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VCE Mathematical Methods $\frac{3}{4}$
SAC 1 Revision V [0.20]
Workshop

Error Logbook:



| New Ideas/Concepts | Didn't Read Question |
|---|--------------------------------------|
| <p>Pg / Q #: _____</p> <p>Notes:</p> | <p>Pg / Q #: _____</p> <p>Notes:</p> |
| Algebraic/Arithmetic/ Calculator Input Mistake | Working Out Not Detailed Enough |
| <p>Pg / Q #: _____</p> <p>Notes:</p> | <p>Pg / Q #: _____</p> <p>Notes:</p> |

Section A: SAC 1 Success

Welcome to the fifth SAC 1 workshop!



Context: SAC 1 Workshops



- SAC 1-50% of SACs, 20% of the study score.
- Will be running all the way till mid-June.
- After that, the workshop will turn into SAC 2 Workshop (Integration).
- Make sure to complete the SAC 1 ~ 8.

Successful SAC



Study Score = How much you know × How much you show

- Answer everything you know.
- Answer without mistakes.
- Time management is **key!**

Analogy: Skipping Questions



- Let's say if you were to fight them and win, you get assigned marks.



- Who would you fight first?
- Skip the hard question with little marks if it doesn't make sense during the reading time.



SAC Proficiency List

Before the SAC

- ☐ Prepare your stationery including a ruler, eraser and your mechanical pencil lead.
- ☐ Skim through the bound reference (if applicable).
- ☐ Do not speak to other people and lock in.
- ☐ TI & Mathematica Only: Check your Contour UDFS.
- ☐ TI Only: Check technology settings.

Document Settings

| | | |
|---------------------|---------|---|
| Display Digits: | Float 6 | ▶ |
| Angle: | Radian | ▶ |
| Exponential Format: | Normal | ▶ |
| Real or Complex: | Real | ▶ |
| Calculation Mode: | Exact | ▶ |
| CAS Mode: | On | ▶ |

OK Cancel

Reading Time

- ☐ **Detailed strategy** on how to exactly solve the question on your technology - Don't just **read**, think about how to solve it and use what technology commands.
- ☐ Identify questions to **skip**.

For difficult SACs, it's not necessarily about getting the 100%. It's about getting better than everyone else. While others are stuck on a hard 1-marker question, you can do an easy 3-marker first.

- ☐ Identify questions to **start first** - You don't have to start from Q1!
- ☐ Look for potential pitfalls - **Units, domain restriction of the unknown, variable and function meaning.**

Writing Time

- ☐ Circle **what the question is asking** for in the question.
- ☐ Spend the first **50%** of the time on all the **easy questions** you identified.
- ☐ Spend the next **25%** of doing the **difficult questions** you left blank.

- ☐ Spend the last 25% of the time on **checking your answers**.
- ☐ Check your answer by reading the question again and see if you answered the question.
- ☐ Check in the order of:

Domino effect (check a, b, c first) > *Questions with high marks* (3+) > *Hard Questions*

- ☐ **TI ONLY:** Use new document - **doc 4, 1**.

After the SAC

- ☐ Think about how each mark loss can be prevented using this proficiency list.
- ☐ Think about the big picture and improve the marks -

Instead of spending 10 minutes on 10c) (1 mark), I should have checked 5a) (3 marks)

Space for Personal Notes

Section B: SAC Questions - Tech-Active (52 Marks)

INSTRUCTION:

➤ 52 Marks. 15 Minutes Reading, 75 Minutes Writing.



Question 1 (9 marks)

Zack Mewton is conducting an experimental synthesis in pursuit of the Philosopher's Stone - a theoretical substance capable of catalysing the transmutation of lead into gold. The reaction chamber begins at a temperature of 24°C and is heated at a controlled rate of 2°C per minute using a calibrated thermal source.

- a.
- Derive an equation for the temperature T , in $^{\circ}\text{C}$, t minutes after the heat source is activated. (1 mark)

- The initial reagent — a purified suspension of metallic salts — must be added when the temperature reaches 32°C . After how many minutes should this be done? (1 mark)

- b. The addition of the metallic salts induces a non-linear thermal fluctuation. The resulting temperature of the system is given by:

$$T_1(t) = T(t) + R(t), \text{ where } R(t) = 8 - 2e^{4t-8}$$

- i. Perform three iterations of Newton's method with $t_0 = 3$ to find an approximate solution to the equation $T_1(t) = 0$. Give your answer correct to three decimal places. (2 marks)

- ii. To stabilise the system, Zack introduces a compensating agent with inverse behaviour.

Determine the equation of the inverse function $R^{-1}(t)$. (2 marks)

- c. For precision calibration, the reaction is transferred to a digitally stabilised platform. However, this platform introduces time-dependent perturbations in temperature, described by:

$$C(x) = -3x^2 + 12x - 5$$

where x is the time in minutes since the stabiliser was engaged.

- i. Determine the largest value of a such that the domain restriction $[0, a]$ allows $C(x)$ to possess an inverse. (1 mark)

- ii. Find the expression for the inverse function $C^{-1}(x)$ over the restricted domain $[0, a]$. (2 marks)

Space for Personal Notes

Question 2 (22 marks)

Consider the lines L_1 and L_2 defined by the equations:

$$L_1 : kx - 6y = k + 2m \text{ and } L_2 : x + (k + 5)y = 2,$$

where k and m are real constants.

- a.** State the value of k if one of the lines is a vertical line. (1 mark)

- b.** Find the value(s) of k and m if:

- i.** The lines L_1 and L_2 have a unique point of intersection. (3 marks)

- ii.** The equations defining L_1 and L_2 have no solution. (2 marks)

iii. The equations defining L_1 and L_2 have an infinite number of solutions. (1 mark)

c. Find the value(s) of k and m if:

i. The lines L_1 and L_2 are perpendicular to each other. (2 marks)

ii. The acute angle between L_1 and L_2 is equal to 50° . Give your answer(s) correct to three decimal places. (2 marks)

d. The lines L_1 and L_2 intersect at the point $(5, -3)$.

i. Find the value of k and the value of m . (2 marks)

ii. Find the equation of the line that **bisects** the acute angle made between L_1 and L_2 when they intersect at $(5, -3)$. Give your answer in the form $y = mx + c$ where m and c are given correct to four decimal places. (3 marks)

e. Find the value(s) of k and m if:

i. The line L_1 is tangent to the curve $y = \frac{1}{x} + x^2$ at the point where $x = 1$. (3 marks)

ii. The line L_1 passes through the point $(2, -9)$, has a negative x -axis intercept, and is tangent to the curve $y = \frac{1}{x} + x^2$. (3 marks)

Space for Personal Notes

Question 3 (21 marks)

A medical researcher is studying the diffusion of a drug from two injection sites, A and B , placed 24 millimetres apart in a tissue sample. The concentration of the drug, y , in mg/L at a point x millimetres from site A along the straight line between the two sites is modelled by:

$$y = \frac{p}{x-26} - \frac{q}{x+1} + 12, \quad 0 \leq x \leq 24, \quad y \geq 0$$

where p and q are positive constants.

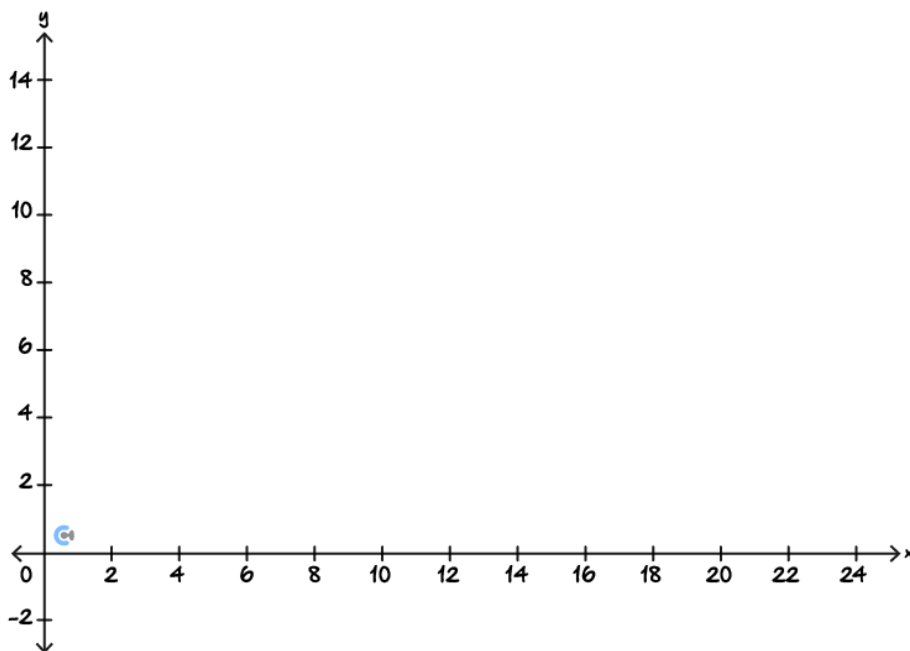
- a. For a particular formulation of the drug, the parameters are $p = 20$ and $q = 8$.
- i. Solve an appropriate equation using algebra to determine the location x (in mm from site A) at which the drug concentration is at a maximum. All working must be shown. (4 marks)

[illegible]

ii. Sketch the graph of the function:

$$y = \frac{20}{x-26} - \frac{8}{x+1} + 12$$

for $0 \leq x \leq 24$. Label the endpoints and the turning point with their coordinates, correct to two decimal places. (2 marks)



iii. At what distance(s) from the injection site B is the drug concentration equal to 7 mg/L ? Give your answer in millimetres, correct to three decimal places (2 marks)

- iv. Determine, correct to one decimal place, the percentage of the distance between A and B over which the concentration y is below 8 mg/L . (2 marks)

- b. The researcher fixes $p = 20$ and considers how varying the drug with different values of q affects the concentration model.

- i. Determine the possible values of q for which the model is defined on the interval $0 \leq x \leq 24$. (2 marks)

- ii. Find, in terms of q , the x -coordinate of the turning point of the function y , when it exists. (2 marks)

iii. Identify the values of q for which the function y does **not** have an inverse. (2 marks)

iv. Determine the value(s) of q for which the **minimum** drug concentration occurs when $x = 24$. (2 marks)

c. The research team designs a formulation such that the drug reaches a maximum concentration of exactly 10 mg/L at the midpoint between the injection sites A and B . Determine the corresponding values of p and q that achieve this. (3 marks)

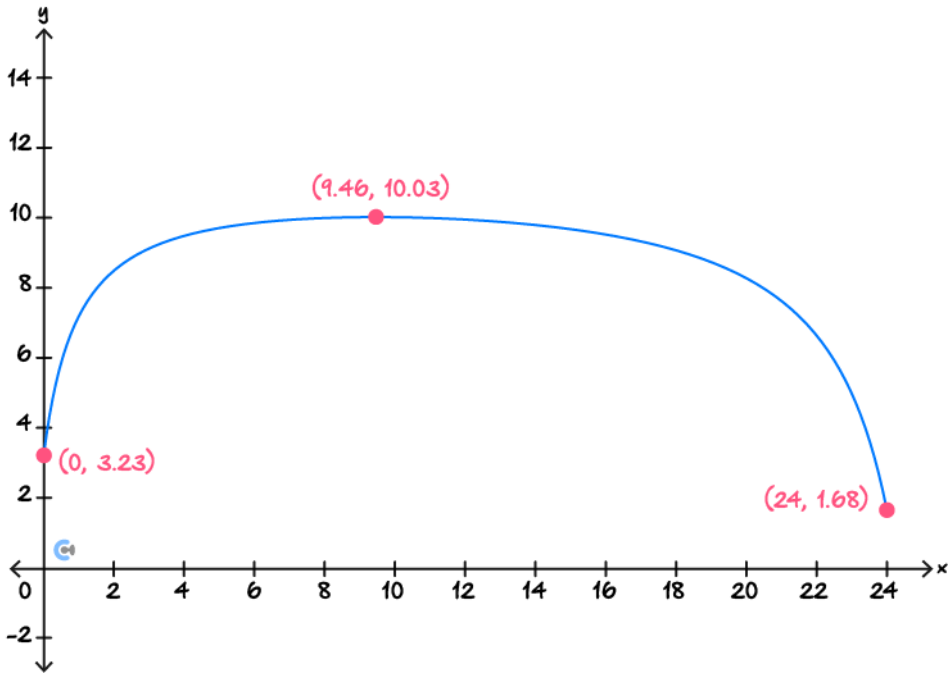
Space for Personal Notes

Section C: Marking Scheme

| Question Number | Solutions |
|-----------------|--|
| 1 a i | $T = 24 + 2t.$ [1A] |
| 1 a ii | $t = 4$ minutes. [1A] |
| 1 b i | Using Newton's method with $t_0 = 3$ we get: $t_1 = 2.836, t_2 = 2.751, t_3 = 2.733$ [1M for one correct value] Our estimate is $t = 2.733$ [1A] |
| 1 b ii | Let $y = R^{-1}(t)$, then $R(y) = t$, so we solve $t = 8 - 2e^{4y-8}$ [1M] $e^{4y-8} = \frac{8-t}{2}$ $4y - 8 = \log_e \left(\frac{8-t}{2} \right)$ $y = \frac{1}{4} \log_e \left(\frac{8-t}{2} \right) + 2$ So $R^{-1}(t) = \frac{1}{4} \log_e \left(\frac{8-t}{2} \right) + 2.$ [1A] |
| 1 c i | Turning point at $(2, 7)$ so $a = 2.$ [1M] |
| 1 c ii | Let $y = C^{-1}(x)$. Then we solve $C(y) = x$. This yields $y = 2 \pm \sqrt{\frac{1}{3}(7-x)}.$ [1M] But the range must be $[0, 2]$ so we have $C^{-1}(x) = 2 - \sqrt{\frac{1}{3}(7-x)}$ [1A] |
| 2 a | Vertical line is of the form $x = a$. This is only possible if $k = -5.$ [1A] |

| | |
|---------|--|
| 2 b i | <p>The gradient of the two lines must differ for a unique point of intersection. [1M]</p> <p>Gradients are the same if $\frac{k}{-6} = \frac{1}{k+5} \implies k = -3, -2$ [1M]</p> <p>So there is a unique point of intersection if</p> $k \in \mathbb{R} \setminus \{-3, -2\}, m \in \mathbb{R} \quad [1A]$ |
| 2 b ii | <p>No solution if the lines are parallel and not the same.</p> <p>If $k = -3$ then if $m = -\frac{3}{2}$ the lines are the same.</p> <p>So no solution if $k = -3, m \in \mathbb{R} \setminus \{-\frac{3}{2}\}$ [1A]</p> <p>If $k = -2$ then if $m = -1$ the lines are the same.</p> <p>So no solution if $k = -2, m \in \mathbb{R} \setminus \{-1\}$ [1A]</p> |
| 2 b iii | $k = -3 \text{ and } m = -\frac{3}{2}$ <p>OR $k = -2 \text{ and } m = -1.$ [1A]</p> |
| 2 c i | <p>Perpendicular if product of the gradients is -1.</p> <p>Thus $\frac{k}{6} \times \frac{-1}{k+5} = -1$ [1M] $\implies k = -6.$</p> <p>$k = -6, m \in \mathbb{R}.$ [1A]</p> |
| 2 c ii | <p>We require that</p> $\left \frac{\frac{k}{6} + \frac{1}{k+5}}{1 + \frac{k}{6} \left(\frac{-1}{k+5} \right)} \right = \tan(50^\circ) \quad [1M]$ <p>$\implies k = -4.996, k = 5.955$</p> <p>So $k = -4.996, m \in \mathbb{R}$ or $k = 5.955, m \in \mathbb{R}.$ [1A]</p> |
| 2 d i | <p>Sub the point into both equations to get the system</p> $18 + 5k = k + 2m$ $5 - 3(5 + k) = 2 \quad [1M]$ <p>solving gives $k = -4$ and $m = 1$ [1A]</p> |

| | |
|----------------------|--|
| <p>2 d ii</p> | <p>The two lines make an acute angle of $\theta = \arctan\left(\frac{1}{5}\right) \approx 11.3099^\circ$ [1M]</p> <p>Take one line to be $-4x - 6y = -2 \implies y = \frac{1}{3} - \frac{2x}{3}$.</p> <p>The bisecting line passes through $(5, -3)$ and makes an acute angle of $\approx 5.65^\circ$ with the above line.</p> <p>This line must have gradient of $m \approx -0.8198$. [1M]</p> <p>The line then has equation $y = 1.0990 - 0.8198x$ [1A]</p> |
| <p>2 e i</p> | <div style="border: 1px solid #ccc; padding: 10px; background-color: #f9f9f9;"> <p>Let $f(x) = \frac{1}{x} + x^2$. Then $f'(1) = 1$.</p> <p>So line L_1 has gradient of 1 and passes through $(1, 2)$. [1M]</p> $y = 1 + x$ [1M] <p>so now $\frac{k}{6} = 1$ and $\frac{k + 2m}{-6} = 1 \implies k = 6, m = -6$ [1A]</p> </div> |
| <p>2 e ii</p> | <p>Tangent to $f(x) = \frac{1}{x} + x^2$ at $x = a$ is given by $y = -\frac{a^4 - 2a^3x - 2a + x}{a^2}$ [1M]</p> <p>This line passes through $(2, -9)$ so we get the equation</p> $-\frac{a^4 - 4a^3 - 2a + 2}{a^2} = -9$ $\implies a = -1, 3 \pm \sqrt{7}$ [1M] <p>Only $a = -1$ will give a negative x-axis intercept.</p> <p>Thus the line is $y = -3 - 3x$.</p> <p>So $k = -18$ and $m = 18$. [1A]</p> |
| <p>3 a i</p> | <div style="border: 1px solid #ccc; padding: 10px; background-color: #f9f9f9;"> <p>Let $y(x) = \frac{20}{x - 26} - \frac{8}{x + 1} + 12$. To find a maximum we must solve</p> $y'(x) = \frac{8}{(x + 1)^2} - \frac{20}{(x - 26)^2} = 0$ [1M] $\implies -\frac{12(x^2 + 38x - 449)}{(x - 26)^2(x + 1)^2} = 0$</div> <p>so we may solve $x^2 + 38x - 449 = 0$ [1M]</p> $(x + 19)^2 - 810 = 0$ $x + 19 = \pm 9\sqrt{10}$ $x = -19 \pm 9\sqrt{10}$ [1M] <p>But we require that $x \geq 0$ so the only solution is $x = -19 + 9\sqrt{10}$. [1A, must say why we reject value]</p> |

| | |
|---------|--|
| 3 a ii |  <p>[1A shape, 1A coordinates]</p> |
| 3 a iii | <p>Solve $y(x) = 7 \implies x = 0.903, 21.697$. These are distances from site A. [1M] Therefore distance from site B are 23.097 and 2.303 millimetres. [1A]</p> |
| 3 a iv | <p>We solve $y(x) = 8 \implies x = 1.51317, 20.4868$. [1M] So the percentage is $\frac{1.51317 + (24 - 20.4868)}{24} \times 100 = 20.9\%$ [1A].</p> |
| 3 b i | <p>We require that $y(0) \geq 0$ and that $y(24) \geq 0$. [1M] Also $q > 0$ since we said that it is a positive constant. So $0 < q \leq \frac{146}{13}$. [1A]</p> |
| 3 b ii | <p>We solve $y'(x) = 0 \implies x = 26 - \frac{54(\sqrt{5q} - 10)}{q - 20}, \frac{54(\sqrt{5q} + 10)}{q - 20} + 26$ [1M] But we need the value that is between 0 and 24 on the domain of q. So $x = 26 - \frac{54(\sqrt{5q} - 10)}{q - 20}$ or equivalent. [1A]</p> |

| | |
|---------|--|
| 3 b iii | <p>No inverse if the function has a turning point in the interval $0 < x < 24$. Note we already have the restriction $0 < q \leq \frac{146}{13}$ Using above part b.ii we also solve $0 < 26 - \frac{54(\sqrt{5q} - 10)}{q - 20} \implies q > \frac{5}{169}$ [1M] Combining our restrictions gives no inverse if</p> $\frac{5}{169} < q \leq \frac{146}{13} \quad [1A]$ |
| 3 b iv | <p>We will require that $y(0) \geq y(24)$ [1M] Thus $0 < q \leq \frac{125}{13}$ [1A] .</p> |
| 3 c | <p>We must have a max of 10 when $x = 12$. So must have $\frac{dy}{dx} = 0$ when $x = 12$. [1M] We solve $y'(12) = 0$ and $y(12) = 10$ simultaneously to get $p = \frac{392}{27}$ [1M] and $q = \frac{338}{27}$ [1A] .</p> |

Space for Personal Notes

Section D: Mathematica Solutions

| Question Number | Solutions | | | | | | | | | | | | | | | | | | | | |
|-----------------|---|-----------|-------------------|-----------|-------------------|---|-----------|-----------|----------|---|-----------|----------|----------|---|----------|----------|----------|---|----------|----------|---------|
| 1 | <div><p>Q1.</p><pre>In[57]:= T[t_] := 24 + 2 t In[58]:= Solve[T[t] == 32] Out[58]= {{t -> 4}}</pre><p>In[59]:= R[t_] := 8 - 2 Exp[4 t - 8]</p><pre>In[60]:= Solve[R[y] == t, y, Reals]</pre><p>Out[60]= $\left\{ \left\{ y \rightarrow 2 + \frac{1}{4} \operatorname{Log}\left[\frac{8-t}{2}\right] \text{ if } t < 8 \right\} \right\}$</p><pre>In[61]:= T1[t_] := 24 + 2 t + 8 - 2 Exp[4 t - 8] In[62]:= NSolve[T1[t] == 0, t, Reals]</pre><p>Out[62]= $\{ \{t \rightarrow -16.\}, \{t \rightarrow 2.73257\} \}$</p><pre>In[63]:= n[x_] := x - \frac{T1[x]}{T1'[x]} In[64]:= n[3.0] Out[64]= 2.83625</pre><pre>In[65]:= n[n[3.0]] Out[65]= 2.75155</pre><pre>In[66]:= n[n[n[3.0]]] Out[66]= 2.73328</pre><pre>In[67]:= NewtonMethod[T1[x], 3, 4]</pre><table><thead><tr><th>Iteration</th><th>x_n</th><th>x_{n+1}</th><th>$x_{n+1} - x_n$</th></tr></thead><tbody><tr><td>0</td><td>3.0000000</td><td>2.8362495</td><td>0.163751</td></tr><tr><td>1</td><td>2.8362495</td><td>2.751546</td><td>0.084704</td></tr><tr><td>2</td><td>2.751546</td><td>2.733277</td><td>0.018268</td></tr><tr><td>3</td><td>2.733277</td><td>2.732567</td><td>0.00071</td></tr></tbody></table><pre>In[68]:= c[x_] := -3 x^2 + 12 x - 5 In[69]:= Solve[c[y] == x, y] // FullSimplify</pre><p>Out[69]= $\left\{ \left\{ y \rightarrow 2 - \frac{\sqrt{7-x}}{\sqrt{3}} \right\}, \left\{ y \rightarrow 2 + \frac{\sqrt{7-x}}{\sqrt{3}} \right\} \right\}$</p></div> | Iteration | x_n | x_{n+1} | $ x_{n+1} - x_n $ | 0 | 3.0000000 | 2.8362495 | 0.163751 | 1 | 2.8362495 | 2.751546 | 0.084704 | 2 | 2.751546 | 2.733277 | 0.018268 | 3 | 2.733277 | 2.732567 | 0.00071 |
| Iteration | x_n | x_{n+1} | $ x_{n+1} - x_n $ | | | | | | | | | | | | | | | | | | |
| 0 | 3.0000000 | 2.8362495 | 0.163751 | | | | | | | | | | | | | | | | | | |
| 1 | 2.8362495 | 2.751546 | 0.084704 | | | | | | | | | | | | | | | | | | |
| 2 | 2.751546 | 2.733277 | 0.018268 | | | | | | | | | | | | | | | | | | |
| 3 | 2.733277 | 2.732567 | 0.00071 | | | | | | | | | | | | | | | | | | |

Q2.

```
In[70]:= Solve[ $\frac{k}{-6} == \frac{1}{k+5}$ , k] // FullSimplify
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```
Out[70]= {{k → -3}, {k → -2}}
```

```
In[71]:= Solve[k x - 6 y == k + 2 m /. k → -3, y] // Expand
```

```
Out[71]= {{y →  $\frac{1}{2} - \frac{m}{3} - \frac{x}{2}$ }}
```

```
In[72]:= Solve[x + (k + 5) y == 2 /. k → -3, y] // Expand
```

```
Out[72]= {{y →  $1 - \frac{x}{2}$ }}
```

```
In[73]:= Solve[1 == 1/2 - m/3]
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```
Out[73]= {{m →  $-\frac{3}{2}$ }}
```

```
In[74]:= Solve[k x - 6 y == k + 2 m /. k → -2, y] // Expand
```

```
Out[74]= {{y →  $\frac{1}{3} - \frac{m}{3} - \frac{x}{3}$ }}
```

```
In[75]:= Solve[x + (k + 5) y == 2 /. k → -2, y] // Expand
```

```
Out[75]= {{y →  $\frac{2}{3} - \frac{x}{3}$ }}
```

```
In[76]:= Solve[1/3 - m/3 == 2/3]
```

```
Out[76]= {{m → -1}}
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```
In[77]:= Solve[ $\frac{k}{6} \left( \frac{-1}{k+5} \right) == -1$ , k]
```

```
Out[77]= {{k → -6}}
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```
In[78]:= Solve[Abs[ $\frac{k/6 + 1/(k+5)}{1 + k/6(-1/(k+5))}$ ] == Tan[50 Degree], k, Reals] // N
```

```
Out[78]= {{k → -4.99624}, {k → 5.955}}
```

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In[79]:= k x - 6 y == k + 2 m /. {x → 5, y → -3}
```

```
Out[79]= 18 + 5 k == k + 2 m
```

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In[80]:= x + (k + 5) y == 2 /. {x → 5, y → -3}
```

```
Out[80]= 5 - 3 (5 + k) == 2
```

```
In[81]:= Solve[18 + 5 k == k + 2 m && 5 - 3 (5 + k) == 2]
```

```
Out[81]= {{k → -4, m → 1}}
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2

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In[82]:= Abs[ $\frac{k/6 + 1/(k+5)}{1 + k/6(-1/(k+5))}$ ] /. k -> -4
Out[82]=  $\frac{1}{5}$ 

In[83]:= ArcTan[ $\frac{1}{5}$ ]/Degree // N
Out[83]= 11.3099

In[84]:= Solve[-4 x - 6 y == -2, y] // Expand
Out[84]=  $\left\{ \left\{ y \rightarrow \frac{1}{3} - \frac{2x}{3} \right\} \right\}$ 

In[85]:= 11.309932474020215^2 / 2
Out[85]= 5.65497

In[86]:= Solve[Abs[ $\frac{m+2/3}{1+m(-2/3)}$ ] == Tan[1/2 * ArcTan[ $\frac{1}{5}$ ]], m, Reals] // N
Out[86]= {{m -> -0.532496}, {m -> -0.819804}}

In[87]:= Solve[y + 3 == -0.819803902718557 (x - 5), y] // Expand
Out[87]= {{y -> 1.09902 - 0.819804 x}}

In[88]:= (* Check and see that the line found with m=-0.53 does not bisect the angle*)
In[89]:= Solve[y + 3 == -0.5324955213946582 (x - 5), y] // Expand
Out[89]= {{y -> -0.337522 - 0.532496 x}}

In[90]:= f[x_] := 1/x + x^2
In[91]:= f'[1]
Out[91]= 1

In[92]:= f[1]
Out[92]= 2

In[93]:= TangentLine[f[x], x, 1]
Out[93]= 1 + x

In[94]:= Solve[ $\frac{k}{6} == 1$  &&  $\frac{k+2m}{-6} == 1$ ]
Out[94]= {{k -> 6, m -> -6}}

In[95]:= TangentLine[f[x], x, a] // FullSimplify
Out[95]=  $-\frac{-2a + a^4 + x - 2a^3x}{a^2}$ 

In[96]:=  $-\frac{-2a + a^4 + x - 2a^3x}{a^2}$  /. x -> 2
Out[96]=  $-\frac{2 - 2a - 4a^3 + a^4}{a^2}$ 

In[97]:= Solve[ $-\frac{2 - 2a - 4a^3 + a^4}{a^2} == -9$ , a]
Out[97]= {{a -> -1}, {a -> -1}, {a -> 3 -  $\sqrt{7}$ }, {a -> 3 +  $\sqrt{7}$ }}

In[98]:= Solve[ $-\frac{2 - 2a - 4a^3 + a^4}{a^2} == -9$ , a] // N
Out[98]= {{a -> -1.}, {a -> -1.}, {a -> 0.354249}, {a -> 5.64575}}

In[99]:= TangentLine[f[x], x, -1]
Out[99]= -3 - 3 x

In[100]:= Solve[ $\frac{k}{6} == -3$  &&  $\frac{k+2m}{-6} == -3$ ]
Out[100]= {{k -> -18, m -> 18}}

```

3

Q3.

$$\text{In[101]}:= f[x_] := \frac{20}{x-26} - \frac{8}{x+1} + 12$$

$$\text{In[102]}:= f'[x]$$

$$\text{Out[102]}= -\frac{20}{(-26+x)^2} + \frac{8}{(1+x)^2}$$

$$\text{In[103]}:= \text{Solve}[f'[x] == 0, x]$$

$$\text{Out[103]}= \left\{ \left\{ x \rightarrow -19 - 9\sqrt{10} \right\}, \left\{ x \rightarrow -19 + 9\sqrt{10} \right\} \right\}$$

$$\text{In[104]}:= -\frac{20}{(-26+x)^2} + \frac{8}{(1+x)^2} // \text{Together}$$

$$\text{Out[104]}= -\frac{12(-449 + 38x + x^2)}{(-26+x)^2(1+x)^2}$$

$$\text{In[105]}:= \text{CompleteTheSquare}[-449 + 38x + x^2, x]$$

$$\text{Out[105]}= -810 + (19+x)^2$$

$$\text{In[106]}:= \sqrt{810}$$

$$\text{Out[106]}= 9\sqrt{10}$$

$$\text{In[107]}:= f[24] // N$$

$$\text{Out[107]}= 1.68$$

$$\text{In[108]}:= f[0] // N$$

$$\text{Out[108]}= 3.23077$$

$$\text{In[109]}:= \text{Solve}[f[x] == 7, x] // N$$

$$\text{Out[109]}= \left\{ \left\{ x \rightarrow 0.903366 \right\}, \left\{ x \rightarrow 21.6966 \right\} \right\}$$

$$\text{In[110]}:= 24 - 0.9033659293019256$$

$$\text{Out[110]}= 23.0966$$

$$\text{In[111]}:= 24 - 21.696634070698078$$

$$\text{Out[111]}= 2.30337$$

$$\text{In[112]}:= \text{Solve}[f[x] == 8, x] // N$$

$$\text{Out[112]}= \left\{ \left\{ x \rightarrow 1.51317 \right\}, \left\{ x \rightarrow 20.4868 \right\} \right\}$$

$$\text{In[113]}:= \frac{1.5131670194948619 + (24 - 20.486832980505138)}{24} * 100$$

$$\text{Out[113]}= 20.9431$$

$$\text{In[114]:= } g[x_] := \frac{20}{x-26} - \frac{q}{x+1} + 12$$

$$\text{In[115]:= } \text{Solve}[g[0] == 0]$$

$$\text{Out[115]= } \left\{ \left\{ q \rightarrow \frac{146}{13} \right\} \right\}$$

$$\text{In[116]:= } \text{Solve}[g[24] == 0]$$

$$\text{Out[116]= } \left\{ \left\{ q \rightarrow 50 \right\} \right\}$$

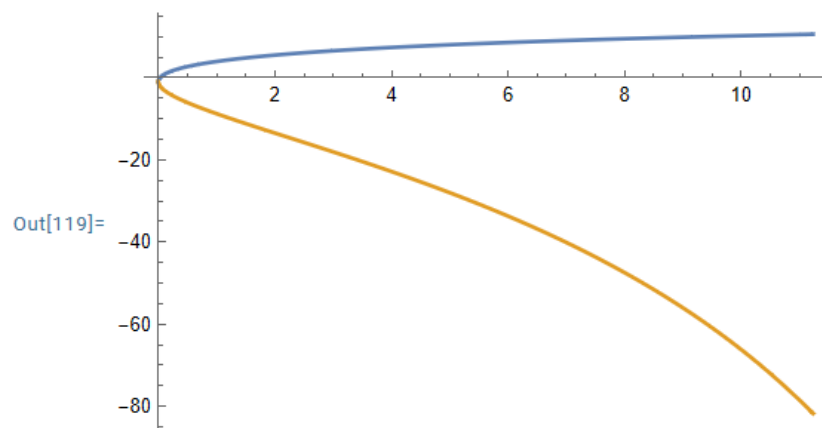
$$\text{In[117]:= } \text{Solve}[g[0] == 0] // N$$

$$\text{Out[117]= } \left\{ \left\{ q \rightarrow 11.2308 \right\} \right\}$$

$$\text{In[118]:= } \text{Assuming}[q > 0, \text{Apart}[\text{Solve}[g'[x] == 0, x]]]$$

$$\text{Out[118]= } \left\{ \left\{ x \rightarrow 26 - \frac{54(-10 + \sqrt{5}\sqrt{q})}{-20 + q} \right\}, \left\{ x \rightarrow 26 + \frac{54(10 + \sqrt{5}\sqrt{q})}{-20 + q} \right\} \right\}$$

$$\text{In[119]:= } \text{Plot}\left[\left\{ 26 - \frac{54(-10 + \sqrt{5}\sqrt{q})}{-20 + q}, 26 + \frac{54(10 + \sqrt{5}\sqrt{q})}{-20 + q} \right\}, \{q, 0, 146/13\}\right]$$



$$\text{In[120]:= } \text{Reduce}\left[0 < 26 - \frac{54(-10 + \sqrt{5}\sqrt{q})}{-20 + q} < 24, q\right]$$

$$\text{Out[120]= } \frac{5}{169} < q < 20 \mid \mid 20 < q < 3125$$

$$\text{In[121]:= } \text{Reduce}[g[0] \geq g[24], q]$$

$$\text{Out[121]= } q \leq \frac{125}{13}$$

$$\text{In[122]:= } h[x_] := \frac{p}{x-26} - \frac{q}{x+1} + 12$$

$$\text{In[123]:= } \text{Solve}[h'[12] == 0 \&\& h[12] == 10]$$

$$\text{Out[123]= } \left\{ \left\{ p \rightarrow \frac{392}{27}, q \rightarrow \frac{338}{27} \right\} \right\}$$

Section E: Casio Solutions

| Question Number | Solutions |
|-----------------|---|
| 1 | <pre> Define T(t)=24+2t solve(T(t)=32, t Define R(t)=8-2e^{4t-8} solve(R(y)=t, y Define T1(t)=T(t)+R(t) solve(T1(t)=0, t Define n(t)=t-T1(t) / (d/dt(T1(t))) n(3) n(ans) n(ans) Define C(x)=-3x^2+12x-5 fmax(C(x), x solve(C(y)=x, y </pre> <p>done</p> <p>{t=4}</p> <p>done</p> $\left\{ y = -\frac{\ln\left(\frac{-t}{2} + 4\right)}{4} + 2 \right\}$ <p>done</p> <p>{t=-16, t=2.732565875}</p> <p>done</p> <p>2.836249486</p> <p>2.75154571</p> <p>2.733277251</p> <p>done</p> <p>{MaxValue=7, x=2}</p> $\left\{ y = \frac{-\sqrt{-3 \cdot (x-7)}}{3} + 2, y = \frac{\sqrt{-3 \cdot (x-7)}}{3} + 2 \right\}$ |

2

$$\text{solve}\left(\frac{k}{-6} = \frac{1}{k+5}, k\right)$$

$$\{k=-3, k=-2\}$$

$$\text{solve}(k \cdot x - 6y = k + 2m \mid k = -3, y)$$

$$\left\{y = \frac{-x}{2} - \frac{m}{3} + \frac{1}{2}\right\}$$

$$\text{solve}(x + (k+5) \cdot y = 2 \mid k = -3, y)$$

$$\left\{y = \frac{-x}{2} + 1\right\}$$

$$\text{solve}(1 = \frac{1}{2} - m/3, m)$$

$$\left\{m = -\frac{3}{2}\right\}$$

$$\text{solve}(k \cdot x - 6y = k + 2m \mid k = -2, y)$$

$$\left\{y = \frac{-x}{3} - \frac{m}{3} + \frac{1}{3}\right\}$$

$$\text{solve}(x + (k+5) \cdot y = 2 \mid k = -2, y)$$

$$\left\{y = \frac{-x}{3} + \frac{2}{3}\right\}$$

$$\text{solve}(1/3 - m/3 = 2/3, m)$$

$$\{m = -1\}$$

$$\text{solve}\left(\frac{k}{6} \cdot \frac{-1}{k+5} = -1, k\right)$$

$$\{k = -6\}$$

$$\text{solve}\left(\left|\frac{k/6 + \frac{1}{k+5}}{1 + \frac{k}{6} \cdot \frac{-1}{k+5}}\right| = \tan(50^\circ), k\right)$$

$$\{k = 5.9550042, k = -4.996236237\}$$

$$k \cdot x - 6y = k + 2m \mid x = 5 \mid y = -3$$

$$5 \cdot k + 18 = k + 2 \cdot m$$

$$x + (k+5) \cdot y = 2 \mid x = 5 \mid y = -3$$

$$-3 \cdot (k+5) + 5 = 2$$

$$\begin{cases} 5 \cdot k + 18 = k + 2 \cdot m \\ -3 \cdot (k+5) + 5 = 2 \end{cases} \mid k, m$$

$$\{k = -4, m = 1\}$$

$$\left|\frac{k/6 + \frac{1}{k+5}}{1 + \frac{k}{6} \cdot \frac{-1}{k+5}}\right| \mid k = -4$$

$$\frac{1}{5}$$

```

tan-1(ans)
11.30993247

solve(-4x-6y=-2, y
{y=-2*x/3+1/3}

11.30993247/2
5.654966235

solve(|m+2/3|/|1+m*-2/3|=tan(5.654966235), m
{m=-0.5324955214, m=-0.8198039027}

solve(y+3=-0.8198039027(x-5), y
{y=-0.8198039027*x+1.099019514}

solve(y+3=-0.5324955214(x-5), y
{y=-0.5324955214*x-0.337522393}

Define f(x)=1/x+x^2
done

d/dx (f(x)) | x=1
1

f(1)
2

tanLine(f(x), x, 1)
x+1

{k/6=1 | k+2m=-6=1} | k, m
{k=6, m=-6}

tanLine(f(x), x, a)
a^2+x*(2*a-1/a^2)-a*(2*a-1/a^2)+1/a

simplify(ans)
-a^4-2*a^3*x+x-2*a^2

ans | x=2
-a^4-4*a^3-2*a+2

simplify(ans)
-a^4-4*a^3-2*a+2

solve(ans=-9, a
{a=-1, a=-sqrt(7)+3, a=sqrt(7)+3}

ans
{a=-1, a=0.3542486889, a=5.645751311}

tanLine(f(x), x, -1)
-3*x-3

{k/6=-3 | k+2m=-6=-3} | k, m
{k=-18, m=18}

```

3

```

Define f(x)=20/(x-26)-8/(x+1)+12
done

d/dx (f(x))
-12*x^2+456*x-5388/((x+1)^2*(x-26)^2)

solve(ans=0, x
{x=-9*sqrt(10)-19, x=9*sqrt(10)-19}

f(0)
3.230769231

fmax(f(x), x, 0, 24
{MaxValue=10.0259918, x=9.460498942}

f(24)
1.68

solve(f(x)=7, x
{x=0.9033659293, x=21.69663407}

24-ans
{-x+24=23.09663407, -x+24=2.303365929}

solve(f(x)<8, x)
{-1<x<1.513167019, 20.48683298<x<26}

1.513167019+24-20.4868329
24
*100
20.94305883

Define y(x)=p/(x-26)-q/(x+1)+12
done

solve(y(0)=0 | p=20, q
{q=146/13}

solve(y(24)=0 | p=20, q
{q=50}

{q=146/13}
{q=11.23076923}

solve(d/dx (y(x))=0 | p=20, x
ERROR:Insufficient Memory

d/dx (y(x))=0 | p=20
q*x^2-20*x^2-52*q*x-40*x+676*q-20=0
(x+1)^2*(x-26)^2

simplify(ans)
q*(x^2-52*x+676)-20*x^2-40*x-20=0
(x+1)^2*(x-26)^2

solve(ans=0 | q>0, x
ERROR:Insufficient Memory

solve(q*(x^2-52*x+676)-20*x^2-40*x-20=0, x
{x=2*(13*q-27*sqrt(5*q)+10)/(q-20), x=2*(13*q+27*sqrt(5*q)+10)/(q-20)}

propfrac(ans)
{x=-54*sqrt(5*q)/(q-20)+540/(q-20)+26, x=54*sqrt(5*q)/(q-20)+540/(q-20)+26}

solve(-54*sqrt(5*q)/(q-20)+540/(q-20)+26>0, q
{q>5/169 and q!=20}

solve(y(0)>=y(24) | p=20, q
{q<=125/13}

{d/dx (y(x))=0 | x=12
 y(12)=10
p, q
{p=392/27, q=338/27}

```

VCE Mathematical Methods $\frac{3}{4}$ Free 1-on-1 Support



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|--|---|
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