 and L_2 when they intersect at $(5, -3)$. Give your answer in the form $y = mx + c$ where m and c are given correct to four decimal places. (3 marks)

$k=-4$
$m=1$



$L_1: y = -\frac{4}{6}x + \frac{2}{6}$

$y = -\frac{2}{3}x + \frac{1}{3}$

$L_2: y = \frac{-1}{1}x + \frac{2}{1}$

$y = -x+2$

$\tan(\theta) = \left|\frac{-\frac{2}{3}-(-1)}{1+\frac{2}{3}}\right|, 0<\theta<90°$

$\theta = \tan^{-1}(\frac{1}{5})$

Angle $= \frac{1}{2}\tan^{-1}(\frac{1}{5})$

$\tan(\frac{1}{2}\tan^{-1}(\frac{1}{5})) = \left|\frac{m-(-1)}{1+(m\cdot -1)}\right|$, solve for m:

$m \approx -0.8198, -1.2198$

reject $m \approx -1.2198$ as line needs to bisect.

$y-(-3) = -0.8198....(x-5)$

$y = -0.8198x + 1.0990$

e. Find the value(s) of k and m if:

Tangent/Normal: 1. Point 2. Gradient

i. The line L_1 is tangent to the curve $y = \frac{1}{x} + x^2$ at the point where $x = 1$. (3 marks)

$\frac{dy}{dx} = 2x - \frac{1}{x^2}$

At $x=1$

$\frac{dy}{dx} = 2-1 = 1$

At $x=1$, $y = 1+1 = 2$ $(1, 2)$

$y - 2 = 1(x-1)$

$\boxed{y = x+1}$

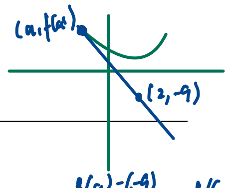
$L_1: \ y = \frac{k}{6}x - \frac{(k+2m)}{6}$

$\frac{k}{6} = 1$, $-\frac{(k+2m)}{6} = 1$

$\boxed{k=6}$, $-6-2m=6$

$\boxed{m=-6}$

ii. The line L_1 passes through the point $(2, -9)$, has a negative x-axis intercept, and is tangent to the curve $y = \frac{1}{x} + x^2$. (3 marks)



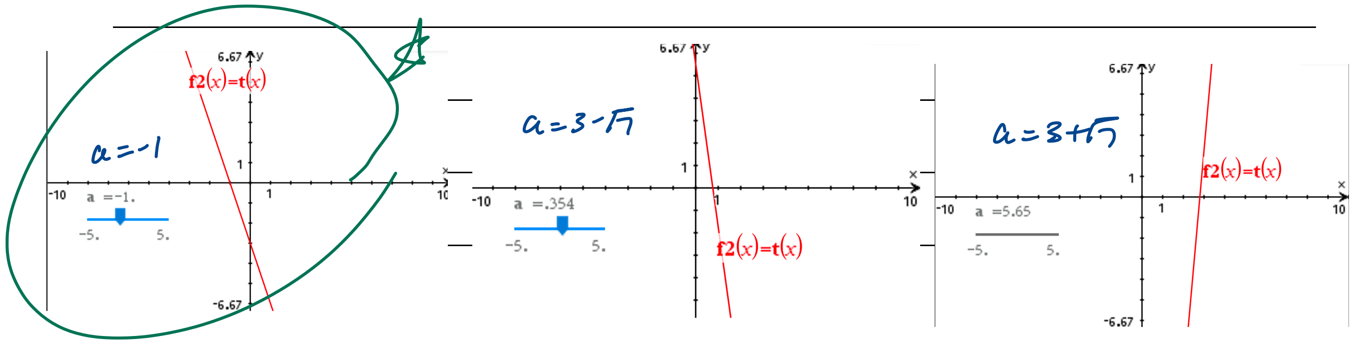
$\frac{f(a)-(-9)}{a-2} = f'(a)$

tangent to $y = \frac{1}{x} + x^2$ at $x = a$

$t(x) = \frac{(2a^3-1)}{a^2}x - \frac{a^3-2}{a}$

$t(2) = -9$, solve for a

$a = -1, \ 3 \pm \sqrt{7}$



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when $a=-1$

$t(x) = -3x-3$

$\frac{k}{6} = -3$, $-\frac{(k+2m)}{6} = -3$

$k = -18$
$m = 18$

Question 3 (21 marks)

A medical researcher is studying the diffusion of a drug from two injection sites, A and B, placed 24 millimetres apart in a tissue sample. The concentration of the drug, y, in mg/L at a point x millimetres from site A along the straight line between the two sites is modelled by:

$$y = \frac{p}{x-26} - \frac{q}{x+1} + 12, \qquad 0 \le x \le 24, \qquad y \ge 0$$

where p and q are positive constants.

a. For a particular formulation of the drug, the parameters are $p = 20$ and $q = 8$.

i. Solve an appropriate equation using algebra to determine the location x (in mm from site A) at which the drug concentration is at a maximum. All working must be shown. (4 marks)

$y = \frac{20}{x-26} - \frac{8}{x+1} + 12$

$\frac{dy}{dx} = \frac{-20}{(x-26)^2} - \left(\frac{-8}{(x+1)^2}\right)$

$\frac{dy}{dx} = \frac{-20}{(x-26)^2} + \frac{8}{(x+1)^2}$

$\frac{dy}{dx} = 0$

$\frac{-20}{(x-26)^2} + \frac{8}{(x+1)^2} = 0$ (1)

$\frac{8}{(x+1)^2} = \frac{20}{(x-26)^2}$

$8(x-26)^2 = 20(x+1)^2$

$8(x-26)^2 - 20(x+1)^2 = 0$

$-12(x^2 + 38x - 449) = 0$

$x^2 + 38x - 449 = 0$ (1)

$x = \frac{-38 \pm \sqrt{38^2 - 4(-449)}}{2}$

$x = -19 \pm 9\sqrt{10}$

As $x \in [0, 24]$ (1)

take $x = -19 + 9\sqrt{10}$ (1)

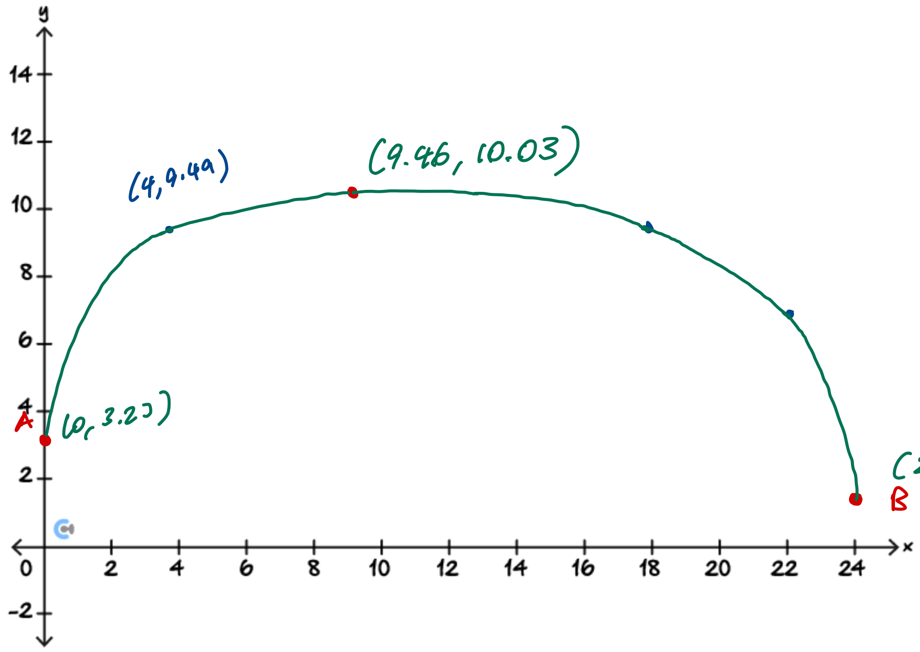
ii. Sketch the graph of the function:

$$y = \frac{20}{x-26} - \frac{8}{x+1} + 12$$

Graph Trace !

menu → 5 → 1

for $0 \le x \le 24$. Label the endpoints and the turning point with their coordinates, correct to two decimal places. (2 marks)



iii. At what distance(s) from the injection site B is the drug concentration equal to $7\ mg/L$? Give your answer in millimetres, correct to three decimal places (2 marks)

$y = 7$

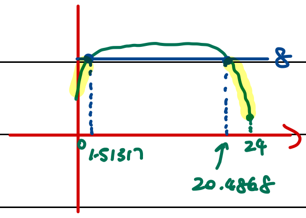
$x \approx 0.903$mm, 21.697mm

$24 - 0.903... \approx \boxed{23.097\text{mm}}$

$24 - 21.697... \approx 2.303$mm

iv. Determine, correct to one decimal place, the percentage of the distance between A and B over which the concentration y is below $8\ mg/L$. (2 marks)

$y = 8$, $x \approx 1.51317$, 20.4868



$\dfrac{1.51317 - 0 + 24 - 20.4868}{24} \times 100\%$

$\approx \boxed{20.9\%}$

b. The researcher fixes $p = 20$ and considers how varying the drug with different values of q affects the concentration model.

$y = \dfrac{20}{x-26} - \dfrac{q}{x+1} + 12$

i. Determine the possible values of q for which the model is defined on the interval $0 < x < 24$. (2 marks)

conc ≥ 0 conditions.

$y(0) \geq 0$ left endpoint

$y(24) \geq 0$ right endpoint

$0 < q \leq \dfrac{146}{13}$

ii. Find, in terms of q, the x-coordinate of the turning point of the function y, when it exists. (2 marks)

$\dfrac{dy}{dx} = 0$

$x = \dfrac{2(27\sqrt{5q} + 13q + 10)}{q-20}, \dfrac{2(-27\sqrt{5q} + 13q + 10)}{q-20}$

$x \in [0, 24]$

Define $x1(q) = \dfrac{2 \cdot (27 \cdot \sqrt{5 \cdot q} + 13 \cdot q + 10)}{q-20}$ *Done*

Define $x2(q) = \dfrac{2 \cdot (-27 \cdot \sqrt{5 \cdot q} + 13 \cdot q + 10)}{q-20}$ *Done*

$x1(1)$ -8.77619319921

$x2(1)$ 3.93408793605

$x = \dfrac{2(-27\sqrt{5q} + 13q + 10)}{q-20}$

iii. Identify the values of q for which the function y does **not** have an inverse. (2 marks)

Turning point to be between $x \in [0, 24]$ not 1-1

$24 > \frac{2(-27\sqrt{5q}+13q+10)}{q-20} > 0$

$3125 > q > \frac{5}{169}$

$\frac{146}{13} > q > \frac{5}{169}$

iv. Determine the value(s) of q for which the **minimum** drug concentration occurs when $x = 24$. (2 marks)

$y(0) \geq y(24)$, solve for q:

left endpoint

right endpoint

$0 < q \leq \frac{125}{13}$

c. The research team designs a formulation such that the drug reaches a maximum concentration of exactly 10 mg/L at the midpoint between the injection sites A and B. Determine the corresponding values of p and q that achieve this. (3 marks)

$x = 12$

$y(12) = 10$ 1

$y'(12) = 0$ 1

$p = \frac{392}{27}$

$q = \frac{338}{27}$ 1

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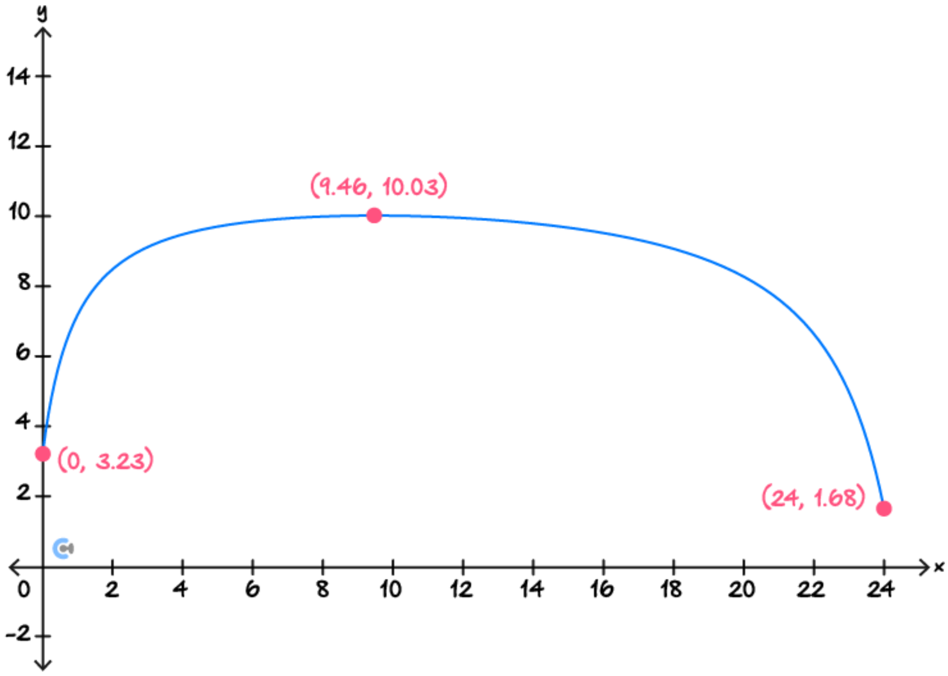
Section C: Marking Scheme

Question Number	Solutions
1 a i	$T = 24 + 2t$. **[1A]**
1 a ii	$t = 4$ minutes. **[1A]**
1 b i	Using Newton's method with $t_0 = 3$ we get: $t_1 = 2.836$, $t_2 = 2.751$, $t_3 = 2.733$ **[1M for one correct value]** Our estimate is $t = 2.733$ **[1A]**
1 b ii	Let $y = R^{-1}(t)$, then $R(y) = t$, so we solve $t = 8 - 2e^{4y-8}$ **[1M]** $e^{4y-8} = \frac{8-t}{2}$ $4y - 8 = \log_e\left(\frac{8-t}{2}\right)$ $y = \frac{1}{4}\log_e\left(\frac{8-t}{2}\right) + 2$ So $R^{-1}(t) = \frac{1}{4}\log_e\left(\frac{8-t}{2}\right) + 2$. **[1A]**
1 c i	Turning point at $(2, 7)$ so $a = 2$. **[1M]**
1 c ii	Let $y = C^{-1}(x)$. Then we solve $C(y) = x$. This yields $y = 2 \pm \sqrt{\frac{1}{3}(7-x)}$. **[1M]** But the range must be $[0, 2]$ so we have $C^{-1}(x) = 2 - \sqrt{\frac{1}{3}(7-x)}$ **[1A]**
2 a	Vertical line is of the form $x = a$. This is only possible if $k = -5$. **[1A]**

2 b i	The gradient of the two lines must differ for a unique point of intersection. **[1M]** Gradients are the same if $\frac{k}{-6} = \frac{1}{k+5} \implies k = -3, -2$ **[1M]** So there is a unique point of intersection if $k \in \mathbb{R} \setminus \{-3, -2\},\ m \in \mathbb{R}$ **[1A]**
2 b ii	No solution if the lines are parallel and not the same. If $k = -3$ then if $m = -\frac{3}{2}$ the lines are the same. So no solution if $k = -3, \quad m \in \mathbb{R} \setminus \left\{-\frac{3}{2}\right\}$ **[1A]** If $k = -2$ then if $m = -1$ the lines are the same. So no solution if $k = -2, \quad m \in \mathbb{R} \setminus \{-1\}$ **[1A]**
2 b iii	$k = -3$ and $m = -\frac{3}{2}$ OR $k = -2$ and $m = -1$. **[1A]**
2 c i	Perpendicular if product of the gradients is -1. Thus $\frac{k}{6} \times \frac{-1}{k+5} = -1$ **[1M]** $\implies k = -6$. $k = -6, \quad m \in \mathbb{R}$. **[1A]**
2 c ii	We require that $$\left\lvert \frac{\frac{k}{6} + \frac{1}{k+5}}{1 + \frac{k}{6}\left(\frac{-1}{k+5}\right)} \right\rvert = \tan(50^\circ) \quad \textbf{[1M]}$$ $\implies k = -4.996, k = 5.955$ So $k = -4.996, \quad m \in \mathbb{R} \quad$ or $\quad k = 5.955, \quad m \in \mathbb{R}$. **[1A]**
2 d i	Sub the point into both equations to get the system $$18 + 5k = k + 2m$$ $$5 - 3(5 + k) = 2 \quad \textbf{[1M]}$$ solving gives $k = -4$ and $m = 1$ **[1A]**

2 d ii	The two lines make an acute angle of $\theta = \arctan\left(\frac{1}{5}\right) \approx 11.3099°$ **[1M]** Take one line to be $-4x - 6y = -2 \implies y = \frac{1}{3} - \frac{2x}{3}$. The bisecting line passes through $(5, -3)$ and makes an acute angle of $\approx 5.65°$ with the above line. This line must have gradient of $m \approx -0.8198$. **[1M]** The line then has equation $y = 1.0990 - 0.8198x$ **[1A]**
2 e i	Let $f(x) = \frac{1}{x} + x^2$. Then $f'(1) = 1$. So line L_1 has gradient of 1 and passes through $(1, 2)$. **[1M]** $$y = 1 + x \quad \textbf{[1M]}$$ so now $\frac{k}{6} = 1$ and $\frac{k+2m}{-6} = 1 \implies k = 6, m = -6$ **[1A]**
2 e ii	Tangent to $f(x) = \frac{1}{x} + x^2$ at $x = a$ is given by $y = -\frac{a^4 - 2a^3x - 2a + x}{a^2}$ **[1M]** This line passes through $(2, -9)$ so we get the equation $$-\frac{a^4 - 4a^3 - 2a + 2}{a^2} = -9$$ $$\implies a = -1, 3 \pm \sqrt{7} \quad \textbf{[1M]}$$ Only $a = -1$ will give a negative x-axis intercept. Thus the line is $y = -3 - 3x$. So $k = -18$ and $m = 18$. **[1A]**
3 a i	Let $y(x) = \frac{20}{x-26} - \frac{8}{x+1} + 12$. To find a maximum we must solve $$y'(x) = \frac{8}{(x+1)^2} - \frac{20}{(x-26)^2} = 0 \quad \textbf{[1M]}$$ $$\implies -\frac{12\left(x^2 + 38x - 449\right)}{(x-26)^2(x+1)^2} = 0$$ so we may solve $x^2 + 38x - 449 = 0$ **[1M]** $$(x+19)^2 - 810 = 0$$ $$x + 19 = \pm 9\sqrt{10}$$ $$x = -19 \pm 9\sqrt{10} \quad \textbf{[1M]}$$ But we require that $x \geq 0$ so the only solution is $x = -19 + 9\sqrt{10}$. **[1A, must say why we reject value]**

3 a ii	 **[1A shape, 1A coordinates]**
3 a iii	Solve $y(x) = 7 \implies x = 0.903, 21.697$. These are distances from site A. **[1M]** Therefore distance froms site B are 23.097 and 2.303 millimetres. **[1A]**
3 a iv	We solve $y(x) = 8 \implies x = 1.51317, 20.4868$. **[1M]** So the percentage is $\frac{1.51317 + (24 - 20.4868)}{24} \times 100 = 20.9\%$ **[1A]** .
3 b i	We require that $y(0) \geq 0$ and that $y(24) \geq 0$. **[1M]** Also $q > 0$ since we said that it is a **positve** constant. So $0 < q \leq \frac{146}{13}$. **[1A]**
3 b ii	We solve $y'(x) = 0 \implies x = 26 - \frac{54\left(\sqrt{5q} - 10\right)}{q - 20}, \frac{54\left(\sqrt{5q} + 10\right)}{q - 20} + 26$ **[1M]** But we need the value that is between 0 and 24 on the domain of q. So $x = 26 - \frac{54\left(\sqrt{5q} - 10\right)}{q - 20}$ or equivalent. **[1A]**

3 b iii	No inverse if the function has a turning point in the interval $0 < x < 24$. Note we already have the restriction $0 < q \leq \frac{146}{13}$ Using above part b.ii we also solve $0 < 26 - \frac{54\left(\sqrt{5q}-10\right)}{q-20} \implies q > \frac{5}{169}$ **[1M]** Combining our restrictions gives no inverse if $$\frac{5}{169} < q \leq \frac{146}{13} \quad \textbf{[1A]}$$
3 b iv	We will require that $y(0) \geq y(24)$ **[1M]** Thus $0 < q \leq \frac{125}{13}$ **[1A]** .
3 c	We must have a max of 10 when $x = 12$. So must have $\frac{dy}{dx} = 0$ when $x = 12$. **[1M]** We solve $y'(12) = 0$ and $y(12) = 10$ simultaneously to get $p = \frac{392}{27}$ **[1M]** and $q = \frac{338}{27}$ **[1A]** .

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